# LLF Finite Volume Scheme Overview

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## Step by step calculation of $\tilde{F}^{LLF}_{j-1/2}$

$\bar{U}_j$ is known at $j = 0, 1, 2, \dots 9$ $(n = 8)$

$\sigma^b_j = \bar{U}_j - \bar{U}_{j-1} \quad \forall\ j = 1, 2, \dots 8$

$S^{left}_{j-1/2} = \text{minmod}\left(\sigma^b_{j-1}, \sigma^b_j\right) \quad \forall\ j = 3, 4, \dots 6$

$U^-_{j-1/2} = \bar{U}_j + \frac{S^{left}_{j-1/2}}{2} \quad \forall\ j = 3, 4, \dots 6$

$S^{right}_{j-1/2} = \text{minmod}\left(\sigma^b_j, \sigma^b_{j+1}\right) \quad \forall\ j = 3, \dots 6$

$U^+_{j-1/2} = \bar{U}_j - \frac{S^{right}_{j-1/2}}{2} \quad \forall\ j = 3, 4, \dots 6$

Next calculate $F\left(U^+_{j-1/2}\right)$, $F\left(U^-_{j-1/2}\right)$ $\forall\ j = 3, 4, \dots 6$

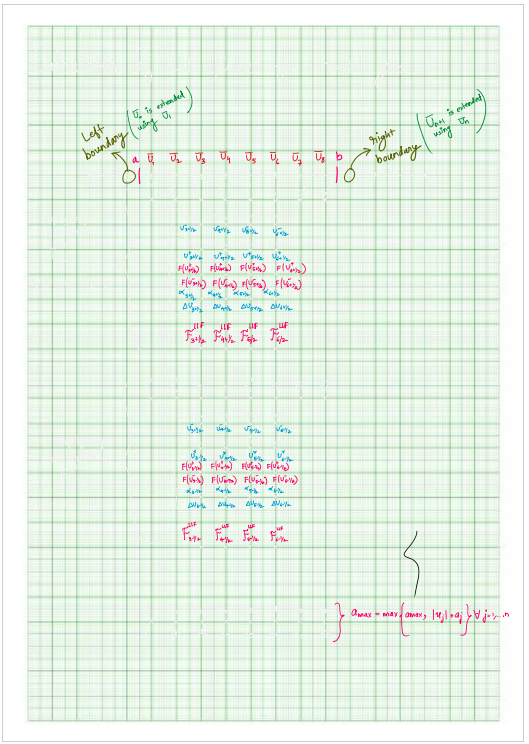
Next calculate coefficient of diffusion (LLF) at $j-1/2$

$\alpha_{j-1/2} = \max\left\{|u_{j-1}| + a_{j-1},\ |u_j| + a_j\right\} \quad \forall\ j = 3, \dots 6$

Next calculate $\Delta U_{j-1/2} = U^+_{j-1/2} - U^-_{j-1/2} \quad \forall\ j = 3, \dots 6$

Now calculate LLF fluxes at $j-1/2$ interface

$\tilde{F}^{LLF}_{j-1/2} = \frac{1}{2}\left(F\left(U^+_{j-1/2}\right) + F\left(U^-_{j-1/2}\right)\right) - \frac{\alpha_{j-1/2}}{2}\left(U^+_{j-1/2} - U^-_{j-1/2}\right) \quad \forall\ j = 3, \dots 6$




Finally after calculating $\tilde{F}^{LLF}_{j+1/2}$ and $\tilde{F}^{LLF}_{j-1/2}$ $\forall\ j = 3, \dots, n-2$ we can calculate the values of conserved variables at next time step. (using Euler Forward Differences)

(i) First calculate $\Delta t$ using CFL conditions :-

$\Delta t = CFL\left(\frac{\Delta x}{a_{max}}\right)$ ; $a_{max} = \max\left\{a_{max},\ |u_j| + a_j\right\} \forall\ j = 1, 2, \dots n$

(ii) Update conserved variables :-

$\bar{U}^{t+1}_j = \bar{U}^t_j - \frac{\Delta t}{\Delta x}\left(\tilde{F}^{LLF}_{j+1/2} - \tilde{F}^{LLF}_{j-1/2}\right) \quad \forall\ j = 3, 4, \dots n-2$

## Step by step calculation of $\tilde{F}^{LLF}_{j+1/2}$

$(N = 8)$

$\bar{U}_j$ is known $\forall\ j = 0, 1, \dots 8, 9$

$\sigma^f_j = \bar{U}_{j+1} - \bar{U}_j \quad \forall\ j = 1, 2, \dots 8$

$S^{left}_{j+1/2} = \text{minmod}\left(\sigma^f_j, \sigma^f_{j-1}\right) \quad \forall\ j = 3, \dots 6$

$U^-_{j+1/2} = U_j + \frac{S^{left}_{j+1/2}}{2} \quad \forall\ j = 3, \dots 6$

$S^{right}_{j+1/2} = \text{minmod}\left(\sigma^f_j, \sigma^f_{j+1}\right) \quad \forall\ j = 3, \dots 6$

$U^+_{j+1/2} = U_{j+1} - \frac{S^{right}_{j+1/2}}{2} \quad \forall\ j = 3, \dots 6$

Next calculate $F\left(U^+_{j+1/2}\right)$, $F\left(U^-_{j+1/2}\right)$ $\forall\ j = 3, 4, \dots 6$

Next calculate coefficient of diffusion (LLF) at $j+1/2$

$\alpha_{j+1/2} = \max\left\{|u_j| + a_j,\ |u_{j+1}| + a_{j+1}\right\} \quad \forall\ j = 3, 4, \dots 6$

Next calculate $\Delta U_{j+1/2} = U^+_{j+1/2} - U^-_{j+1/2} \quad \forall\ j = 3, 4, \dots 6$

Now calculate LLF flux at $j+1/2$ interface

$\tilde{F}^{LLF}_{j+1/2} = \frac{1}{2}\left(F\left(U^+_{j+1/2}\right) - F\left(U^-_{j+1/2}\right)\right) - \frac{\alpha_{j+1/2}}{2}\left(U^+_{j+1/2} - U^-_{j+1/2}\right) \quad \forall\ j = 3, 4, \dots 6$

Issues identified :-

→ if we use $j = 3, 4, \dots n-2$

$\bar{U}_1$, $\bar{U}_2$, $\bar{U}_{n-1}$, $\bar{U}_n$

are never updated and since $\bar{U}_0$, $\bar{U}_{n+1}$ are extended from these, they are also not updated at any time step.

⇒ giving wrong results. !!