Classification

Machine Learning Techniques

Karthik Thiagarajan

Classification

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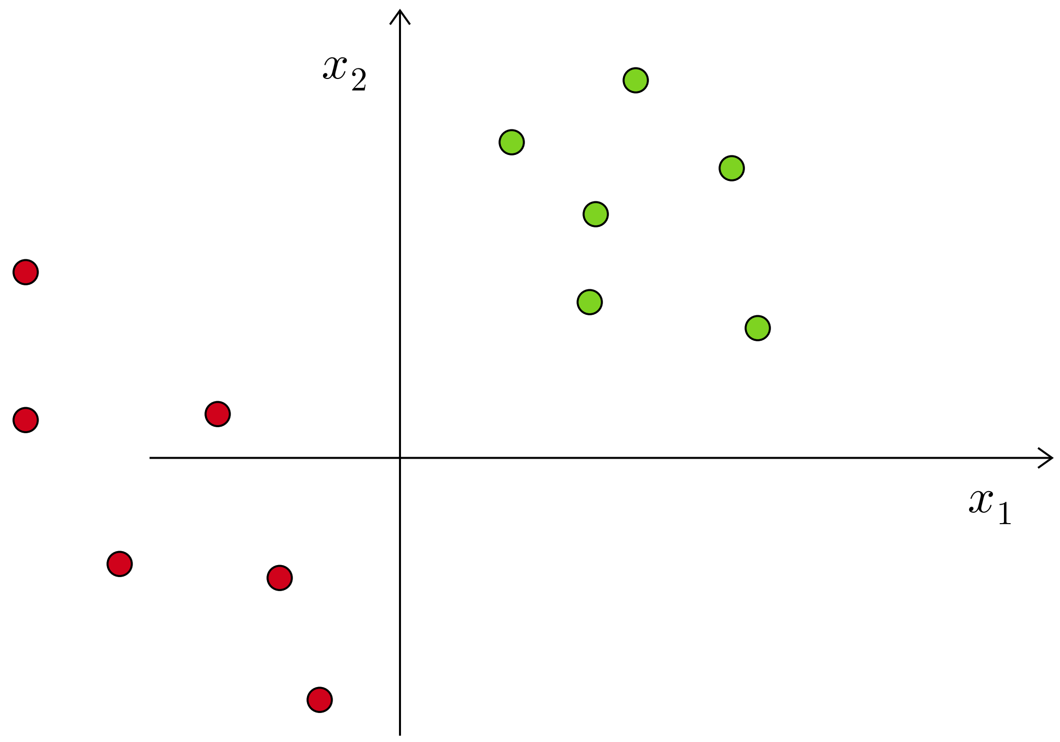
0-1 Loss

$$\text{Loss}(h) = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbf{1}[h(\mathbf{x}_i) \neq y_i]$$

Linear Classifiers

class-1
class-0



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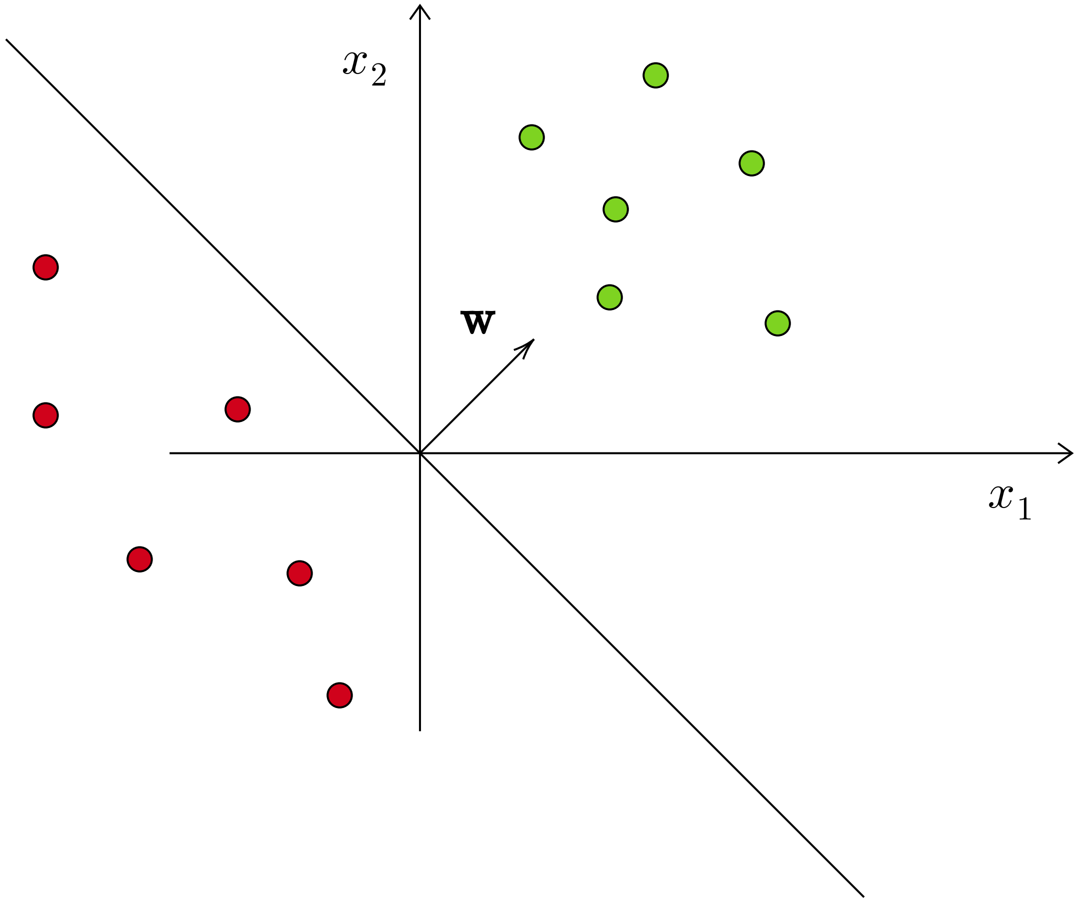

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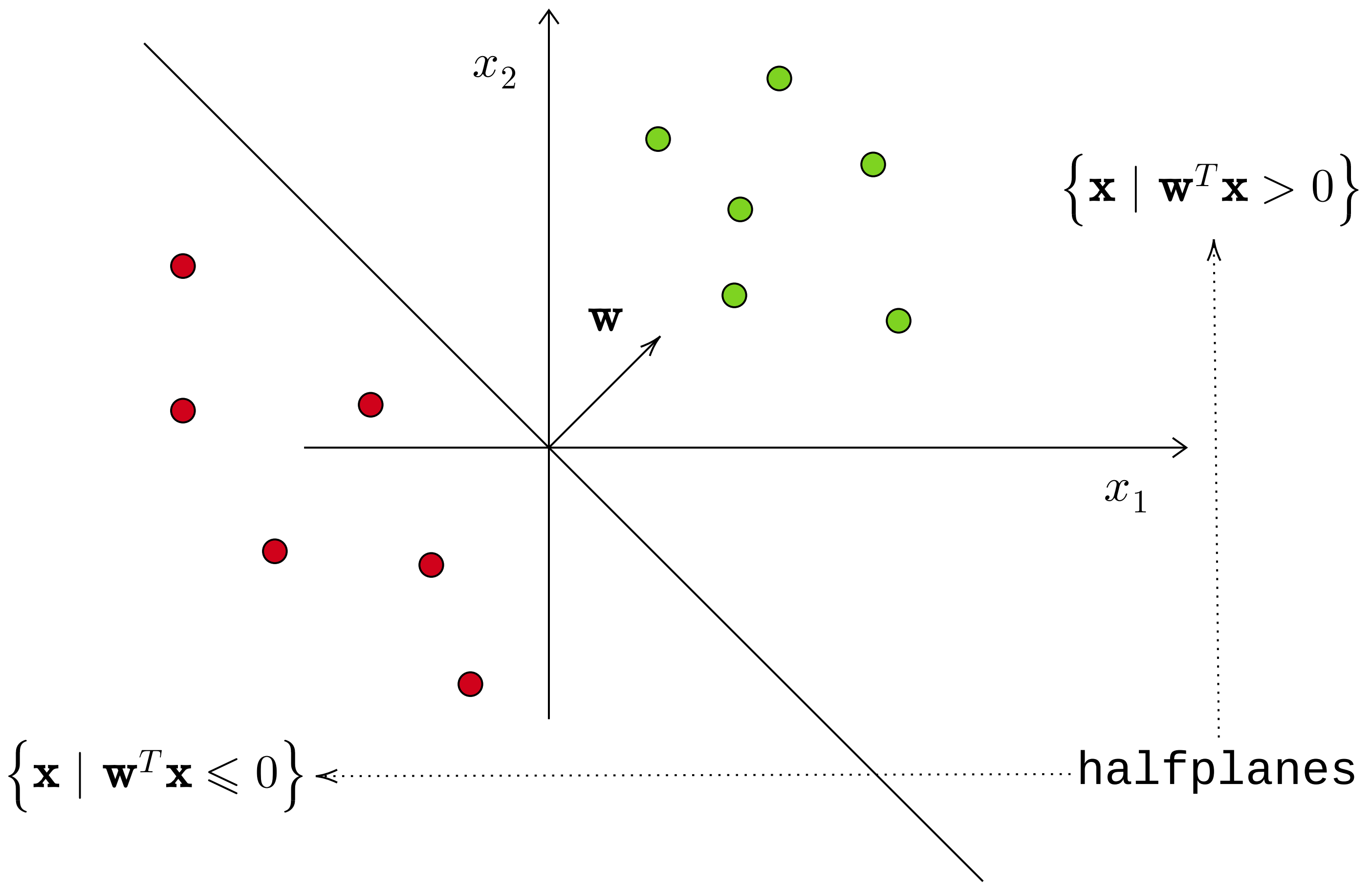

Linear Classifiers

$$h_{\mathbf{w}}(\mathbf{x}) = \mathtt{sign}\left(\mathbf{w}^T\mathbf{x}\right)$$

$$= \begin{cases} 1, & \mathbf{w}^T\mathbf{x} > 0 \\ 0, & \mathbf{w}^T\mathbf{x} \leqslant 0 \end{cases}$$

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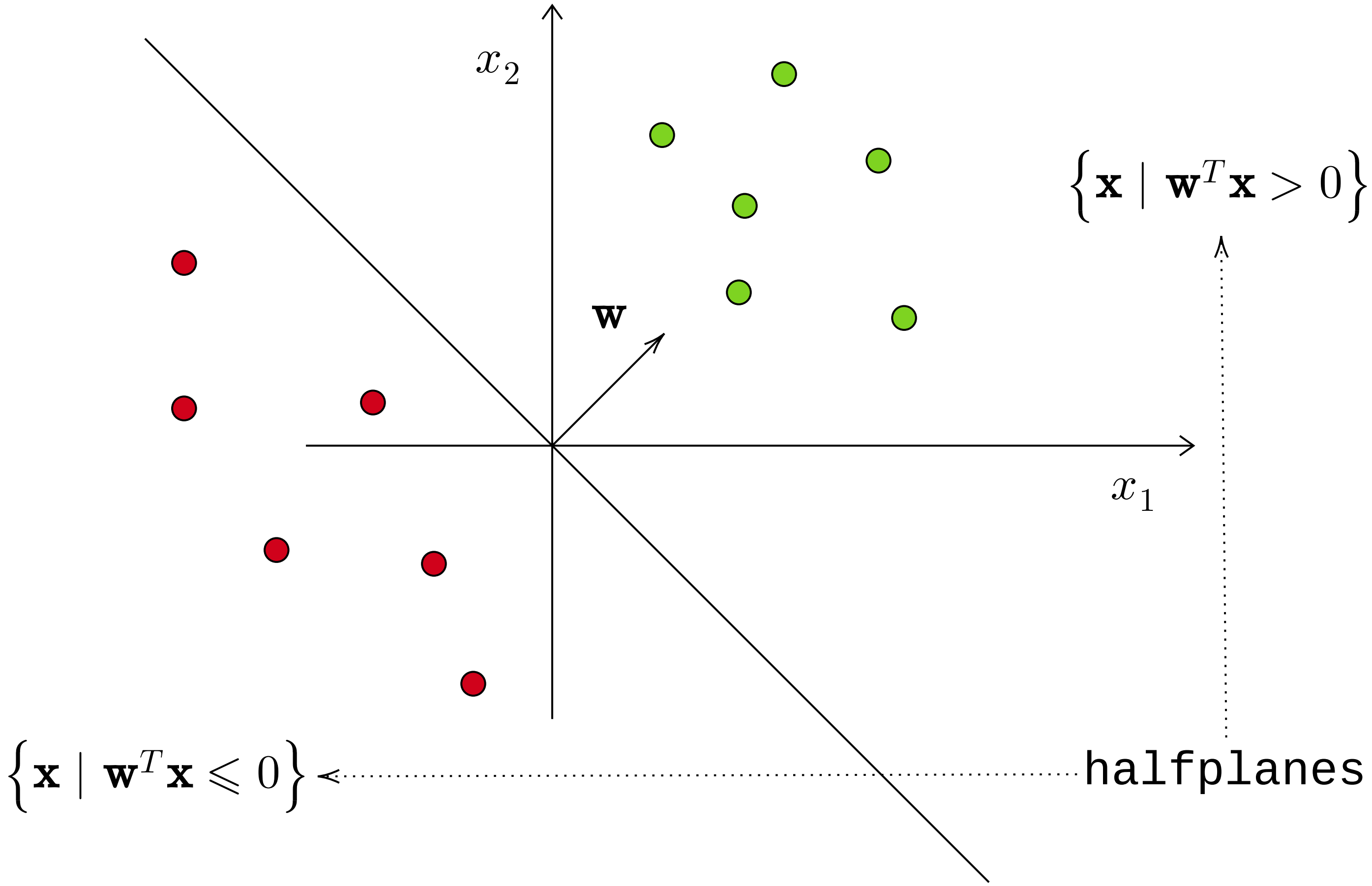

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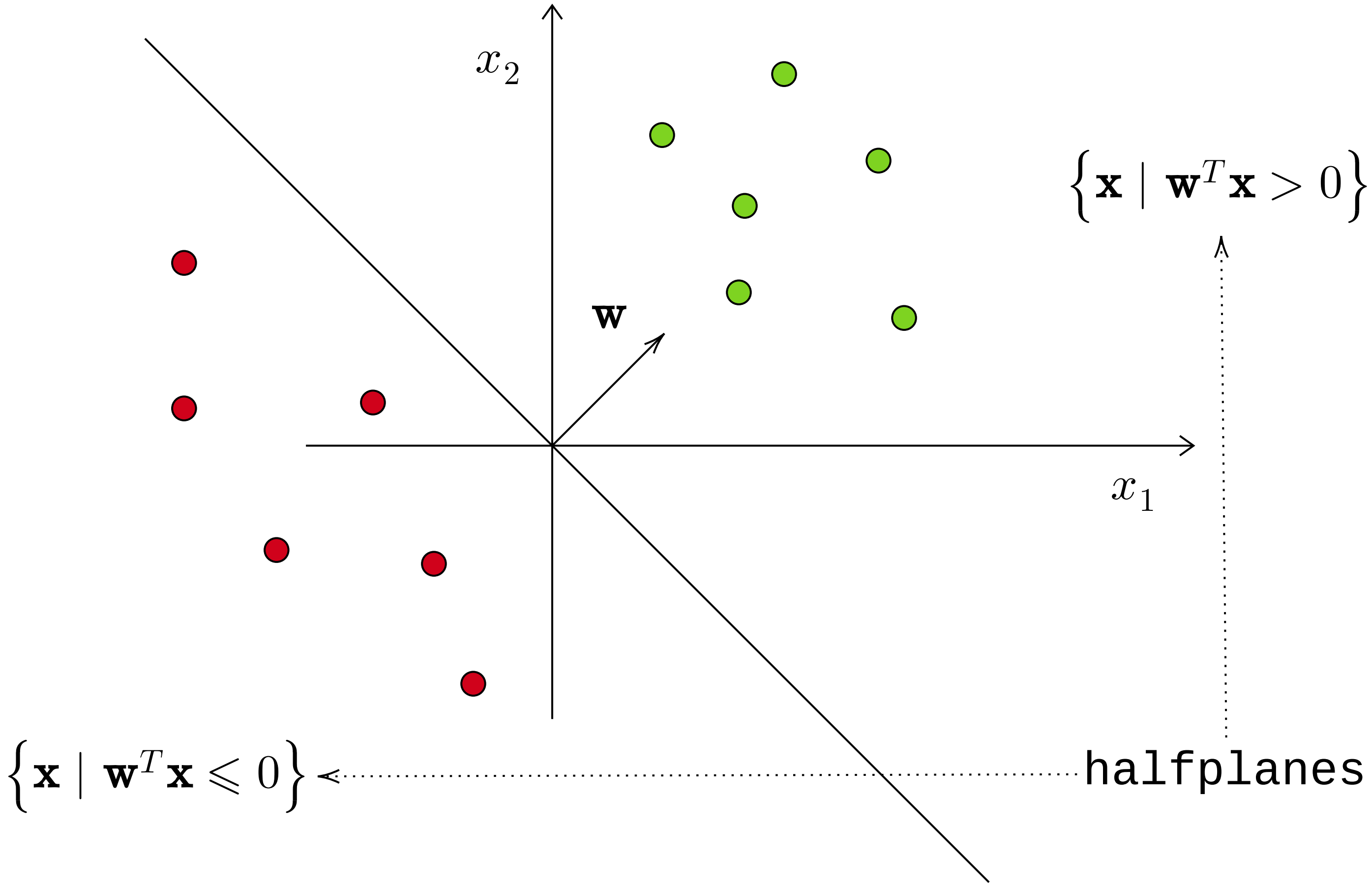

$$\mathcal{H}_{\mathtt{linear}} = \left\{h_{\mathbf{w}} \mid h_{\mathbf{w}}(\mathbf{x}) = \mathtt{sign}\left(\mathbf{w}^T\mathbf{x}\right)\right\}$$

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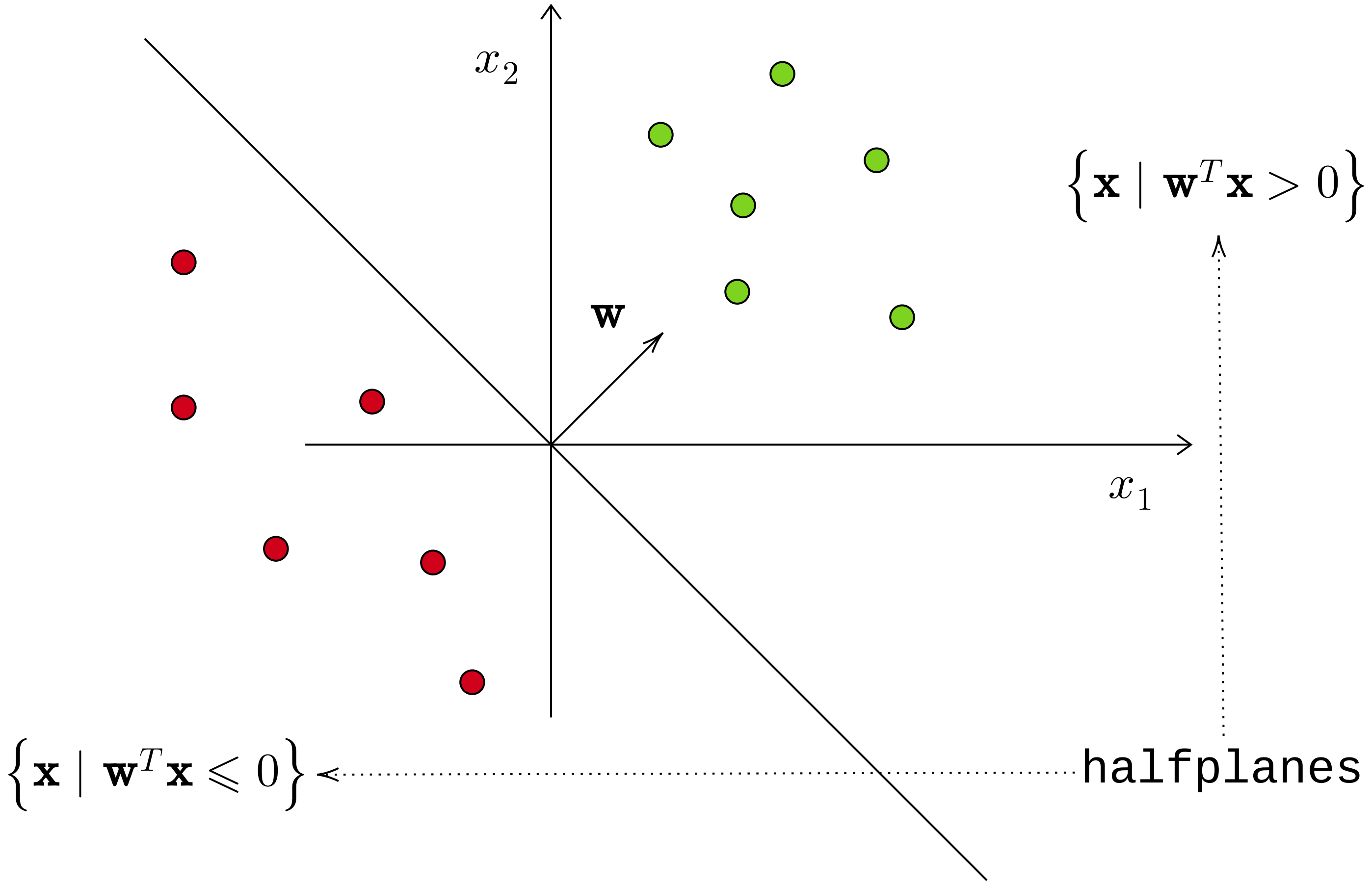

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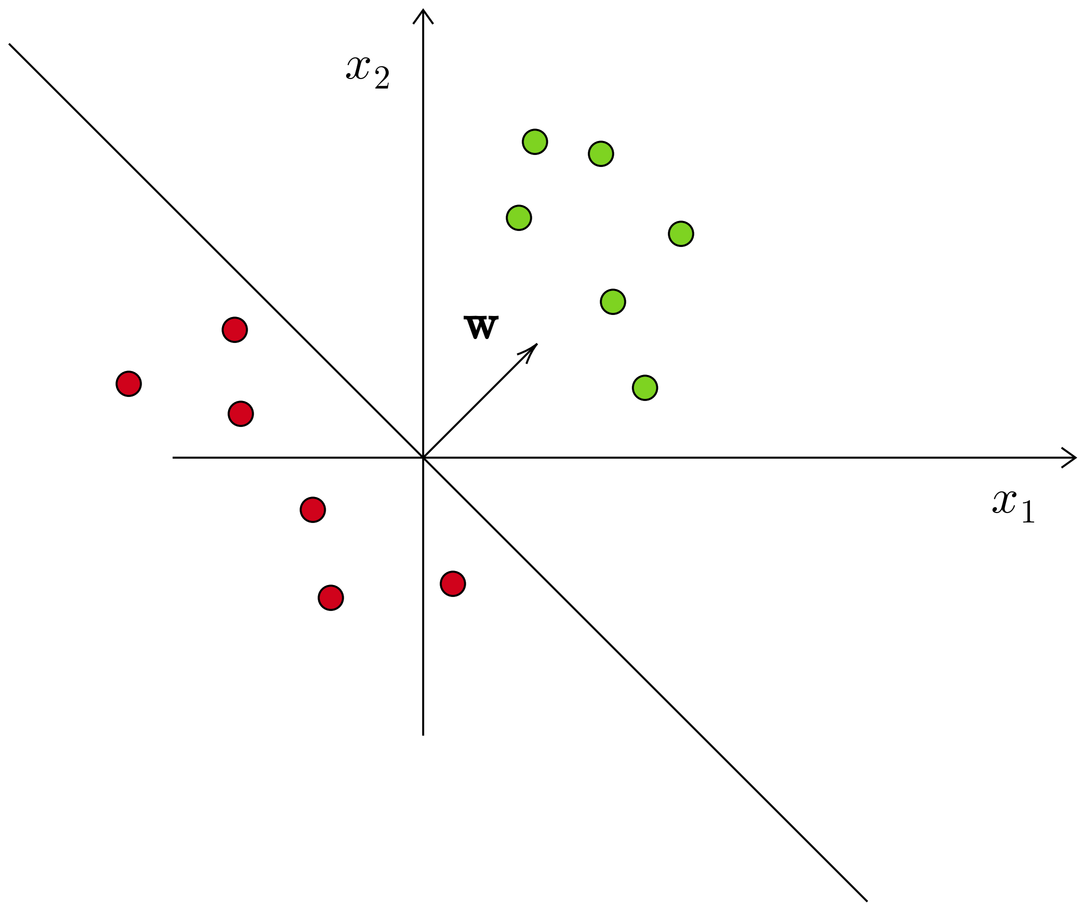
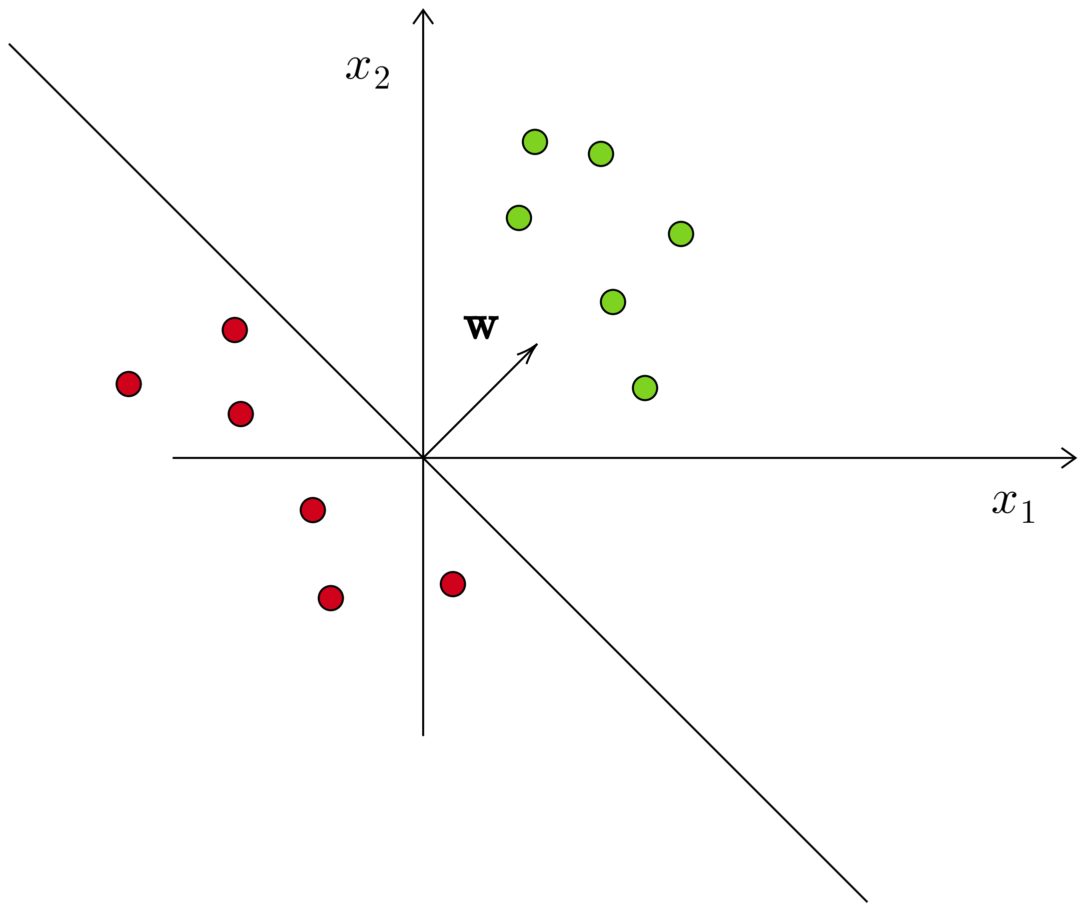
Linear Classifiers

$$\min_{h \in \mathcal{H}_{\mathtt{linear}}} \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbf{1}[h(\mathbf{x}_i) \neq y_i]$$

Classification as Regression



Classification as Regression



$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \ \frac{1}{n} \cdot \sum_{i=1}^{n} \left(\mathbf{w}^T \mathbf{x}_i - y_i\right)^2$$

$$h_{\mathbf{w}^*}(\mathbf{x}) = \mathtt{sign}\left(\mathbf{w}^{*T} \mathbf{x}\right)$$

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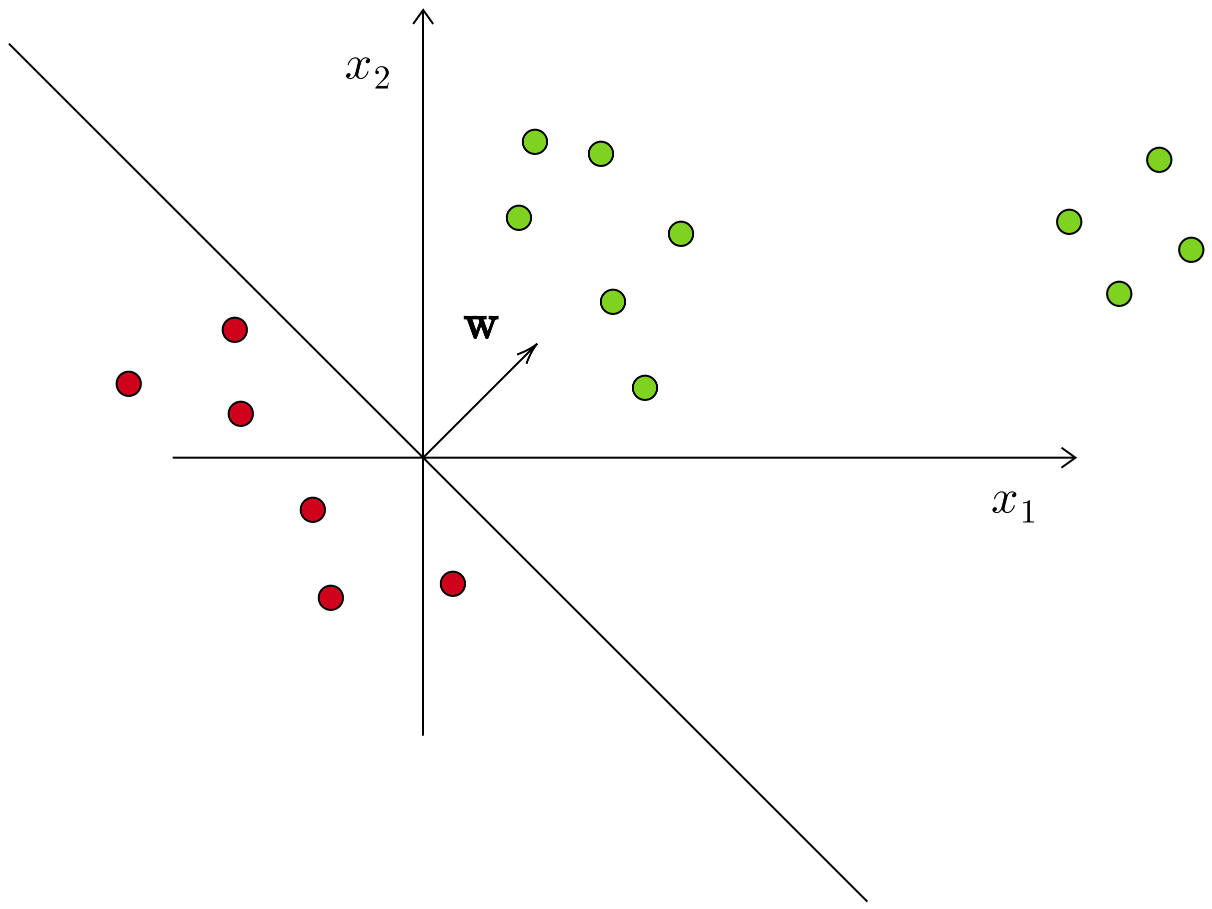

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- Sensitive to outliers

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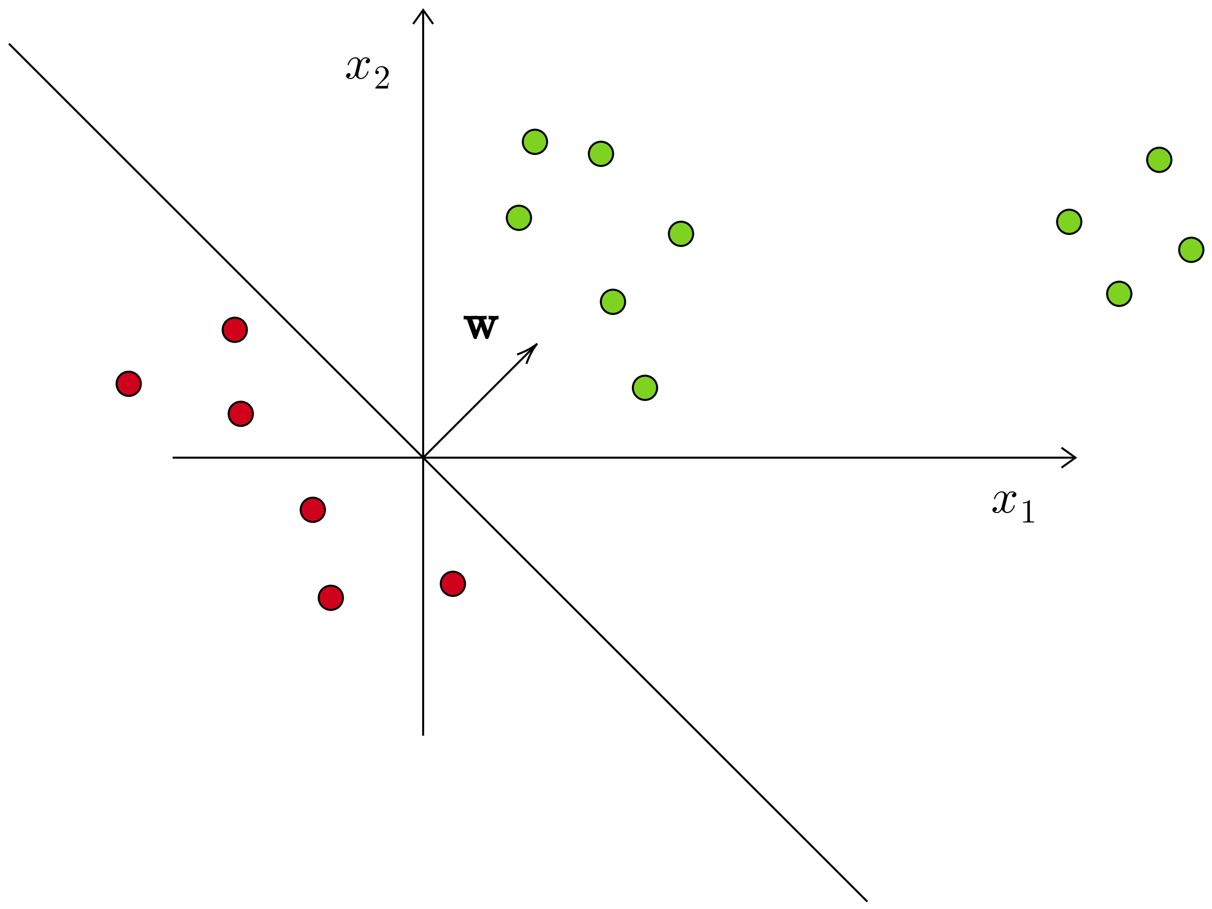

- Sensitive to outliers
- Natural ordering is absent
  - example: spam, work, family