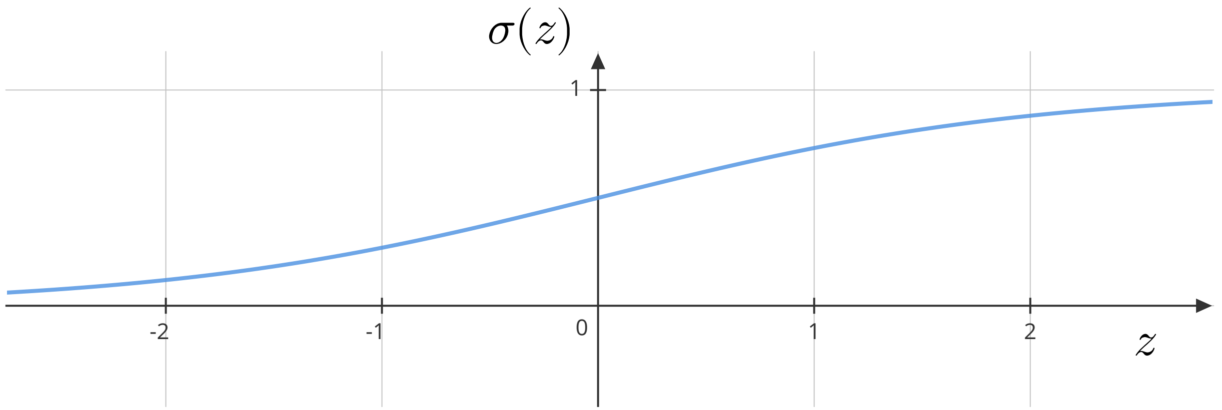
Logistic Regression

Machine Learning Techniques

References and Credits

- The content presented in these slides is derived from professor Arun Rajkumar's lectures and slides in the MLT course. This is the "ground truth" for almost all the content in these slides. As a result, these slides should be viewed as a presentation of the same content in the professor's lectures using a different medium. At the same time, these slides are not meant to be a replacement for the lectures.
- Machine Learning Refined, Second Edition
- The method of incrementally displaying content on slides is borrowed from professor Mitesh Khapra.
- These slides were prepared using the tool mathcha.io.

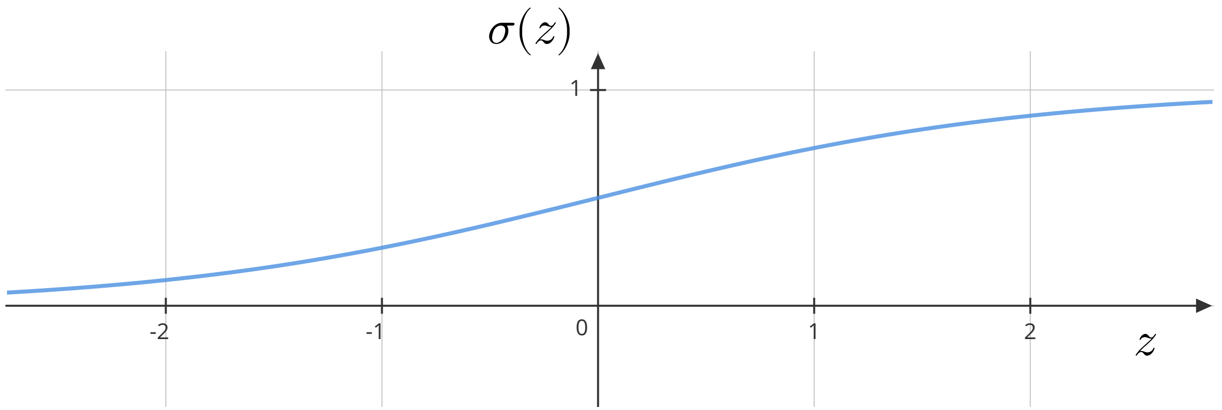
Sigmoid Function



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid Function

$$\sigma(z) = \begin{cases} \geqslant 0.5, & z \geqslant 0 \\ < 0.5 & z < 0 \end{cases}$$



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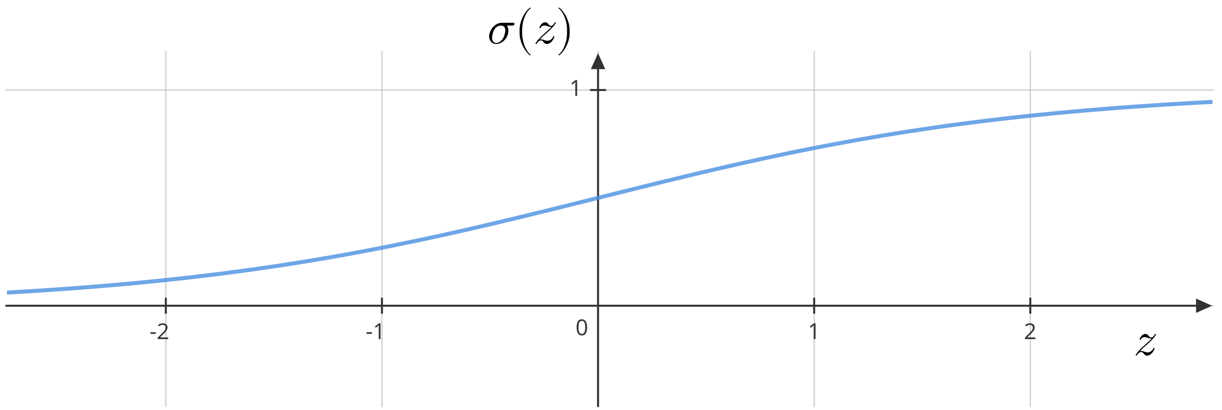

$$\lim_{z \to \infty} \sigma(z) = 1 \qquad \lim_{z \to -\infty} \sigma(z) = 0$$

$$\sigma'(z) =$$

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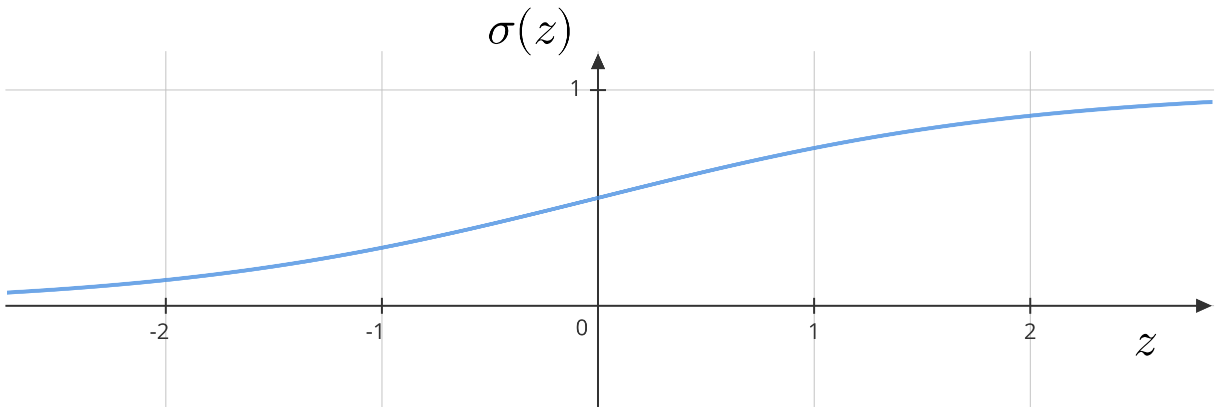

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$$\sigma(z) = \frac{1}{1+e^{-z}}$$

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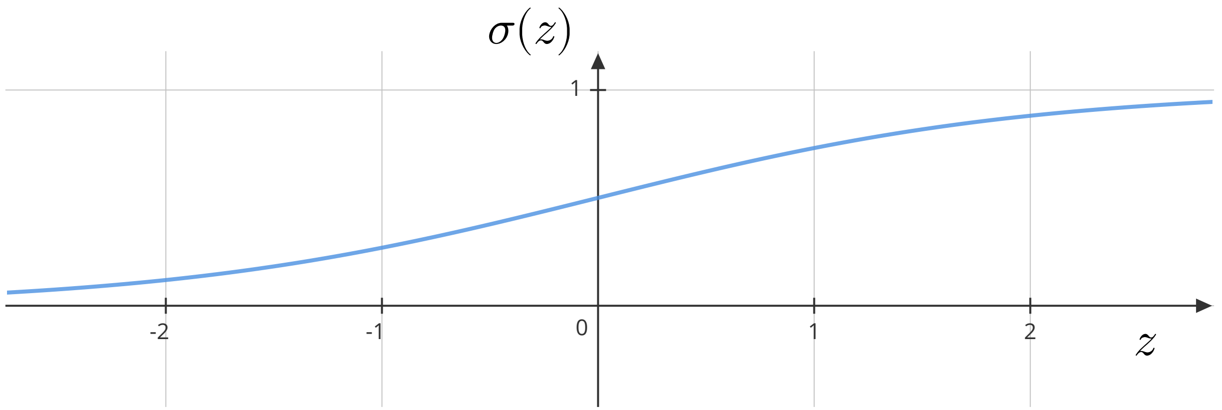

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$$\begin{aligned} \sigma'(z) &= \frac{-1}{\left(1+e^{-z}\right)^2} \cdot e^{-z} \cdot (-1) \\ &= \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} \end{aligned}$$

Sigmoid Function

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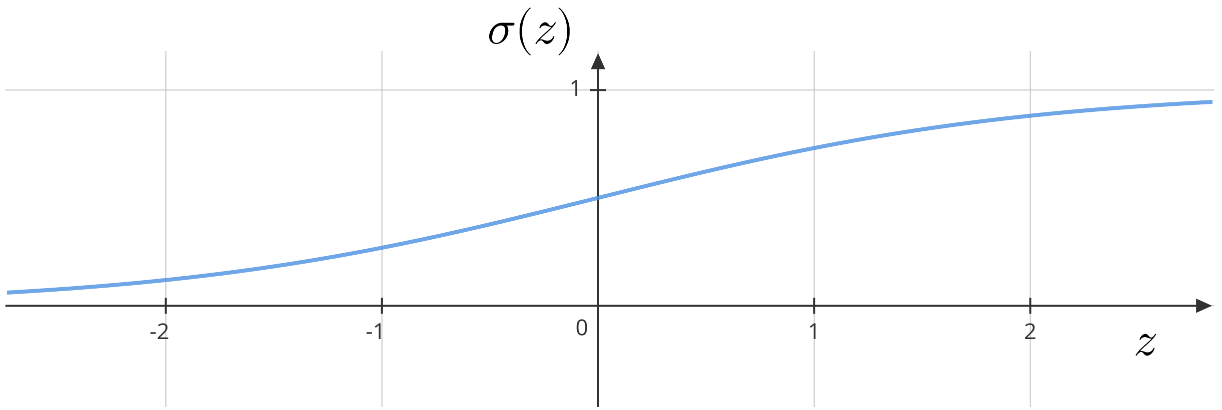

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Logistic Regression

Model

$$P(y = 1 \mid \mathbf{x}) = \sigma\big(\mathbf{w}^T \mathbf{x}\big)$$

Discriminative

$$h : \mathbb{R}^d \to \{0, 1\}$$

$$h(\mathbf{x}) = \begin{cases} 1, & \sigma\big(\mathbf{w}^T \mathbf{x}\big) \geqslant T \\ 0, & \texttt{otherwise} \end{cases}$$

- T is a threshold
- Typically, $T = 0.5$

Logistic Regression

MLE

$$\sigma_i = \sigma\left(\mathbf{w}^T \mathbf{x}_i\right)$$

σ_i is the probability of seeing label 1

$$Y_i \mid \mathbf{x}_i \sim Br(\sigma_i)$$

Likelihood

$$\begin{aligned} L(\mathbf{w}; D) &= P(D \mid \mathbf{w}) \\ &= \prod_{i=1}^{n} \sigma_i^{y_i} \cdot (1 - \sigma_i)^{1-y_i} \end{aligned}$$

$$\max_{\mathbf{w}} \quad l(\mathbf{w}; D)$$

Log-Likelihood

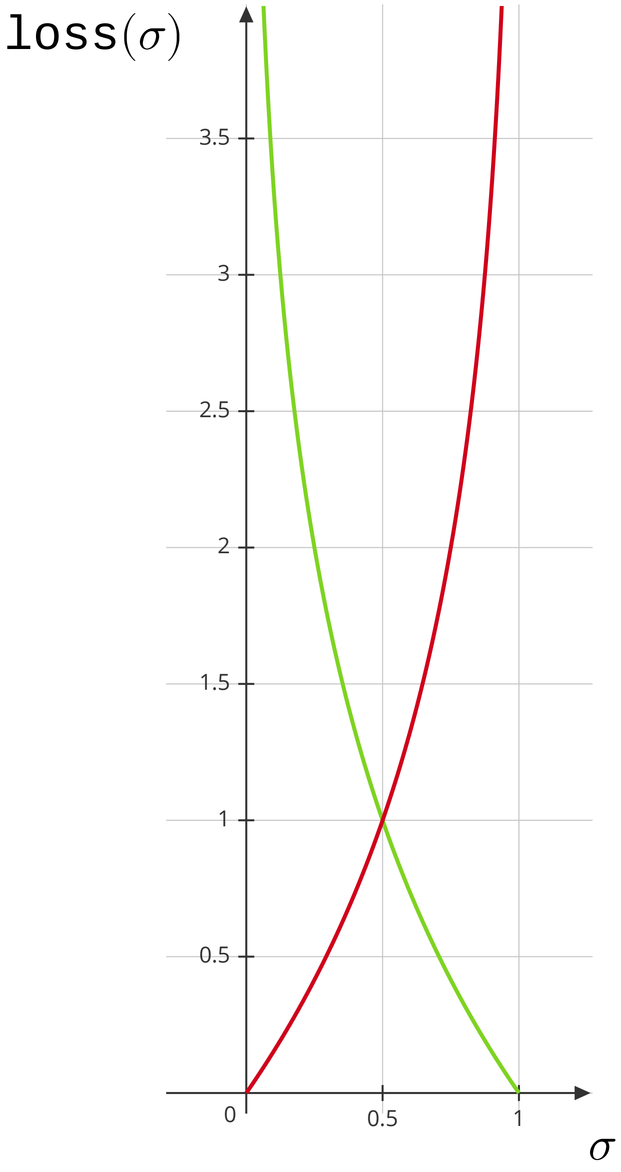
$$\begin{aligned} l(\mathbf{w}; D) &= \log L(\mathbf{w}; D) \\ &= \sum_{i=1}^{n} y_i \log \sigma_i + (1 - y_i)\log(1 - \sigma_i) \\ &= \sum_{i=1}^{n} y_i \log \sigma\left(\mathbf{w}^T \mathbf{x}_i\right) + (1 - y_i)\log\left[1 - \sigma\left(\mathbf{w}^T \mathbf{x}_i\right)\right] \end{aligned}$$

Logistic Regression

Loss

$$\texttt{loss}(\mathbf{w}) = \sum_{i=1}^{n} -y_i \log \sigma_i - (1-y_i)\log(1-\sigma_i)$$

negative log-likelihood



$$\texttt{loss}(\sigma) = \begin{cases} -\log \sigma, & y = 1 \\ -\log(1-\sigma), & y = 0 \end{cases}$$

$$-y \log \sigma - (1-y)\log(1-\sigma)$$

cross-entropy

Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\Big(\mathbf{w}^T \mathbf{x}_i\Big) + (1 - y_i) \log\Big[1 - \sigma\Big(\mathbf{w}^T \mathbf{x}_i\Big)\Big]$$

Optimization

Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

Optimization

$$\nabla l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \cdot \frac{1}{\sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \sigma'\left(\mathbf{w}^T \mathbf{x}_i\right) \cdot \mathbf{x}_i + (1 - y_i) \cdot \frac{1}{1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \left[-\sigma'\left(\mathbf{w}^T \mathbf{x}_i\right)\right] \cdot \mathbf{x}_i$$

Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

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Logistic Regression Optimization

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\Big(\mathbf{w}^T \mathbf{x}_i\Big) + (1 - y_i) \log\Big[1 - \sigma\Big(\mathbf{w}^T \mathbf{x}_i\Big)\Big]$$

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$$\sigma'(z) = \sigma(z)[1 - \sigma(z)]$$

Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\left(\mathbf{w}^T \mathbf{x}_i\right) + (1 - y_i) \log\left[1 - \sigma\left(\mathbf{w}^T \mathbf{x}_i\right)\right]$$

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$$= \sum_{i=1}^{n} y_i \cdot \left[1 - \sigma\left(\mathbf{w}^T \mathbf{x}_i\right)\right] \cdot \mathbf{x}_i - (1 - y_i) \cdot \sigma\left(\mathbf{w}^T \mathbf{x}_i\right) \cdot \mathbf{x}_i$$

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Logistic Regression Optimization

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\left(\mathbf{w}^T \mathbf{x}_i\right) + (1 - y_i) \log\left[1 - \sigma\left(\mathbf{w}^T \mathbf{x}_i\right)\right]$$

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$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

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$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

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Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

Optimization

$$\begin{aligned}
\nabla l(\mathbf{w}; D) &= \sum_{i=1}^{n} y_i \cdot \frac{1}{\sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \sigma'\big(\mathbf{w}^T \mathbf{x}_i\big) \cdot \mathbf{x}_i + (1 - y_i) \cdot \frac{1}{1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \big[-\sigma'\big(\mathbf{w}^T \mathbf{x}_i\big)\big] \cdot \mathbf{x}_i \\
&= \sum_{i=1}^{n} y_i \cdot \big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big] \cdot \mathbf{x}_i - (1 - y_i) \cdot \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) \cdot \mathbf{x}_i \\
&= \sum_{i=1}^{n} y_i \mathbf{x}_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) \cdot \mathbf{x}_i \\
&= \sum_{i=1}^{n} \big[y_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big] \mathbf{x}_i
\end{aligned}$$

$$\sigma'(z) = \sigma(z)[1 - \sigma(z)]$$

$$\begin{aligned}
e_i &= y_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) \\
&= y_i - \sigma_i
\end{aligned}$$

Logistic Regression

$$l(\mathbf{w}; D) = \sum_{i=1}^{n} y_i \log \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) + (1 - y_i) \log\big[1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\big]$$

Optimization

$$\begin{aligned}
\nabla l(\mathbf{w}; D) &= \sum_{i=1}^{n} y_i \cdot \frac{1}{\sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \sigma'\left(\mathbf{w}^T \mathbf{x}_i\right) \cdot \mathbf{x}_i + (1 - y_i) \cdot \frac{1}{1 - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)} \cdot \left[-\sigma'\left(\mathbf{w}^T \mathbf{x}_i\right)\right] \cdot \mathbf{x}_i \\
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&= \sum_{i=1}^{n} \left[y_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\right] \mathbf{x}_i \\
&= \sum_{i=1}^{n} e_i \cdot \mathbf{x}_i
\end{aligned}$$

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$$\begin{aligned}
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Logistic Regression Optimization

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$$= \sum_{i=1}^{n} y_i \mathbf{x}_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) \cdot \mathbf{x}_i$$

$$\begin{aligned} e_i &= y_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big) \\ &= y_i - \sigma_i \end{aligned}$$

$$= \sum_{i=1}^{n} \left[y_i - \sigma\big(\mathbf{w}^T \mathbf{x}_i\big)\right] \mathbf{x}_i$$

$$= \sum_{i=1}^{n} e_i \cdot \mathbf{x}_i$$

$$\boxed{\texttt{gradient} = \texttt{error} \times \texttt{data-point}}$$

LogisticRegression(D):

    $\mathbf{w}^0 = \mathbf{0}$

    converged = False

    while not converged:

        $\mathbf{w}^{t+1} = \mathbf{w}^t + \alpha \sum_{i=1}^{n} \Big(y_i - \sigma\big({\mathbf{w}^t}^T \mathbf{x}_i\big)\Big)\mathbf{x}_i$

        converged = check(criterion)

    return $\mathbf{w}^*$

Gradient Ascent

LogisticRegression(D):

$\quad \mathbf{w}^0 = \mathbf{0}$

$\quad$ converged = False

$\quad$ while not converged:

$\qquad$ for $i = 1$ to n:

$$\mathbf{w}^{t+1} = \mathbf{w}^t + \alpha\Big(y_i - \sigma\big({\mathbf{w}^t}^T \mathbf{x}_i\big)\Big)\mathbf{x}_i$$

$\qquad$ converged = check(criterion)

$\quad$ return $\mathbf{w}^*$

Stochastic Gradient Ascent

Logistic Regression vs Perceptron

LogisticRegression(D) :

    $\mathbf{w}^0 = \mathbf{0}$

    converged = False

    while not converged :

        for $i = 1$ to n :

            $\boxed{\mathbf{w}^{t+1} = \mathbf{w}^t + \alpha\Big(y_i - \sigma\Big({\mathbf{w}^t}^T \mathbf{x}_i\Big)\Big)\mathbf{x}_i}$

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Logistic Regression vs Perceptron

LogisticRegression(D):

 $\mathbf{w}^0 = \mathbf{0}$

 converged = False

 while not converged:

  for $i = 1$ to n:

$$\boxed{\mathbf{w}^{t+1} = \mathbf{w}^t + \alpha\Big(y_i - \sigma\Big({\mathbf{w}^t}^T\mathbf{x}_i\Big)\Big)\mathbf{x}_i}$$

  converged = check(criterion)

 return $\mathbf{w}^*$

Perceptron(D):

 $\mathbf{w}^0 = \mathbf{0}$

 converged = False

 while not converged:

  for $i = 1$ to n:

$$\boxed{\mathbf{w}^{t+1} = \mathbf{w}^t + \frac{1}{2}\Big(y_i - \mathbf{1}\Big[{\mathbf{w}^t}^T\mathbf{x}_i \geqslant 0\Big]\Big)\mathbf{x}_i}$$

  converged = check(criterion)

 return $\mathbf{w}^*$

Logistic Regression vs Perceptron

LogisticRegression(D):

    $\mathbf{w}^0 = \mathbf{0}$

    converged = False

    while not converged:

        for $i = 1$ to n:

            $\boxed{\mathbf{w}^{t+1} = \mathbf{w}^t + \alpha\Big(y_i - \sigma\Big({\mathbf{w}^t}^T\mathbf{x}_i\Big)\Big)\mathbf{x}_i}$

        converged = check(criterion)

    return $\mathbf{w}^*$

Soft Prediction

Perceptron(D):

    $\mathbf{w}^0 = \mathbf{0}$

    converged = False

    while not converged:

        for $i = 1$ to n:

            $\boxed{\mathbf{w}^{t+1} = \mathbf{w}^t + \frac{1}{2}\Big(y_i - \mathbf{1}\Big[{\mathbf{w}^t}^T\mathbf{x}_i \geqslant 0\Big]\Big)\mathbf{x}_i}$

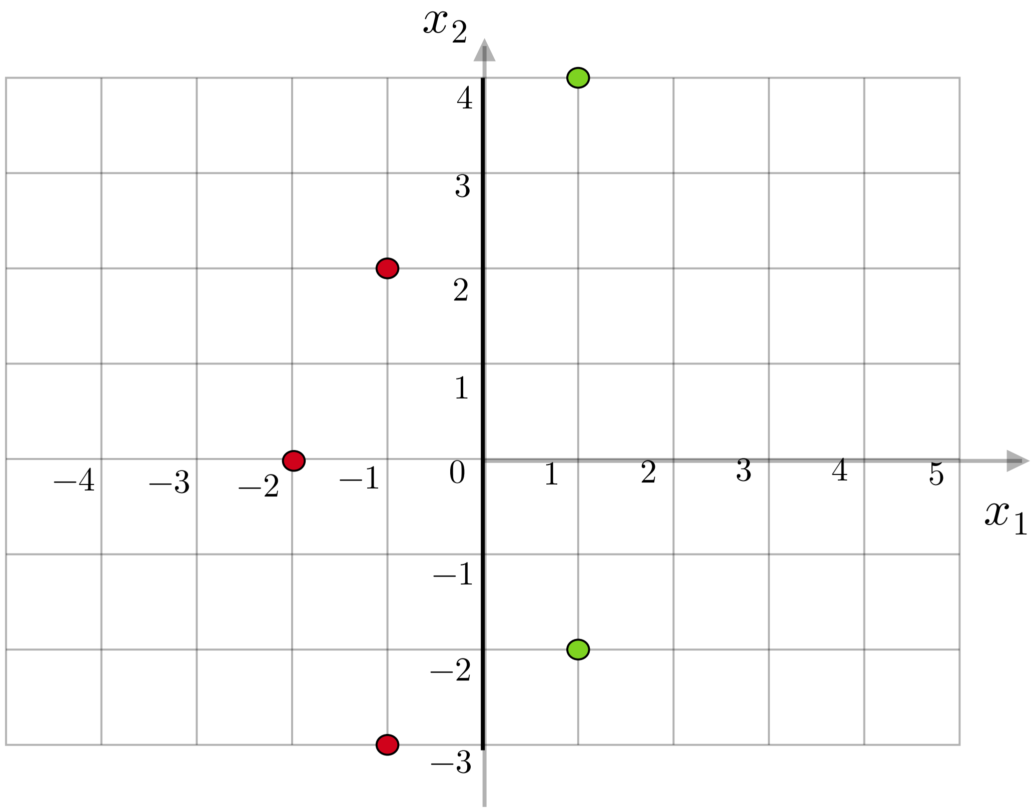
        converged = check(criterion)

    return $\mathbf{w}^*$

Hard Prediction

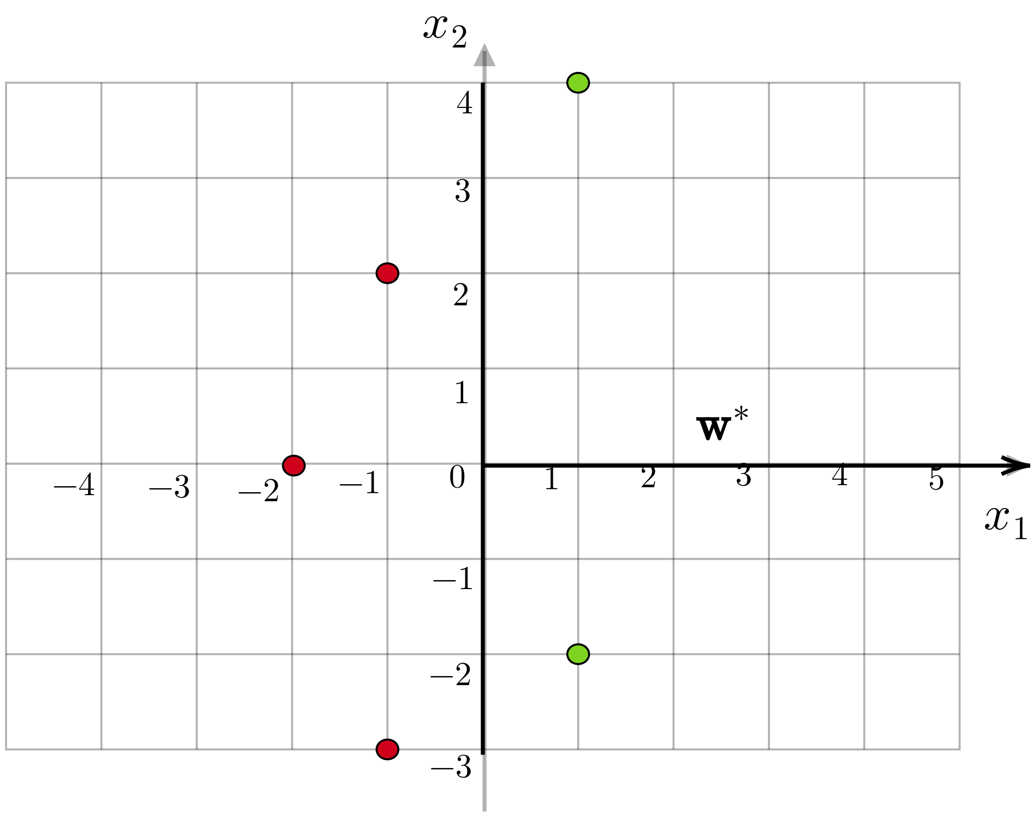
Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & -1 & -1 & -2 \\ 4 & -2 & -3 & 2 & 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

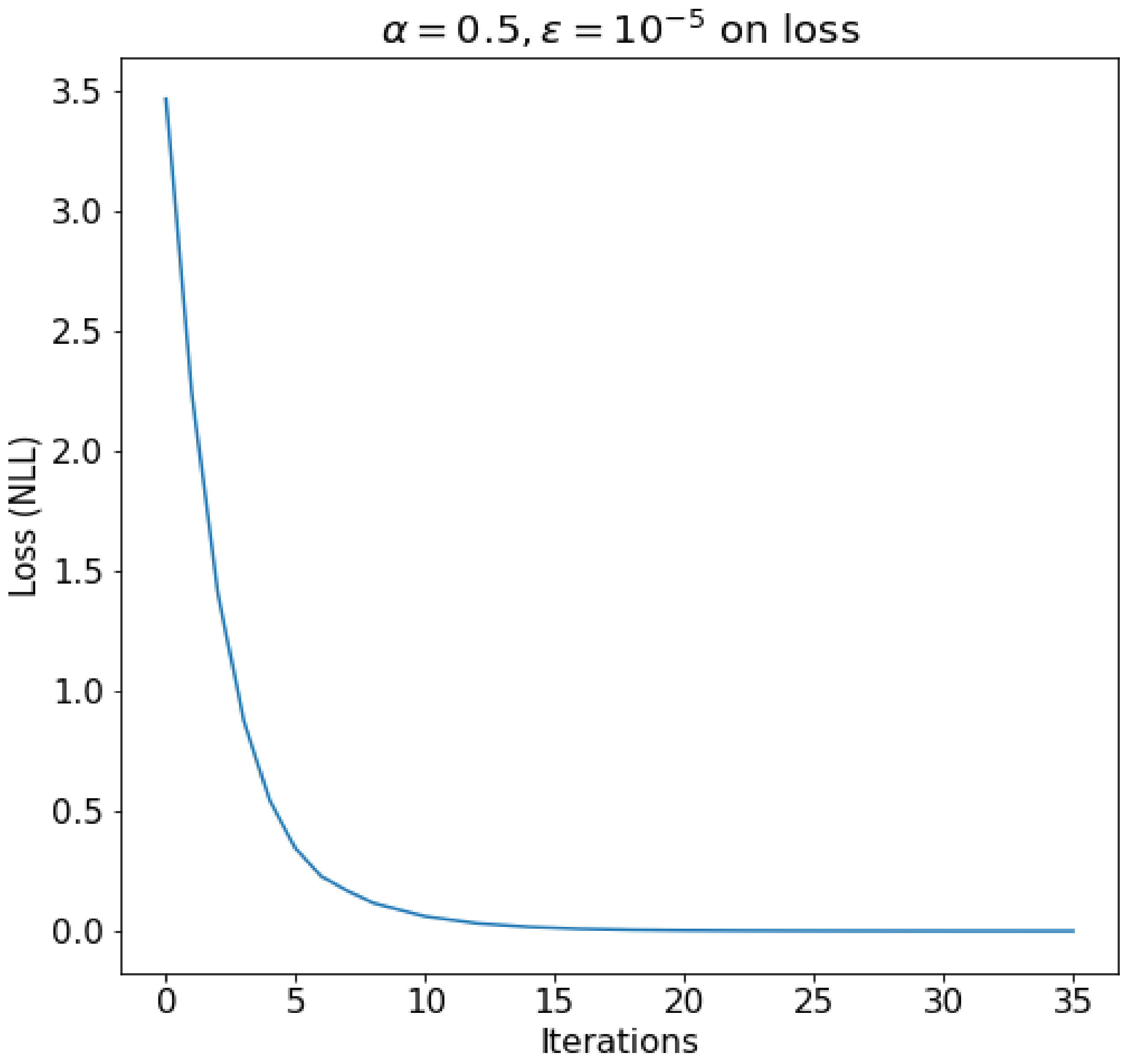


Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & -1 & -1 & -2 \\ 4 & -2 & -3 & 2 & 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



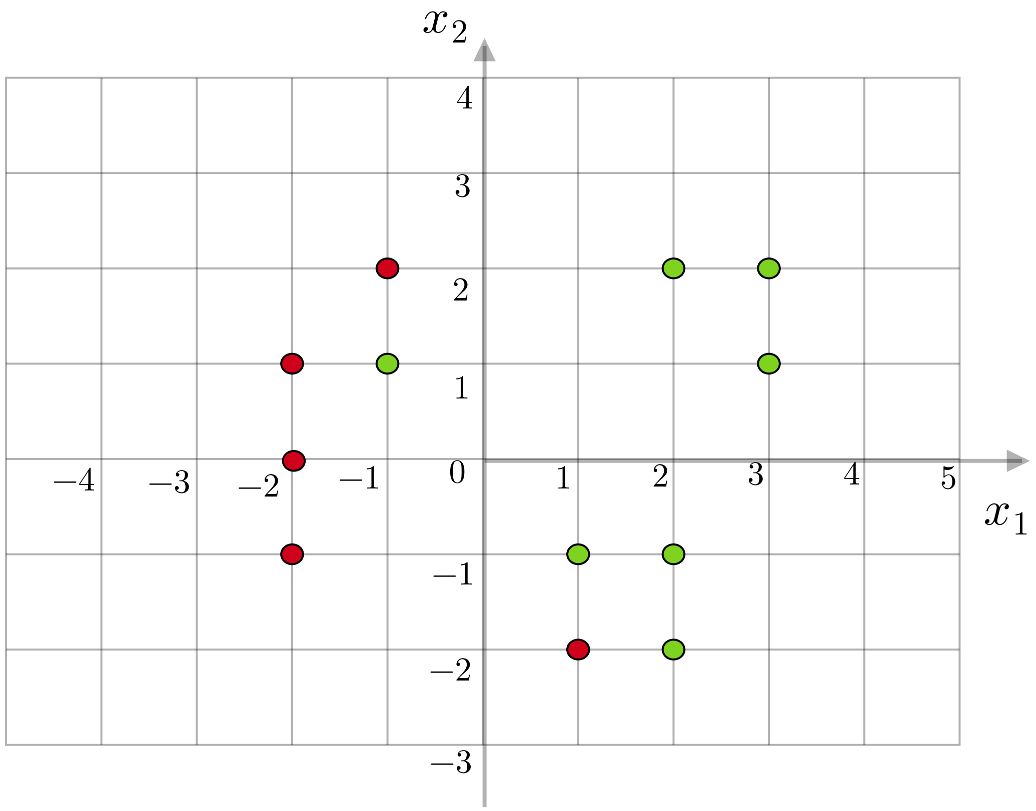
$$\mathbf{w}^* = \begin{bmatrix} 12.5 \\ -0.06 \end{bmatrix}$$



Example-2

$$\mathbf{X} = \begin{bmatrix} 2 & 3 & 3 & 1 & 2 & 2 & -1 & 1 & -1 & -2 & -2 & -2 \\ 2 & 2 & 1 & -1 & -1 & -2 & 1 & -2 & 2 & 1 & 0 & -1 \end{bmatrix},$$

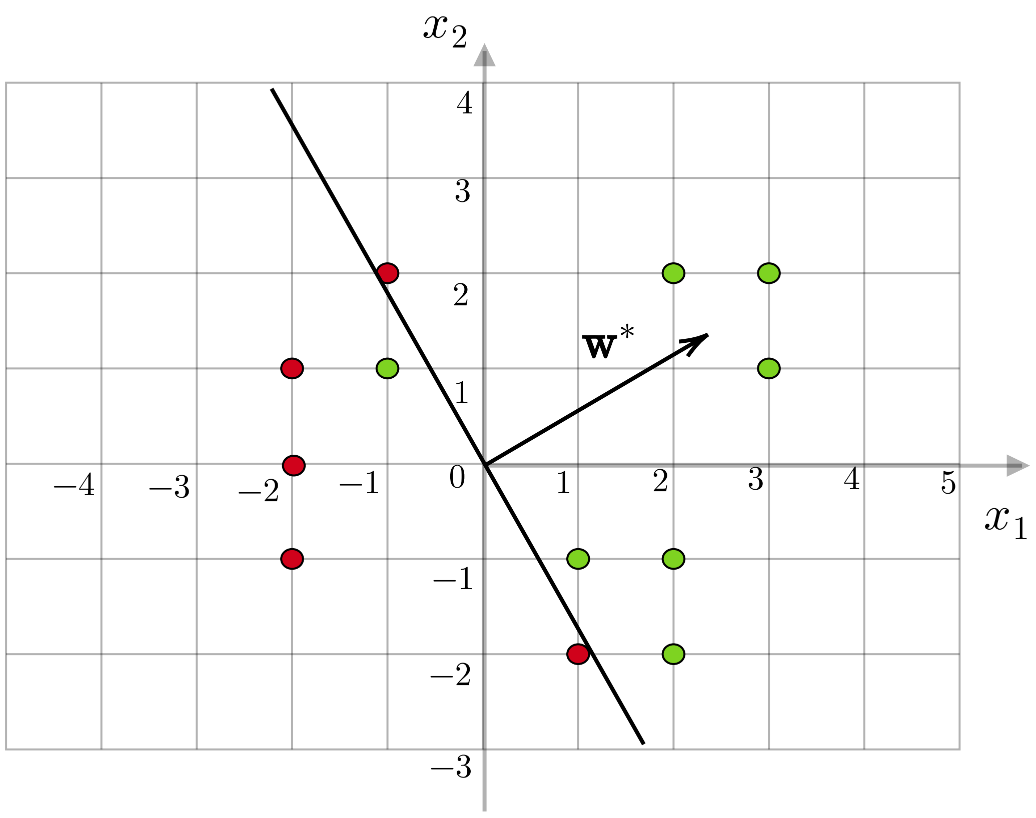
$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$



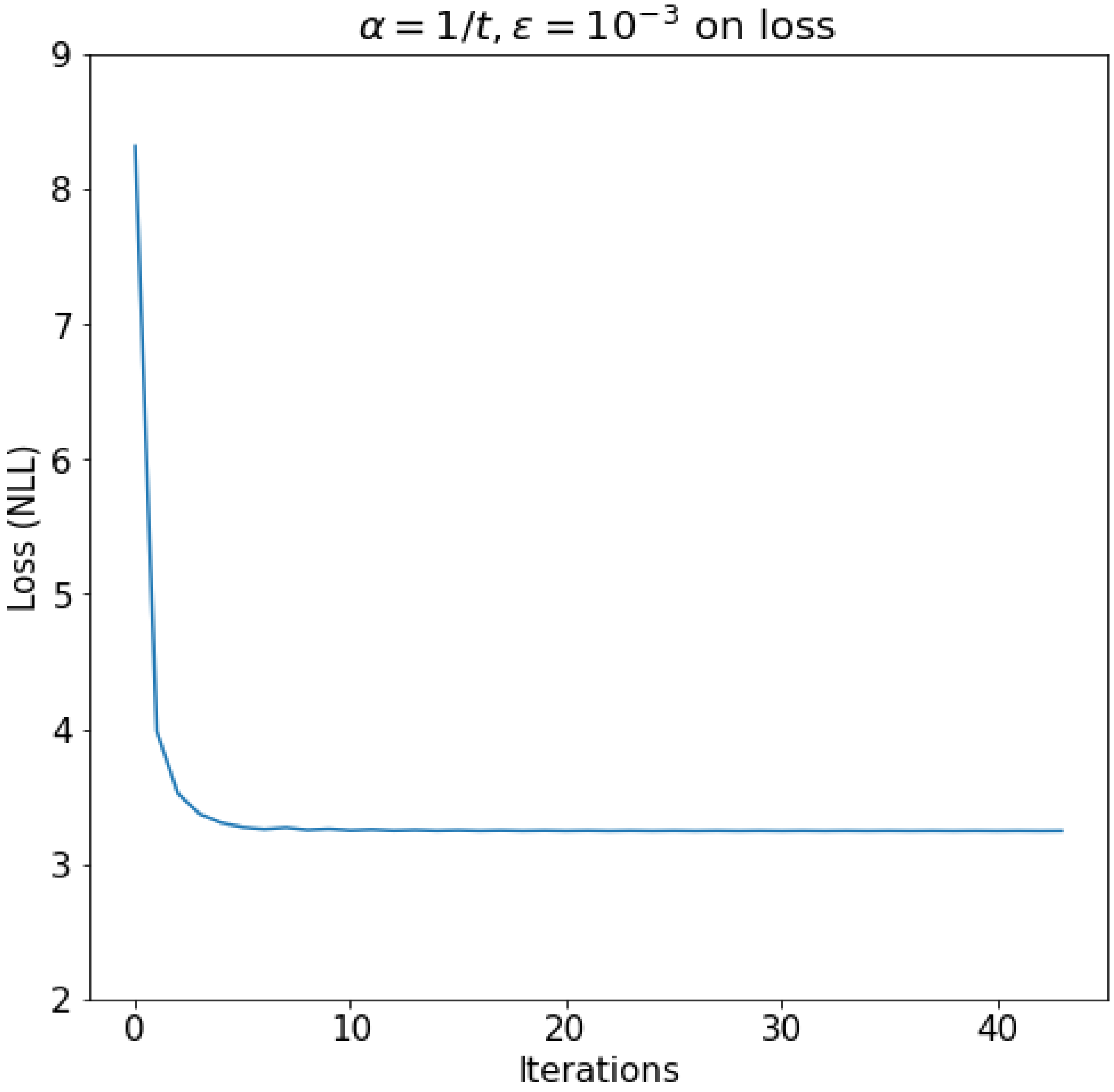
Example-2

$$\mathbf{X} = \begin{bmatrix} 2 & 3 & 3 & 1 & 2 & 2 & -1 & 1 & -1 & -2 & -2 & -2 \\ 2 & 2 & 1 & -1 & -1 & -2 & 1 & -2 & 2 & 1 & 0 & -1 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$



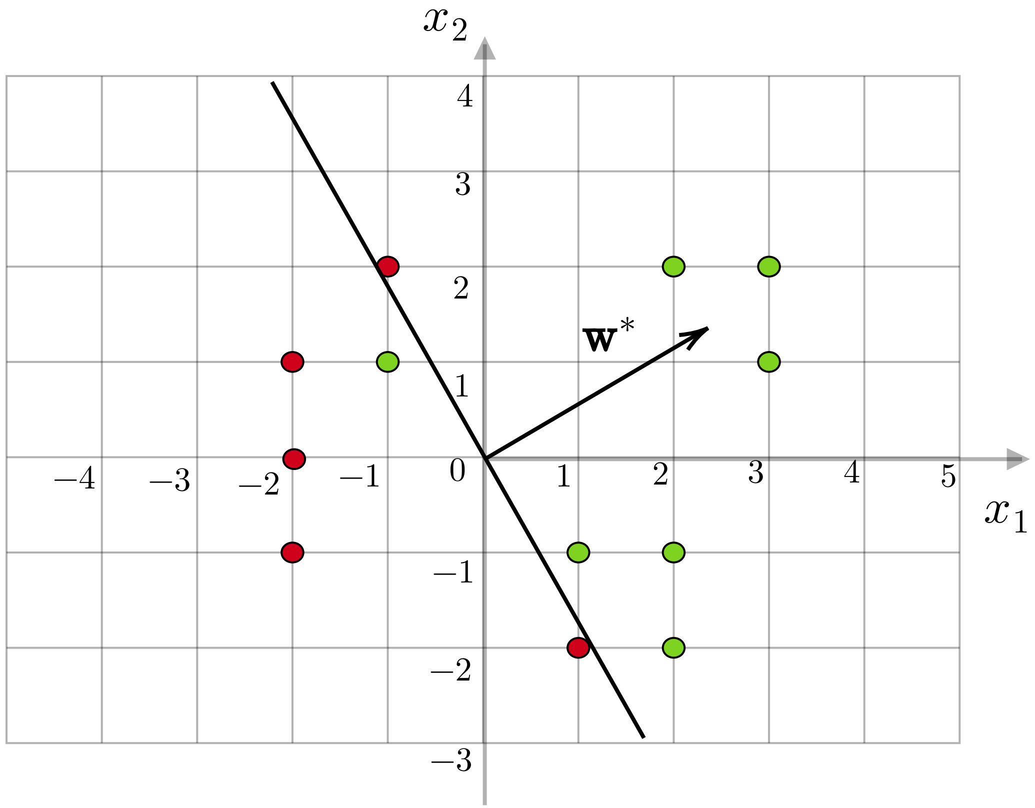
$$\mathbf{w}^* = \begin{bmatrix} 2.30 \\ 1.25 \end{bmatrix}$$



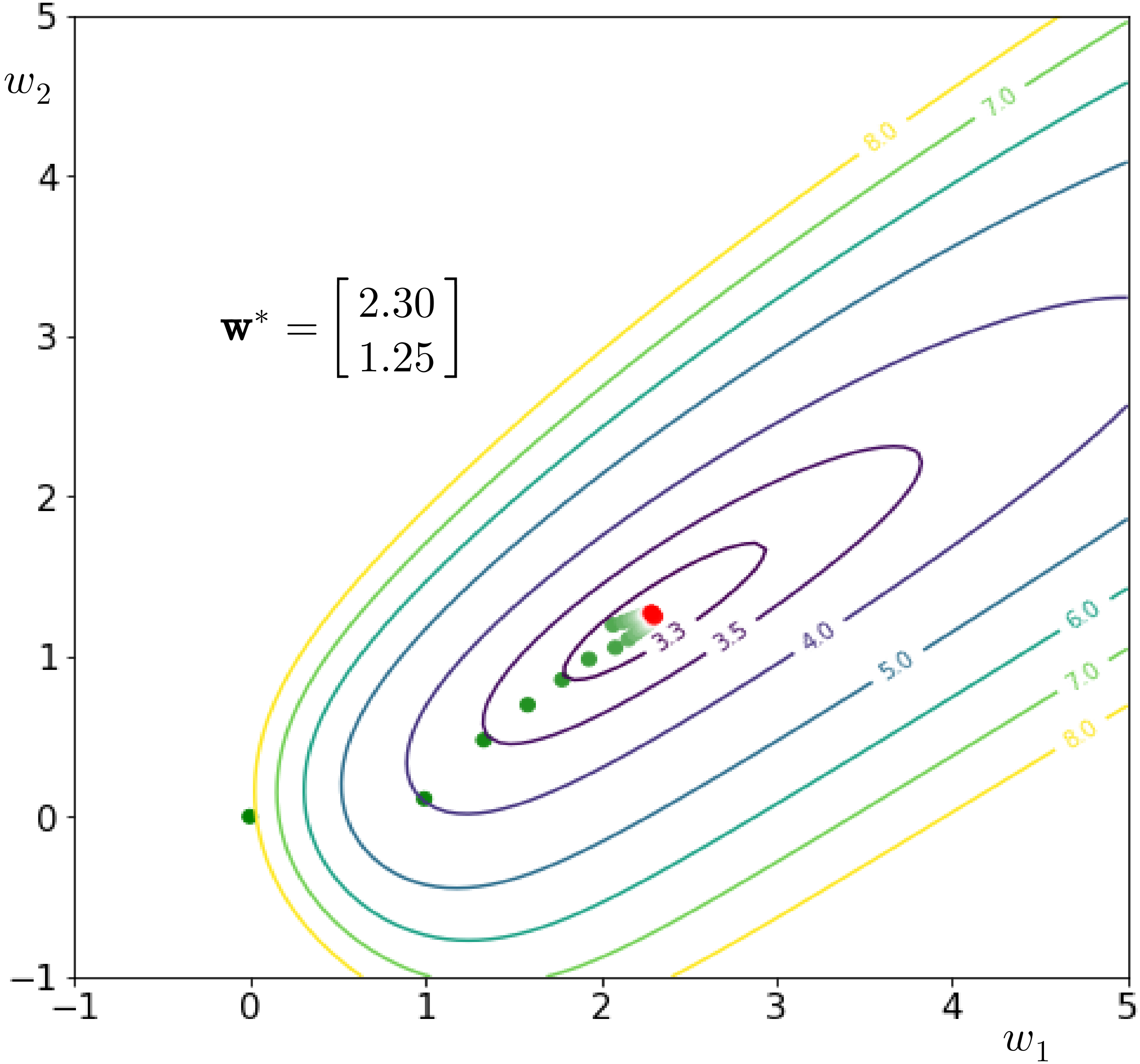
Example-2

$$\mathbf{X} = \begin{bmatrix} 2 & 3 & 3 & 1 & 2 & 2 & -1 & 1 & -1 & -2 & -2 & -2 \\ 2 & 2 & 1 & -1 & -1 & -2 & 1 & -2 & 2 & 1 & 0 & -1 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$



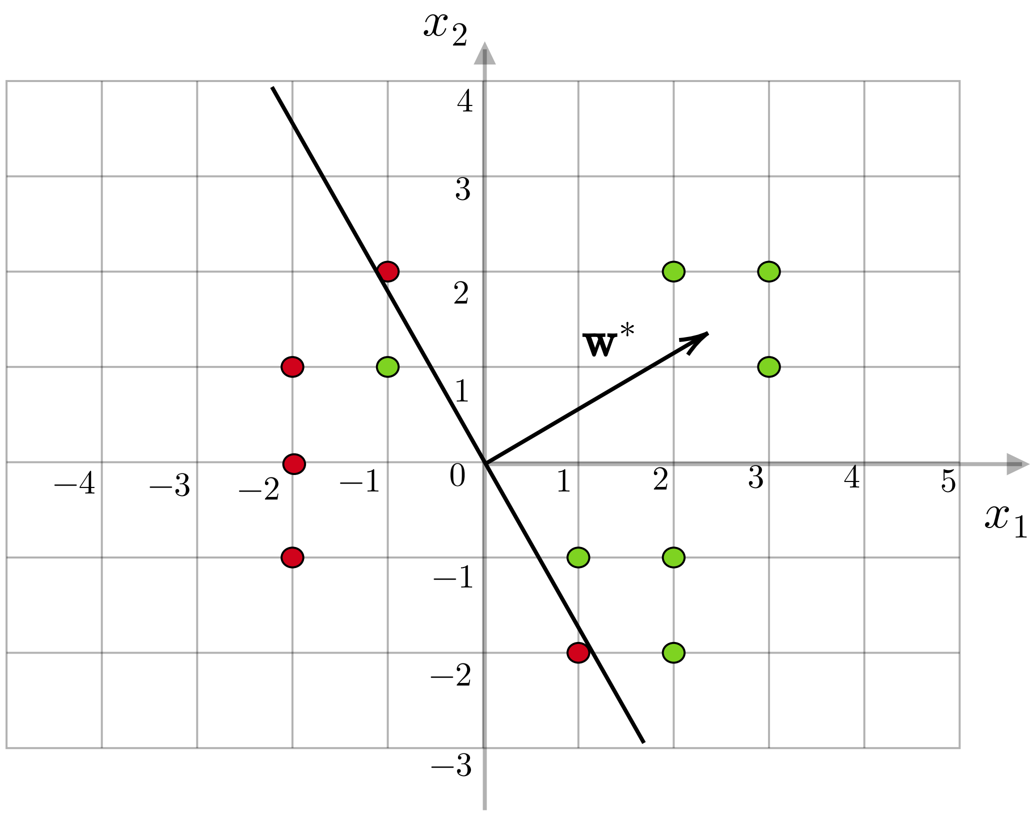
Feature Space



Weight Space

Example-2

$$\mathbf{X} = \begin{bmatrix} 2 & 3 & 3 & 1 & 2 & 2 & -1 & 1 & -1 & -2 & -2 & -2 \\ 2 & 2 & 1 & -1 & -1 & -2 & 1 & -2 & 2 & 1 & 0 & -1 \end{bmatrix},$$
$$\mathbf{y} = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$



$$\mathbf{w}^* = \begin{bmatrix} 2.30 \\ 1.25 \end{bmatrix}$$

x_1	x_2	y	σ	$\widehat{y}$
2	2	1	0.99	1
3	2	1	0.99	1
3	1	1	0.99	1
1	−1	1	0.74	1
2	−1	1	0.96	1
2	−2	1	0.89	1
−1	1	1	0.25	0
1	−2	0	0.45	0
−1	2	0	0.54	1
−2	1	0	0.03	0
−2	0	0	0.01	0
−2	−1	0	0.002	0