Support Vector Machines (examples)

Machine Learning Techniques

Outline

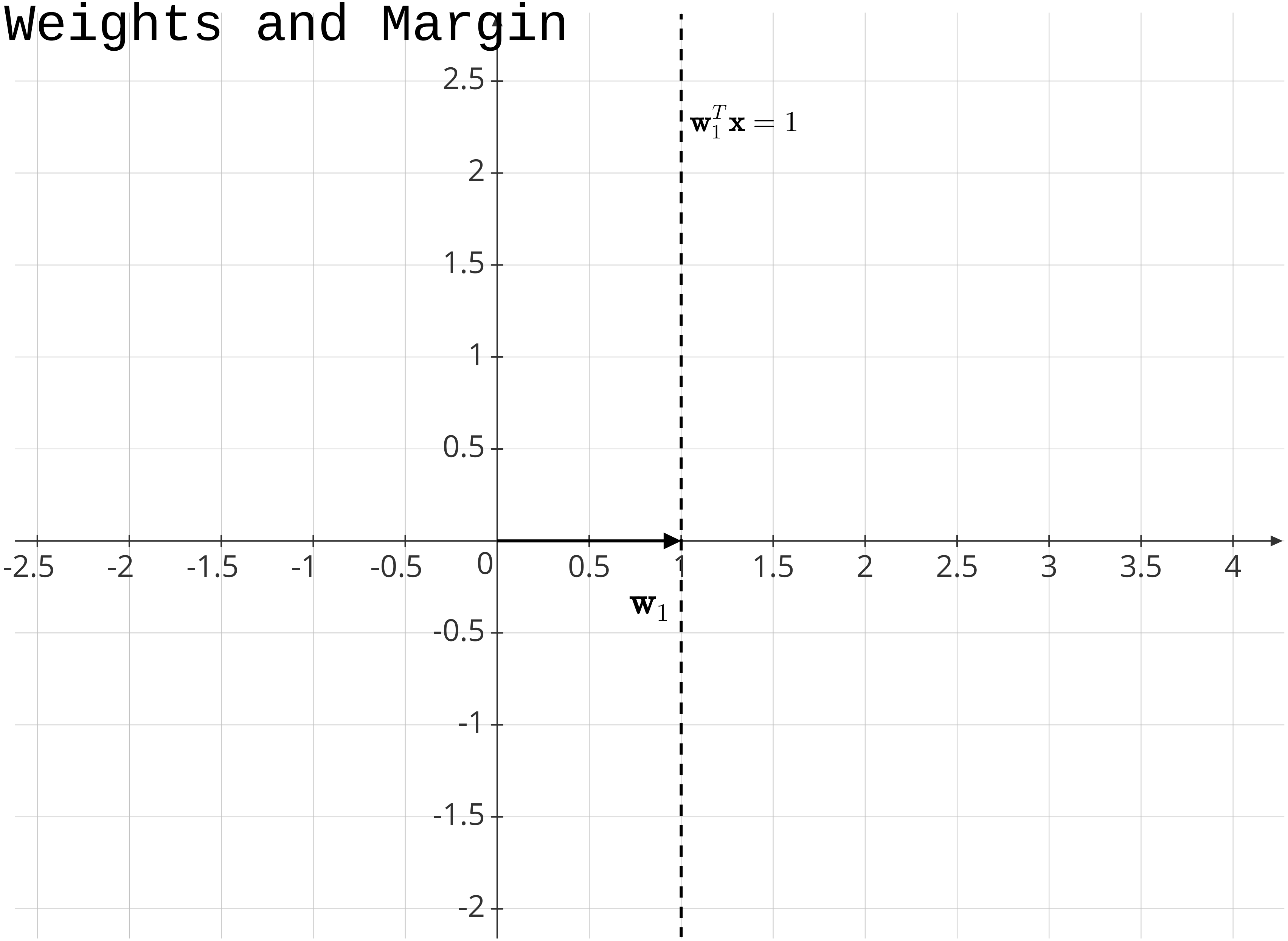
Primal Picture

- Weights and Margin
- Feature space
- Parameter space
- Constraints
- Objective
- Interaction between objective and constraints
- Active and inactive constraints

Dual Picture

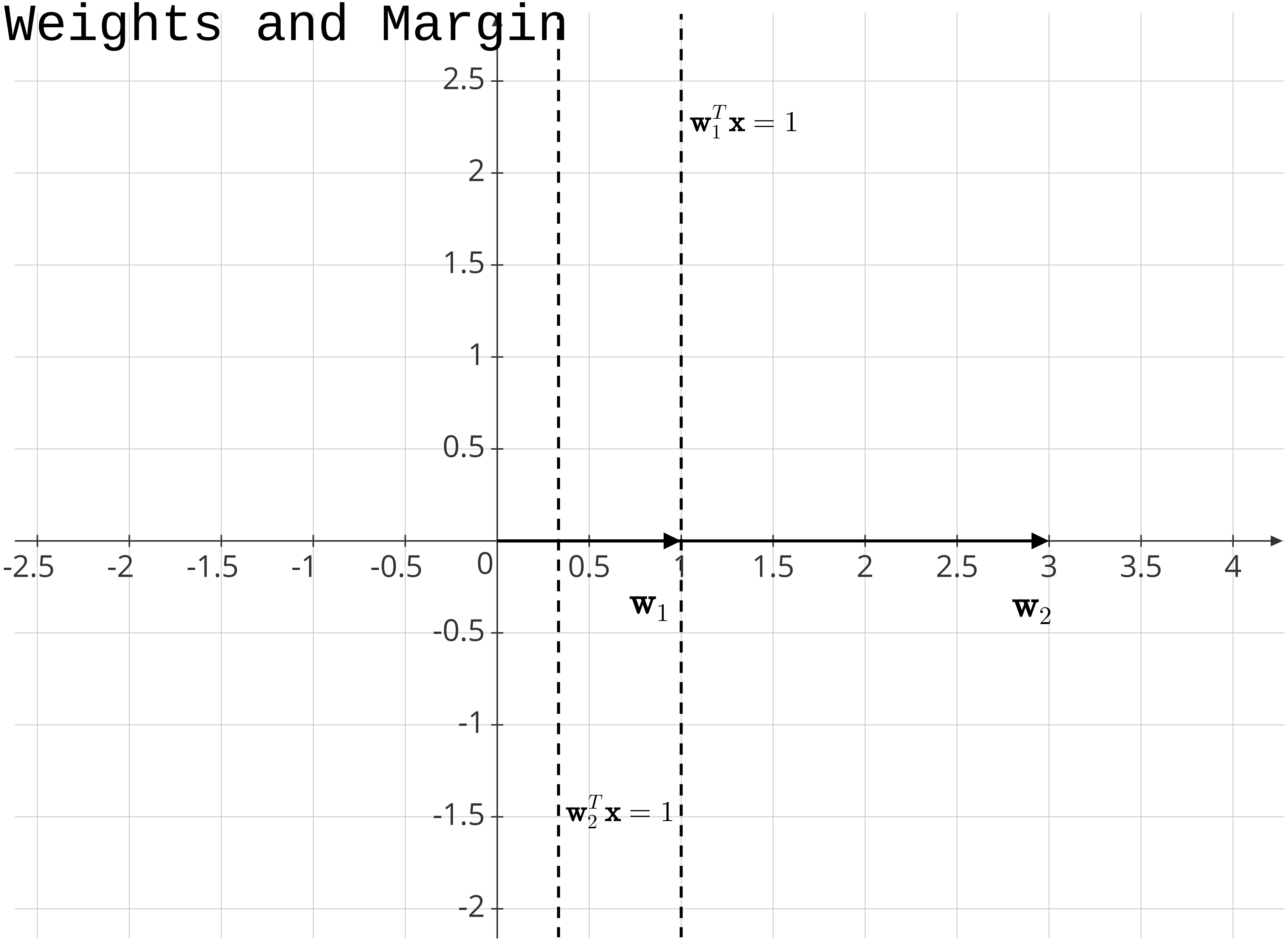
- Lagrange multipliers
- Weight vector

Weights and Margin



$$\mathbf{w}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

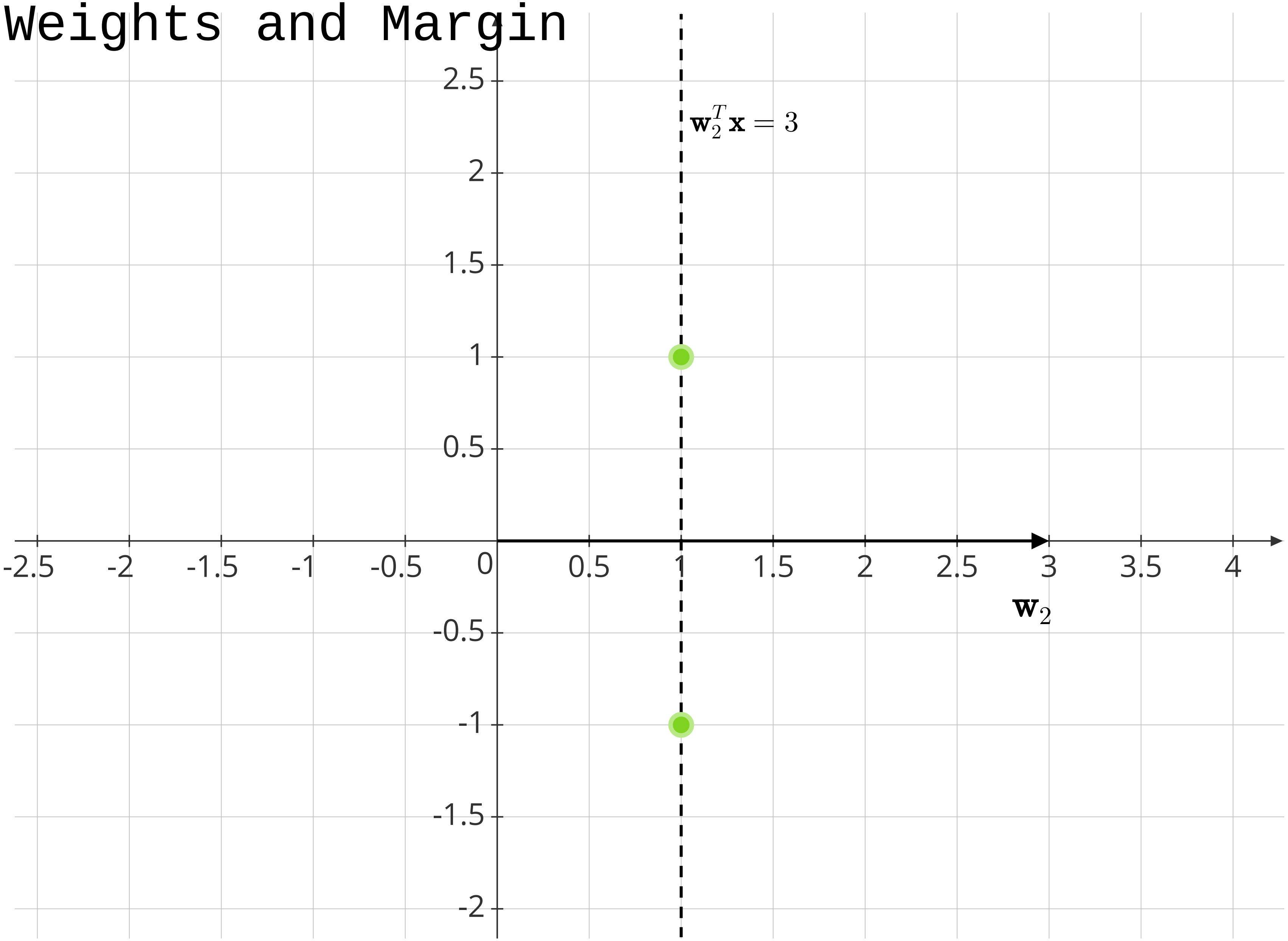
Weights and Margin



$$\mathbf{w}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

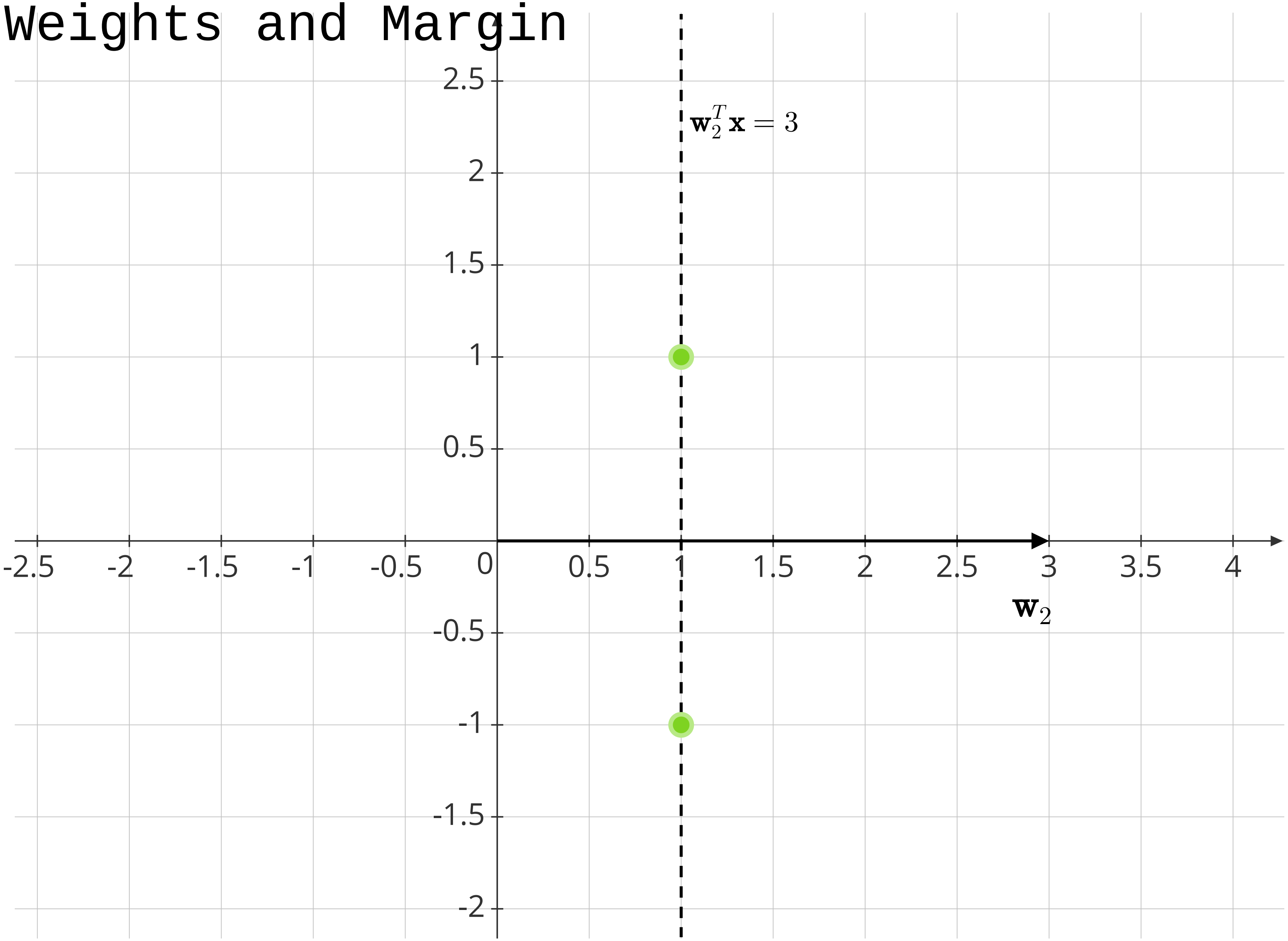
$$\mathbf{w}_2 = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$$

Weights and Margin



$$\mathbf{w}_2 = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$$

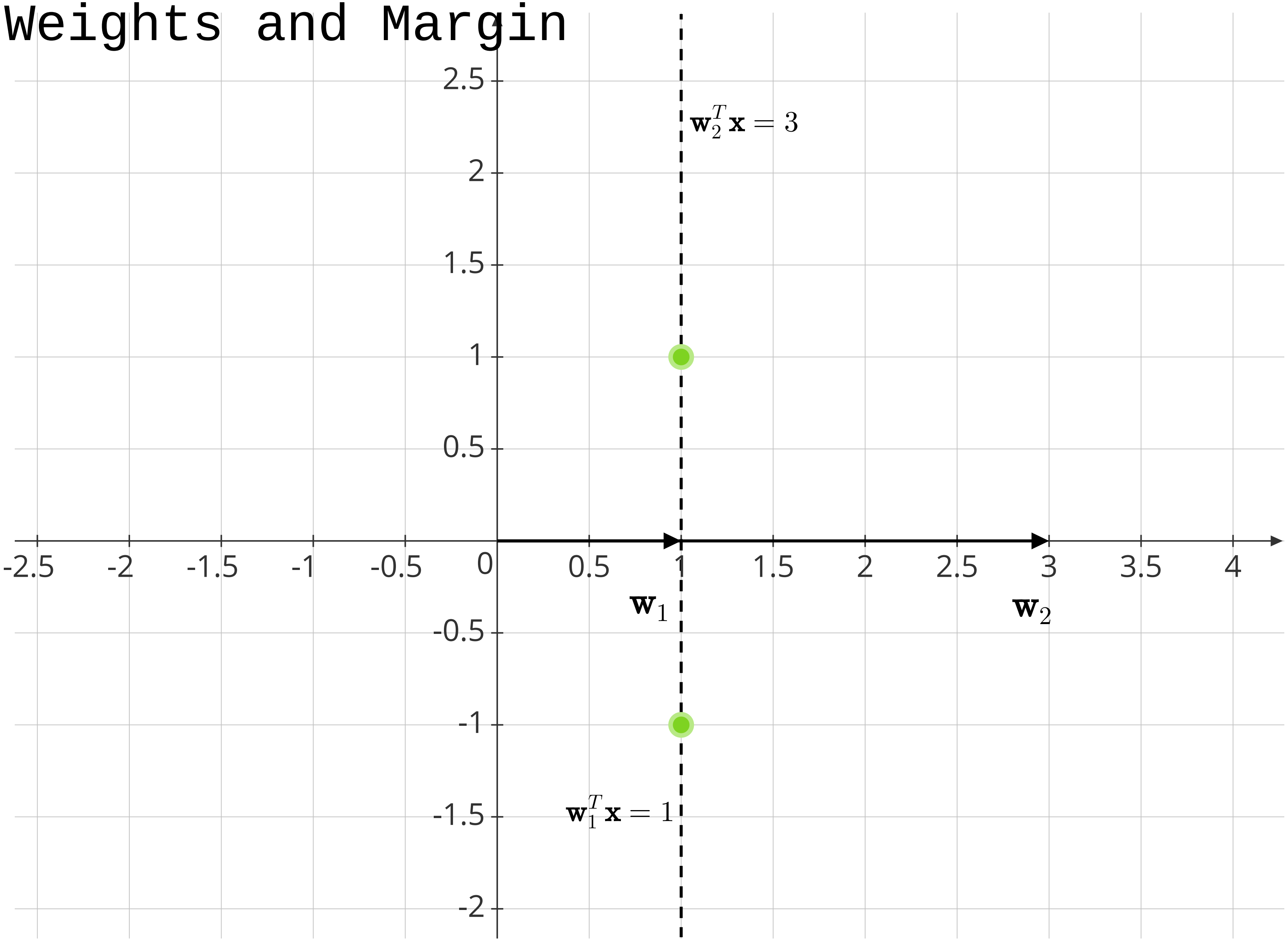
Weights and Margin



$$\mathbf{w}_2 = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$$

$$\mathbf{w}_2^T\mathbf{x} = 3 \implies \frac{1}{3}\left(\mathbf{w}_2^T\mathbf{x}\right) = \frac{1}{3}\cdot 3$$

Weights and Margin



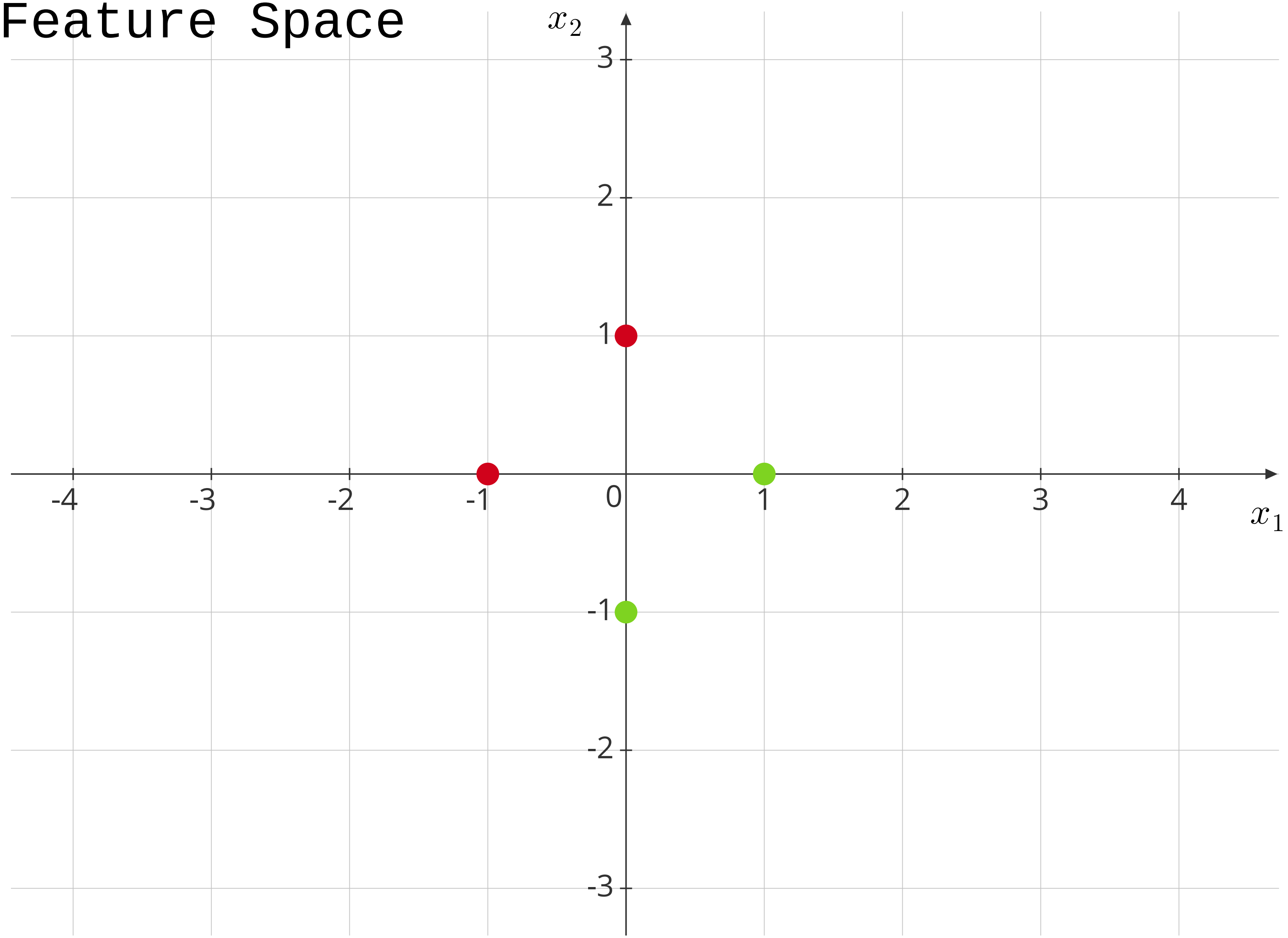
$$\mathbf{w}_2 = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$$

$$\mathbf{w}_2^T\mathbf{x} = 3 \implies \frac{1}{3}\left(\mathbf{w}_2^T\mathbf{x}\right) = \frac{1}{3} \cdot 3$$

$$\implies \left(\frac{\mathbf{w}_2}{3}\right)^T \mathbf{x} = 1 \implies \mathbf{w}_1^T\mathbf{x} = 1$$

$$\frac{\mathbf{w}_2}{3} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T = \mathbf{w}_1$$

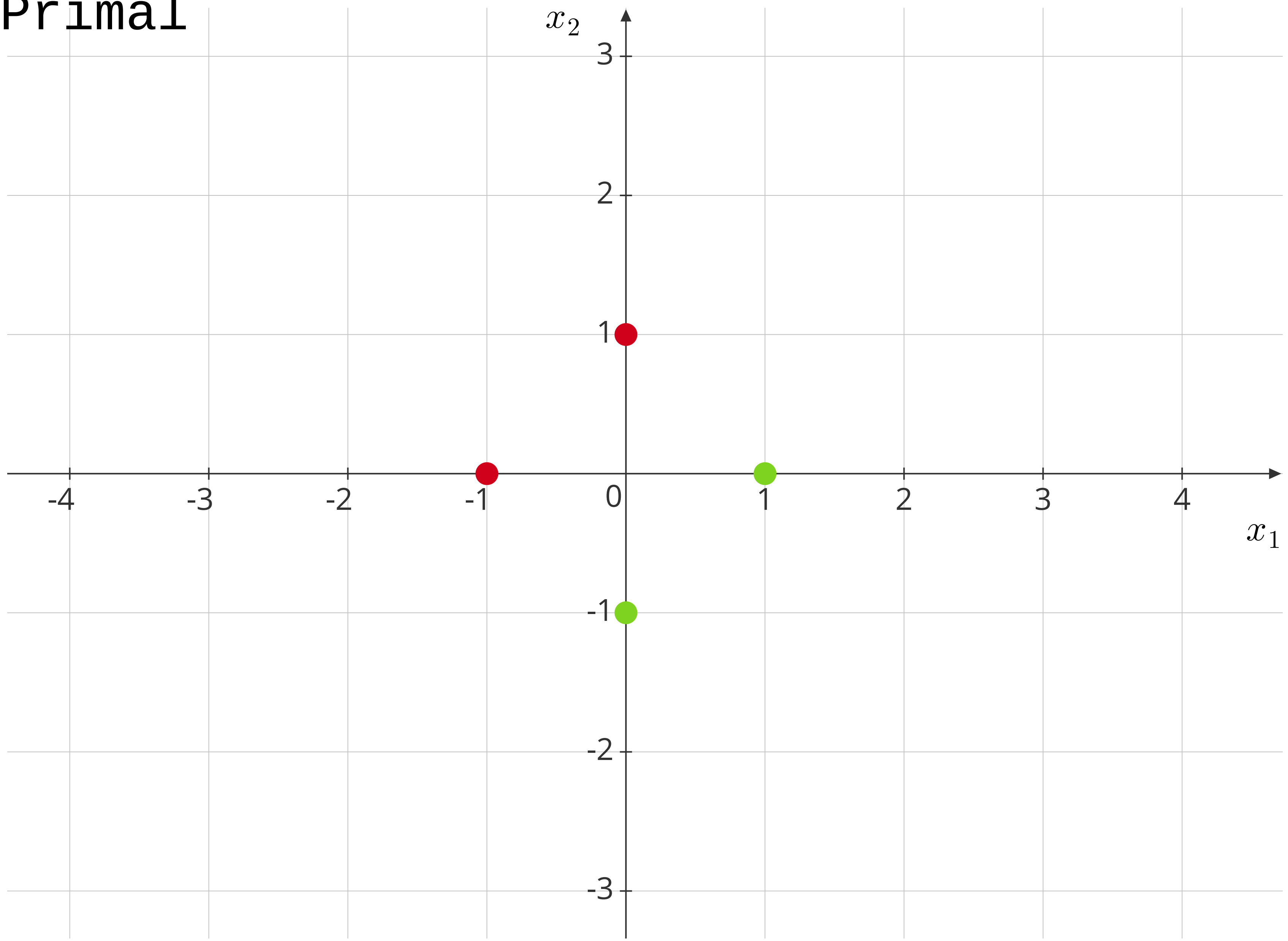
Feature Space



Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Primal



Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{||\mathbf{w}||^2}{2}$$

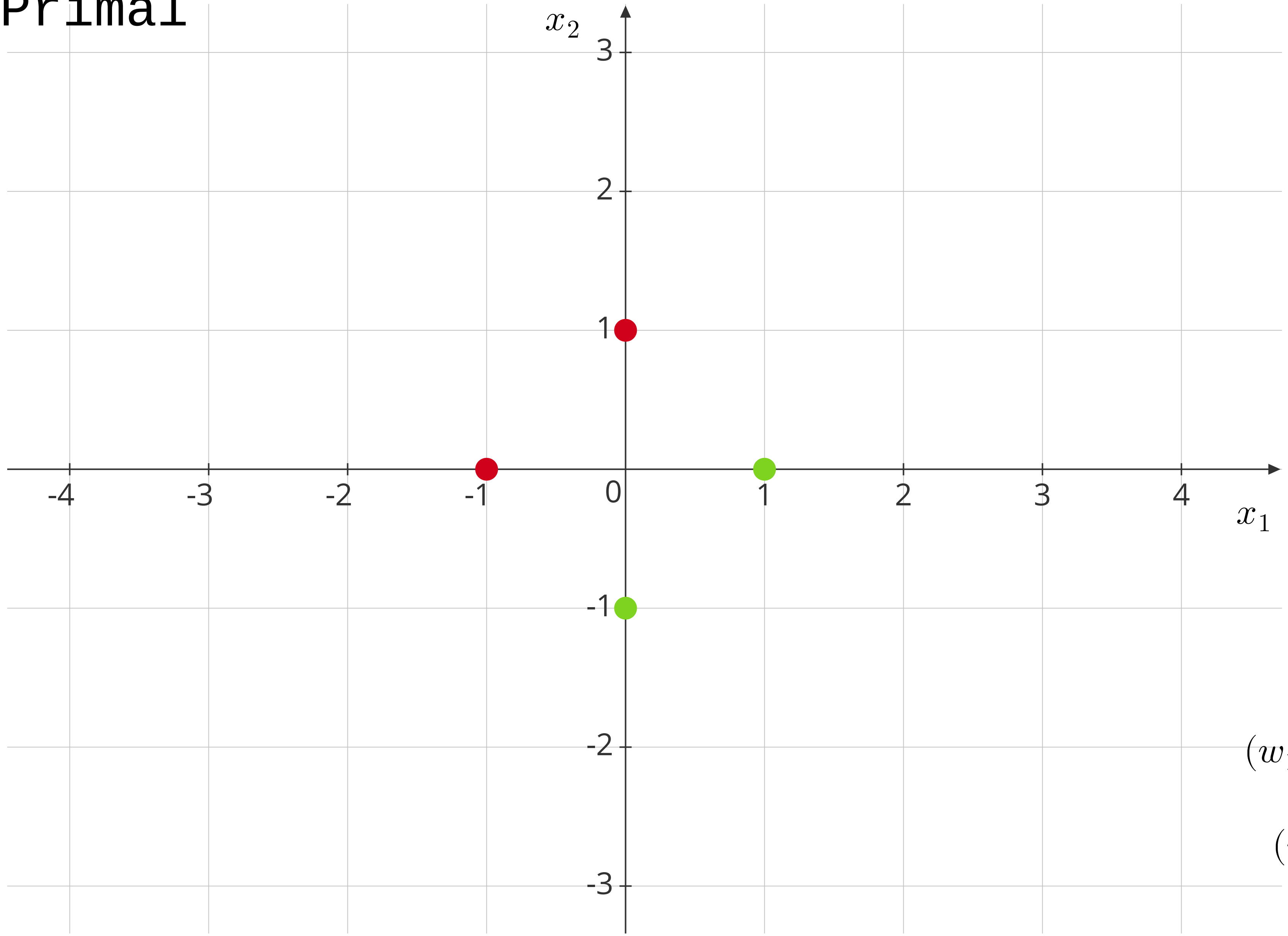
$$\left(\mathbf{w}^T\mathbf{x}_1\right)y_1 \geqslant 1 \quad (1)$$

$$\left(\mathbf{w}^T\mathbf{x}_2\right)y_2 \geqslant 1 \quad (2)$$

$$\left(\mathbf{w}^T\mathbf{x}_3\right)y_3 \geqslant 1 \quad (3)$$

$$\left(\mathbf{w}^T\mathbf{x}_4\right)y_4 \geqslant 1 \quad (4)$$

Primal



Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

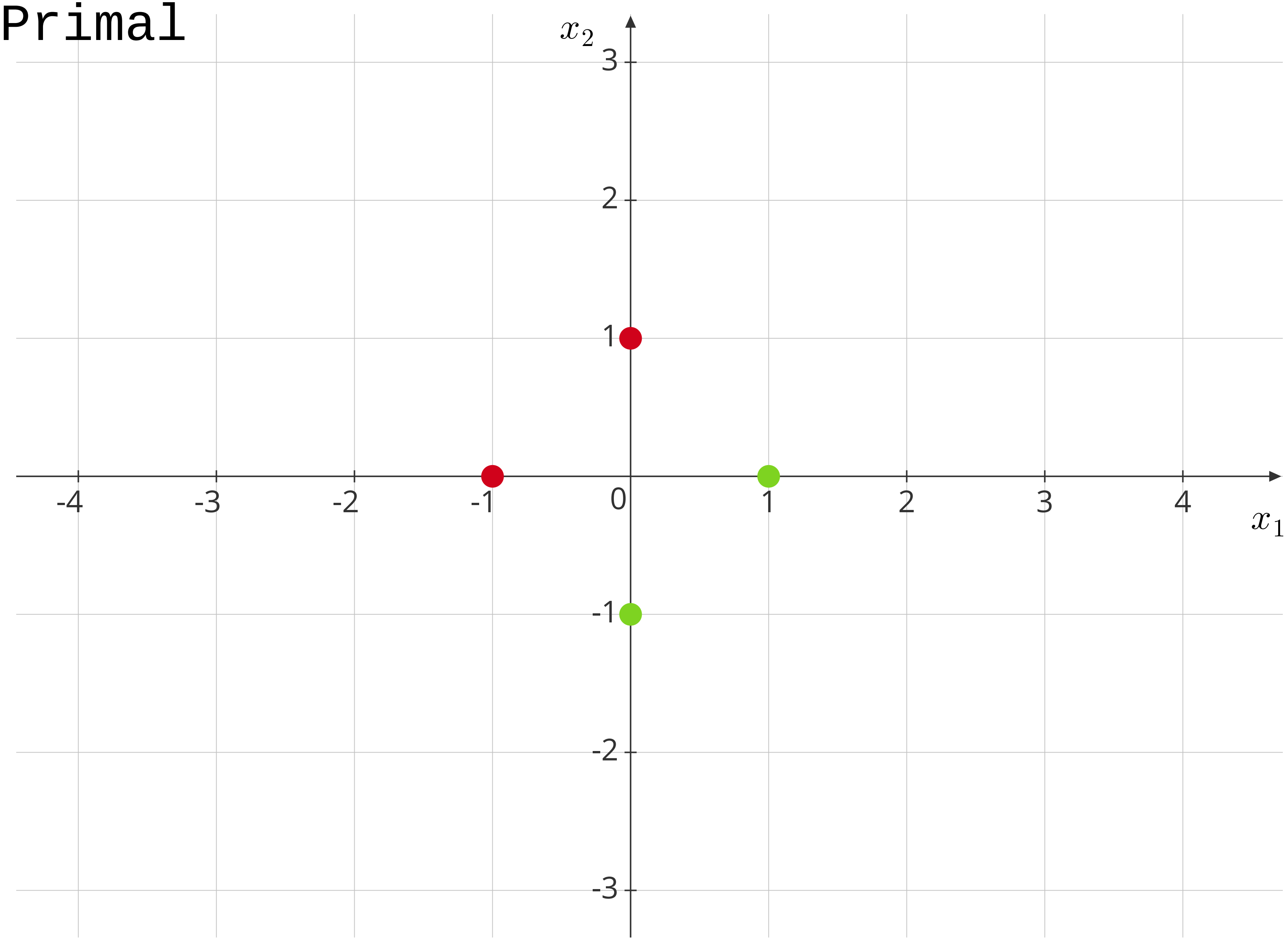
$$(w_1 \times 1 + w_2 \times 0) \cdot 1 \geqslant 1 \quad (1)$$

$$(w_1 \times 0 + w_2 \times -1) \cdot 1 \geqslant 1 \quad (2)$$

$$(w_1 \times (-1) + w_2 \times 0) \cdot (-1) \geqslant 1 \quad (3)$$

$$(w_1 \times (0) + w_2 \times 1) \cdot (-1) \geqslant 1 \quad (4)$$

Primal



Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

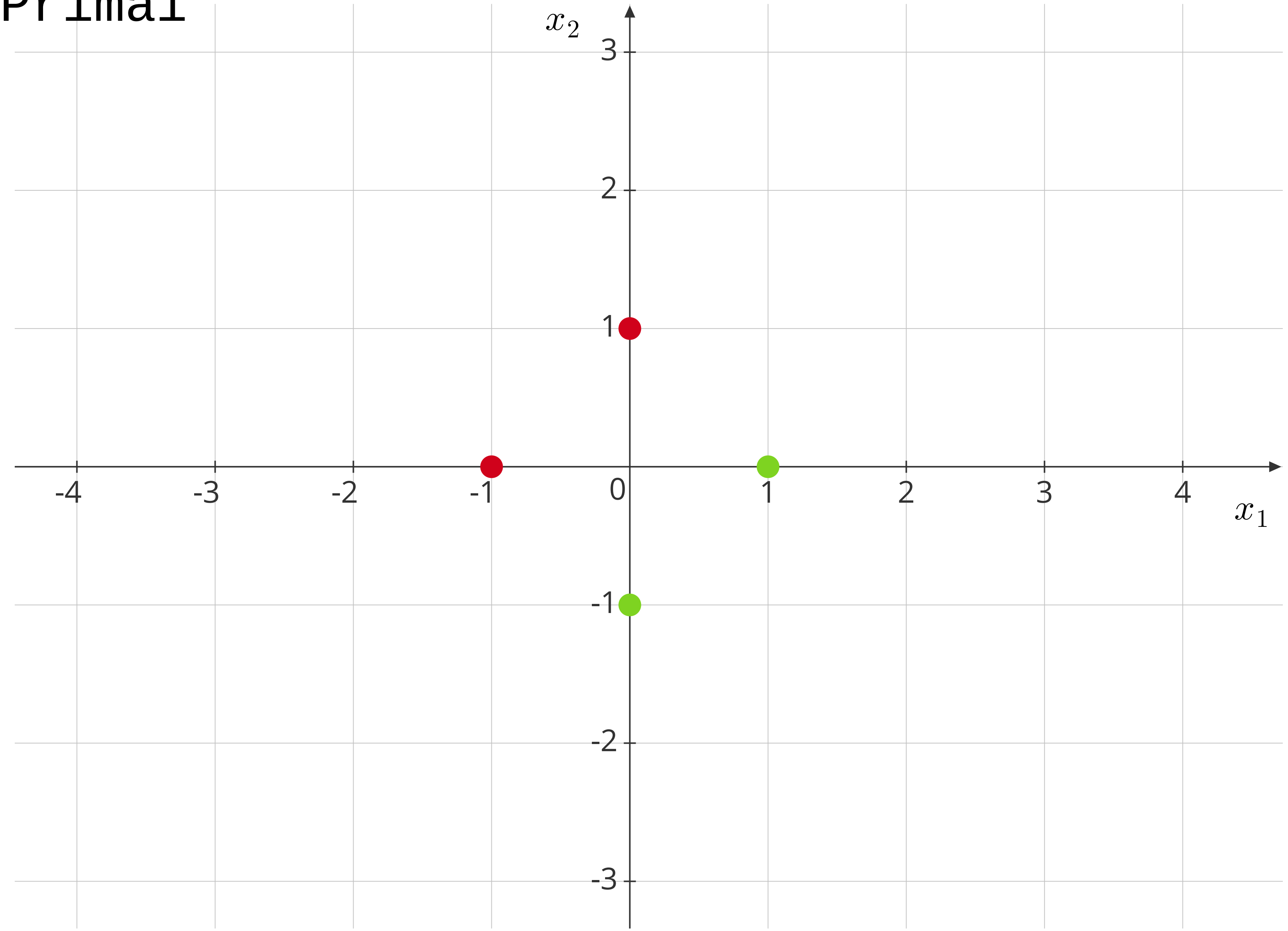
$$w_1 \geqslant 1 \qquad (1)$$

$$-w_2 \geqslant 1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$-w_2 \geqslant 1 \qquad (4)$$

Primal



Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

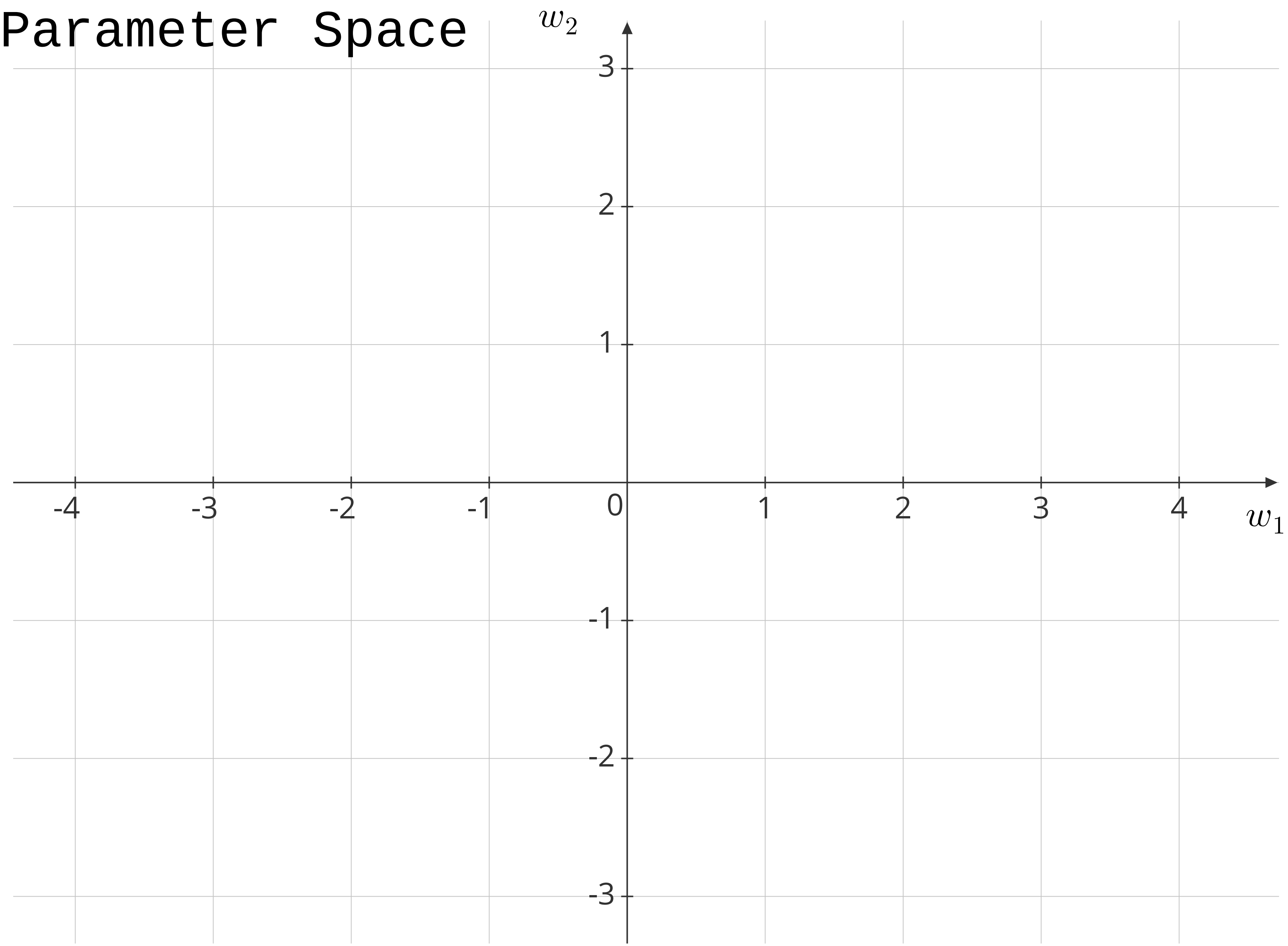
$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

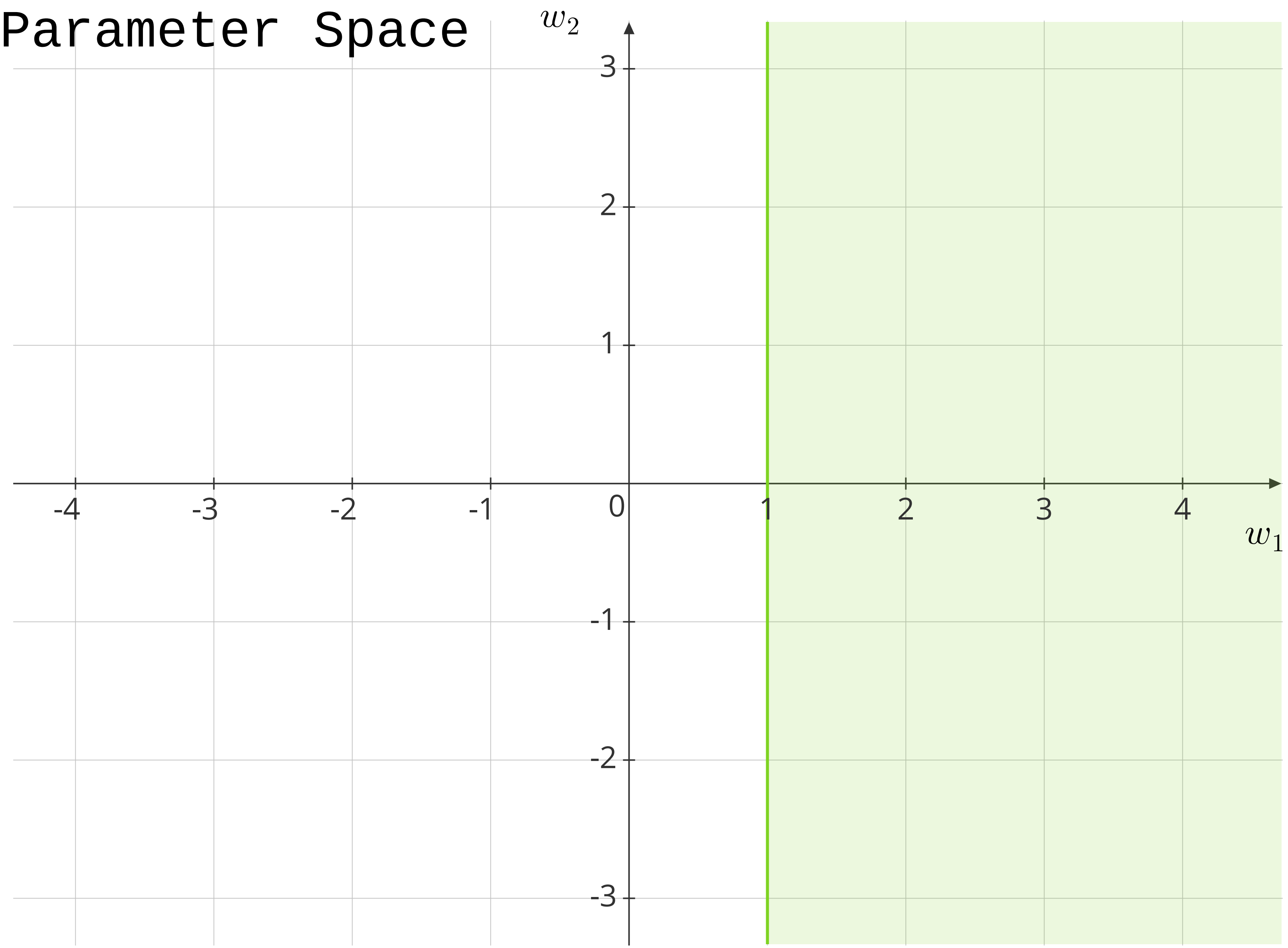
$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

Parameter Space



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$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

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$$w_1 \geqslant 1 \qquad (1)$$

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Parameter Space

Example-1

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

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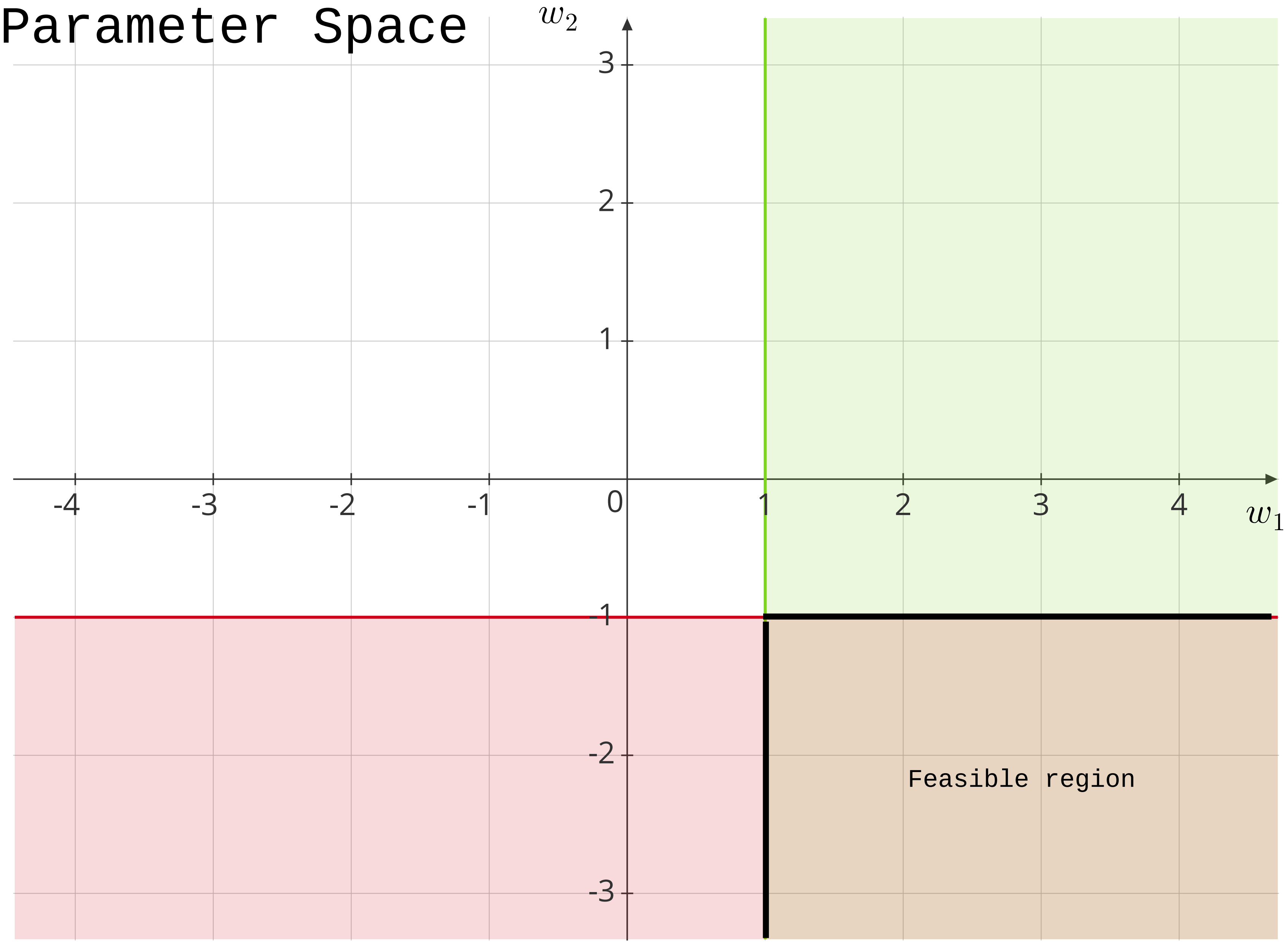
Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

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$$w_1 \geqslant 1 \qquad (3)$$

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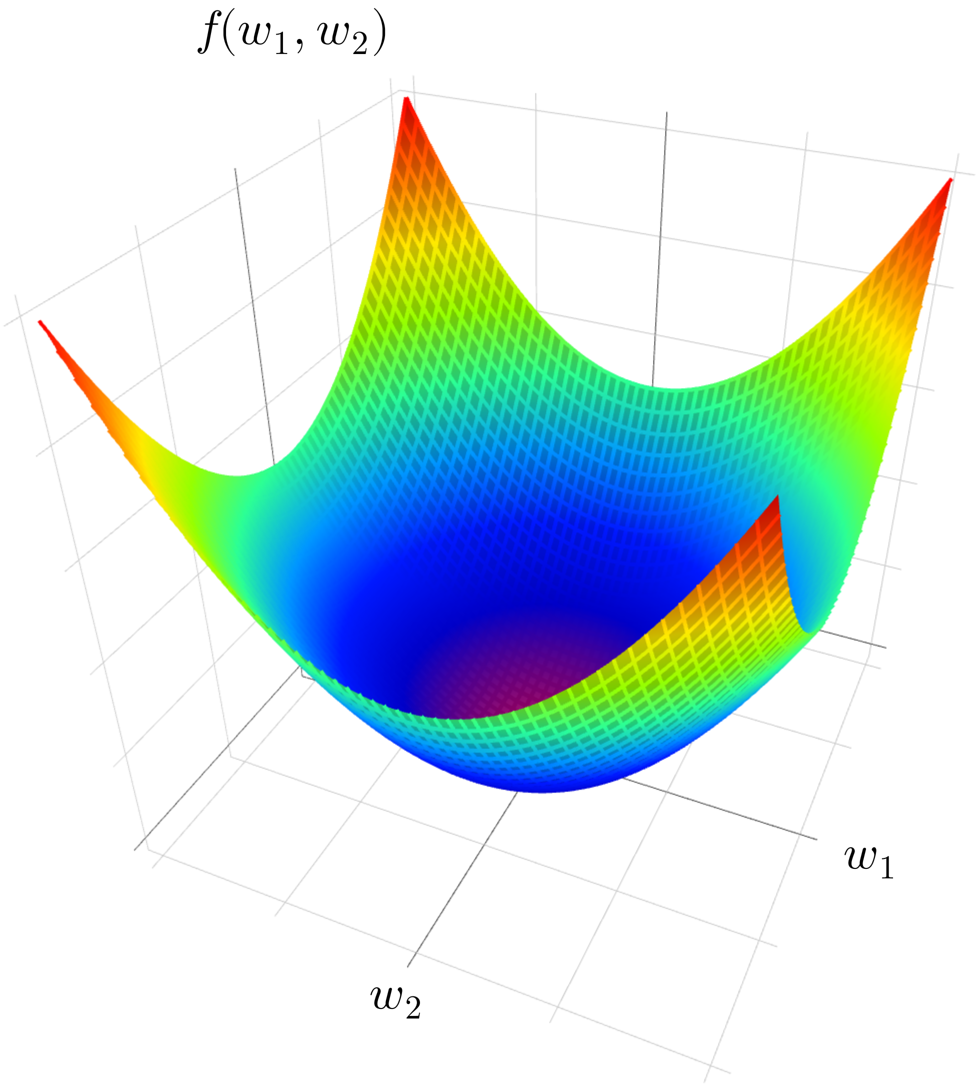
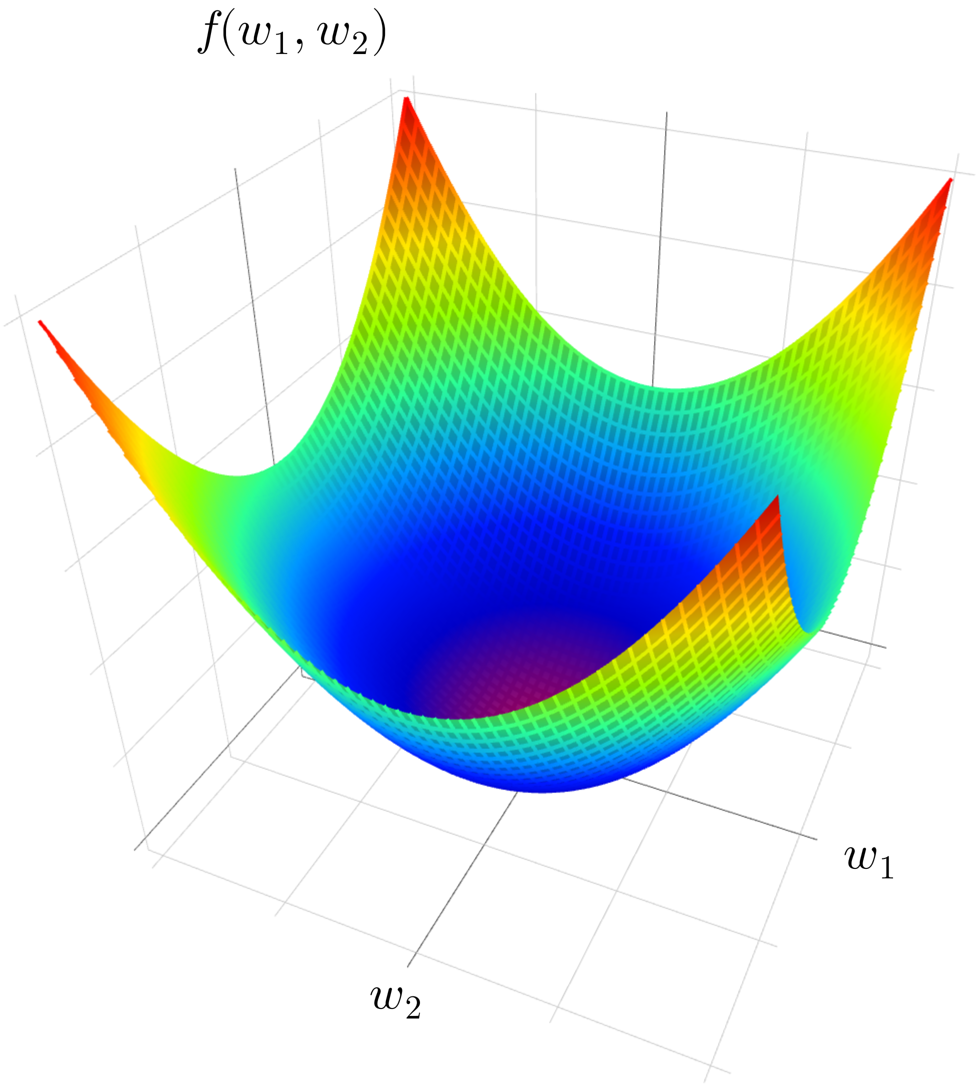
Parameter Space - Objective Surface

Example-1



$$f(w_1, w_2) = \frac{||\mathbf{w}||^2}{2} = \frac{w_1^2 + w_2^2}{2}$$

Parameter Space - Objective Surface   Example-1

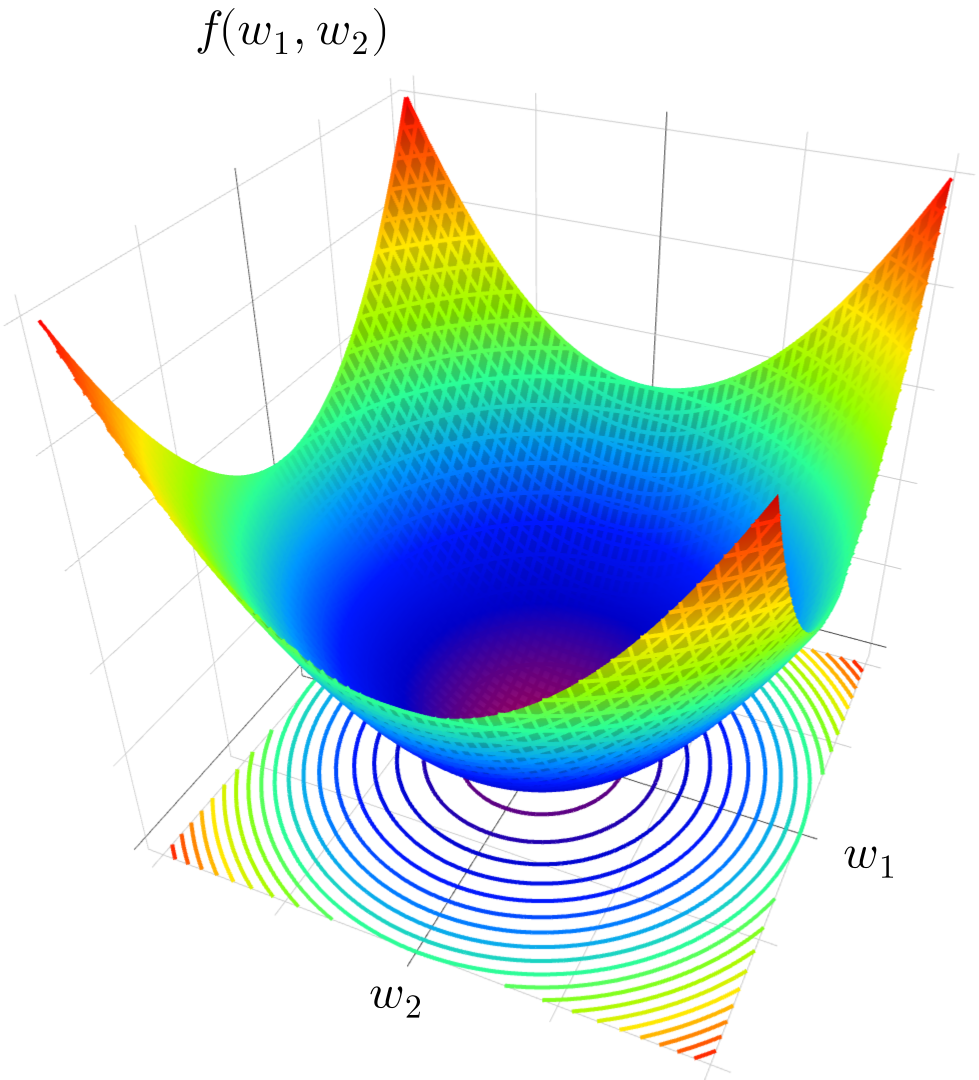


$$f(w_1, w_2) = \frac{||\mathbf{w}||^2}{2} = \frac{w_1^2 + w_2^2}{2}$$

$$f(w_1, w_2) = 0.5 \implies w_1^2 + w_2^2 = 1$$

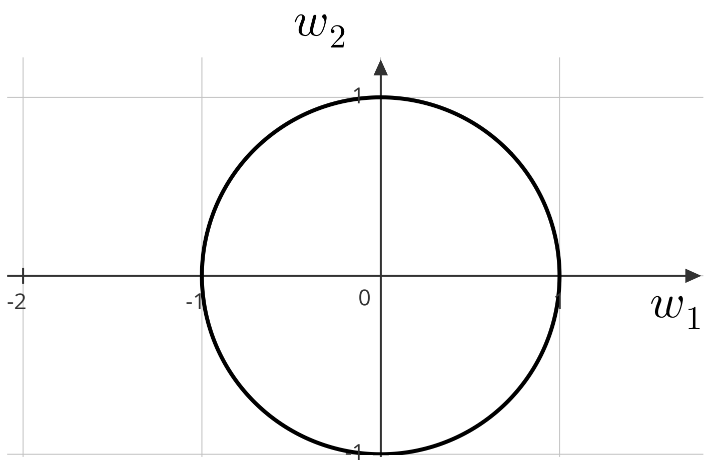
Parameter Space - Objective Surface

Example-1



$$f(w_1, w_2) = \frac{||\mathbf{w}||^2}{2} = \frac{w_1^2 + w_2^2}{2}$$

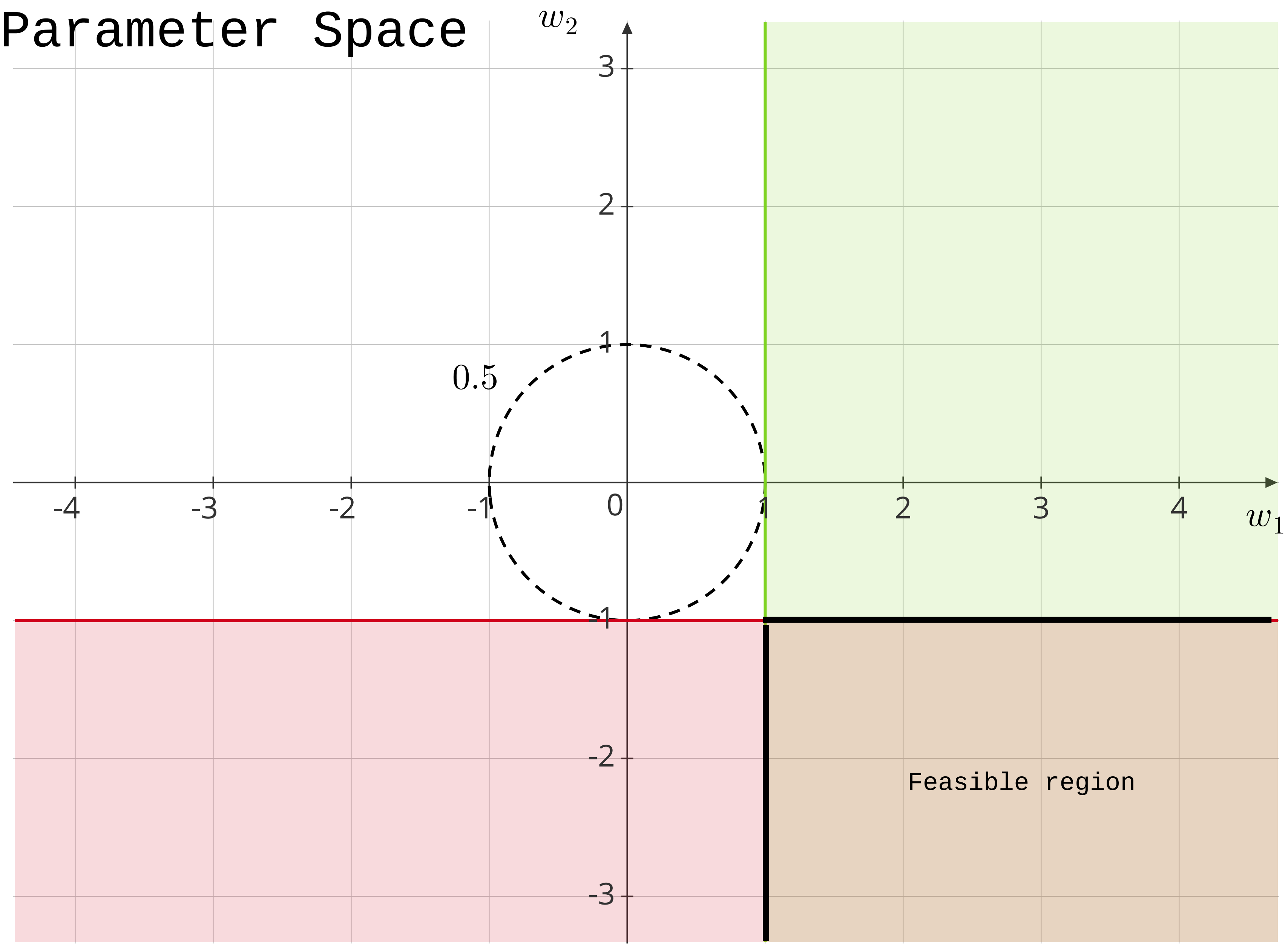
$$f(w_1, w_2) = 0.5 \implies w_1^2 + w_2^2 = 1$$



$$w_1^2 + w_2^2 = 1$$

Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \quad (1)$$

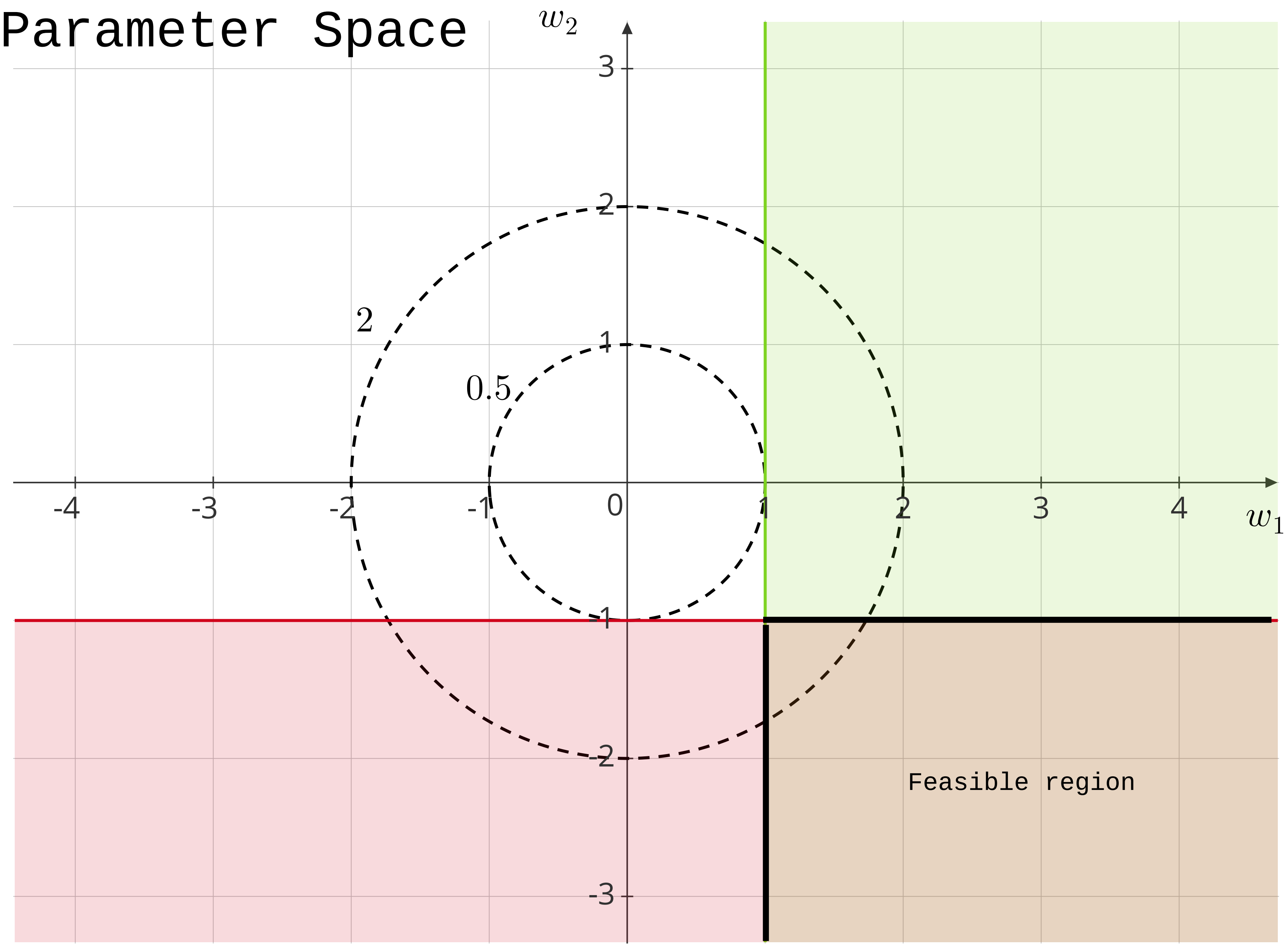
$$w_2 \leqslant -1 \quad (2)$$

$$w_1 \geqslant 1 \quad (3)$$

$$w_2 \leqslant -1 \quad (4)$$

Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

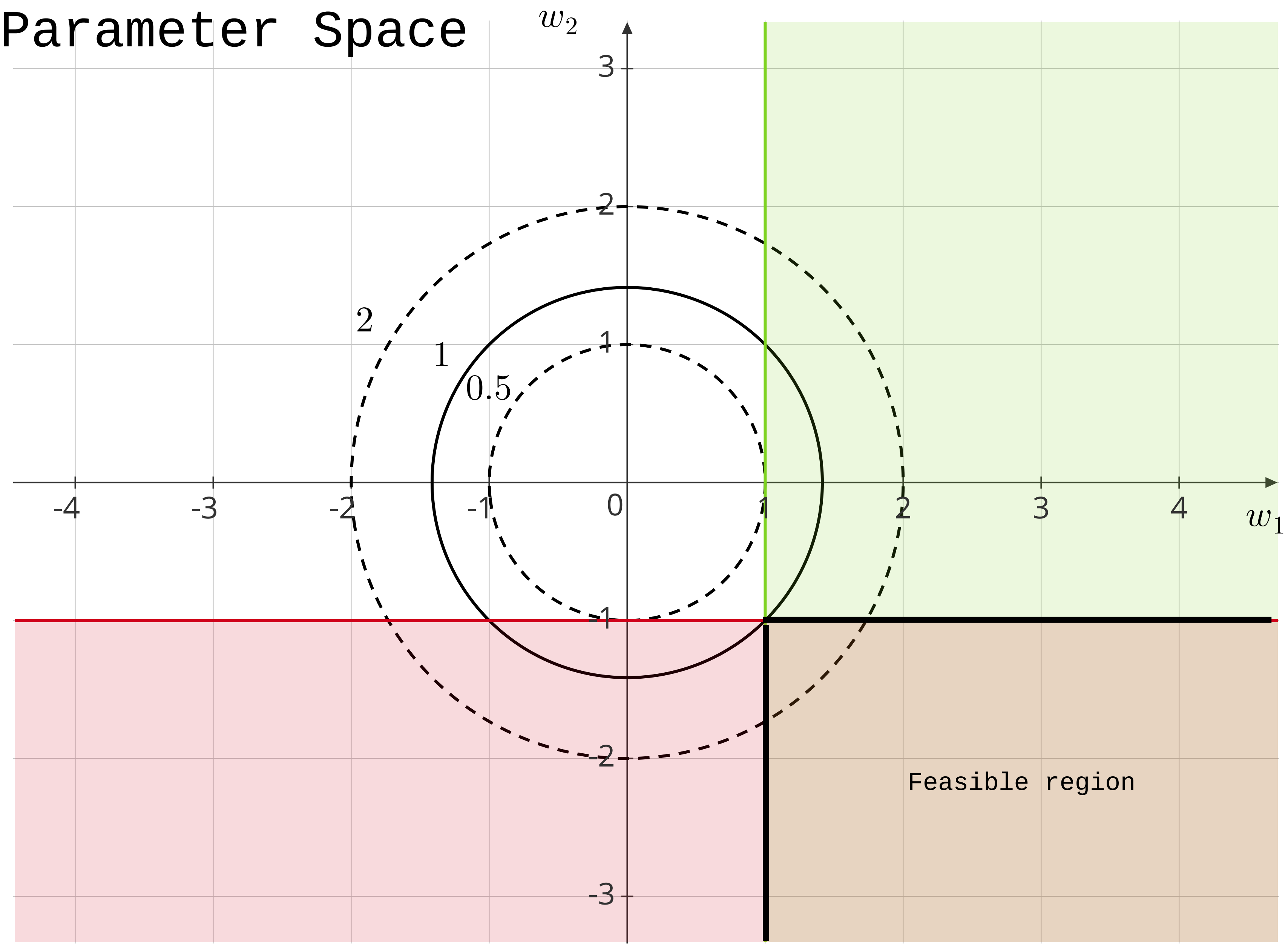
$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

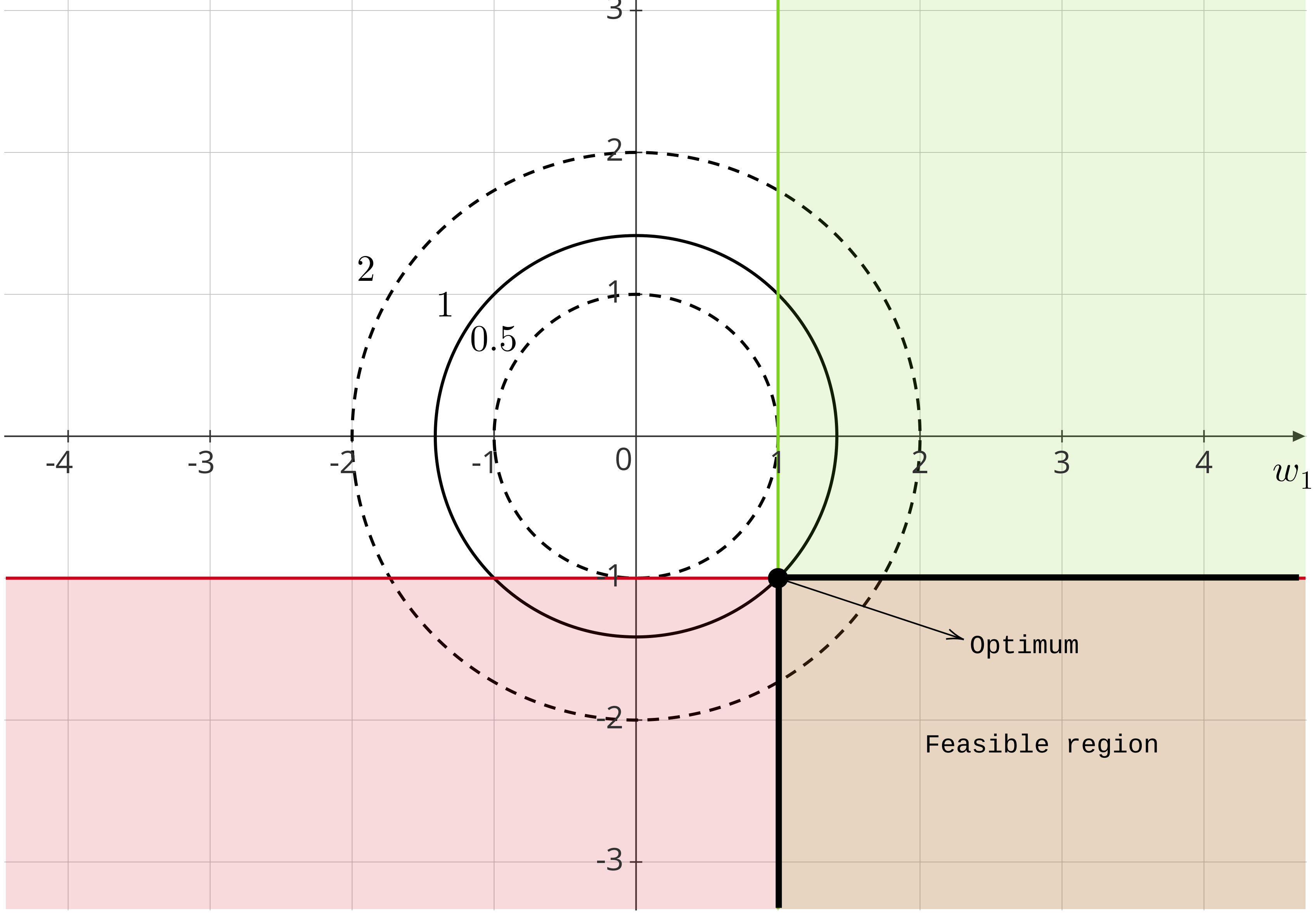
$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

Parameter Space

Example-1



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

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$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

Primal $\rightarrow$ Dual

Example-1

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \ \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

Primal $\rightarrow$ Dual Example-1

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \ \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \ \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Primal $\rightarrow$ Dual

Example-1

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \max_{\boldsymbol{\alpha} \geqslant 0} \; & (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) - \frac{1}{2}\left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2\right) \\ & -(\alpha_1 \alpha_3 + \alpha_2 \alpha_4) \end{aligned}$$

Primal $\rightarrow$ Dual Example-1

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) - \frac{1}{2}\left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2\right) - (\alpha_1 \alpha_3 + \alpha_2 \alpha_4)$$

$$\boldsymbol{\alpha}^* = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}^T$$

Primal $\rightarrow$ Dual Example-1

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$w_1 \geqslant 1 \qquad (3)$$

$$w_2 \leqslant -1 \qquad (4)$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T \mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

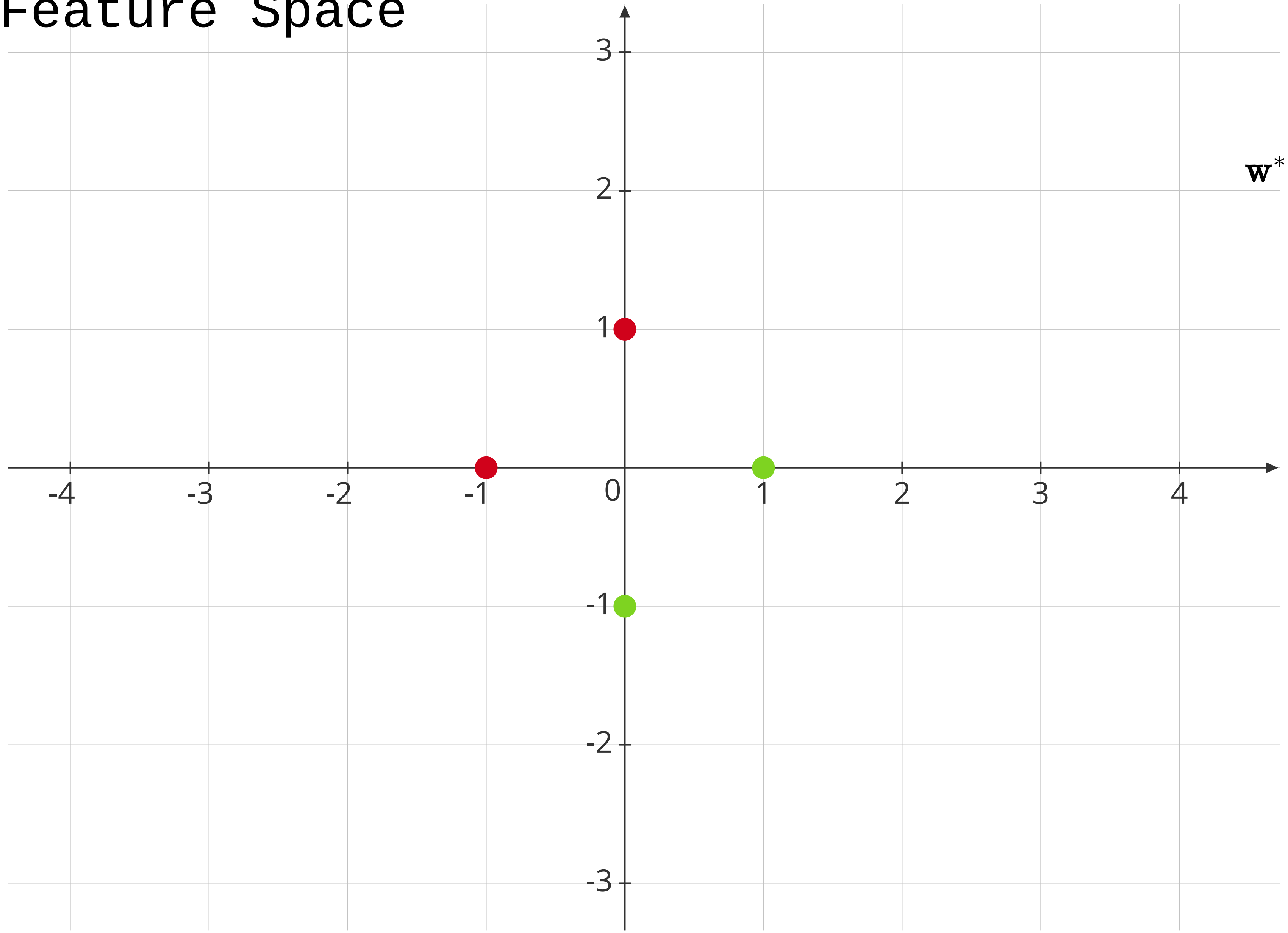
$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) - \frac{1}{2}\left(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2\right) \\ -(\alpha_1 \alpha_3 + \alpha_2 \alpha_4)$$

$\boldsymbol{\alpha}^* = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}^T$ OR $\boldsymbol{\alpha}^* = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$

Feature Space

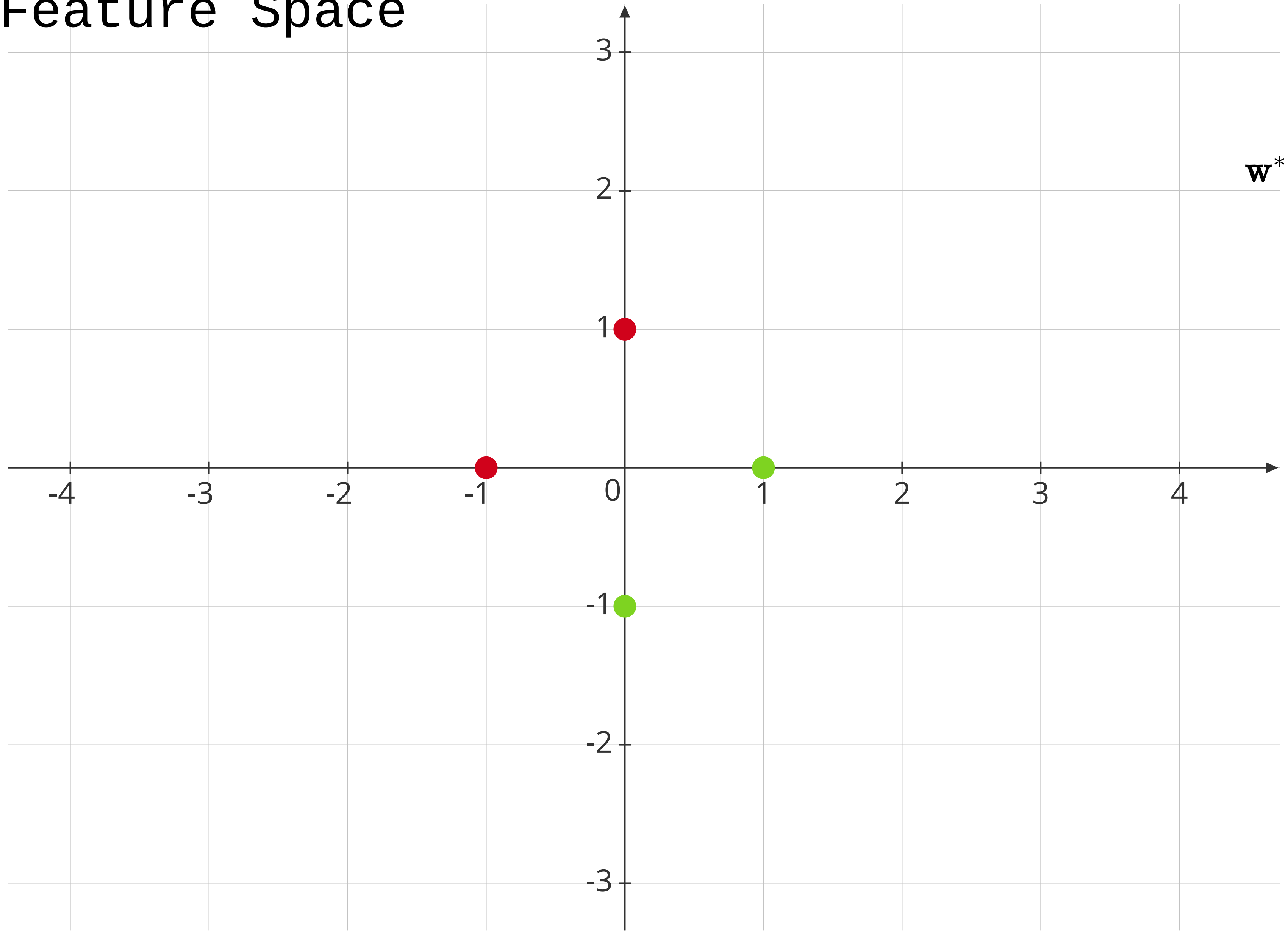
Example-1



$$\mathbf{w}^* = \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i$$

Feature Space

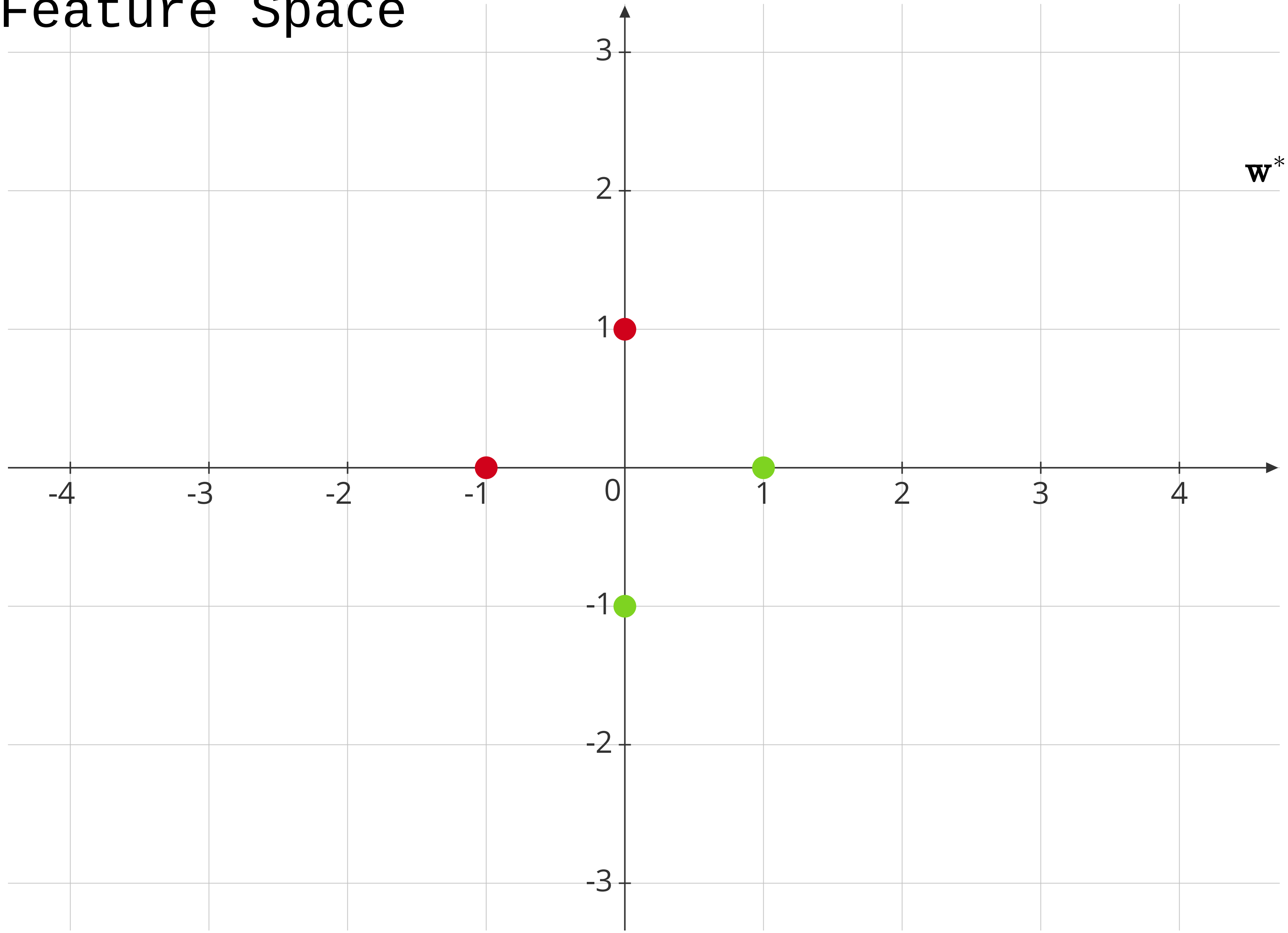
Example-1



$$\mathbf{w}^* = \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i$$
$$= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

Feature Space

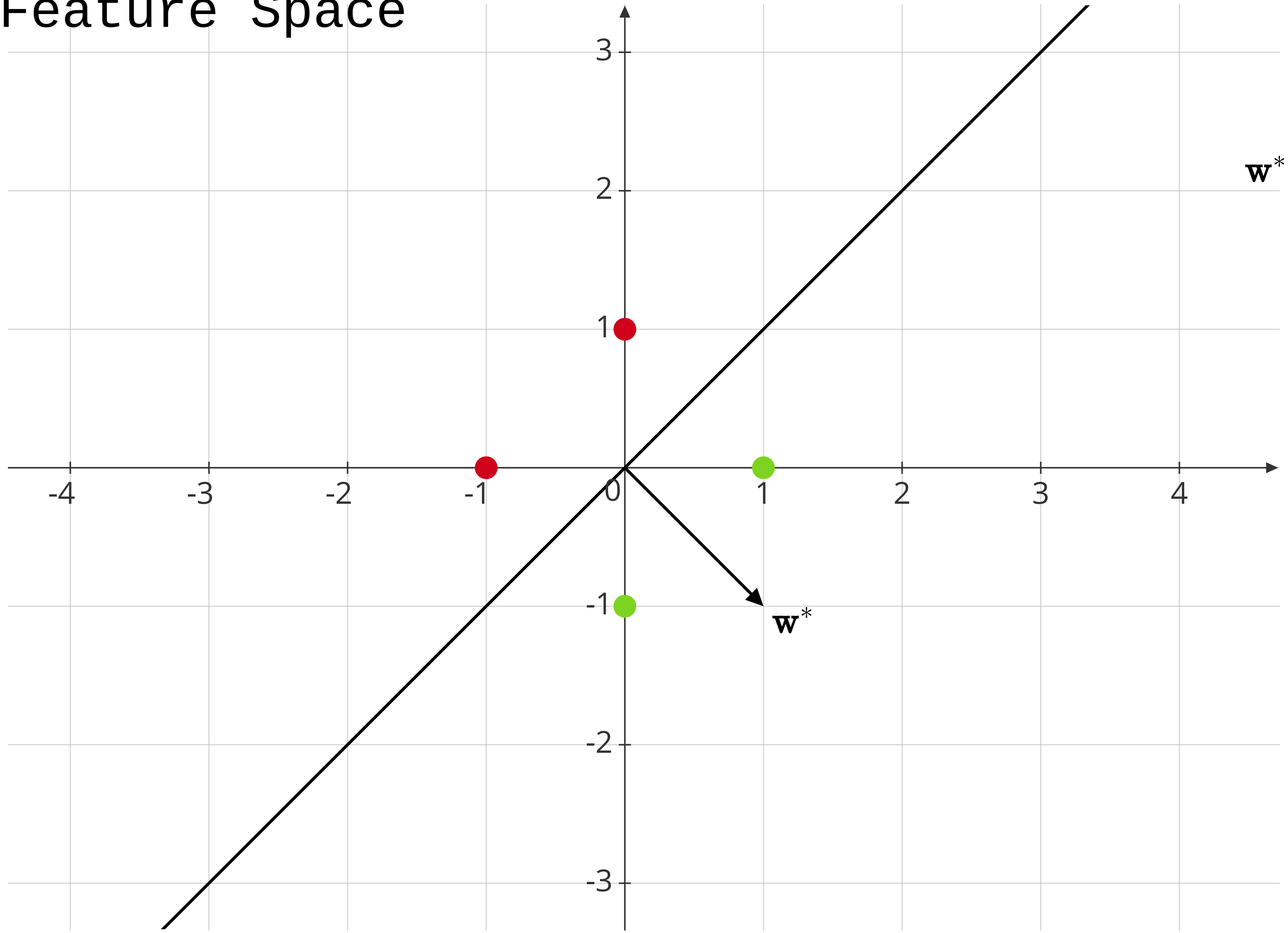
Example-1



$$\mathbf{w}^* = \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i$$
$$= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space

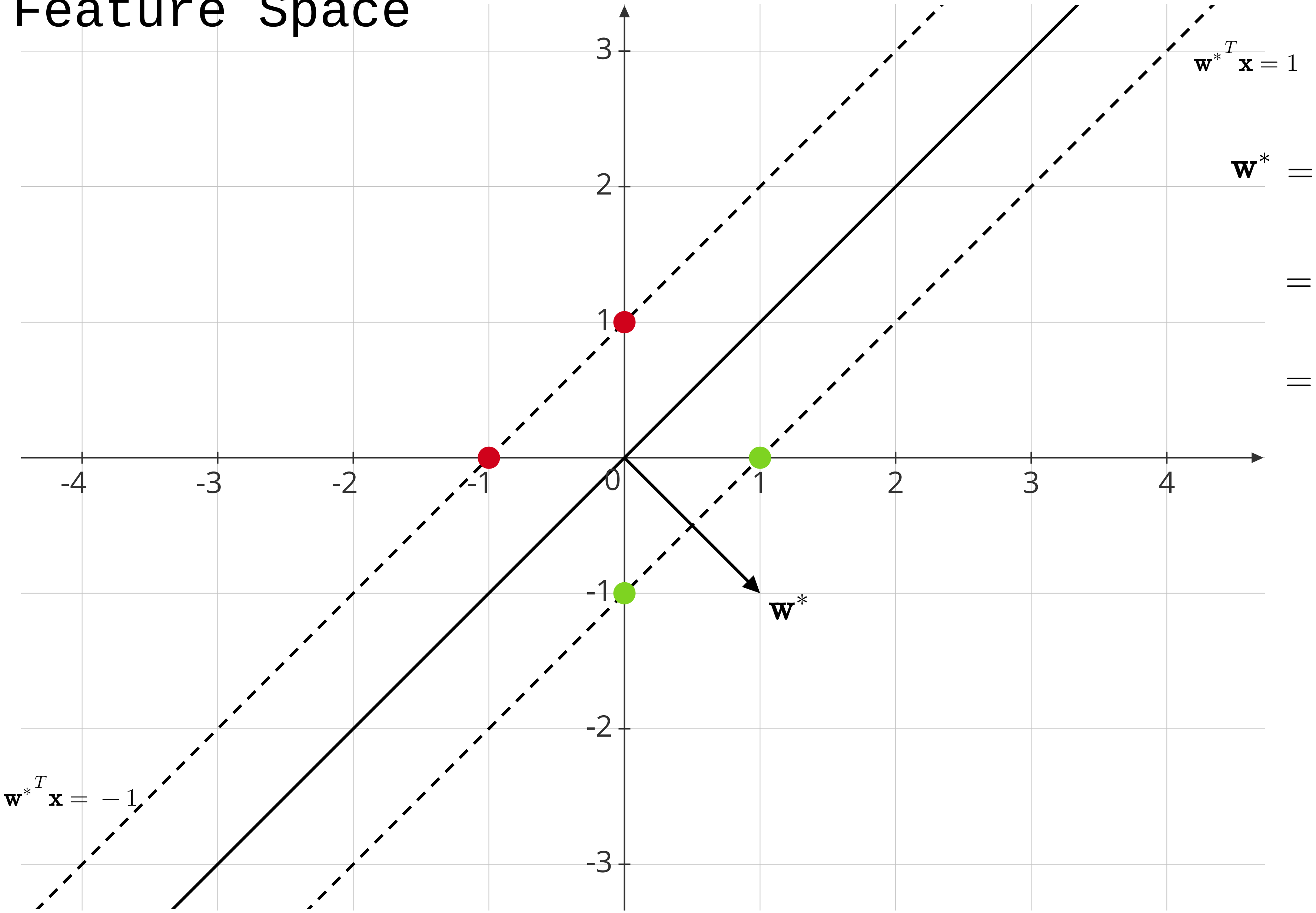
Example-1



$$\mathbf{w}^* = \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i$$
$$= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space

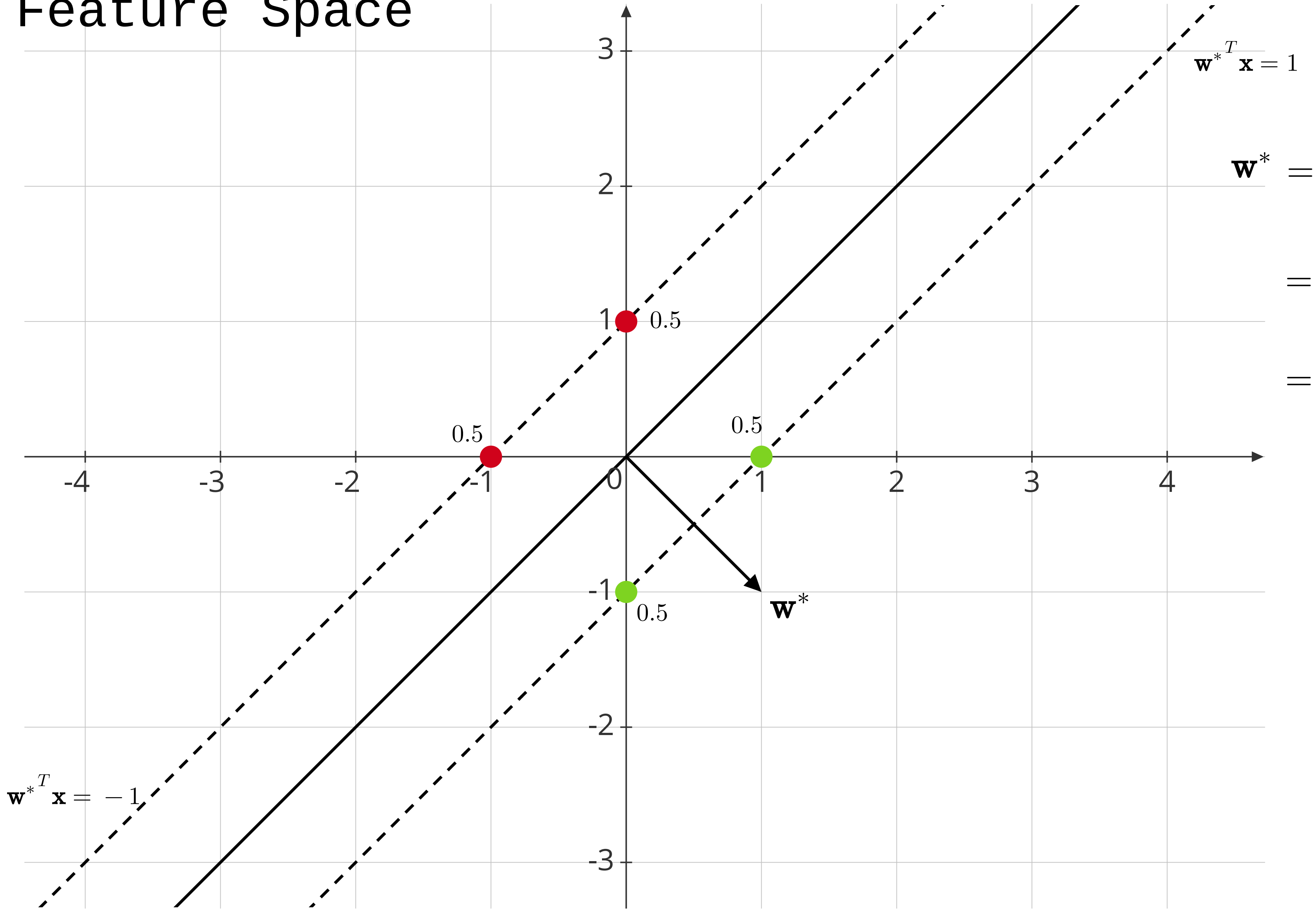
Example-1



$$\begin{aligned}\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\ &= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}\end{aligned}$$

Feature Space

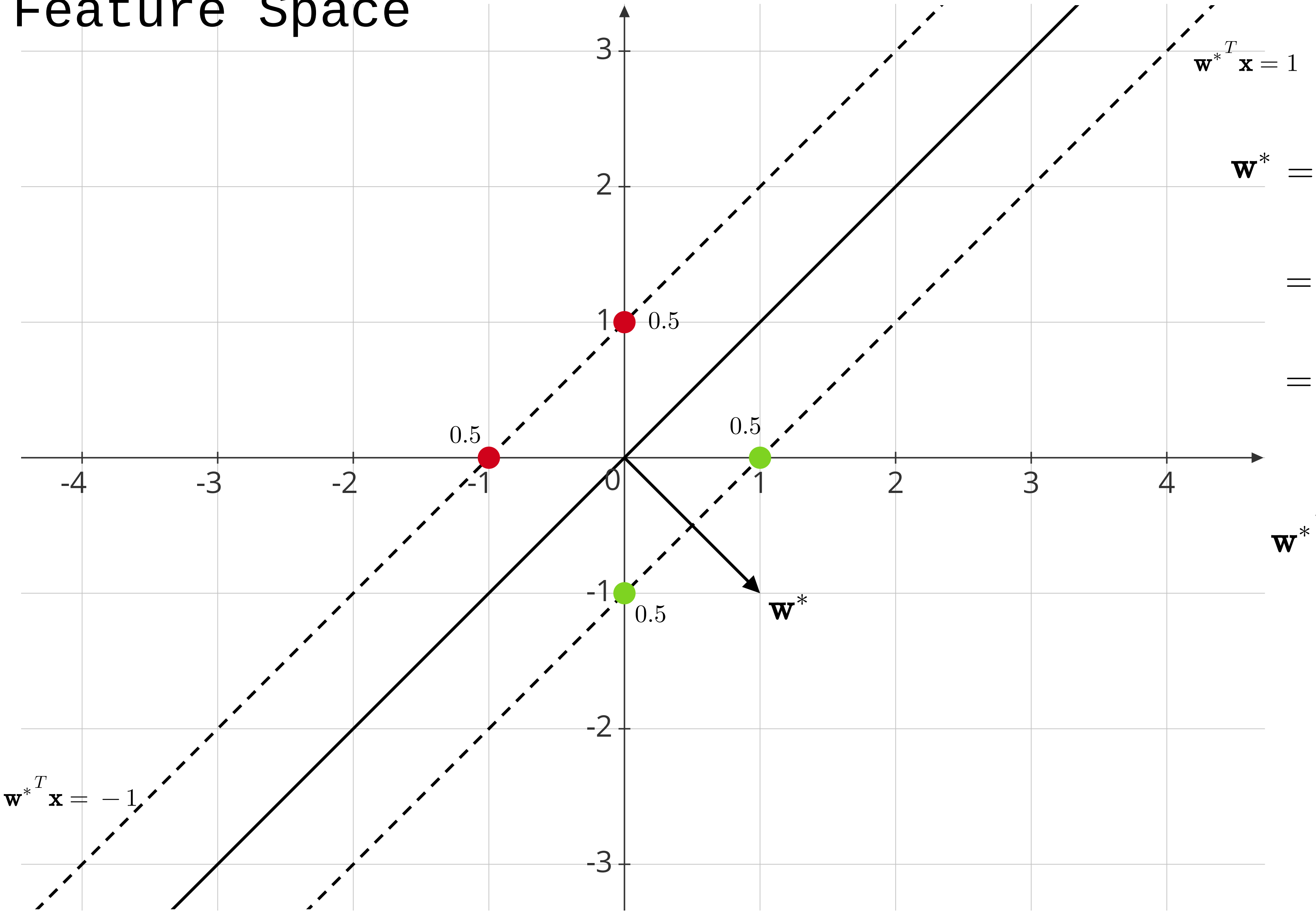
Example-1



$$
\begin{aligned}
\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\
&= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

Feature Space

Example-1

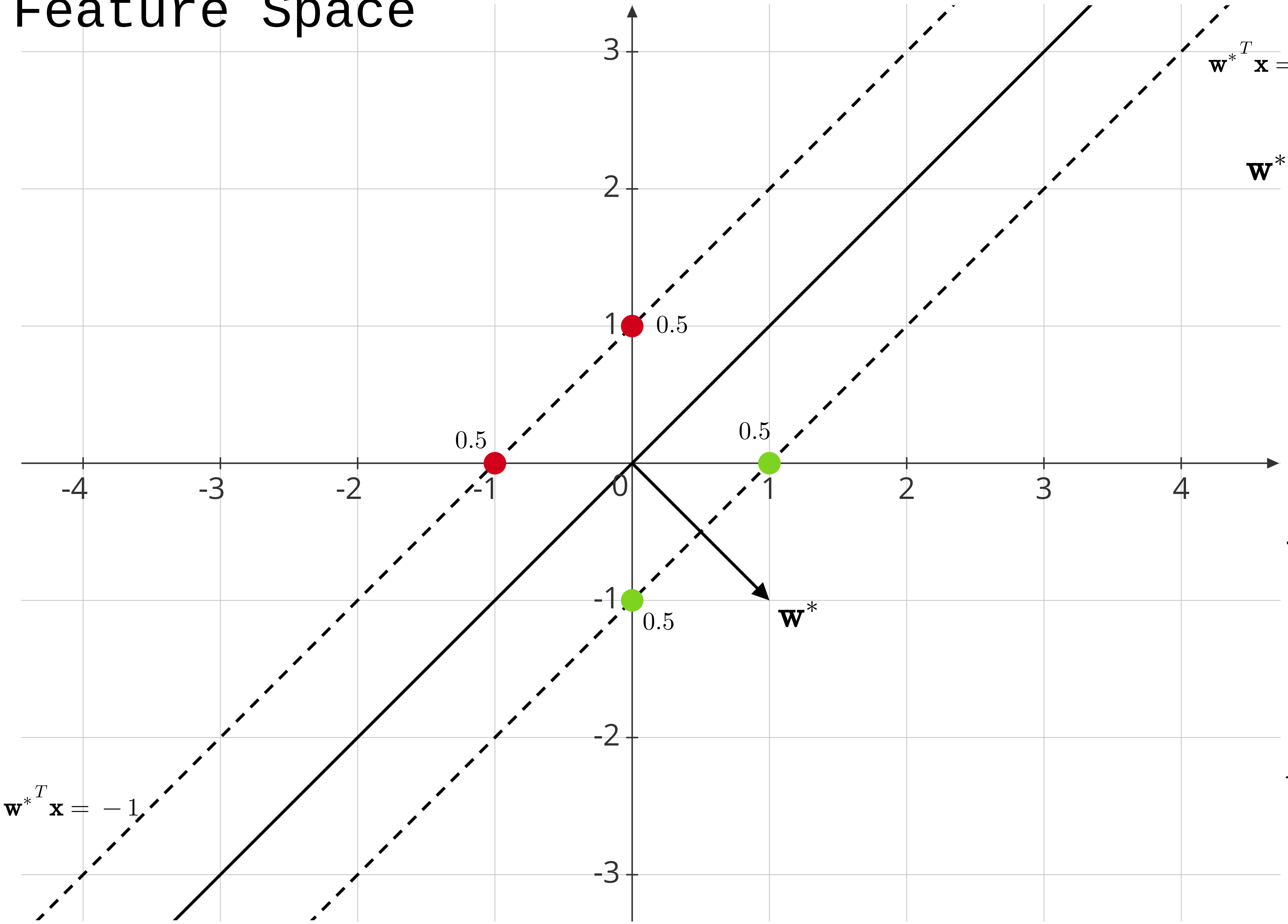


$$
\begin{aligned}
\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\
&= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{w^*}^T \mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n} \alpha_i \cdot \underbrace{\left[(y_i \mathbf{x}_i)^T \mathbf{x}_{\texttt{test}} \right]}_{\texttt{similarity}}
$$

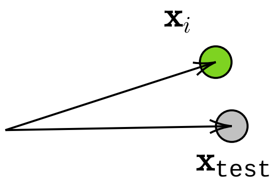
Feature Space

Example-1



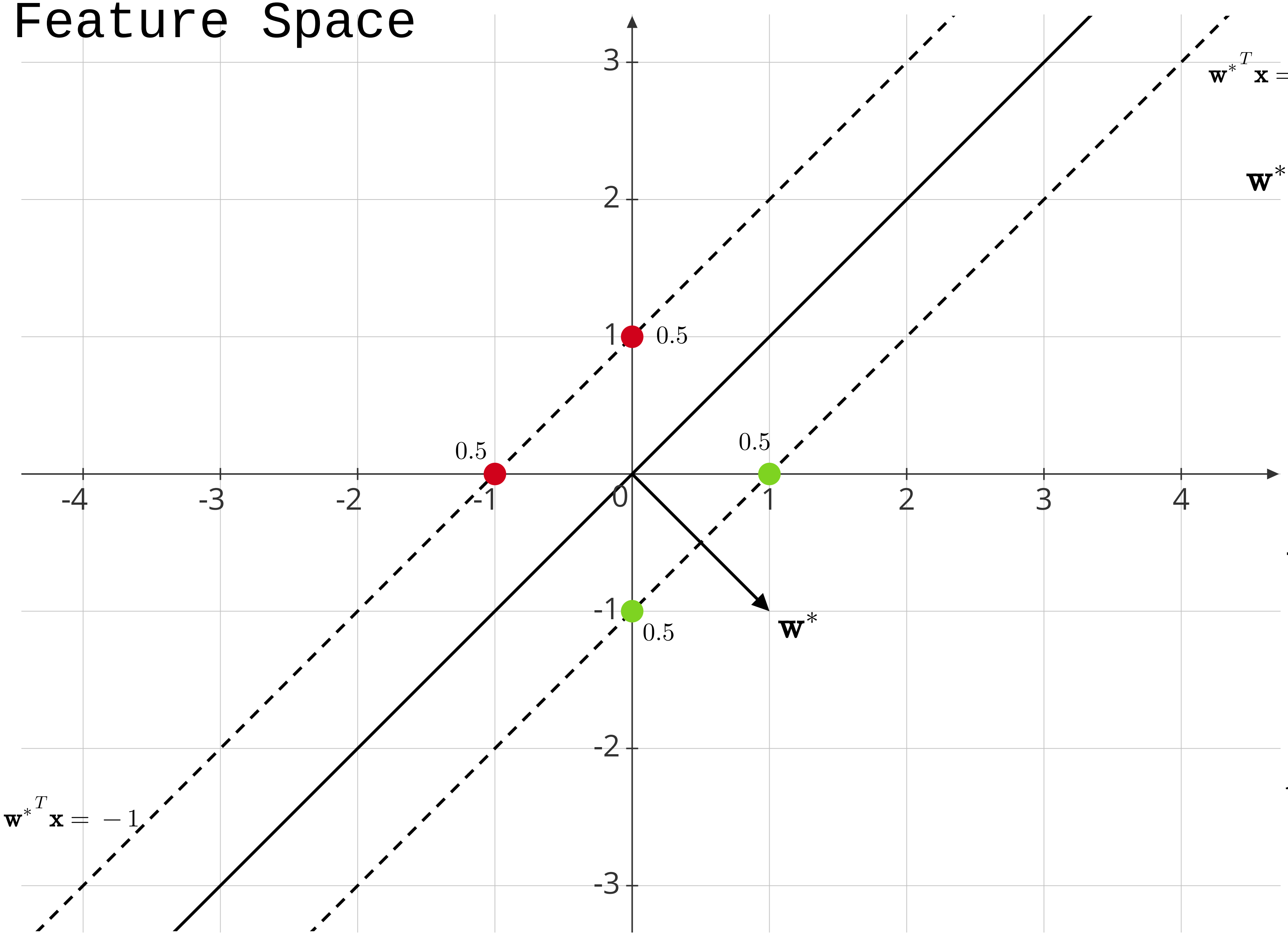
$$
\begin{aligned}
\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\
&= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{w}^{*T}\mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n} \alpha_i \cdot \underbrace{\left[(y_i \mathbf{x}_i)^T \mathbf{x}_{\texttt{test}} \right]}_{\texttt{similarity}}
$$



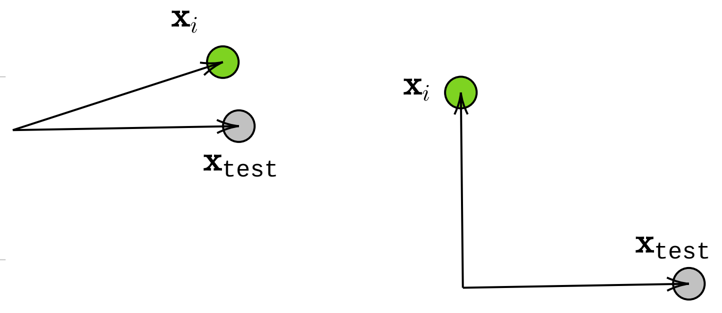
Feature Space

Example-1



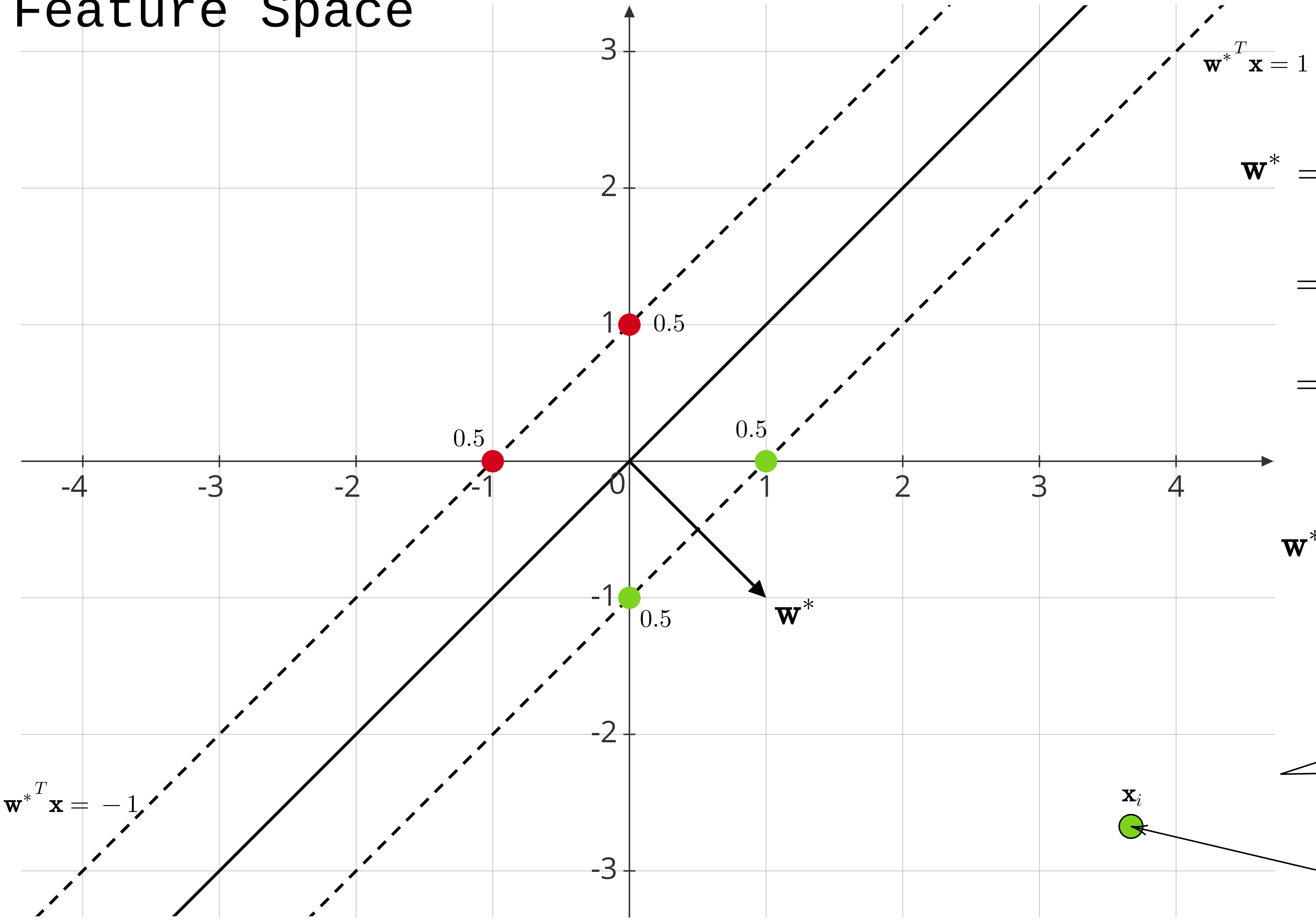
$$
\begin{aligned}
\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\
&= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{w^*}^T \mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n} \alpha_i \cdot \underbrace{\left[(y_i \mathbf{x}_i)^T \mathbf{x}_{\texttt{test}} \right]}_{\texttt{similarity}}
$$



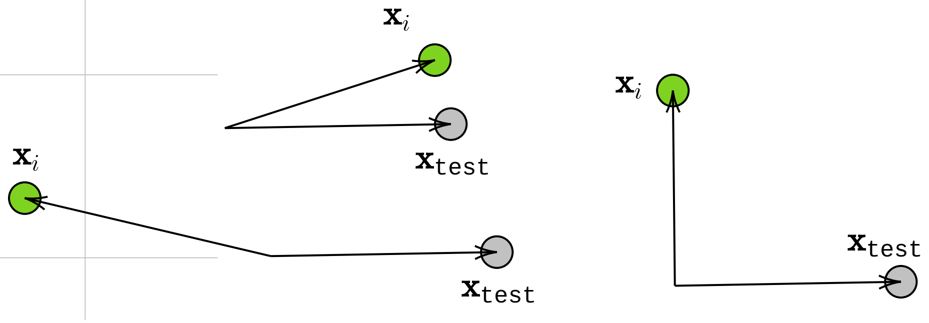
Feature Space

Example-1

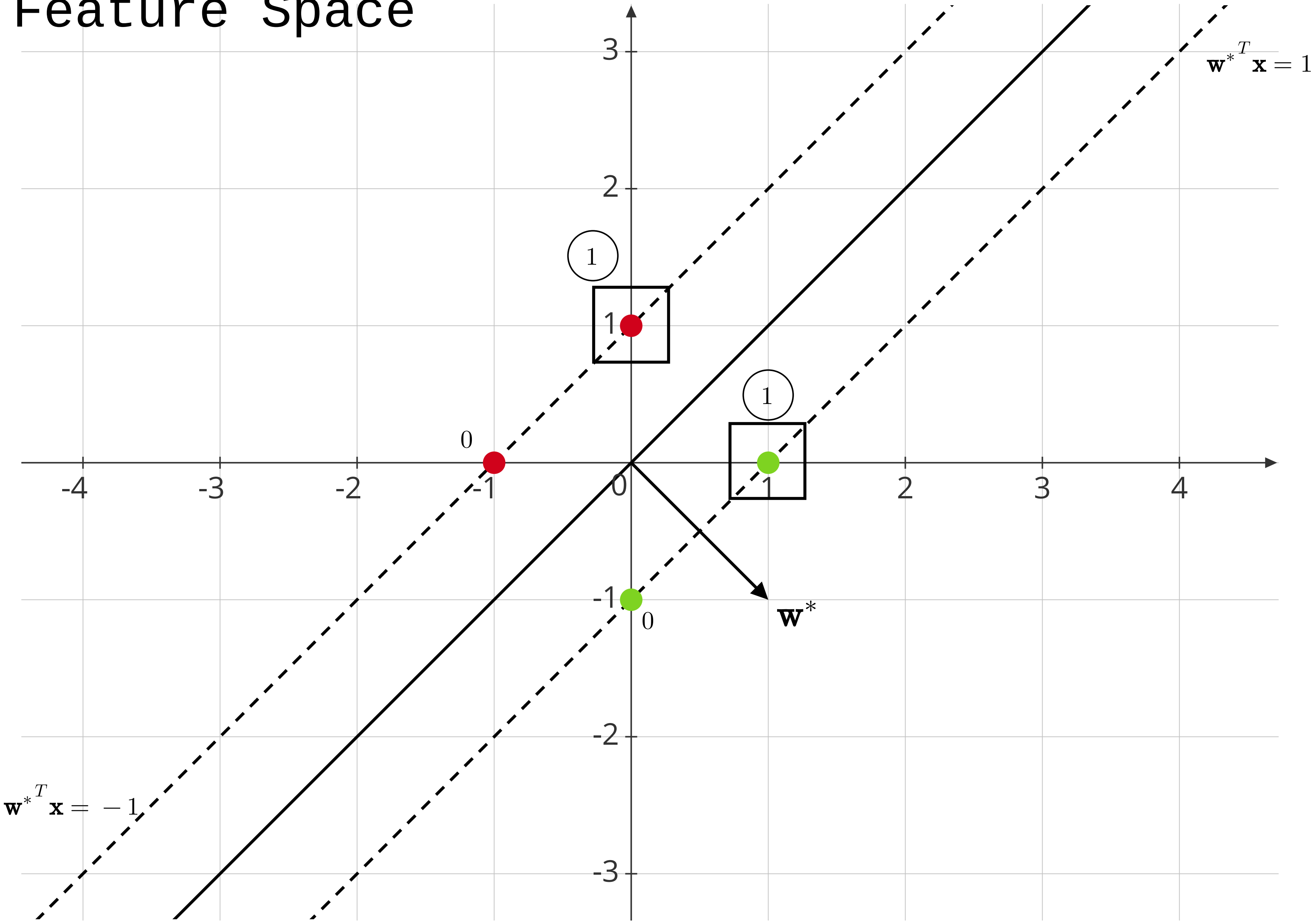


$$
\begin{aligned}
\mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\
&= 0.5 \cdot \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{aligned}
$$

$$
\mathbf{w}^{*T}\mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n} \alpha_i \cdot \underbrace{\left[(y_i \mathbf{x}_i)^T \mathbf{x}_{\texttt{test}} \right]}_{\texttt{similarity}}
$$



Feature Space



Example-1

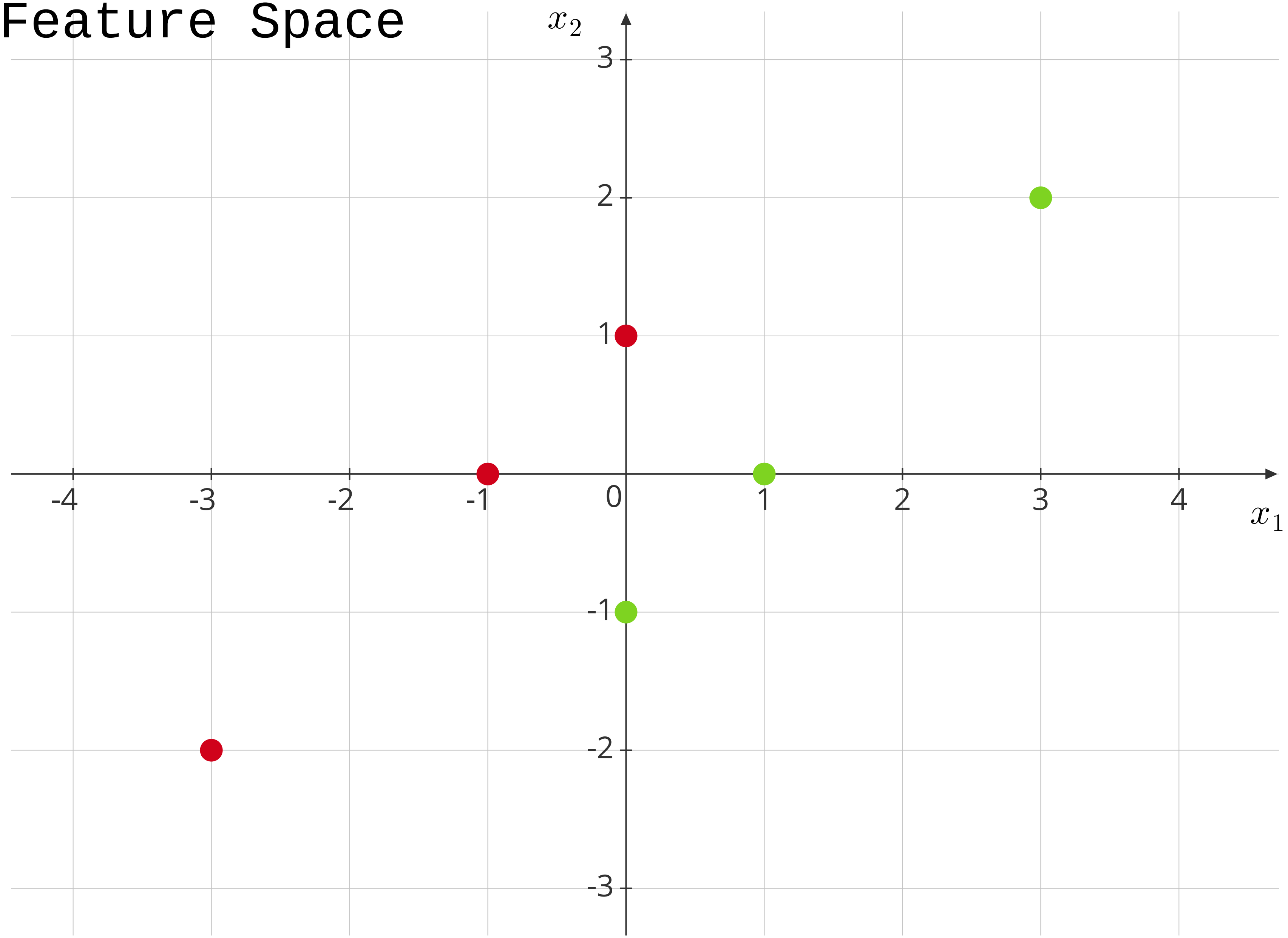
$$\mathbf{X} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\boldsymbol{\alpha}^* = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$$

$$\begin{aligned} \mathbf{w}^* &= \sum_{i=1}^{4} \alpha_i^* \mathbf{x}_i y_i \\ &= 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

Feature Space

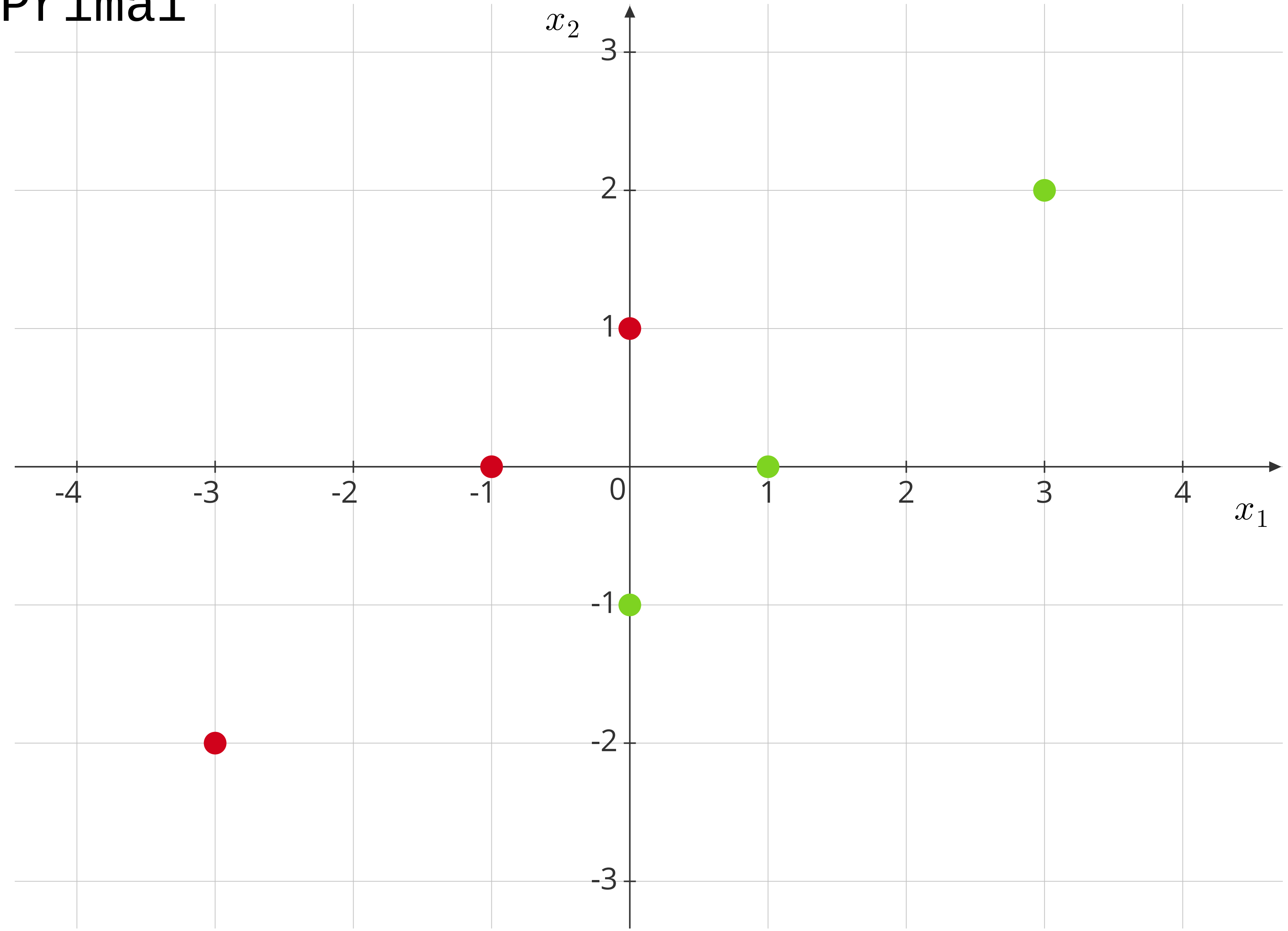
Example-2



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

Primal



Example-2

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$\begin{aligned} w_1 &\geqslant 1 && (1) \\ w_2 &\leqslant -1 && (2) \\ 3w_1 + 2w_2 &\geqslant 1 && (3) \\ w_1 &\geqslant 1 && (4) \\ w_2 &\leqslant -1 && (5) \\ 3w_1 + 2w_2 &\geqslant 1 && (6) \end{aligned}$$

Parameter Space

Example-2

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$3w_1 + 2w_2 \geqslant 1 \qquad (3)$$

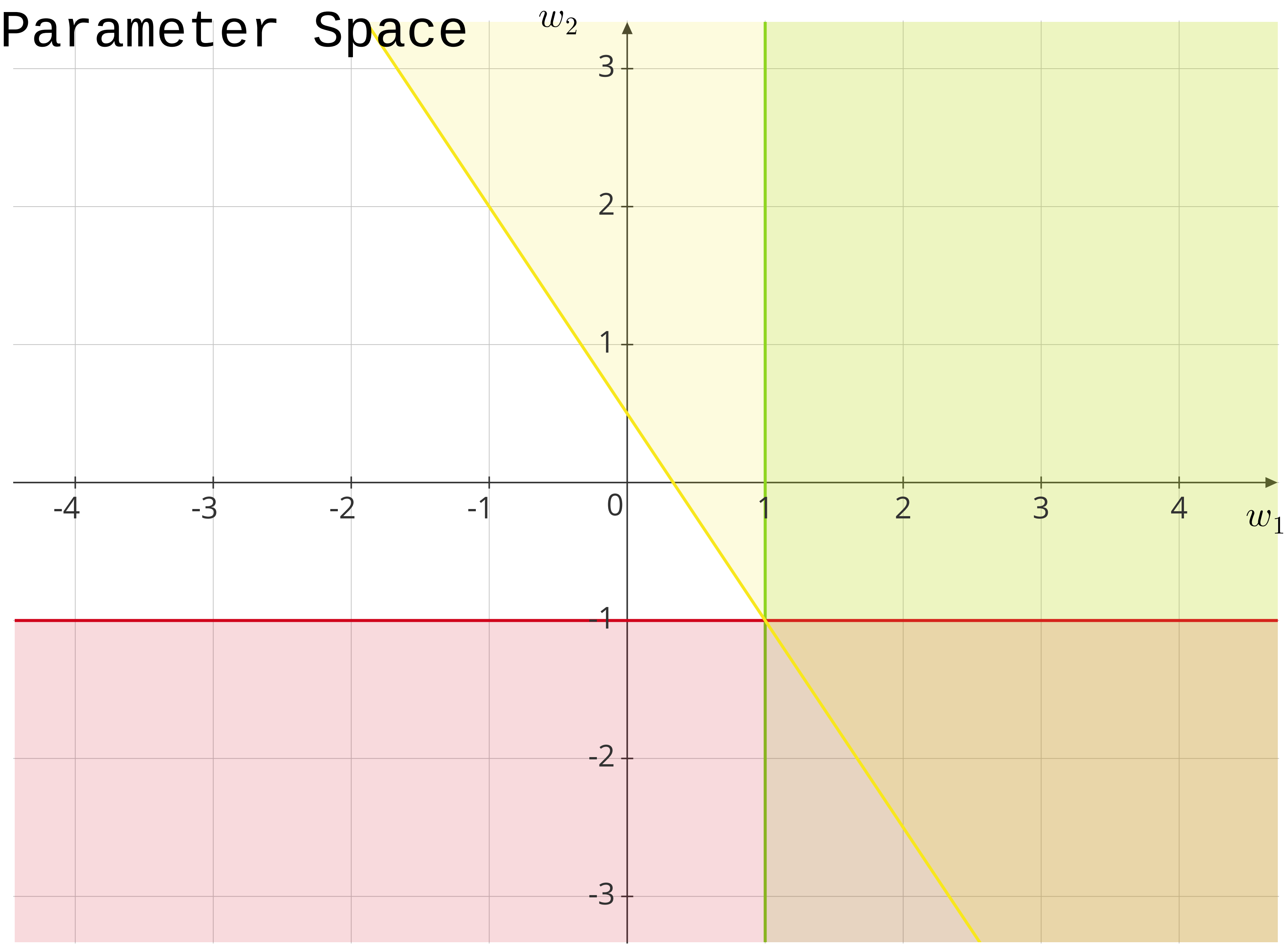
$$w_1 \geqslant 1 \qquad (4)$$

$$w_2 \leqslant -1 \qquad (5)$$

$$3w_1 + 2w_2 \geqslant 1 \qquad (6)$$

Parameter Space

Example-2



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$\begin{aligned} w_1 &\geqslant 1 && (1) \\ w_2 &\leqslant -1 && (2) \\ 3w_1 + 2w_2 &\geqslant 1 && (3) \\ w_1 &\geqslant 1 && (4) \\ w_2 &\leqslant -1 && (5) \\ 3w_1 + 2w_2 &\geqslant 1 && (6) \end{aligned}$$

Parameter Space

Example-2

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$

$$w_2 \leqslant -1 \qquad (2)$$

$$3w_1 + 2w_2 \geqslant 1 \qquad (3)$$

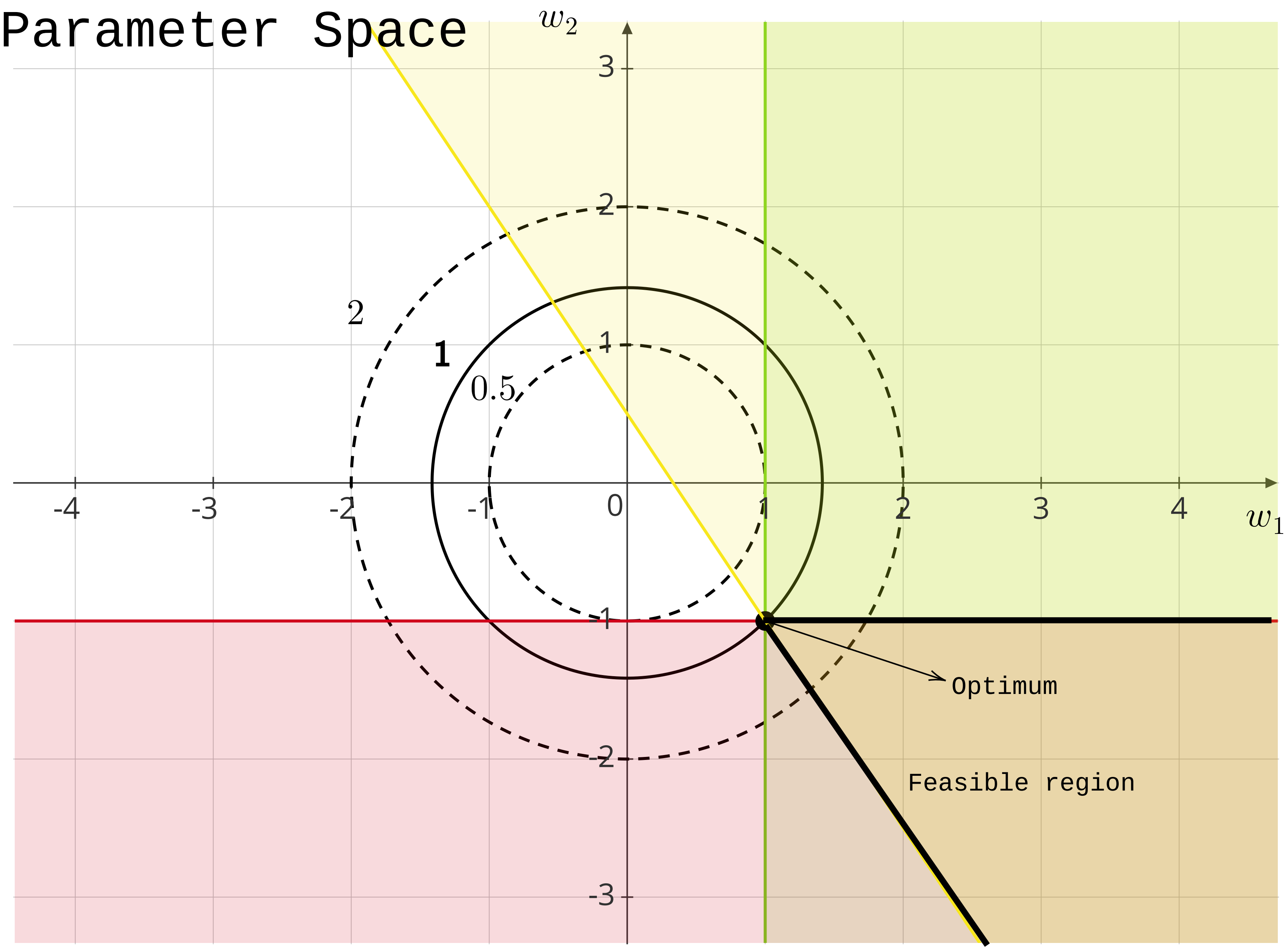
$$w_1 \geqslant 1 \qquad (4)$$

$$w_2 \leqslant -1 \qquad (5)$$

$$3w_1 + 2w_2 \geqslant 1 \qquad (6)$$

Parameter Space

Example-2



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & -1 & 0 & -3 \\ 0 & -1 & 2 & 0 & 1 & -2 \end{bmatrix},$$

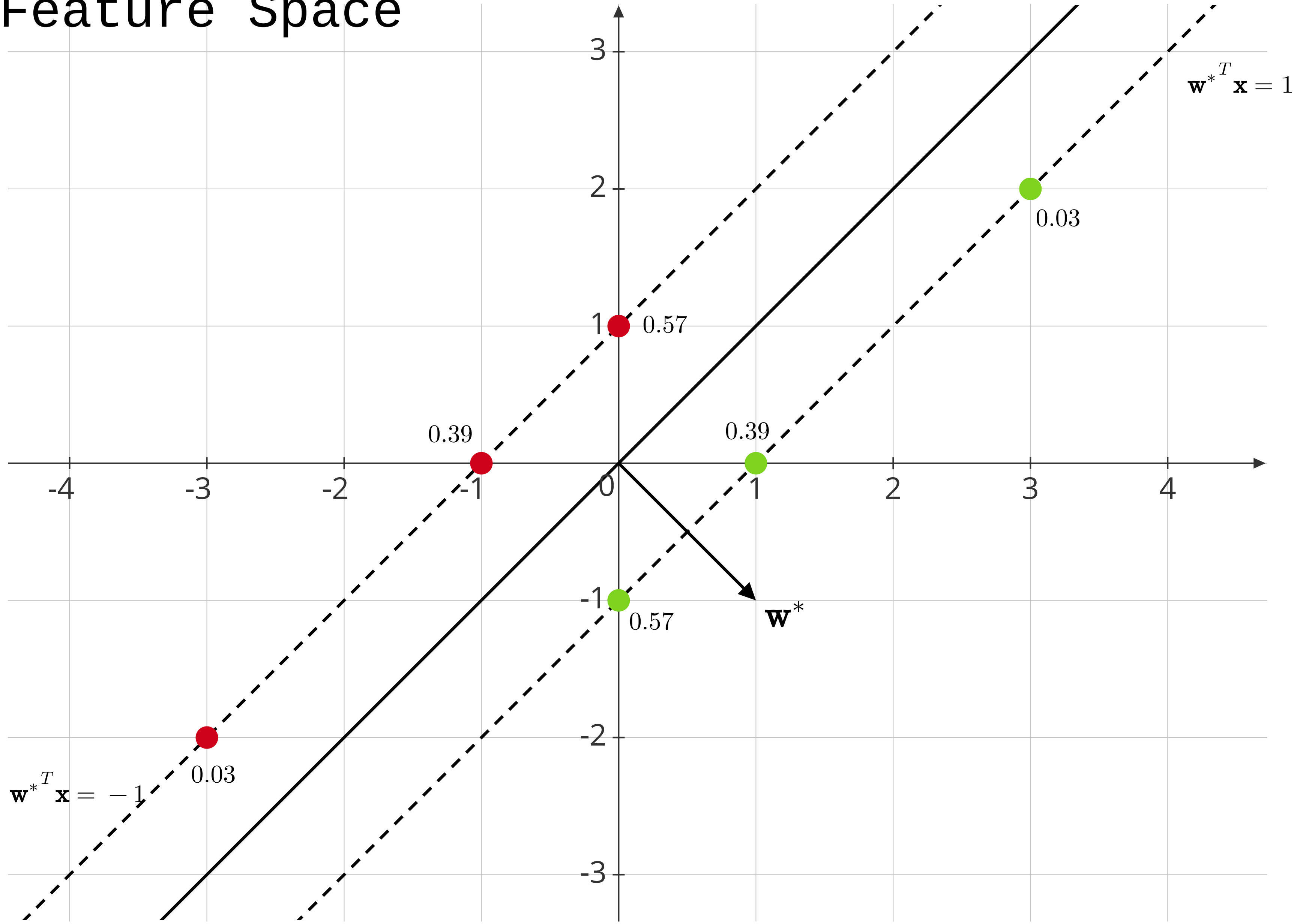
$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$w_1 \geqslant 1 \qquad (1)$$
$$w_2 \leqslant -1 \qquad (2)$$
$$3w_1 + 2w_2 \geqslant 1 \qquad (3)$$
$$w_1 \geqslant 1 \qquad (4)$$
$$w_2 \leqslant -1 \qquad (5)$$
$$3w_1 + 2w_2 \geqslant 1 \qquad (6)$$

Feature Space

Example-2

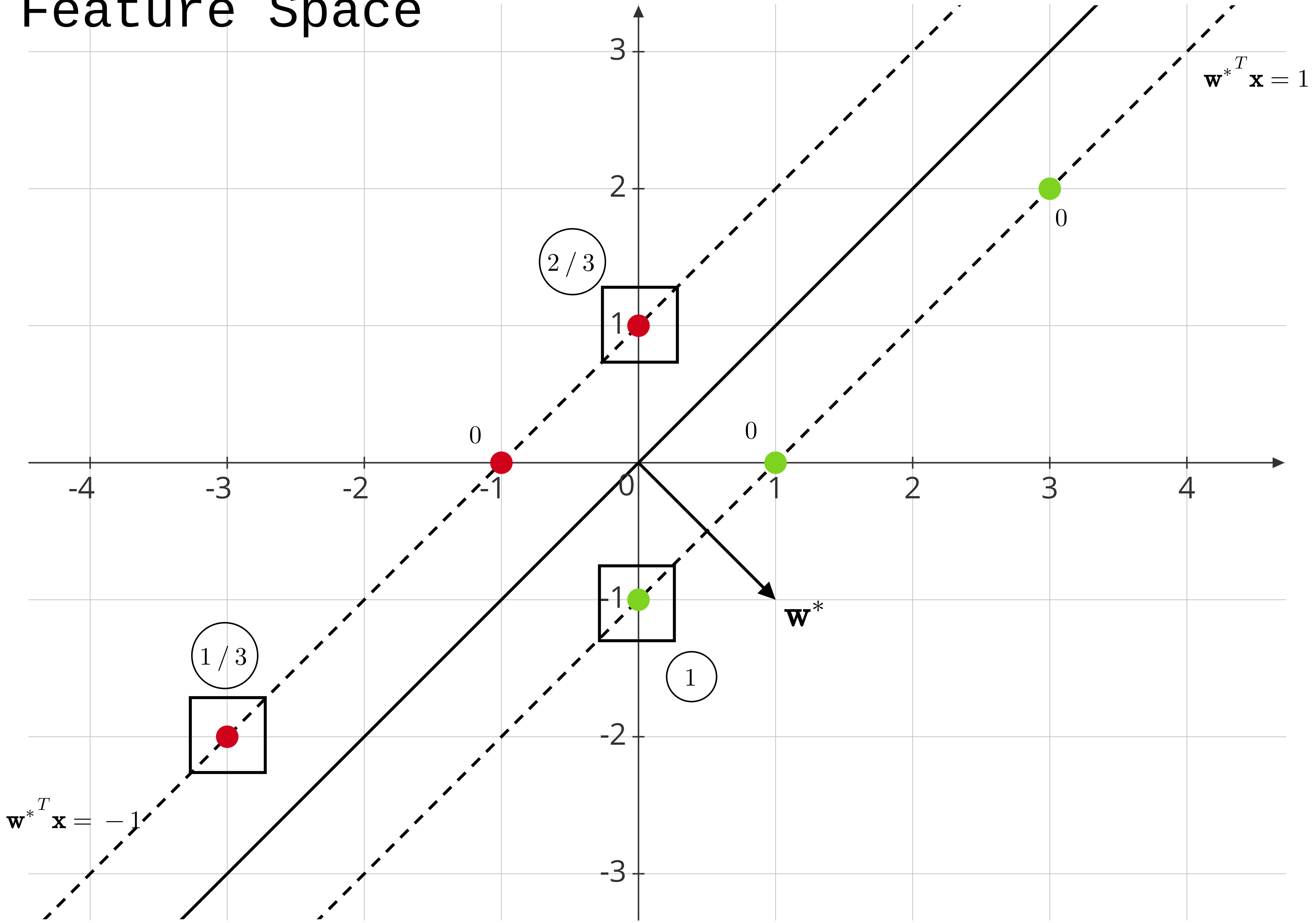


$$\boldsymbol{\alpha}^* = \begin{bmatrix} 0.39 \\ 0.57 \\ 0.03 \\ 0.39 \\ 0.57 \\ 0.03 \end{bmatrix}$$

$$\mathbf{w}^* = \sum_{i=1}^{6} \alpha_i^* \mathbf{x}_i y_i = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space

Example-2

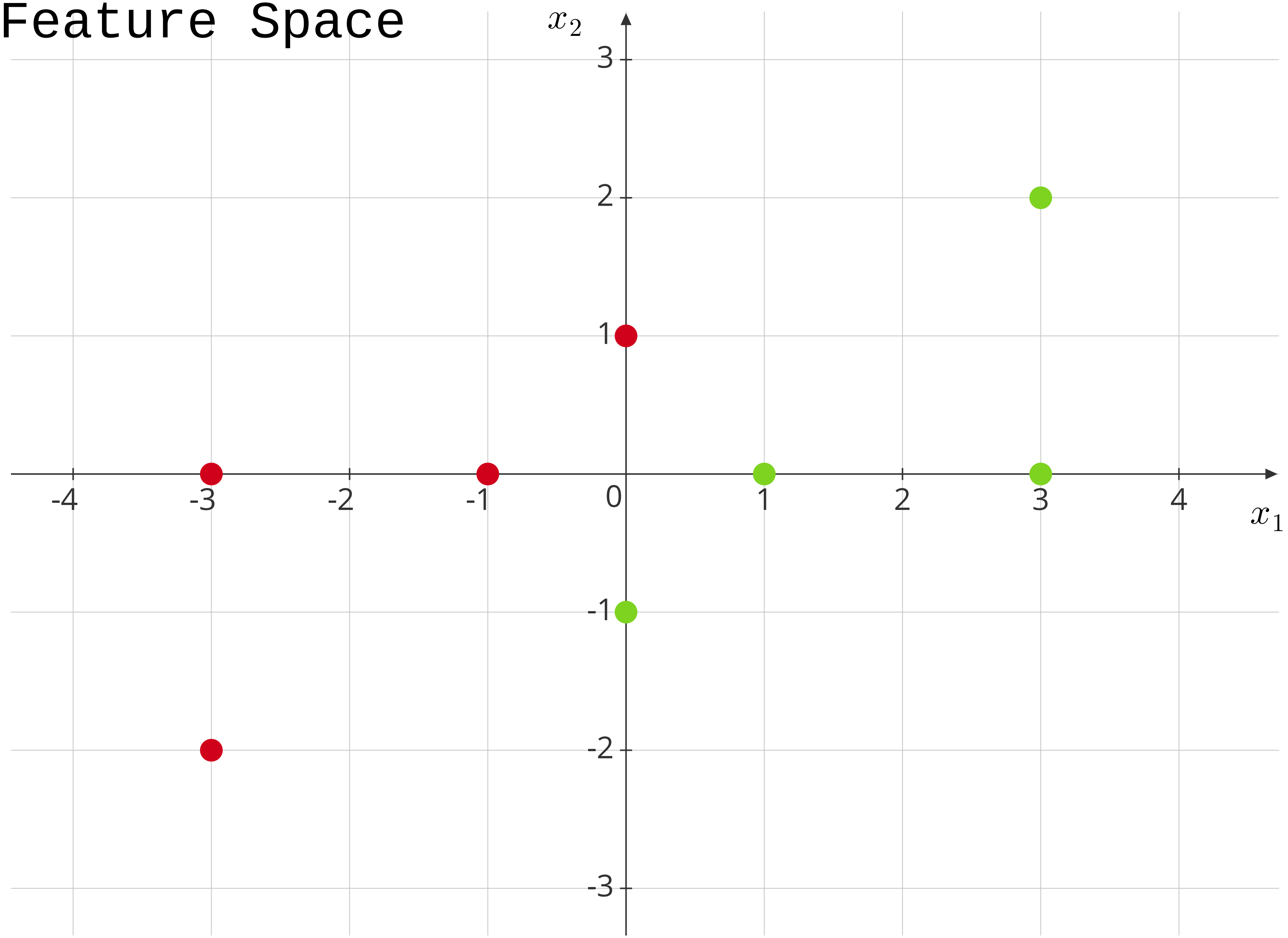


$$\boldsymbol{\alpha}^* = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 2/3 \\ 1/3 \end{bmatrix}$$

$$\mathbf{w}^* = \sum_{i=1}^{6} \alpha_i^* \mathbf{x}_i y_i = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space

Example-3

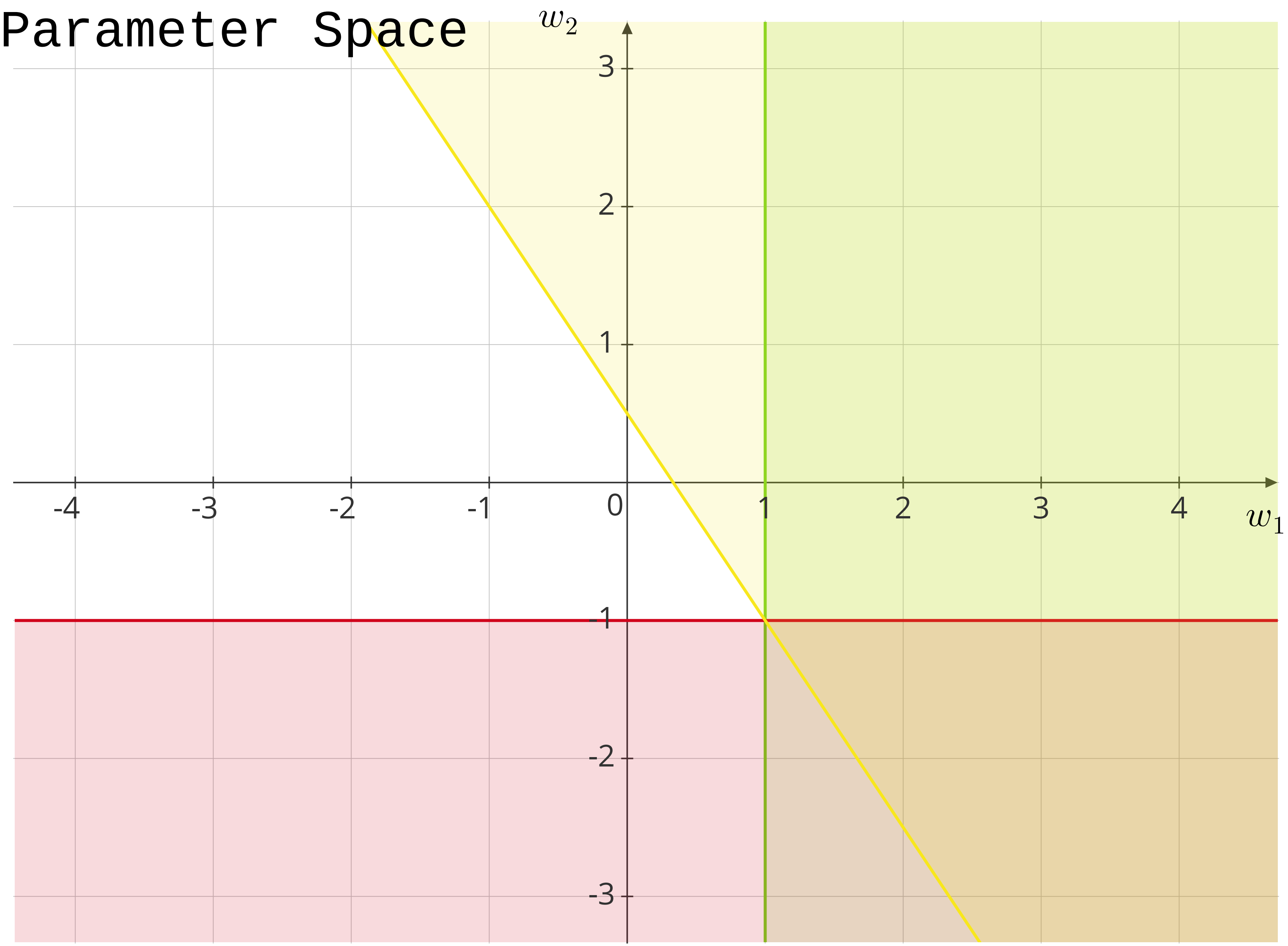


$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & 3 & -1 & 0 & -3 & -3 \\ 0 & -1 & 2 & 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

Parameter Space

Example-3



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & 3 & -1 & 0 & -3 & -3 \\ 0 & -1 & 2 & 0 & 0 & 1 & -2 & 0 \end{bmatrix},$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$\begin{aligned} w_1 &\geqslant 1 && (1) \\ w_2 &\leqslant -1 && (2) \\ 3w_1 + 2w_2 &\geqslant 1 && (3) \\ w_1 &\geqslant 1\,/\,3 && (4) \\ w_1 &\geqslant 1 && (5) \\ w_2 &\leqslant -1 && (6) \\ 3w_1 + 2w_2 &\geqslant 1 && (7) \\ w_1 &\geqslant 1\,/\,3 && (8) \end{aligned}$$

Parameter Space

Example-3

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & 3 & -1 & 0 & -3 & -3 \\ 0 & -1 & 2 & 0 & 0 & 1 & -2 & 0 \end{bmatrix},$$

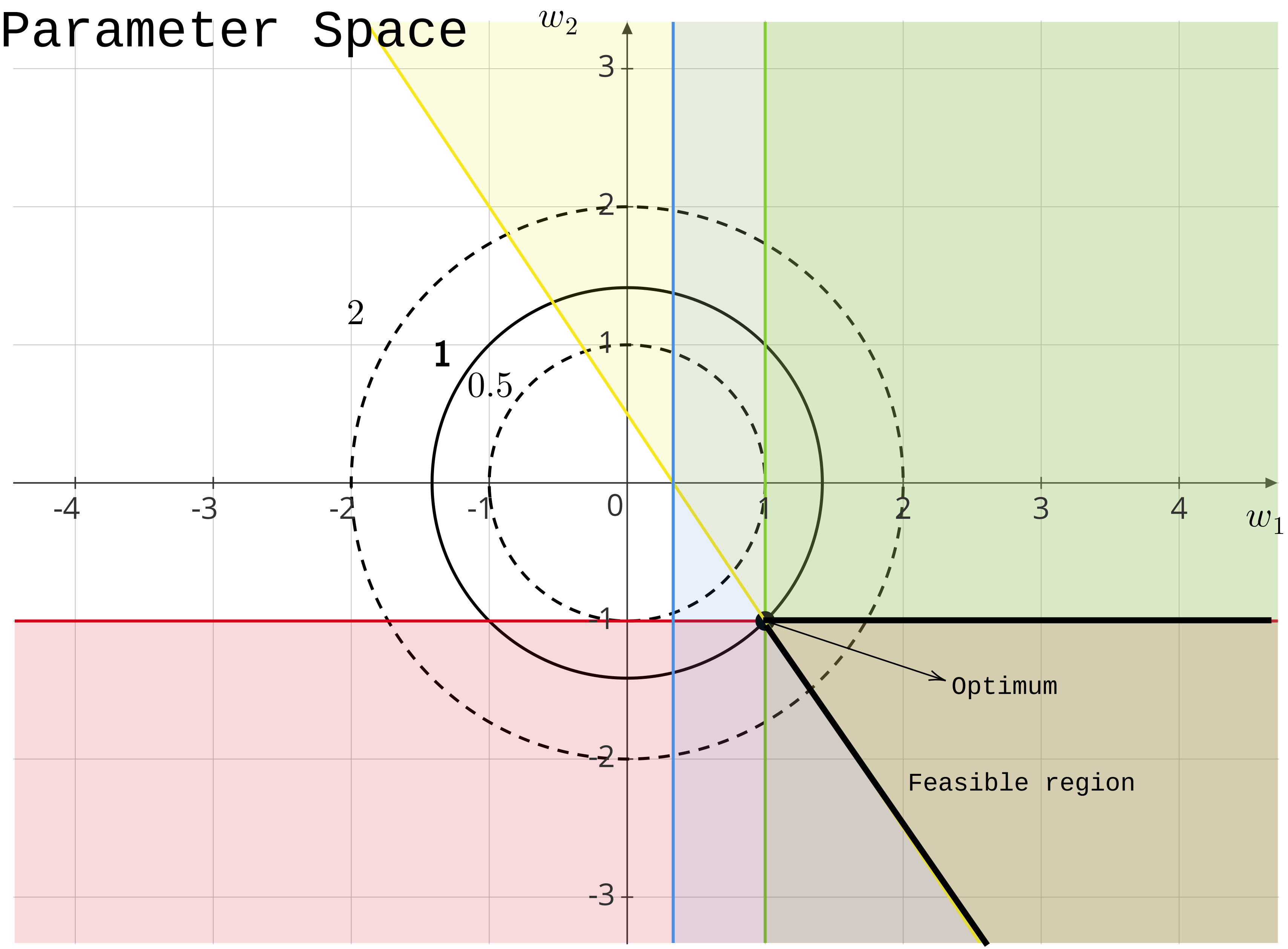
$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$\begin{aligned} w_1 &\geqslant 1 && (1) \\ w_2 &\leqslant -1 && (2) \\ 3w_1 + 2w_2 &\geqslant 1 && (3) \\ w_1 &\geqslant 1\,/\,3 && (4) \\ w_1 &\geqslant 1 && (5) \\ w_2 &\leqslant -1 && (6) \\ 3w_1 + 2w_2 &\geqslant 1 && (7) \\ w_1 &\geqslant 1\,/\,3 && (8) \end{aligned}$$

Parameter Space

Example-3



$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 3 & 3 & -1 & 0 & -3 & -3 \\ 0 & -1 & 2 & 0 & 0 & 1 & -2 & 0 \end{bmatrix},$$

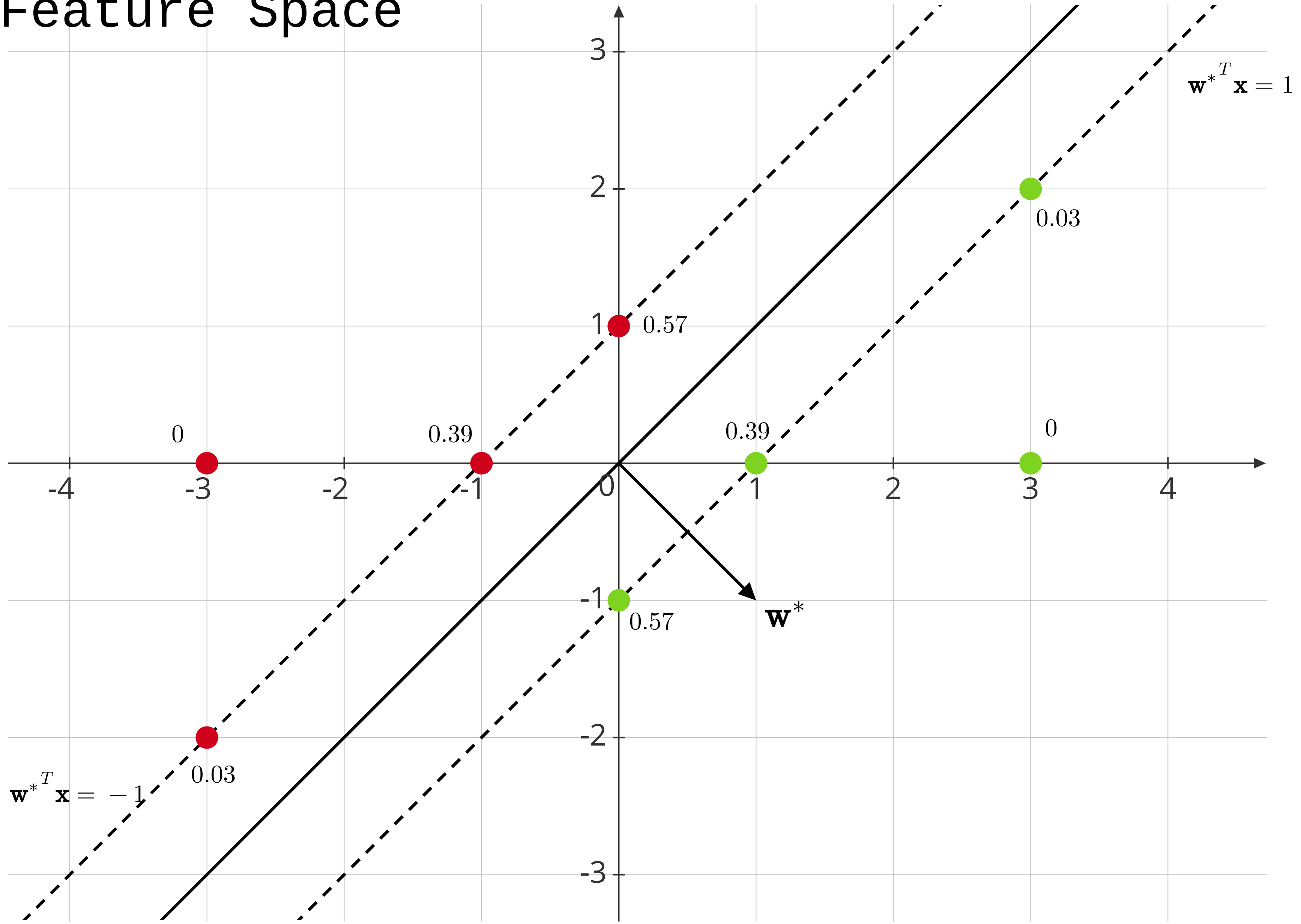
$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$\min_{\mathbf{w}} \quad \frac{w_1^2 + w_2^2}{2}$$

$$\begin{aligned} w_1 &\geqslant 1 && (1) \\ w_2 &\leqslant -1 && (2) \\ 3w_1 + 2w_2 &\geqslant 1 && (3) \\ w_1 &\geqslant 1\,/\,3 && (4) \\ w_1 &\geqslant 1 && (5) \\ w_2 &\leqslant -1 && (6) \\ 3w_1 + 2w_2 &\geqslant 1 && (7) \\ w_1 &\geqslant 1\,/\,3 && (8) \end{aligned}$$

Feature Space

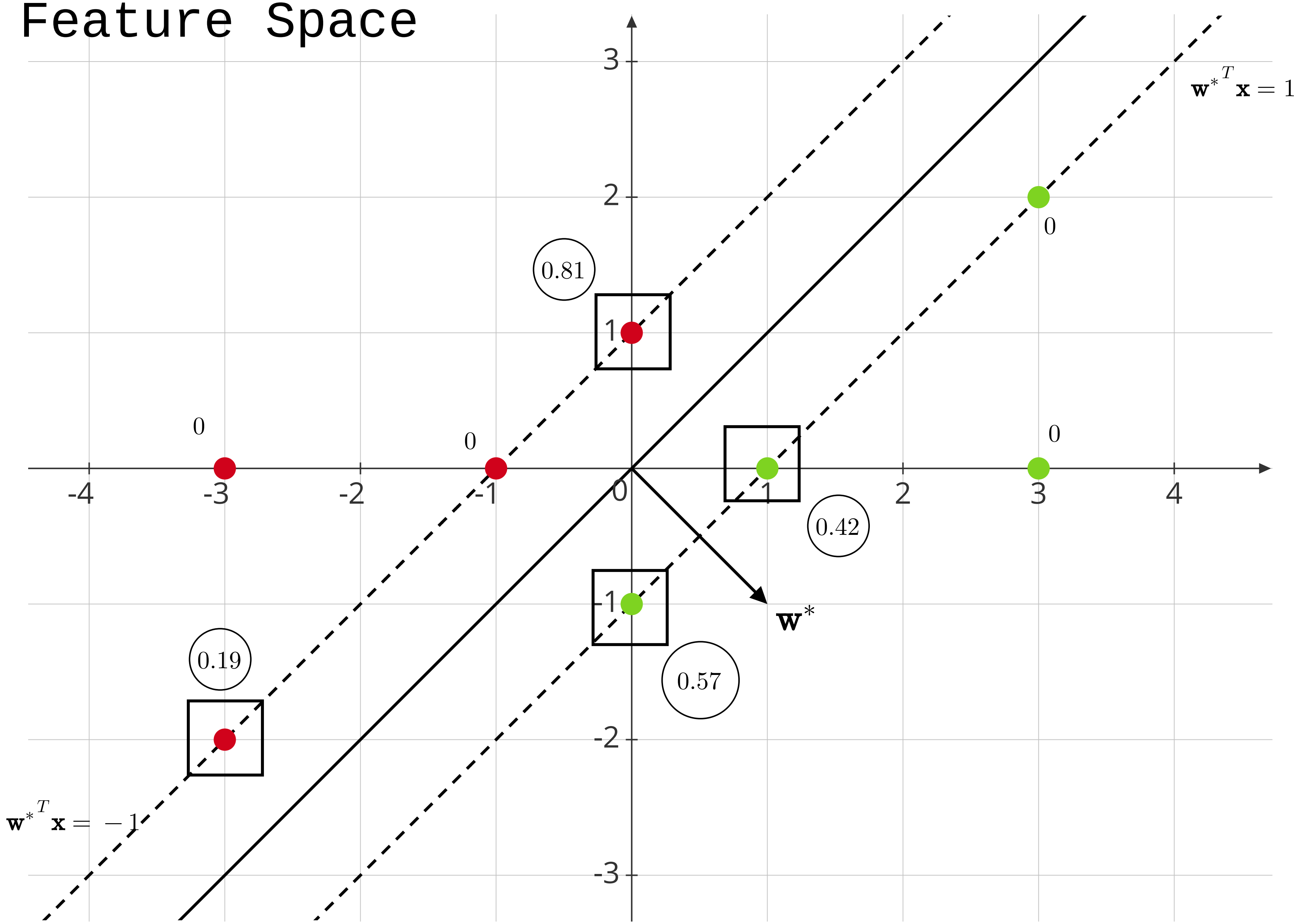
Example-2



$$\boldsymbol{\alpha}^* = \begin{bmatrix} 0.39 \\ 0.57 \\ 0.03 \\ 0 \\ 0.39 \\ 0.57 \\ 0.03 \\ 0 \end{bmatrix}$$

$$\mathbf{w}^* = \sum_{i=1}^{6} \alpha_i^* \mathbf{x}_i y_i$$
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space



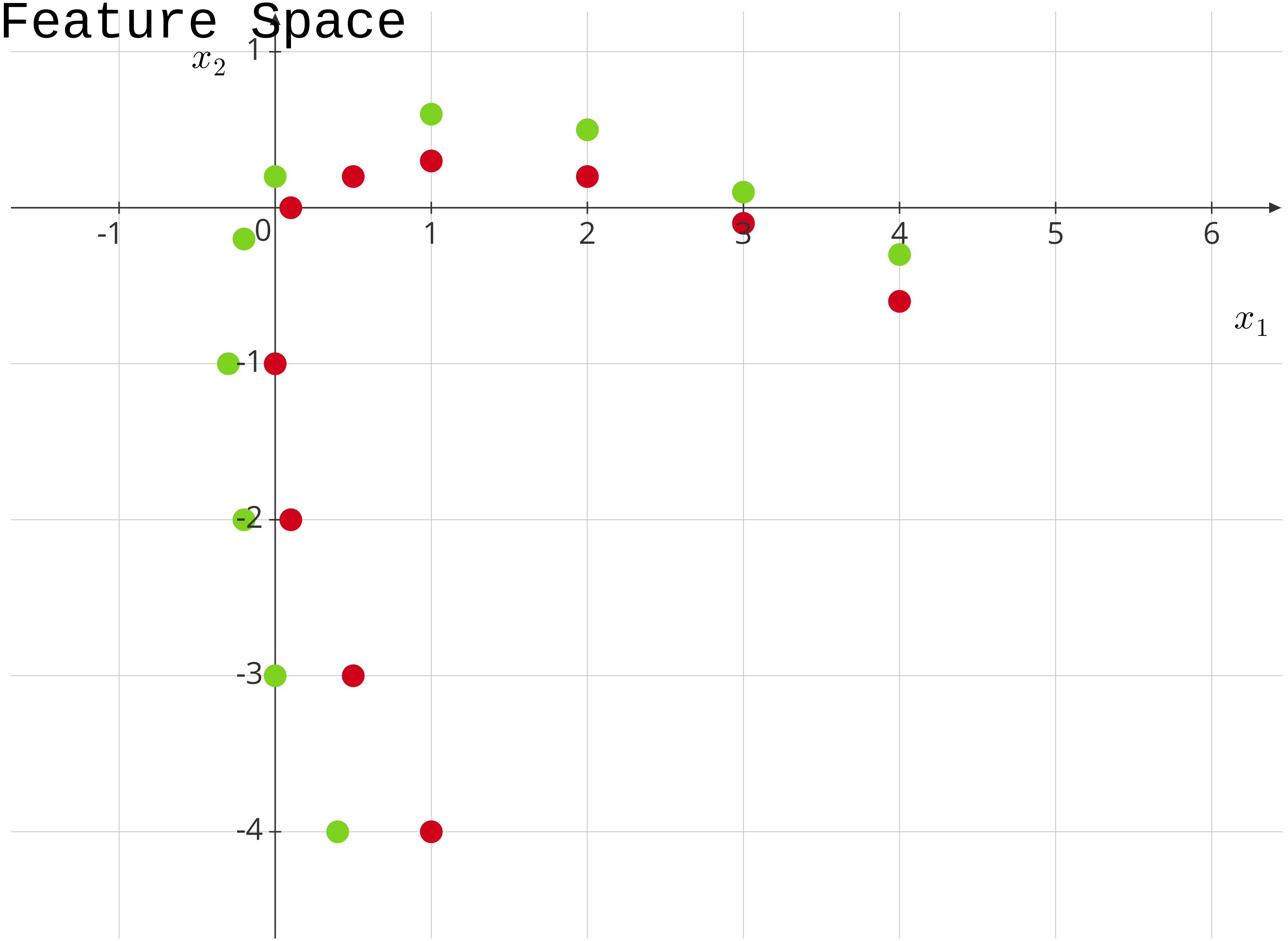
Example-3

$$\boldsymbol{\alpha}^* = \begin{bmatrix} 0.42 \\ 0.57 \\ 0 \\ 0 \\ 0 \\ 0.81 \\ 0.19 \\ 0 \end{bmatrix}$$

$$\mathbf{w}^* = \sum_{i=1}^{6} \alpha_i^* \mathbf{x}_i y_i = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Feature Space

Example-4



Linear-SVM

Kernel-SVM

$$\mathbf{x}_i$$

$$\mathbf{X}$$

$$\mathbf{X}^T\mathbf{X}$$

$$\max_{\boldsymbol{\alpha} \geqslant 0} \; \boldsymbol{\alpha}^T\mathbf{1} - \frac{1}{2} \cdot \boldsymbol{\alpha}^T\mathbf{Y}^T\mathbf{X}^T\mathbf{X}\mathbf{Y}\boldsymbol{\alpha}$$

$$\mathbf{w}^* = \sum_{i=1}^{n} \alpha_i^* \cdot \mathbf{x}_i \cdot y_i$$

$$\mathbf{w^*}^T\mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n} \alpha_i \cdot \underbrace{\left[(y_i\mathbf{x}_i)^T\mathbf{x}_{\texttt{test}}\right]}_{\texttt{similarity}}$$

Linear-SVM

$$\mathbf{x}_i$$

$$\mathbf{X}$$

$$\mathbf{X}^T\mathbf{X}$$

$$\max_{\boldsymbol{\alpha}\geqslant 0}\ \boldsymbol{\alpha}^T\mathbf{1} - \frac{1}{2}\cdot\boldsymbol{\alpha}^T\mathbf{Y}^T\mathbf{X}^T\mathbf{X}\mathbf{Y}\boldsymbol{\alpha}$$

$$\mathbf{w}^* = \sum_{i=1}^{n}\alpha_i^*\cdot\mathbf{x}_i\cdot y_i$$

$$\mathbf{w^*}^T\mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n}\alpha_i\cdot\underbrace{\left[(y_i\mathbf{x}_i)^T\mathbf{x}_{\texttt{test}}\right]}_{\texttt{similarity}}$$

Kernel-SVM

$$\phi(\mathbf{x}_i)$$

$$\phi(\mathbf{X})$$

$$\mathbf{K}$$

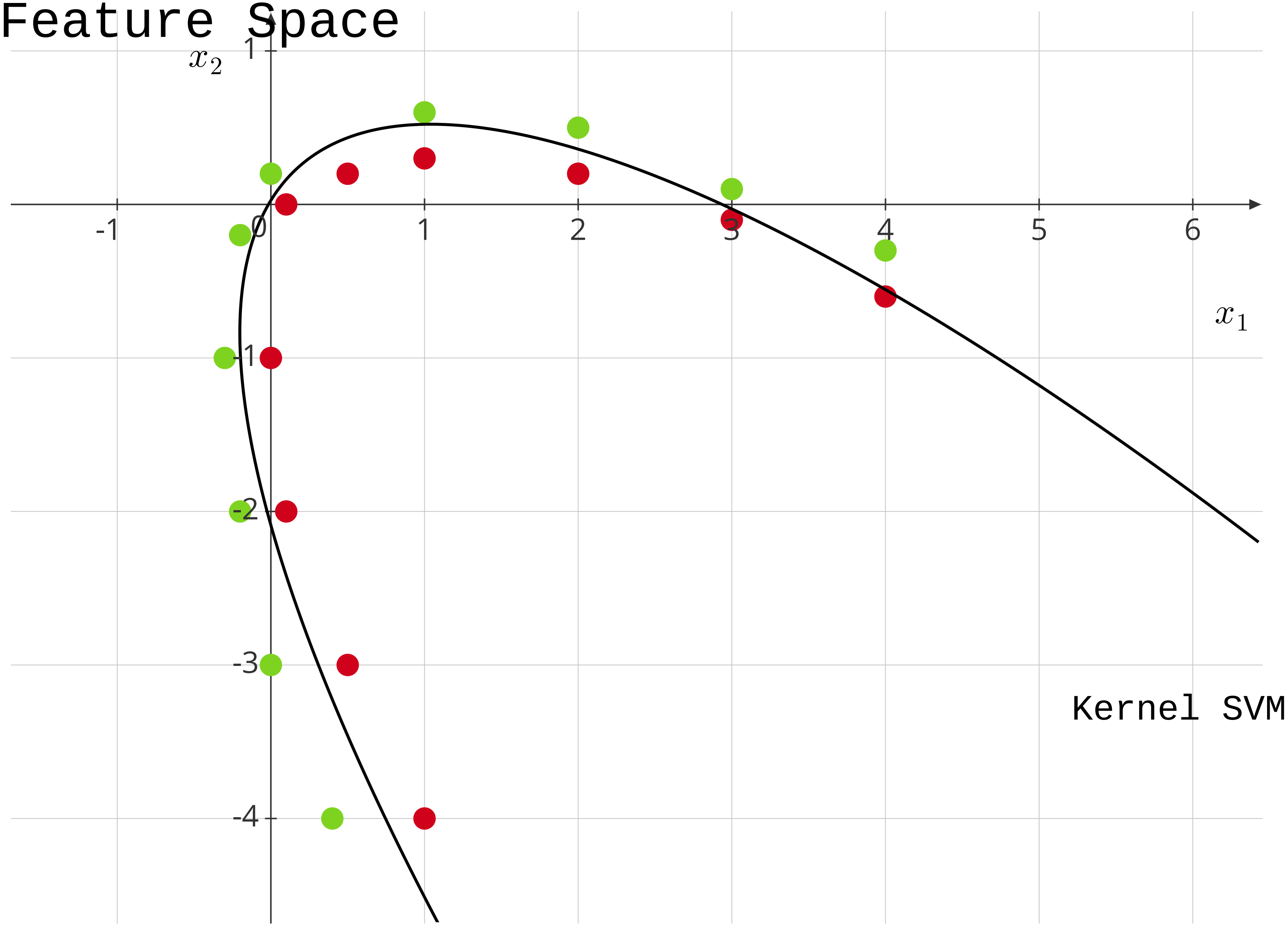
$$\max_{\boldsymbol{\alpha}\geqslant 0}\ \boldsymbol{\alpha}^T\mathbf{1} - \frac{1}{2}\cdot\boldsymbol{\alpha}^T\mathbf{Y}^T\mathbf{K}\mathbf{Y}\boldsymbol{\alpha}$$

$$\mathbf{w}^* = \sum_{i=1}^{n}\alpha_i^*\cdot\phi(\mathbf{x}_i)\cdot y_i$$

$$\mathbf{w^*}^T\mathbf{x}_{\texttt{test}} = \sum_{i=1}^{n}\alpha_i\cdot\underbrace{y_i\cdot k(\mathbf{x}_i,\mathbf{x}_{\texttt{test}})}_{\texttt{similarity}}$$

Feature Space

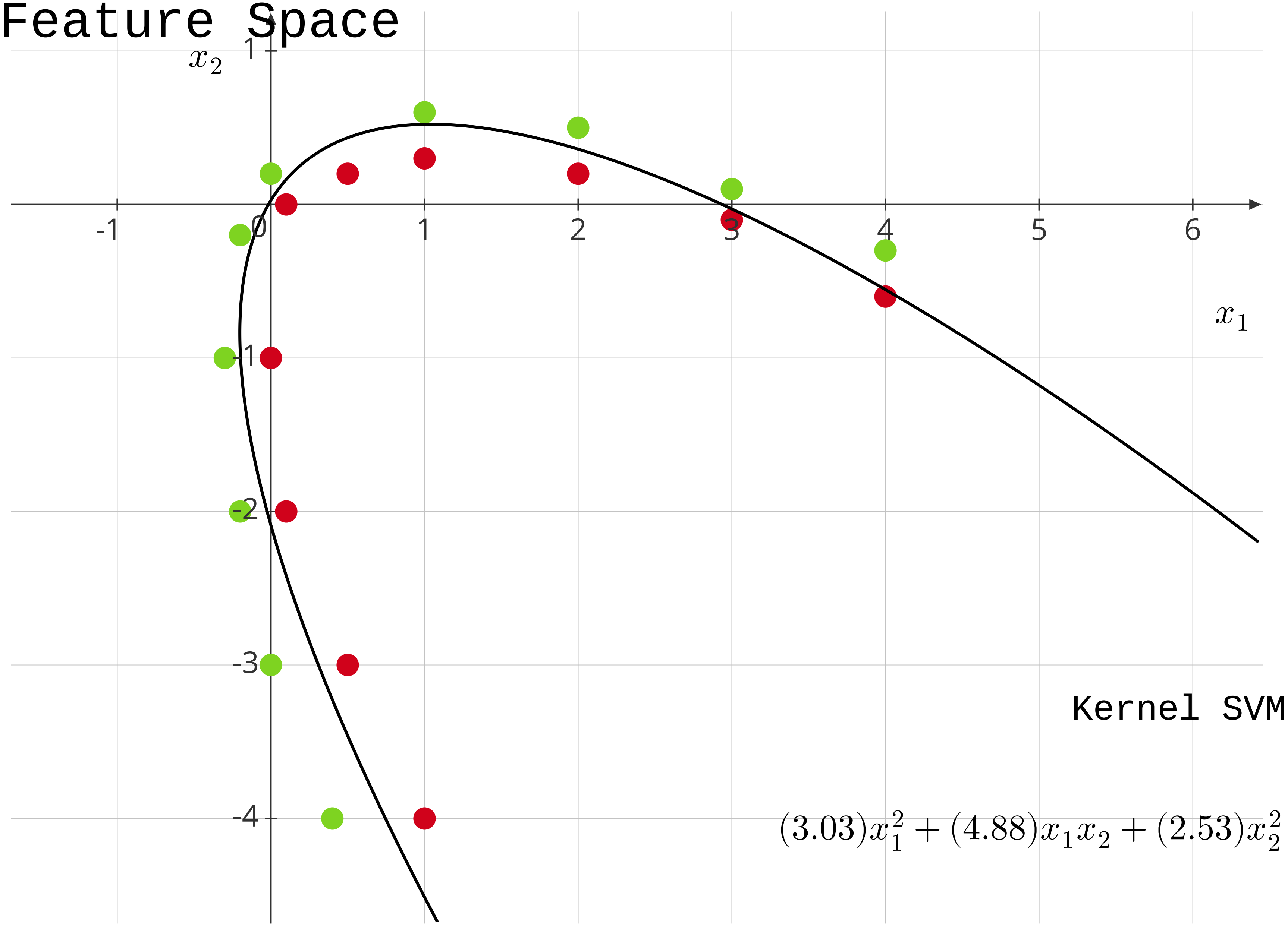
Example-4



Kernel SVM (quadratic)

Feature Space

Example-4

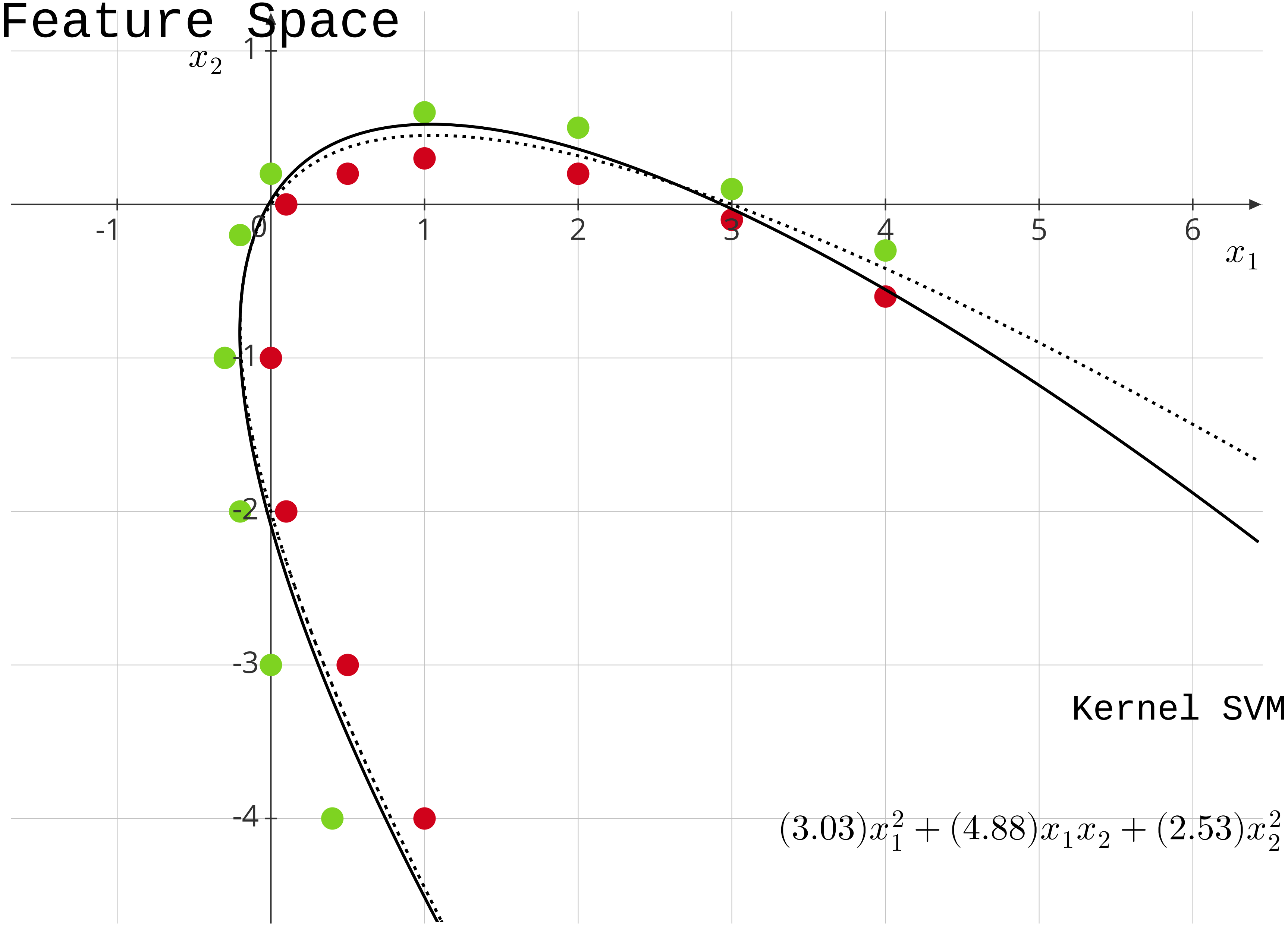


Kernel SVM (quadratic)

$$(3.03)x_1^2 + (4.88)x_1x_2 + (2.53)x_2^2 - (8.85)x_1 + (5.22)x_2 - (0.14) = 0$$

Feature Space

Example-4



Source

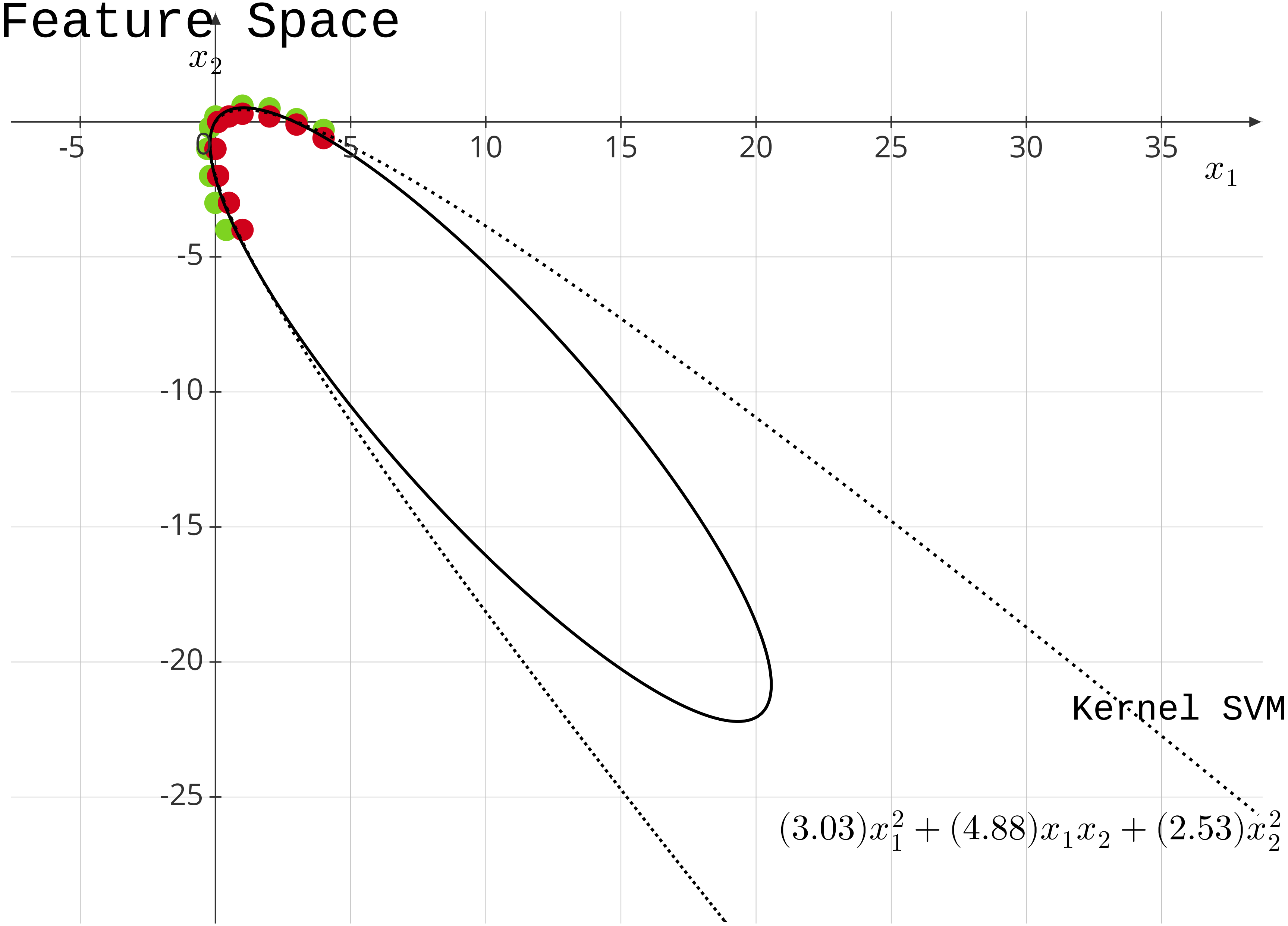
$$x_1^2 + 2x_1x_2 + x_2^2 - 3x_1 + 2x_2 = 0$$

Kernel SVM (quadratic)

$$(3.03)x_1^2 + (4.88)x_1x_2 + (2.53)x_2^2 - (8.85)x_1 + (5.22)x_2 - (0.14) = 0$$

Feature Space

Example-4



Source

$$x_1^2 + 2x_1x_2 + x_2^2 - 3x_1 + 2x_2 = 0$$

Kernel SVM (quadratic)

$$(3.03)x_1^2 + (4.88)x_1x_2 + (2.53)x_2^2 - (8.85)x_1 + (5.22)x_2 - (0.14) = 0$$