GMM

Machine Learning Techniques

Karthik Thiagarajan

GMM

Notation:

- K - number of components
- k - index of the k^{th} component
- X - r.v for the observation
- Z - r.v for the latent variable
- π_k - prior probability of the k^{th} component
- μ_k - mean of the k^{th} component
- σ_k^2 - variance of the k^{th} component
- $f_X(x)$ - density of the GMM
- $f_{X,Z}(x, z)$ - joint density of X and Z
- $f_{X|Z}(x \mid z)$ - conditional density of a component
- $f_Z(z)$ - prior PMF of the latent variable
- $f_{Z|X}(z \mid x)$ - posterior PMF of the latent variable

Example-1

$$\pi_1 = 0.5,\ \pi_2 = 0.5$$
$$\mu_1 = -1,\ \sigma_1^2 = \frac{1}{20}$$
$$\mu_2 = 1,\ \sigma_2^2 = \frac{1}{4}$$

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$$n = 1,000,000$$

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[ 1.41, -0.95,  1.17, ...,
-0.99, 1.11 , -0.82]
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(2) Sample a point from
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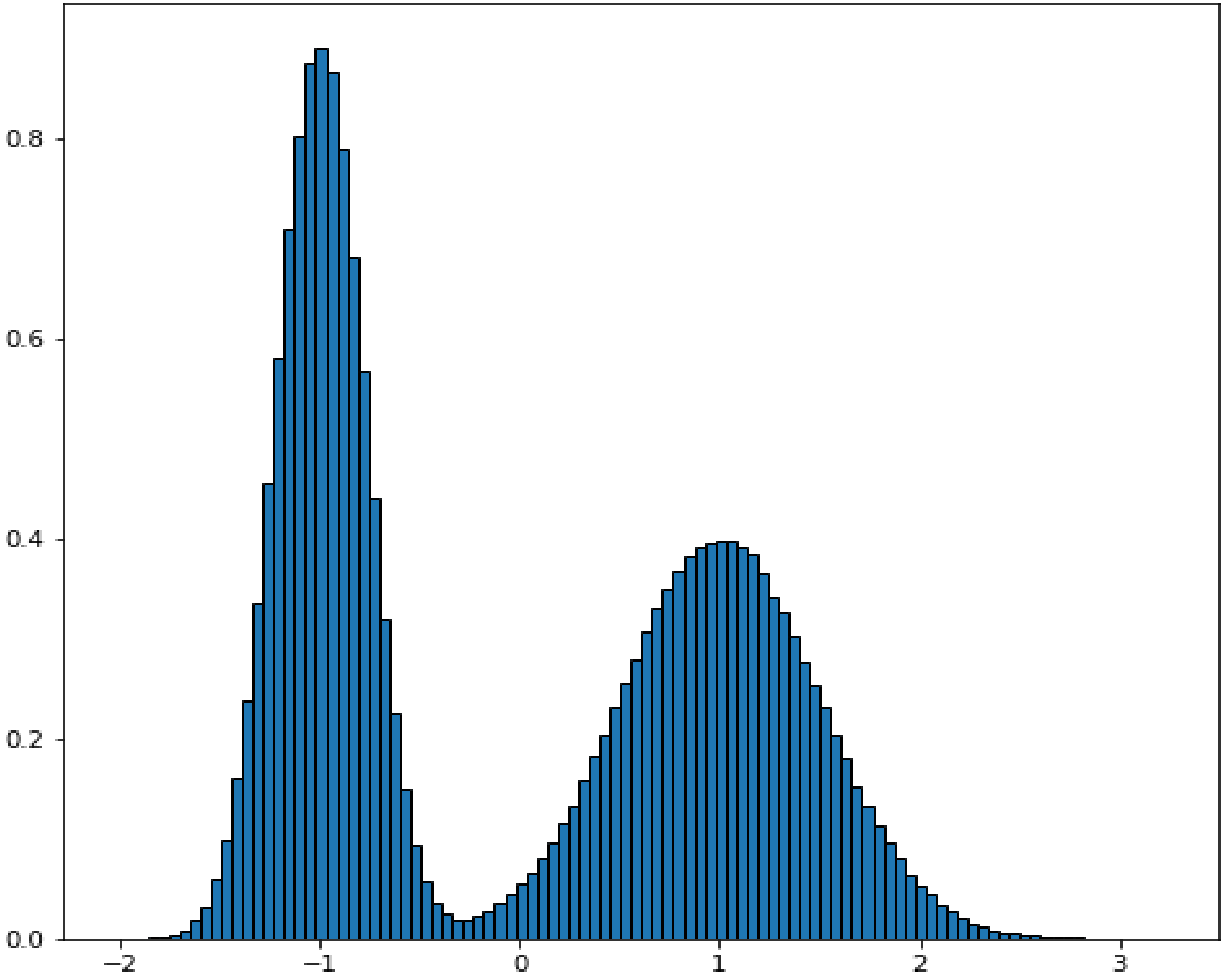

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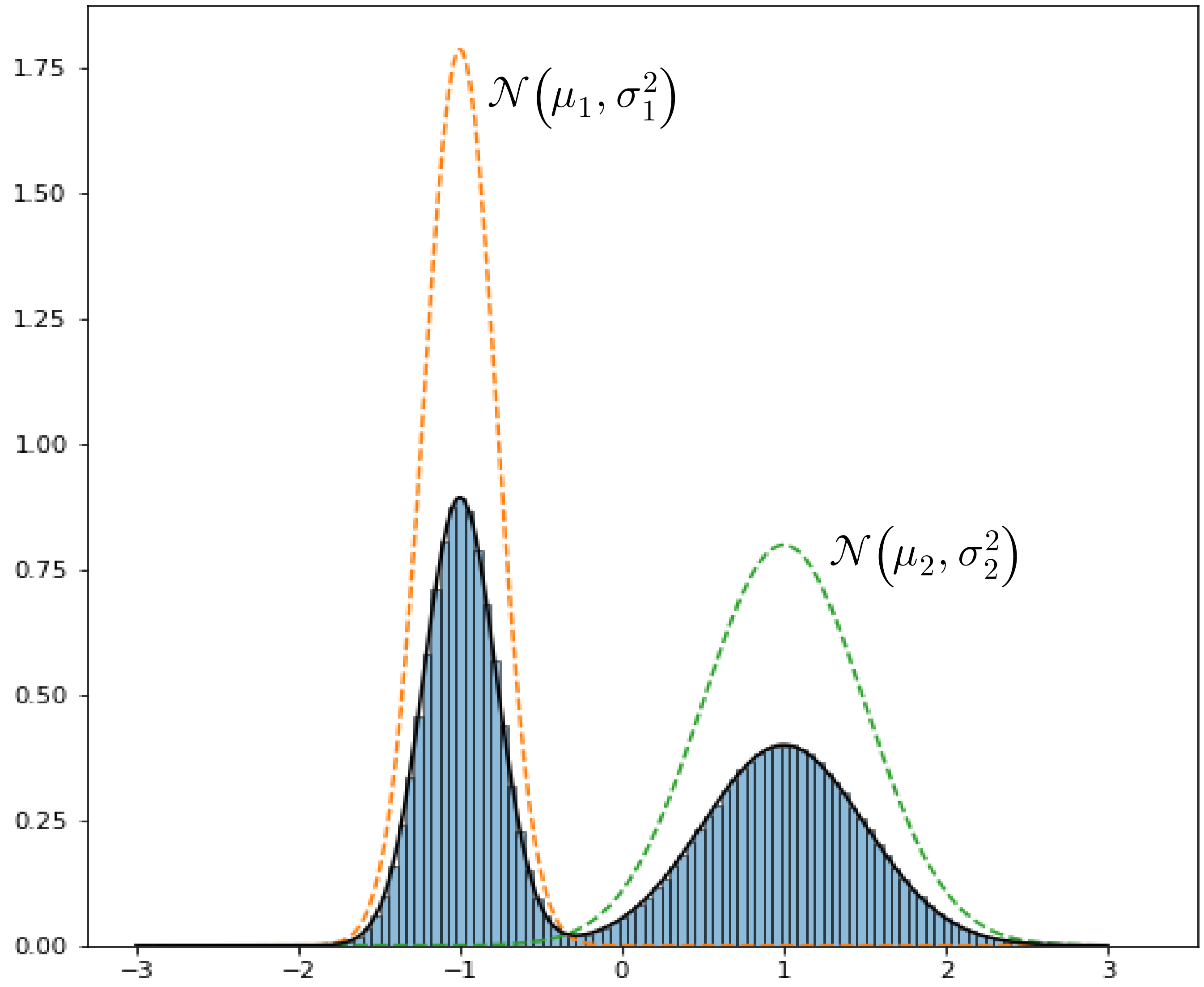

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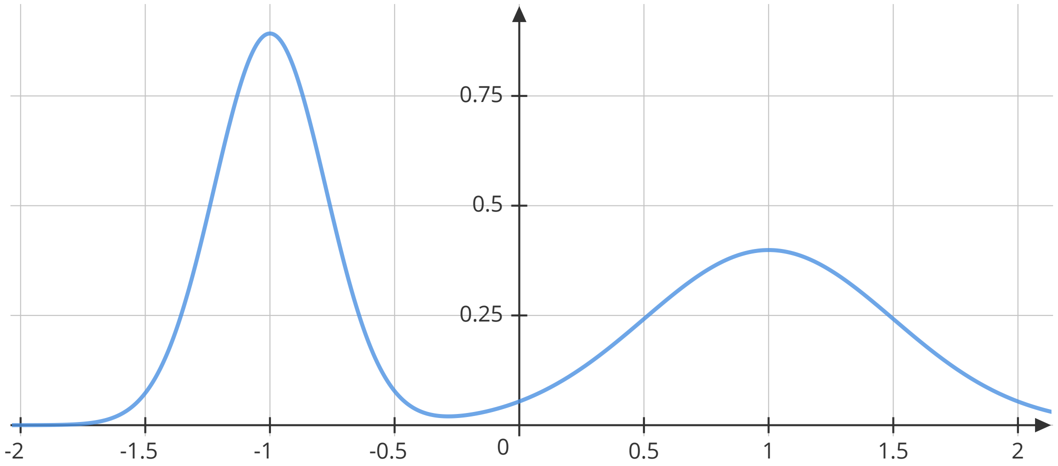

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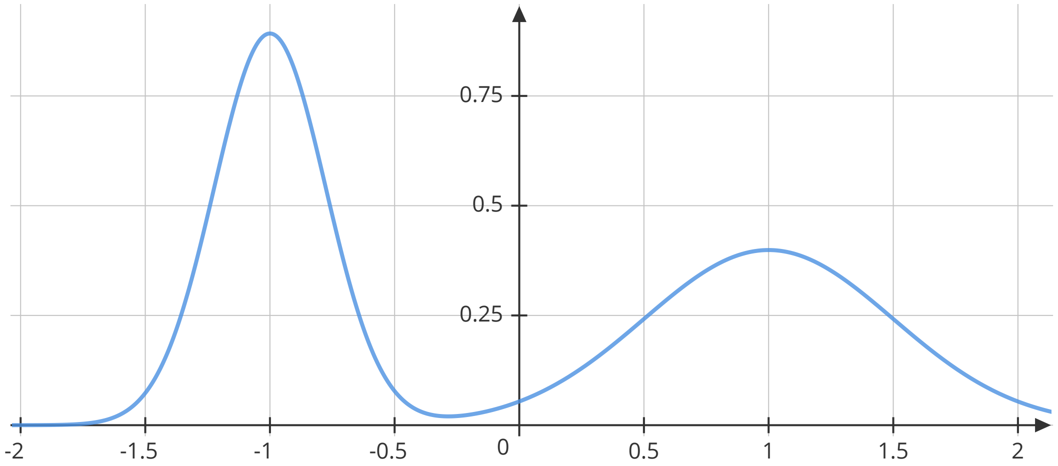

$$f_X(x) = \pi_1 \cdot \mathcal{N}\left(x;\; \mu_1, \sigma_1^2\right) + \pi_2 \cdot \mathcal{N}\left(x;\; \mu_2, \sigma_2^2\right)$$

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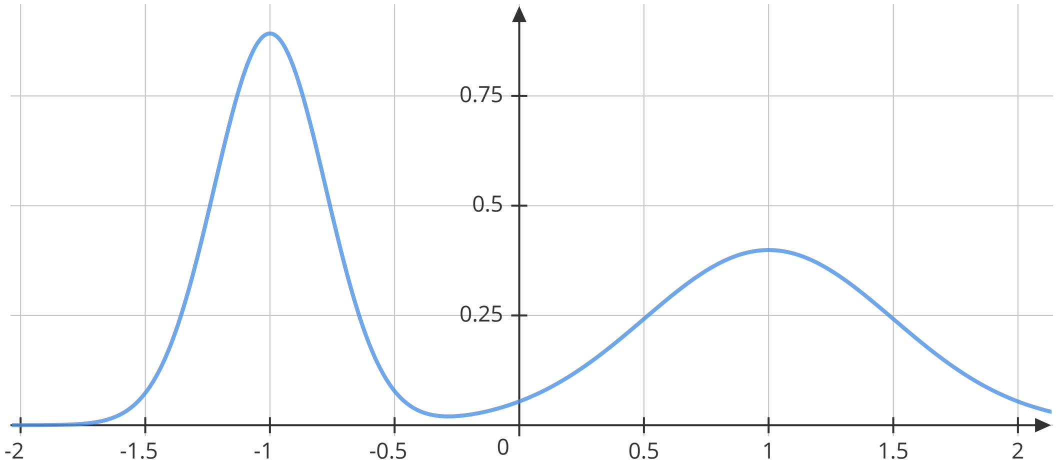

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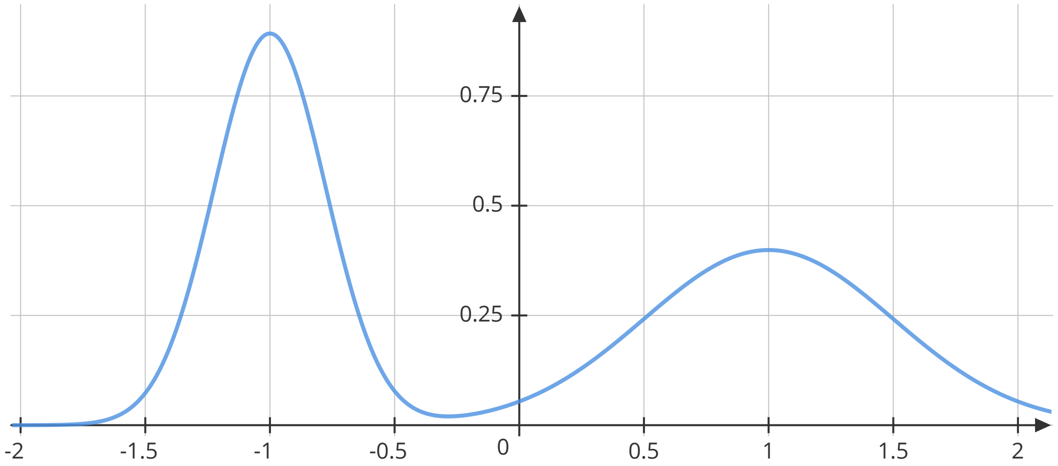

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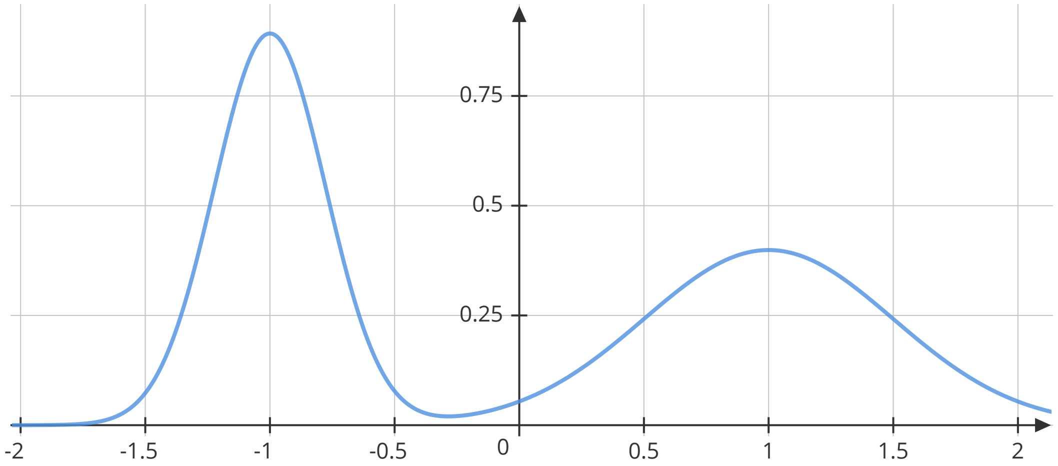

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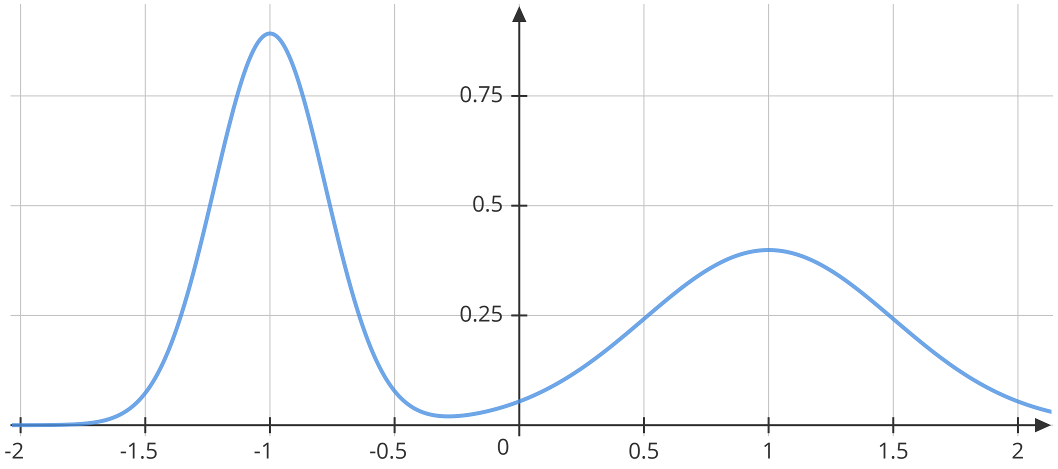

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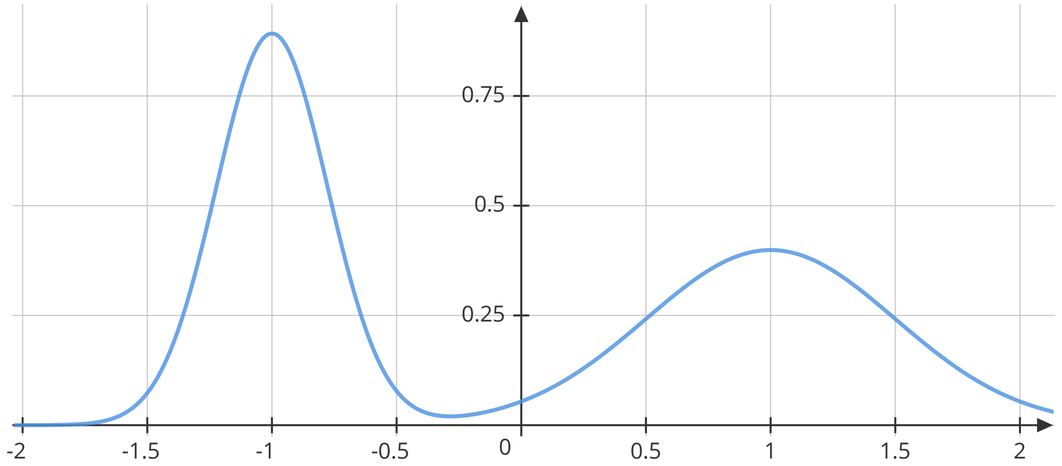

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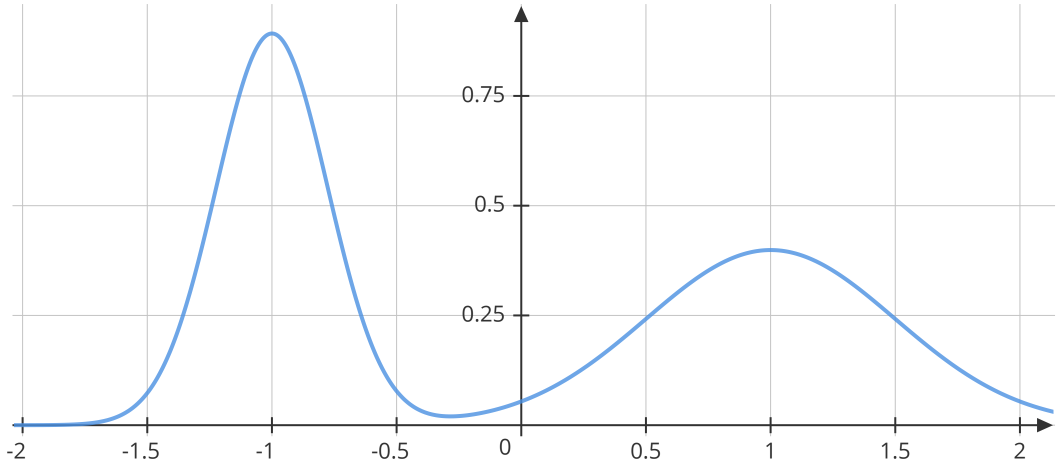

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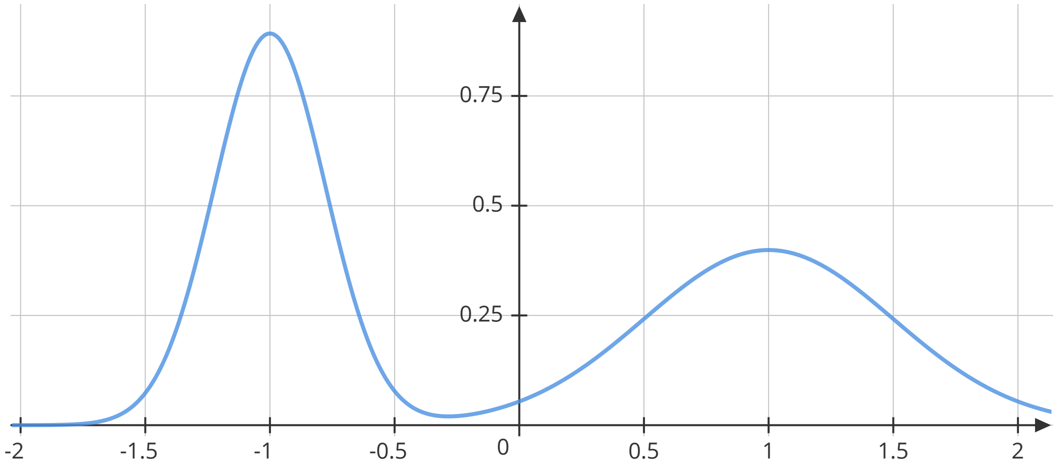

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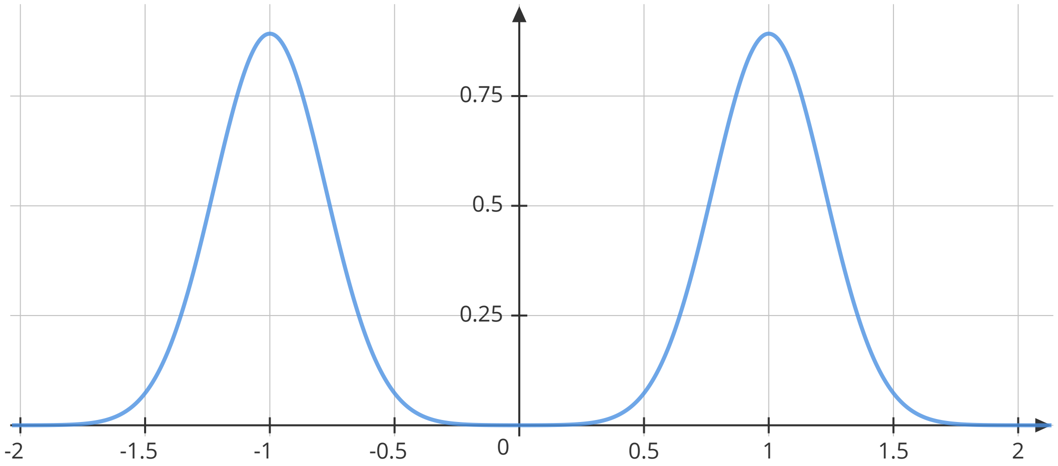

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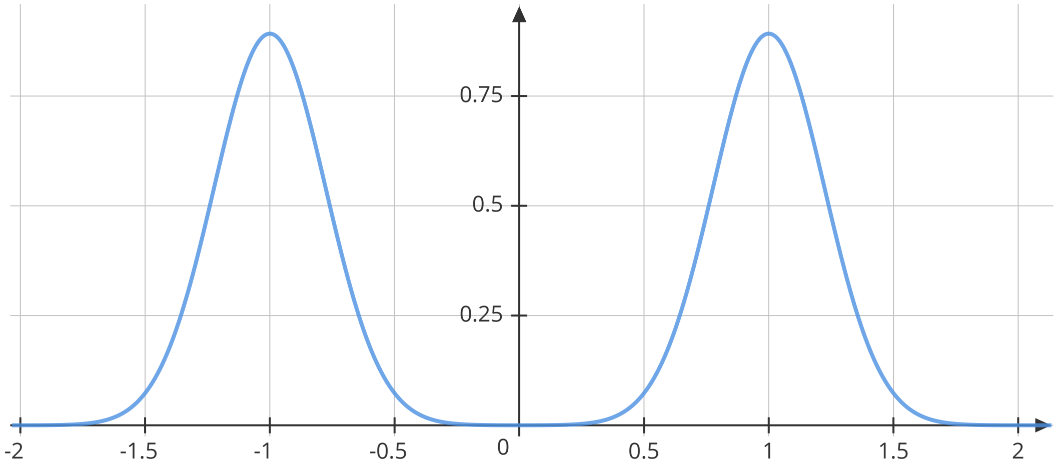

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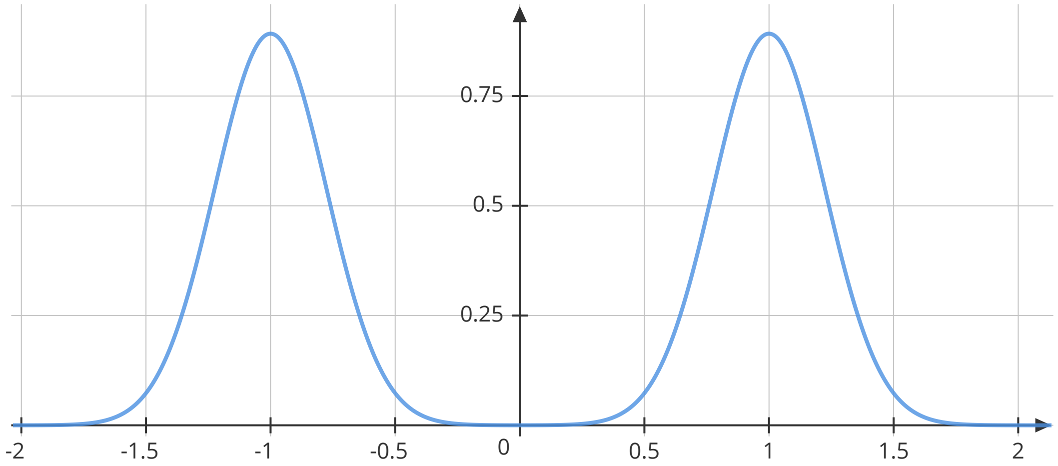

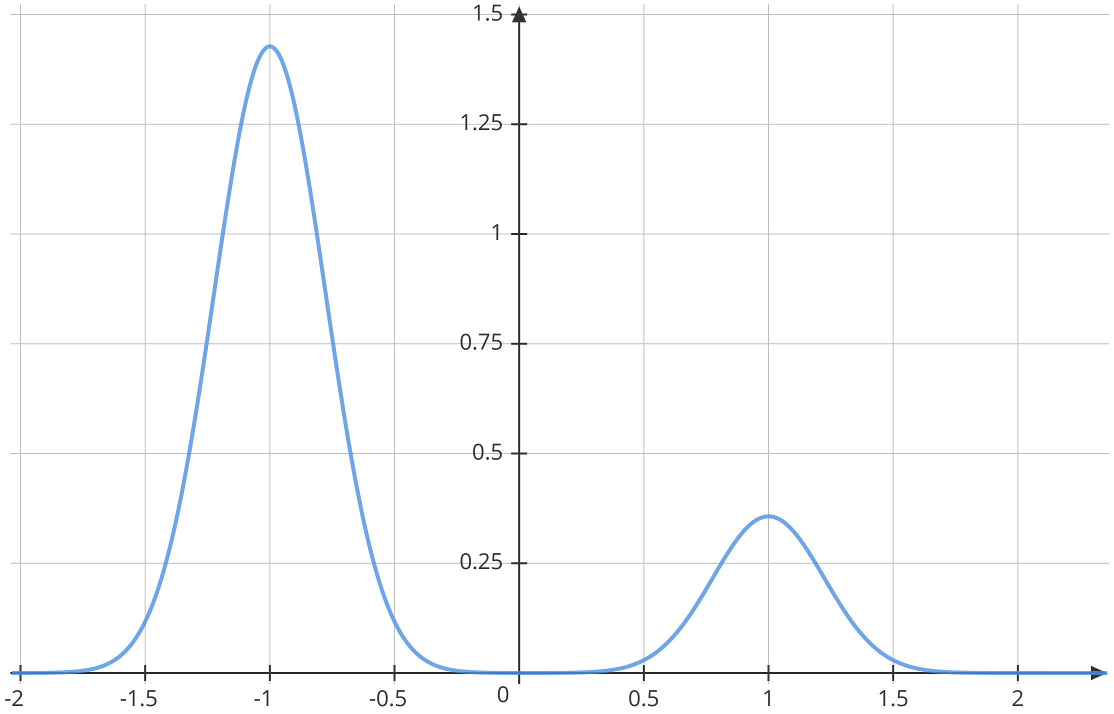

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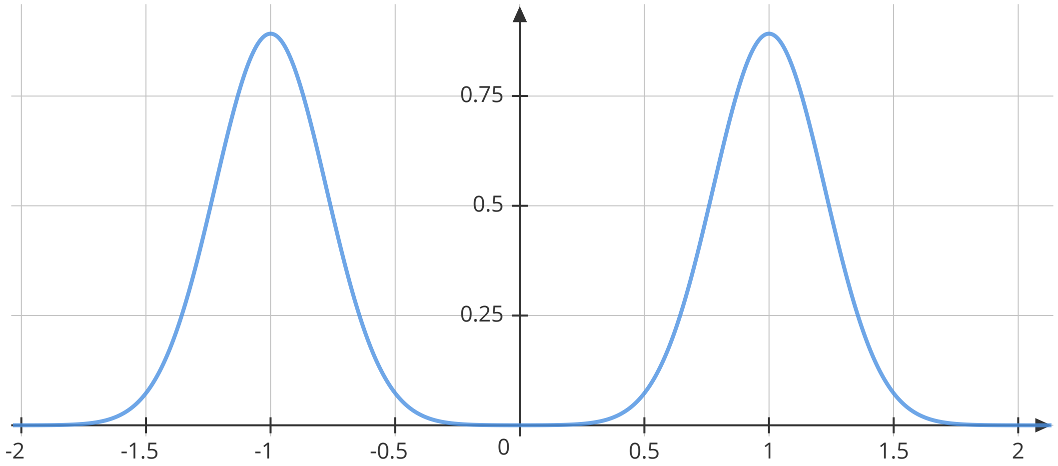

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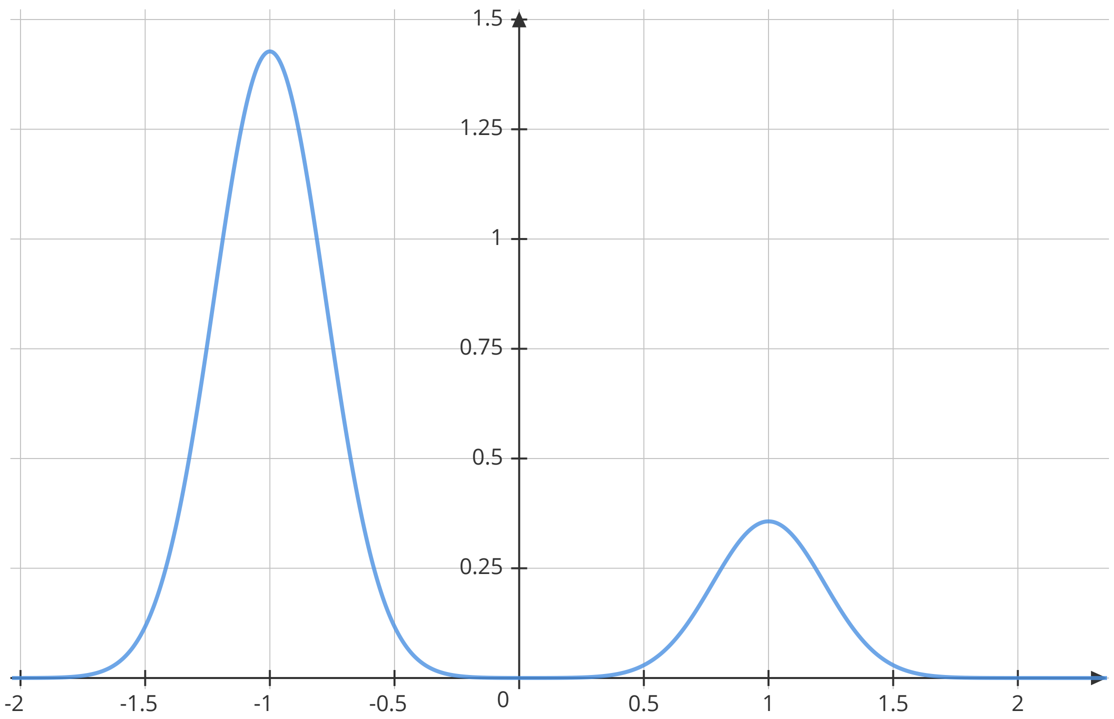

$$\pi_1 = 0.8,\ \ \pi_2 = 0.2$$

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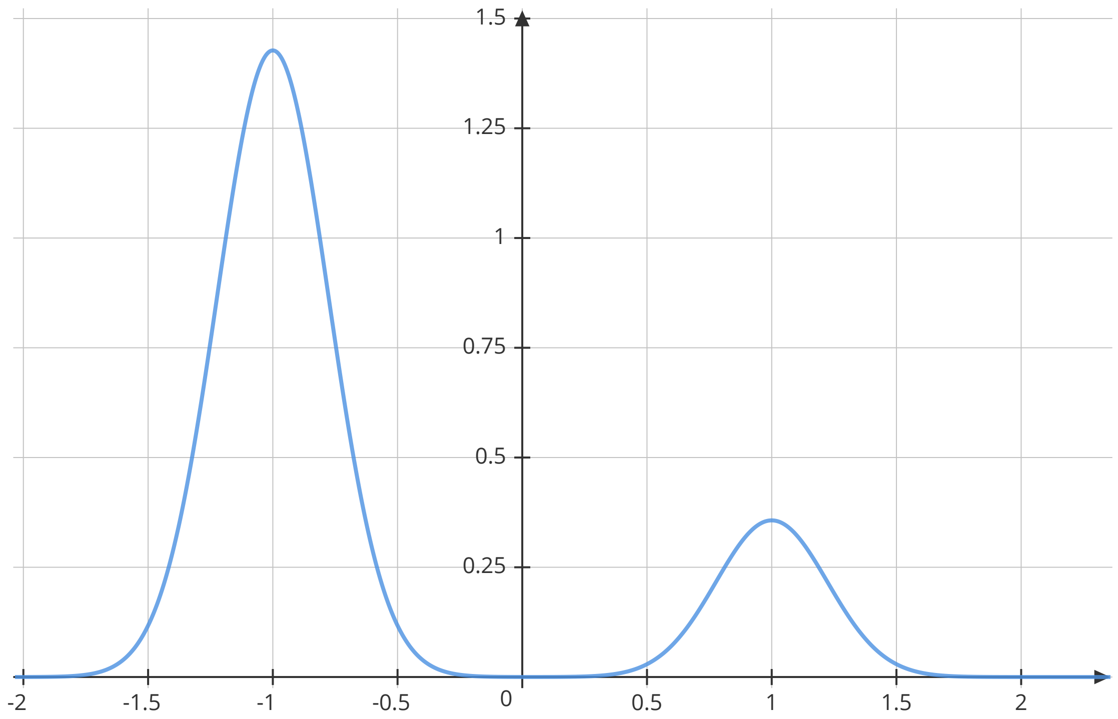

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$$\pi_1 = 0.99, \;\; \pi_2 = 0.01$$
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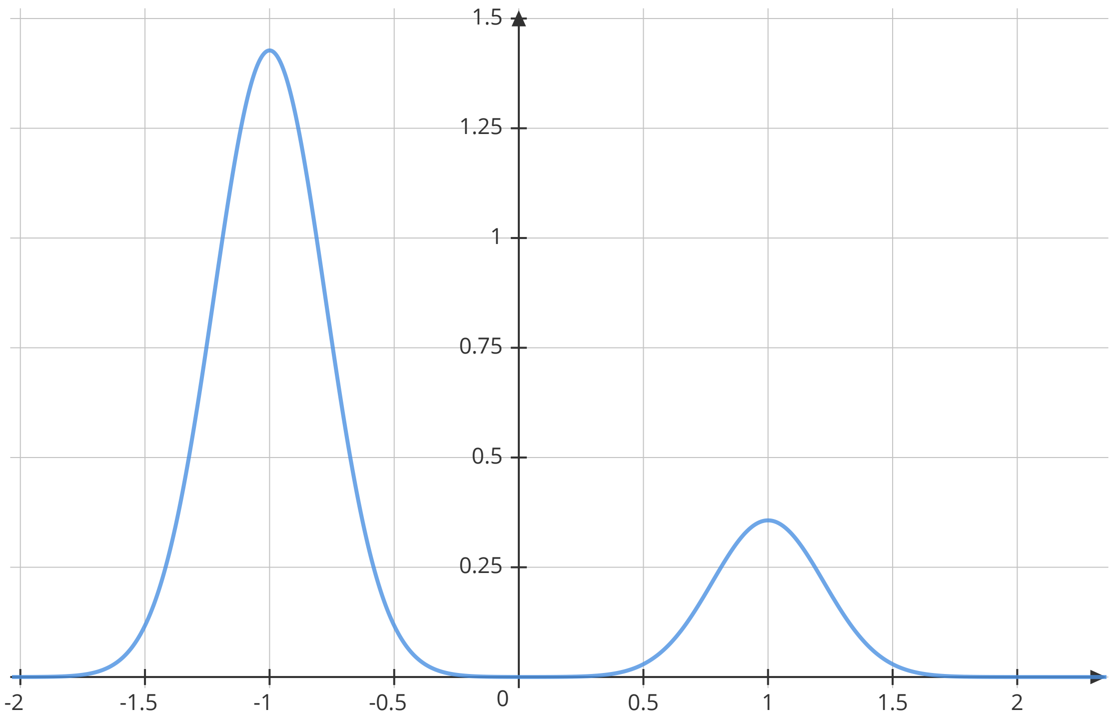

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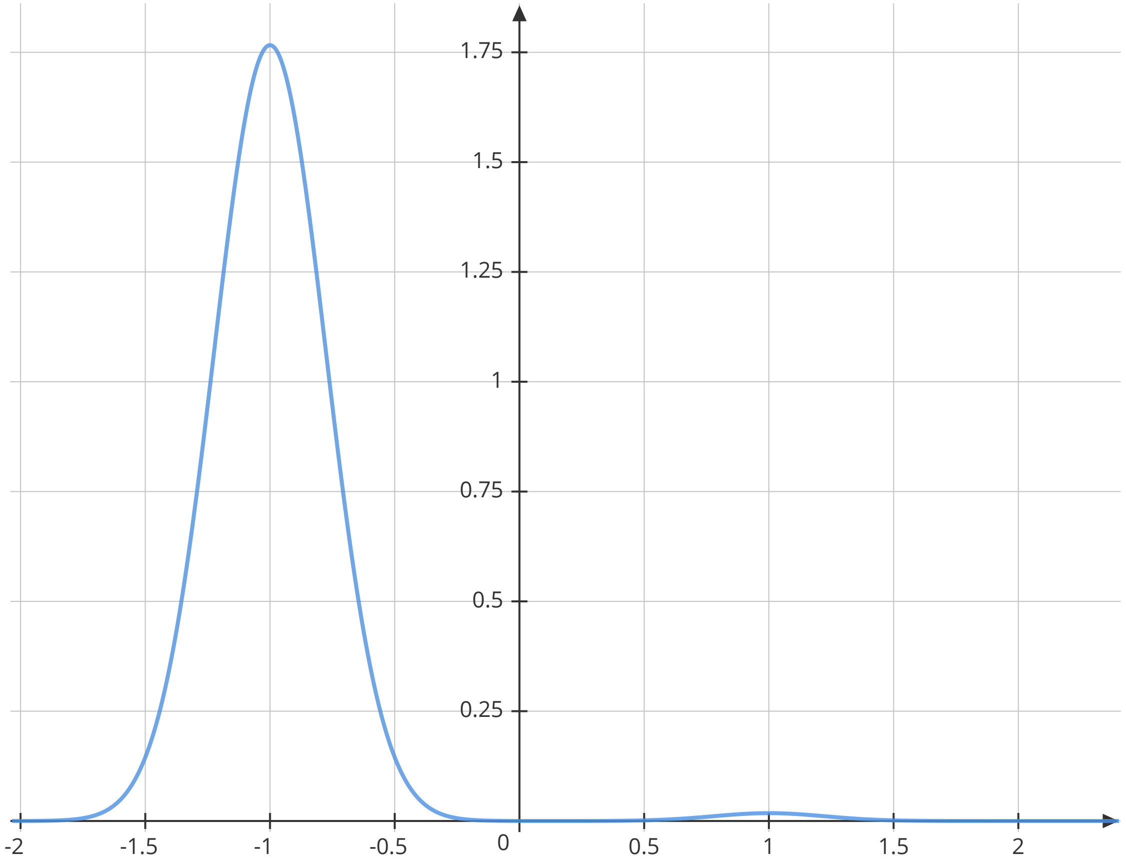

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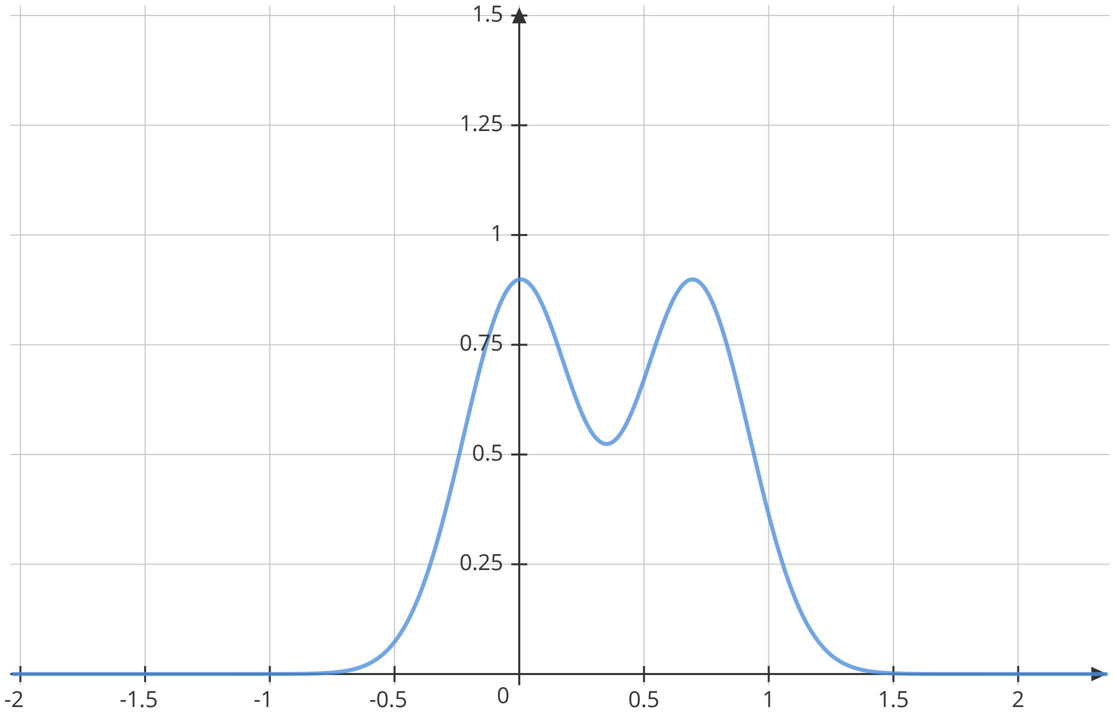

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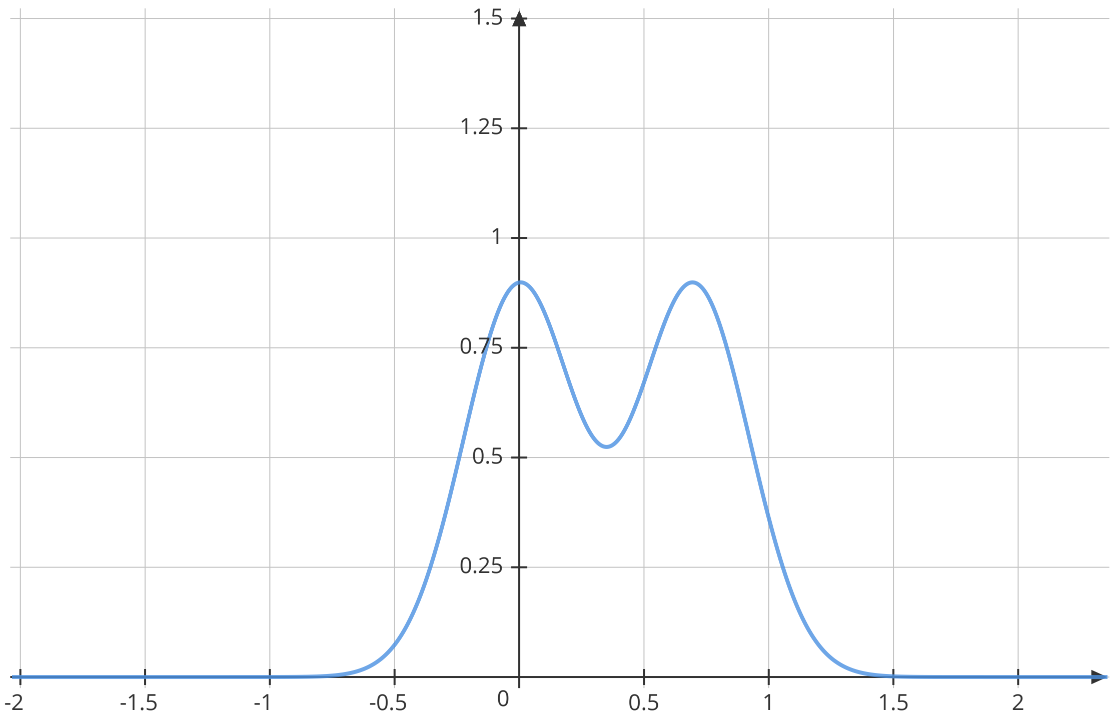

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$$\mu_2 = 0.7,\ \ \sigma_2^2 = \frac{1}{20}$$

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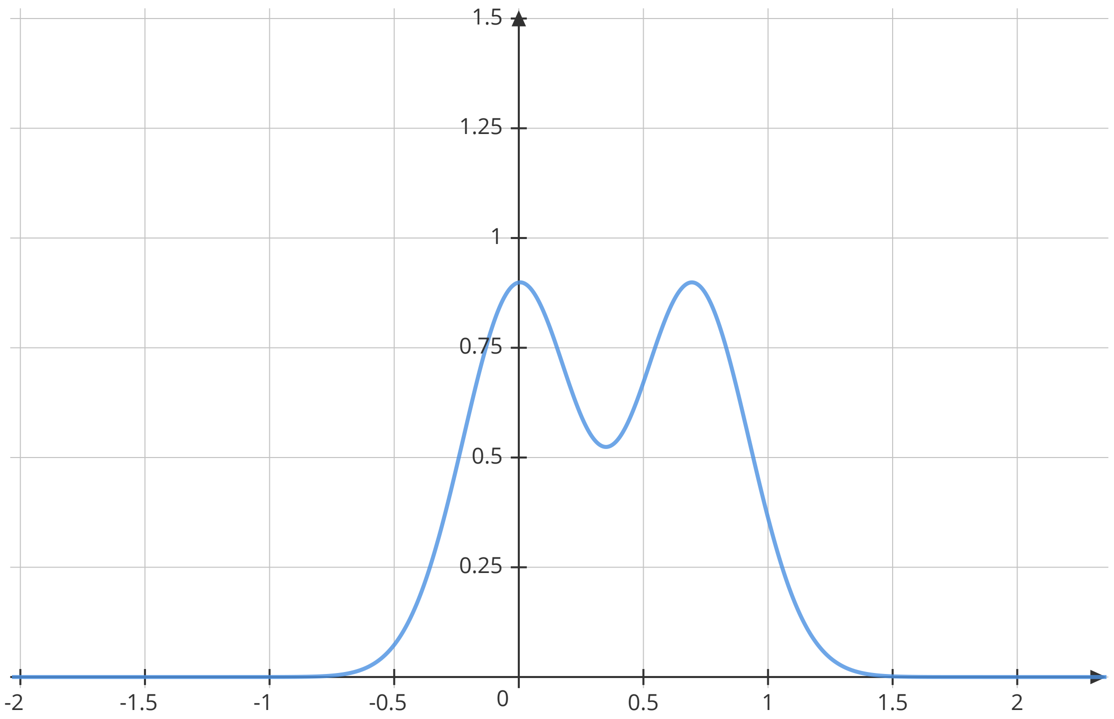

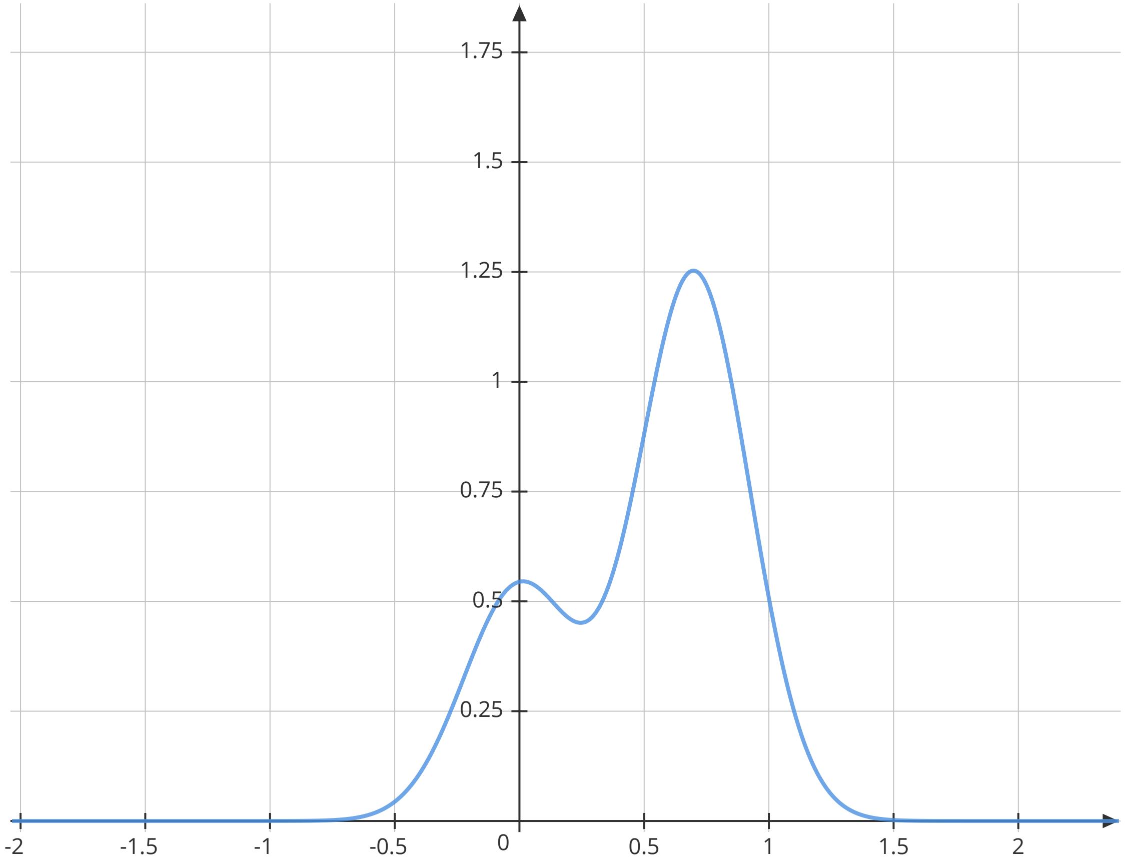

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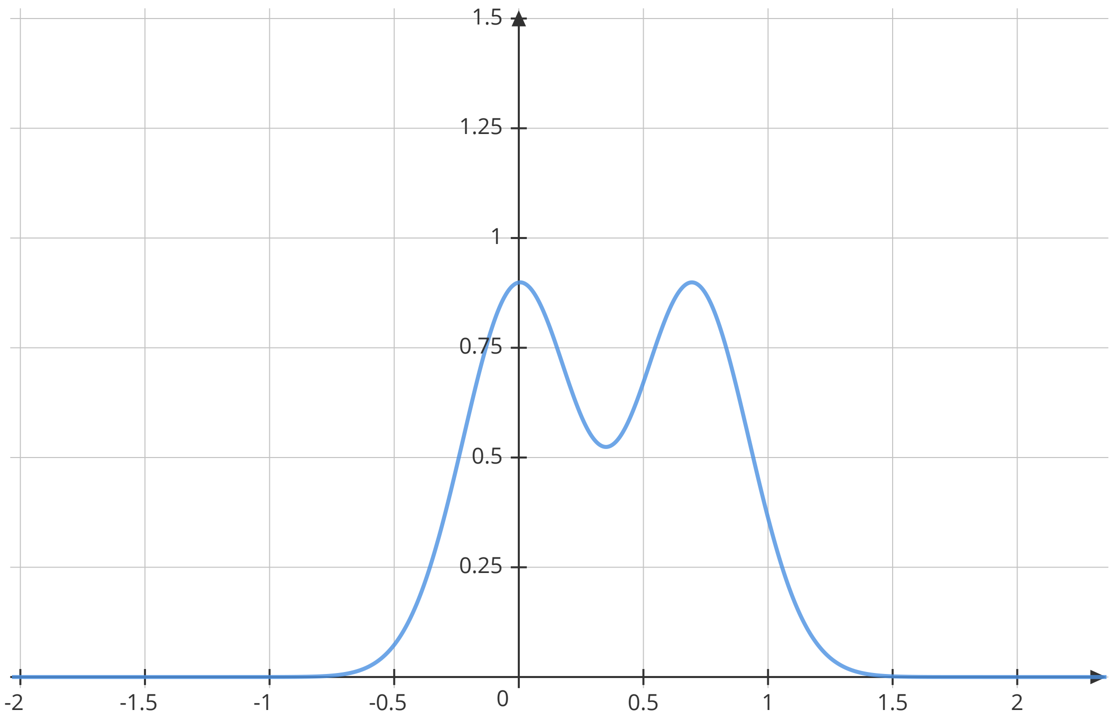

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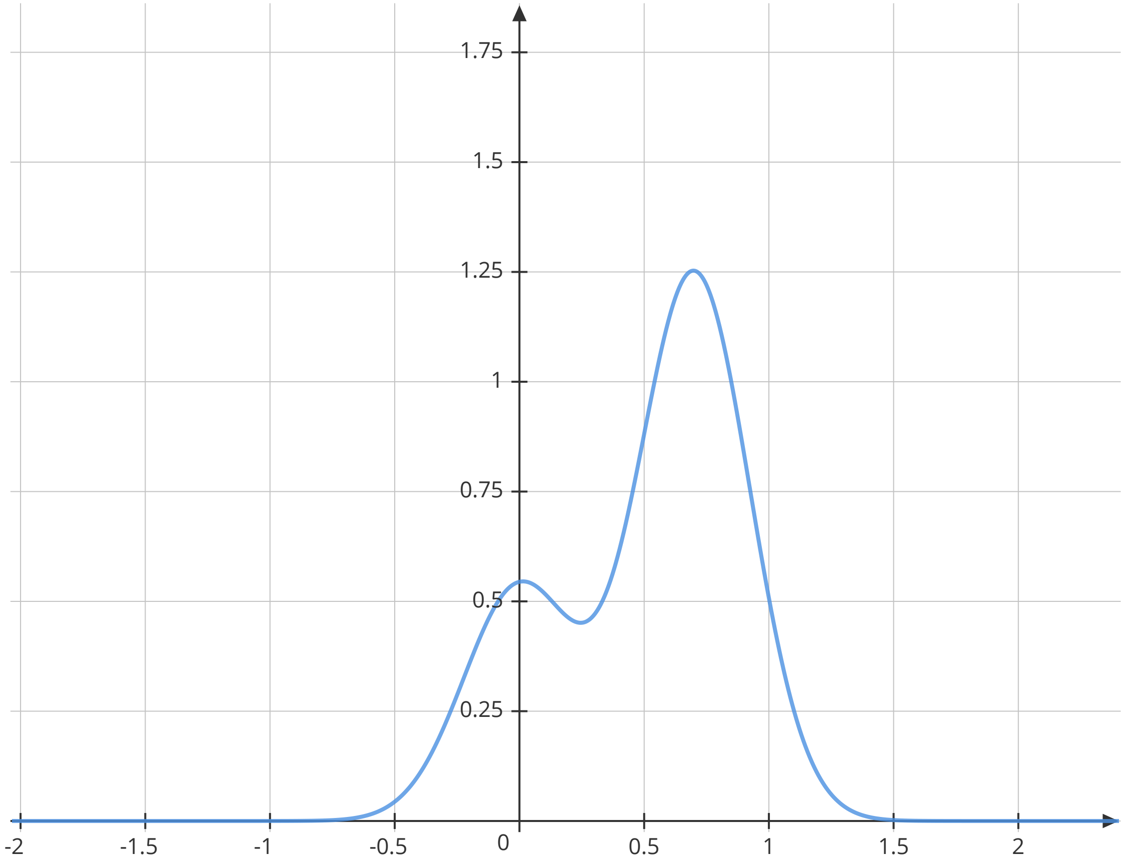

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Z: Prior and Posterior

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$$f_Z(1) = \pi_1$$
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$$f_{Z|X}(k \mid x)$$

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Z: Prior and Posterior

$$f_{Z|X}(k \mid x) = \frac{f_{X,Z}(x,k)}{f_X(x)}$$

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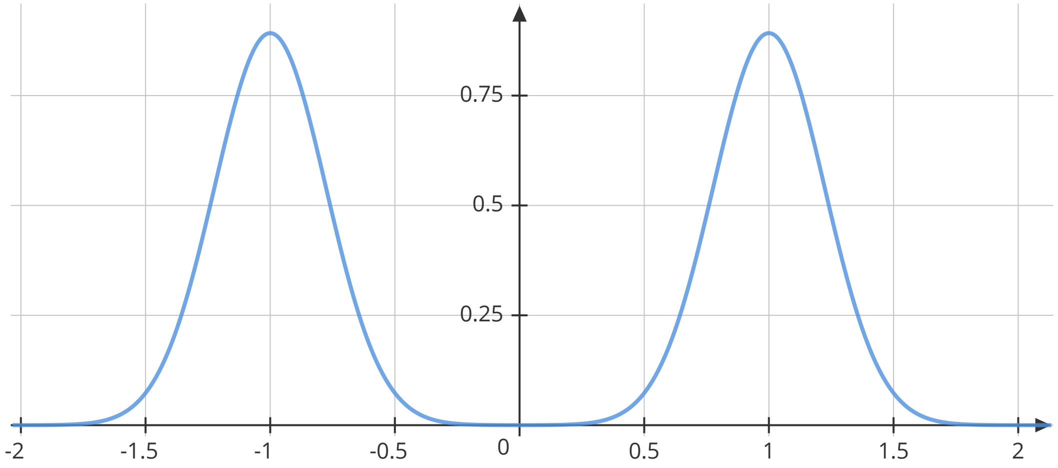

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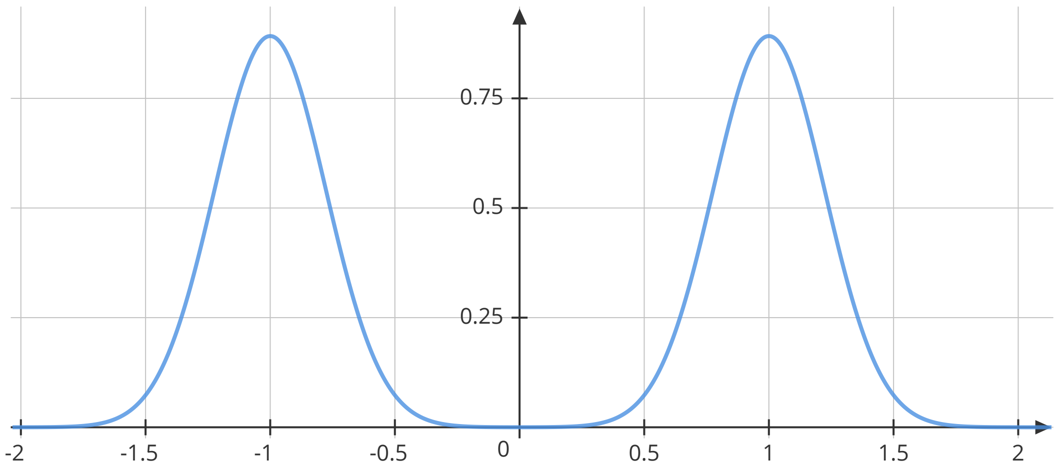

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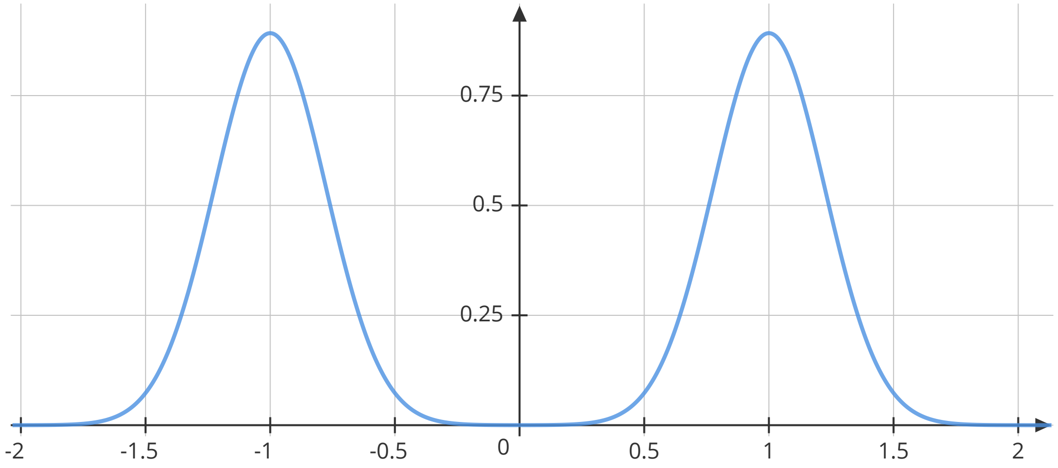
-0.1	
0	
0.1	

Z: Prior and Posterior



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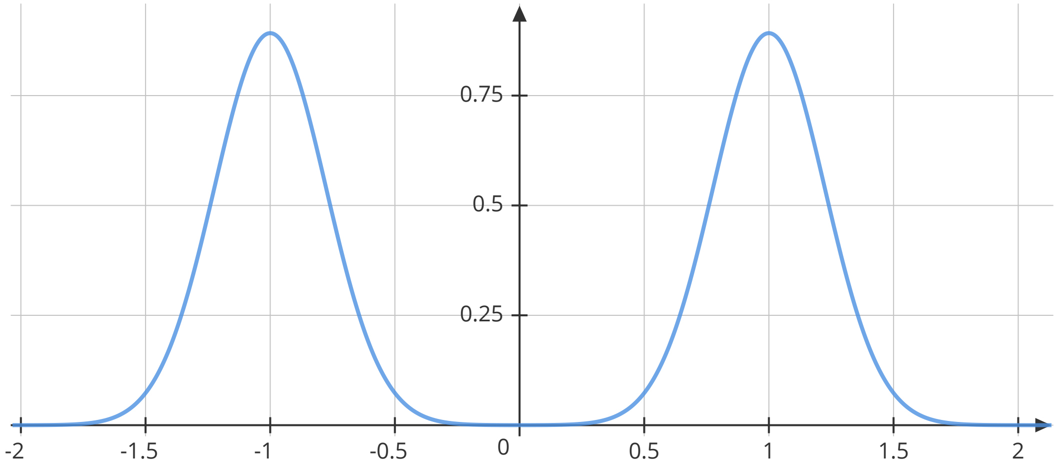
-0.1	0.98
0	
0.1	

Z: Prior and Posterior



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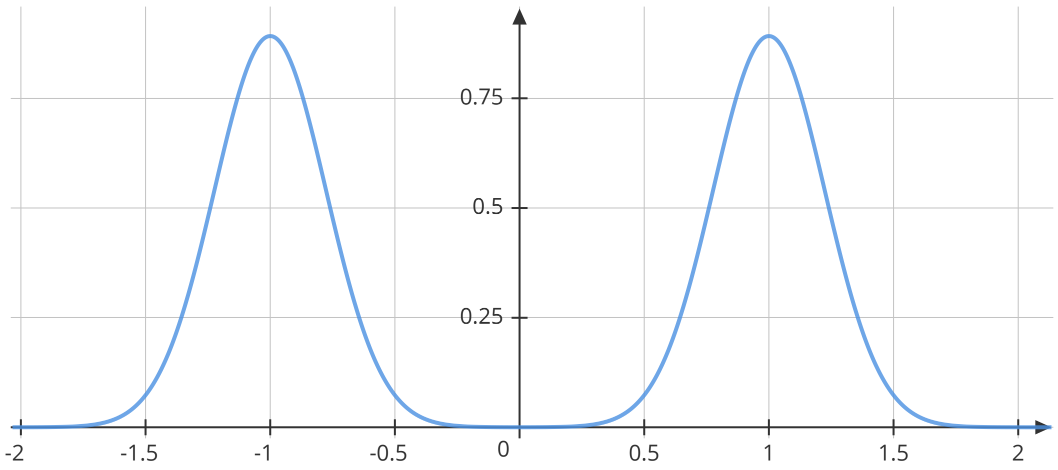
0	0.5
0.1	

Z: Prior and Posterior



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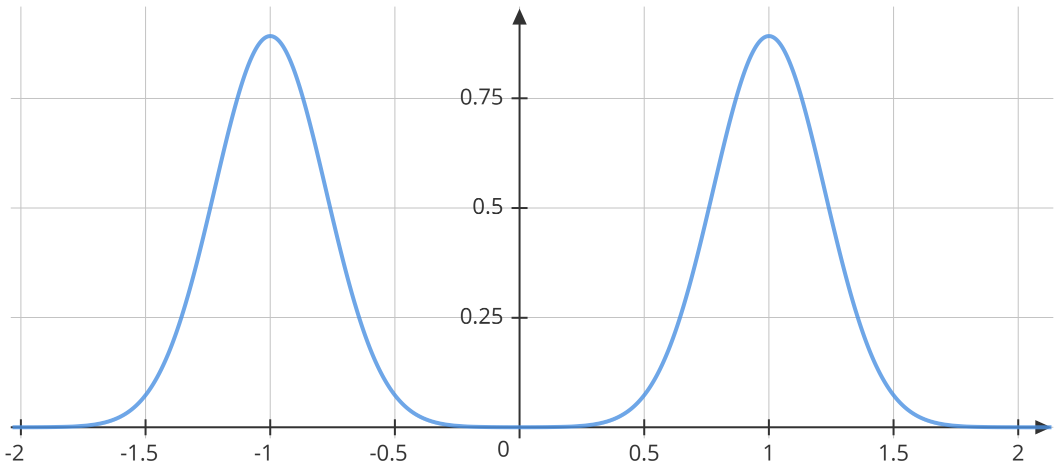
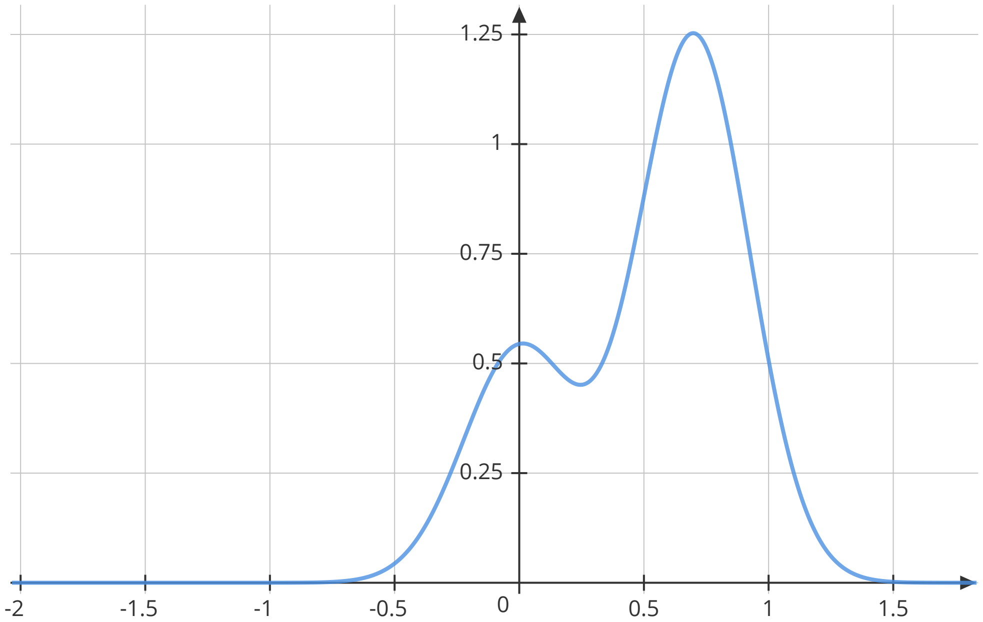
$$\mu_2 = 1,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02

Z: Prior and Posterior





$$\pi_1 = 0.5,\ \ \pi_2 = 0.5$$

$$\mu_1 = -1,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 1,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02

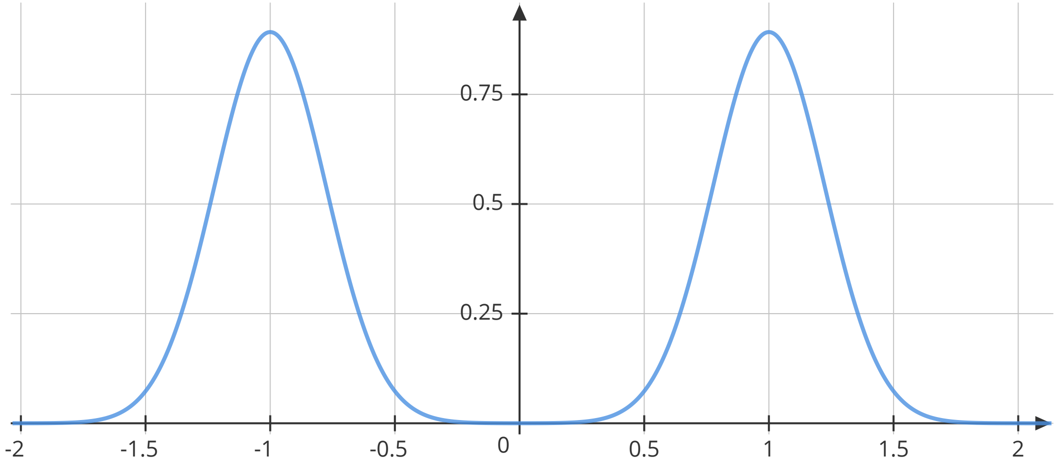
$$\pi_1 = 0.3,\ \ \pi_2 = 0.7$$

$$\mu_1 = 0,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
0.1	
0.2	
0.3	
0.4	
0.5	

Z: Prior and Posterior

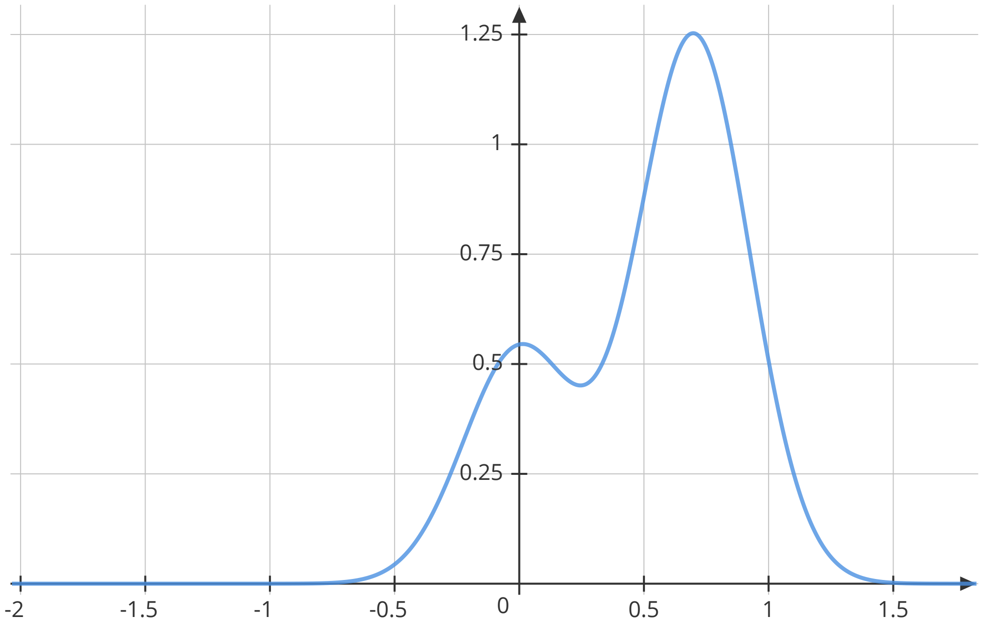


$$\pi_1 = 0.5,\ \ \pi_2 = 0.5$$

$$\mu_1 = -1,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 1,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02



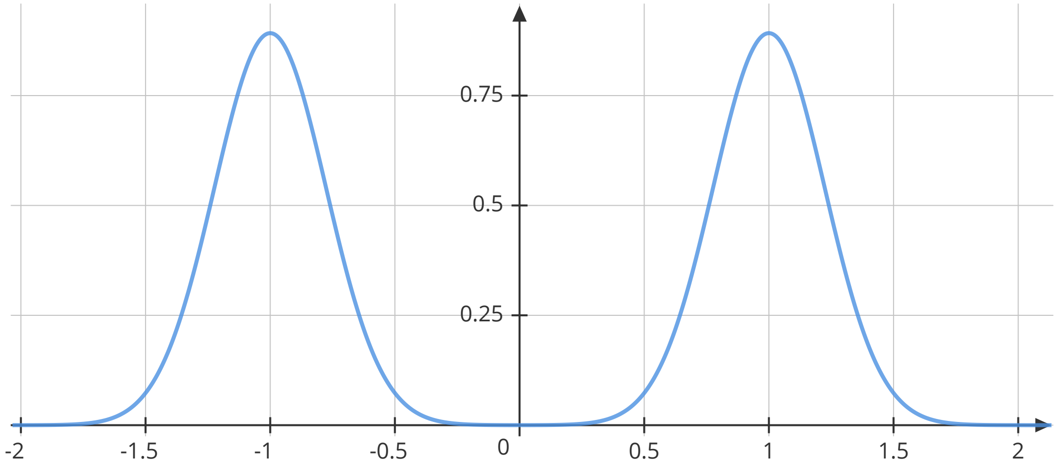
$$\pi_1 = 0.3,\ \ \pi_2 = 0.7$$

$$\mu_1 = 0,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
0.1	0.93
0.2	
0.3	
0.4	
0.5	

Z: Prior and Posterior

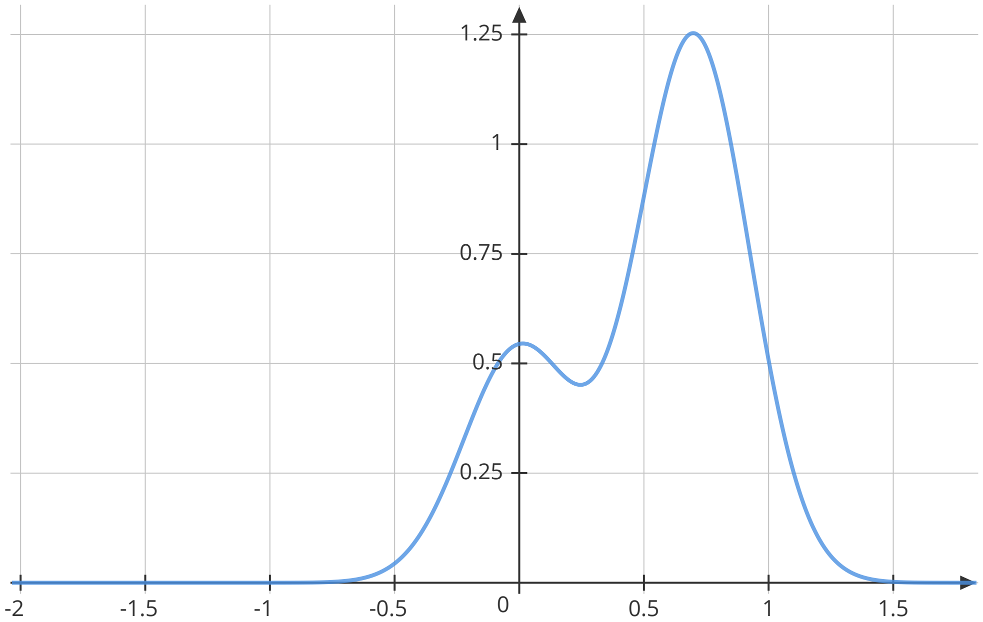


$$\pi_1 = 0.5,\ \pi_2 = 0.5$$

$$\mu_1 = -1,\ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 1,\ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\|X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02



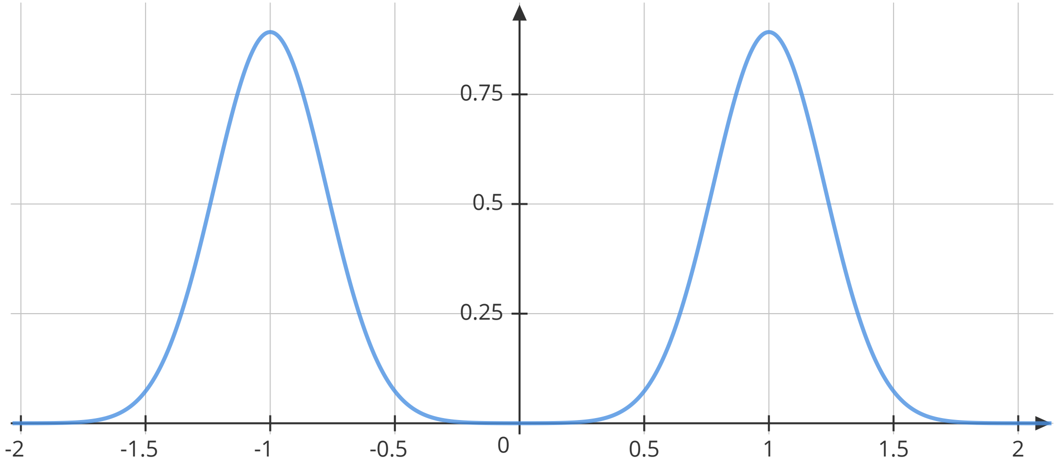
$$\pi_1 = 0.3,\ \pi_2 = 0.7$$

$$\mu_1 = 0,\ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7,\ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\|X}(1 \mid x)$
0.1	0.93
0.2	0.78
0.3	
0.4	
0.5	

Z: Prior and Posterior



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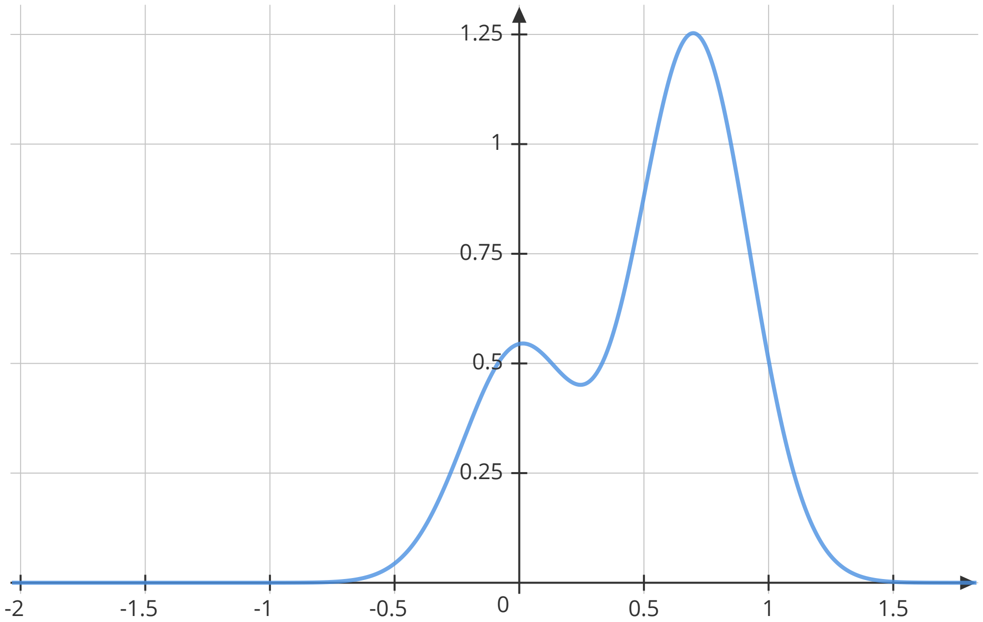
$$\mu_2 = 1, \;\; \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\|X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02



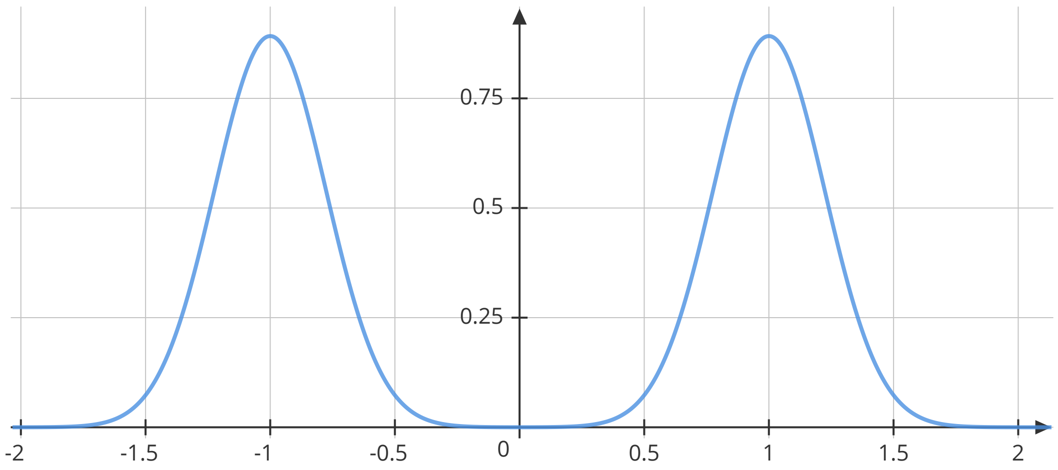
$$\pi_1 = 0.3, \;\; \pi_2 = 0.7$$

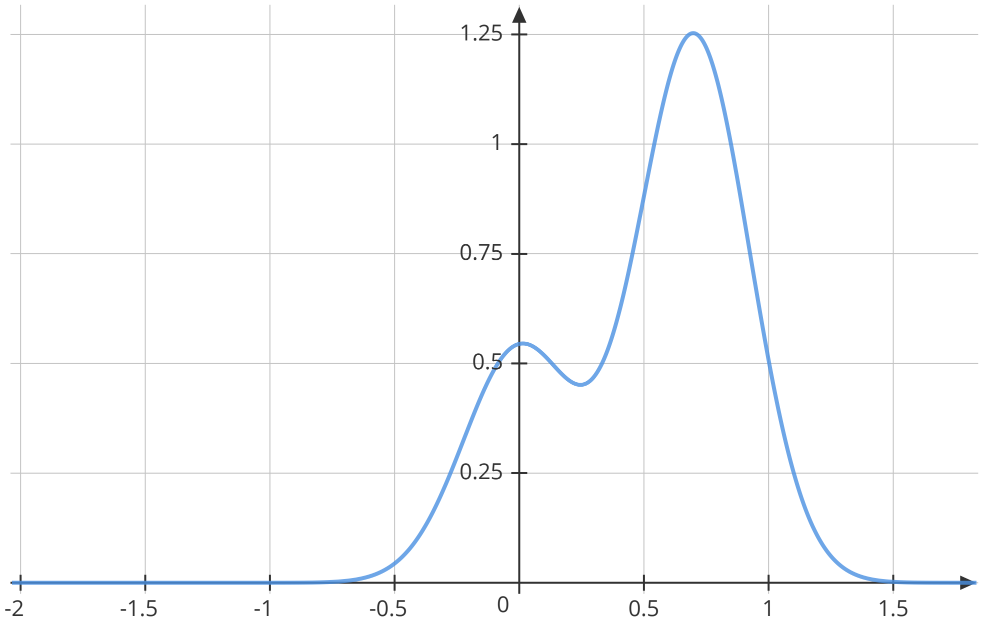
$$\mu_1 = 0, \;\; \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7, \;\; \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\|X}(1 \mid x)$
0.1	0.93
0.2	0.78
0.3	0.46
0.4	
0.5	

Z: Prior and Posterior





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x	$f_{Z\|X}(1 \mid x)$
-0.1	0.98
0	0.5
0.1	0.02

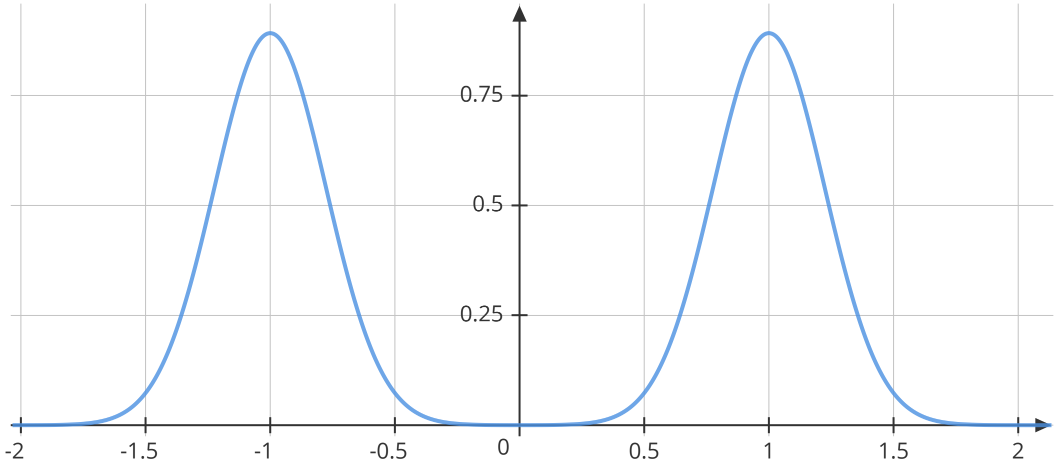
$$\pi_1 = 0.3,\ \ \pi_2 = 0.7$$

$$\mu_1 = 0,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\|X}(1 \mid x)$
0.1	0.93
0.2	0.78
0.3	0.46
0.4	0.18
0.5	

Z: Prior and Posterior



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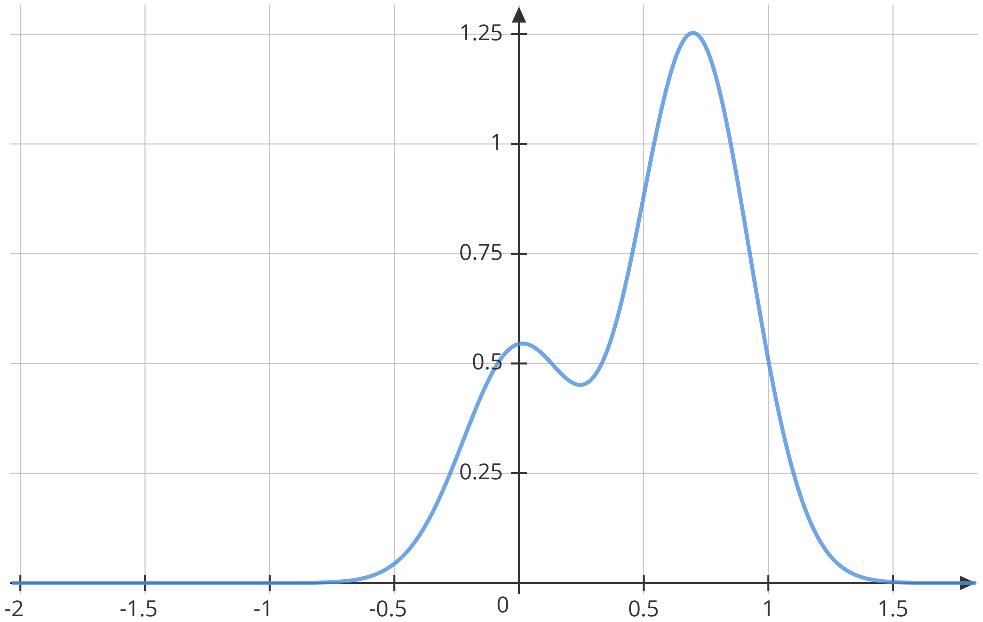
0.1	0.02



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$$\mu_1 = 0,\ \ \sigma_1^2 = \frac{1}{20}$$

$$\mu_2 = 0.7,\ \ \sigma_2^2 = \frac{1}{20}$$

x	$f_{Z\mid X}(1 \mid x)$
0.1	0.93
0.2	0.78
0.3	0.46
0.4	0.18
0.5	0.05

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$$

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$ $3K - 1$ free parameters

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$ $3K-1$ free parameters

$$f_X(x) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x;\ \mu_k, \sigma_k^2\right)$$

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

$$l(\boldsymbol{\theta};\ D)$$

i.i.d samples from a GMM
with K components

$$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$$

$3K - 1$ free parameters

$$f_X(x) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x;\ \mu_k, \sigma_k^2\right)$$

Log-Likelihood

$$D = \{x_1, \, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$$l(\boldsymbol{\theta};\; D) = \log \prod_{i=1}^{n} f_X(x_i;\; \boldsymbol{\theta})$$

$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$ $3K-1$ free parameters

$$f_X(x) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x;\; \mu_k, \sigma_k^2\right)$$

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$$l(\boldsymbol{\theta};\ D) = \log \prod_{i=1}^{n} f_X(x_i;\ \boldsymbol{\theta})$$

$$= \sum_{i=1}^{n} \log f_X(x_i;\ \boldsymbol{\theta})$$

$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$ $3K-1$ free parameters

$$f_X(x) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x;\ \mu_k, \sigma_k^2\right)$$

Log-Likelihood

$$D = \{x_1, \cdots, x_n\}$$

i.i.d samples from a GMM
with K components

$$\boldsymbol{\theta} = [\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\sigma}]$$

$3K-1$ free parameters

$$f_X(x) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x;\ \mu_k, \sigma_k^2\right)$$

$$\begin{aligned} l(\boldsymbol{\theta};\ D) &= \log \prod_{i=1}^{n} f_X(x_i;\ \boldsymbol{\theta}) \\ &= \sum_{i=1}^{n} \log f_X(x_i;\ \boldsymbol{\theta}) \\ &= \sum_{i=1}^{n} \log\left[\sum_{k=1}^{K} \pi_k \cdot \mathcal{N}\left(x_i;\ \mu_k, \sigma_k^2\right)\right] \end{aligned}$$