# Database Management Systems

## Module 38: Algorithms and Data Structures/3: Data Structures

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# Module Recap





- Introduced Data Structures
- Defined Linear Data Structure
- Reviewed array, list, stack, queue
- Reviewed linear and binary search

# Module Objectives





- Introducing Non-linear Data Structures - graph, tree, hash table
- Exploring Binary Search Tree
- Comparing Linear and Non-Linear Data Structures

# Module Outline



- Non-linear Data Structures
- Binary Search Trees
- Comparison of Linear and Non-Linear Data Structures

# Data Structure



- **Data structure**: A data structure specifies the way of organizing and storing **in-memory** data that enables **efficient access** and **modification** of the data.
  - **Linear Data Structures**
  - **Non-linear Data Structures**
- Most data structure has a container for the data and typical operations that its needs to perform
- For applications relating to data management, the key operations are:
  - Create
  - Insert
  - Delete
  - Find / Search
  - Close
- Efficiency is measured in terms of time and space taken for these operations

# Non-linear Data Structures

# Non-linear Data Structures: Why?





- From the study of **Linear data structures** in the last module, we can make the following summary observations:
  - All of them have the space complexity $O(n)$, which optimal. However, the actual used space may be lower in array while linked list has an overhead of 100% (double)
  - All of them have complexities that are identical for Worst as well as Average case
  - All of them offer satisfactory complexity for some operations while being unsatisfactory on the others

| | Array | | Linked List | |
|---|---|---|---|---|
| | Unordered | Ordered | Unordered | Ordered |
| Access | $O(1)$ | $O(1)$ | $O(n)$ | $O(n)$ |
| Insert | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ |
| Delete | $O(n)$ | $O(n)$ | $O(1)$ | $O(1)$ |
| Search | $O(n)$ | $O(\lg n)$ | $O(n)$ | $O(n)$ |

- **Non-Linear data structures** can be used to trade-off between extremes and achieve a balanced good performance for all

# Non-linear Data Structures (2)



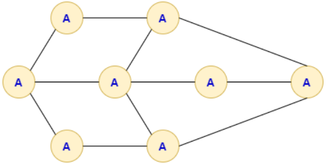


- **Nonlinear data structures** are those data structures in which data items are not arranged in a sequence and each element may have multiple paths to connect to other elements.
- Unlike linear data structures, in which each element is directly connected with utmost two neighbouring elements (previous and next elements), non-linear data structures may be connected with more than two elements.
- The elements don't have a single path to connect to the other elements but have multiple paths. Traversing through the elements is not possible in one run as the data is non-linearly arranged.
- Common Non-Linear Data Structures include:
  - **Graph**: Undirected or Directed, Unweighted or Weighted, and variants
  - **Tree**: Rooted or Unrooted, Binary or n-ary, Balanced or Unbalanced, and variants
  - **Hash Table**: Array with lists (coalesced chains) and one or more hash functions
  - **Skip List**: Multi-layered interconnected linked lists
  - and so on

# Non-linear Data Structures (3): Graph



- **Graphs**: Graph $G$ is a collection of vertices $V$ (store the elements) and connecting edges (links) $E$ between vertices: $G = < V, E >$ where $E \subseteq V \times V$



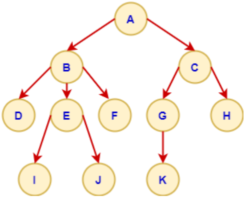

- A graph may be:
  - Undirected or Directed
  - Unweighted or Weighted
  - Cyclic or Acyclic
  - Disconnected or Connected
  - and so on
- Examples of a graph include:
  - ER Diagram
  - Network: Electrical, Water
  - Friendships in Facebook
  - Knowledge Graph

# Non-linear Data Structures (4): Tree





- **Tree**: Is a connected acyclic graph representing hierarchical relationship

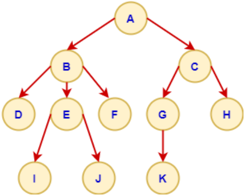



- A tree may be:
  - Rooted or Unrooted
  - Binary or n-ary
  - Balanced or Unbalanced
  - Disconnected (forest) or Connected
  - and so on
- Examples of a tree include:
  - Composite Attributes
  - Family Genealogy
  - Search Trees
  - and so on

# Non-linear Data Structures (5): Tree



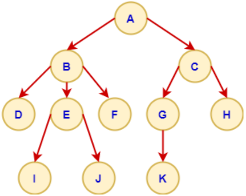





- **Root**: The node at the top of the tree is called root. There is only one root per tree and one path from the root node to any node. **A** is the root node.
- **Parent**: The node which is a predecessor of any node is called parent node. In the given tree, **B** is the parent of **E**. Every node, except the Root, has a unique parent
- **Child**: A node which is the descendant of a node: **D**, **E** and **F** are the child nodes of **B**
- **Leaf**: A node which does not have any child node: **I**, **J** and **K** are leaf nodes

# Non-linear Data Structures (6): Tree





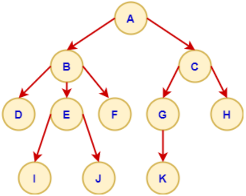

- **Internal Nodes**: The node which has at least one child is called internal Node
- **Subtree**: Subtree represents the tree rooted at that node
- **Path**: Path refers to the sequence of nodes along the edges of a tree
- **Siblings**: Nodes having the same parents: **D**, **E** and **F** are the siblings.
- **Arity**: Number of children of a node. **B** has arity 3, **E** has arity 2, **G** has arity 1, and **D** has arity 0 (**Leaf**)

  Maximum arity of a node is defined as the arity of the tree.

# Non-linear Data Structures (7): Tree







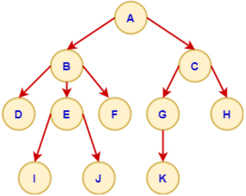

- **Levels**: The root node is said to be at Level 0 and the children of the root node are at Level 1 and the children of the nodes which are at Level 1 will be at Level 2 and so on.

  Level is the length of the path (number of links) or distance of a node from the root node. So, level of **A** is 0, level of **C** is 1, level of **G** is 2, and level of **J** is 3.
- **Height**: Maximum level in a tree
- **Binary Tree**: is a tree, where each node can have at most 2 children. It has arity 2.

# Non-linear Data Structures (8): Tree







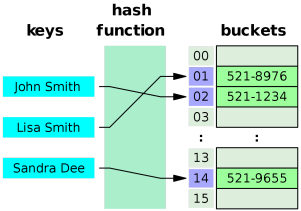

- **Fact 1**: A tree with $n$ nodes has $n-1$ edges
- **Fact 2**: The maximum number of nodes at level $l$ of a binary tree is $2^l$.
- **Fact 3**: If $h$ is the height of a binary tree of $n$ nodes, then:
  - $h+1 \leq n \leq 2^{h+1}-1$
  - $\lceil \lg(n+1) \rceil - 1 \leq h \leq n-1$
  - $O(\lg n) \leq h \leq O(n)$
  - For a $k$-ary tree, $O(\lg_k n) \leq h \leq O(n)$

# Non-linear Data Structures (9): Hash Table (Module 44)





- **Hash Table (Hash Map)**: implements an associative array abstract data type, a structure that can map keys to values by using a hash function to compute an index (hash code), into an array of buckets or slots, from which the desired value can be found




- A hash table may be using:
  - Static or Dynamic Schemes
  - Open Addressing
  - 2-Choice Hashing
  - and so on

- Examples of a hash table include:
  - Associative arrays
  - Database indexing
  - Caches
  - and so on

# Binary Search Tree

# Binary Search and Binary Search Tree





- During the study of linear data structure, we observed that
  - Binary search is efficient in search of a key: $O(\lg n)$. However,
    - it needs to be performed on a sorted array, and
    - the array makes insertion and deletion expensive at $O(n)$
  - The linked list, on the other hand is efficient in insertion and deletion at $O(1)$, while it makes the search expensive at $O(n)$.
    - *$O(1)$ insert / delete is possible because we just need to manipulate pointers and not physically move data*
- Using the non-linearity, specifically (binary) trees, we can combine the benefits of both
- Note that once an array is sorted, we know the order in which its elements may be checked (for any key) during a search
- As the binary search splits the array, we can conceptually consider the *Middle Element* to be the *Root* of a tree and the *left (right) sub-array* to be its *left (right) sub-tree*
- Progressing recursively, we have a **Binary Search Tree**

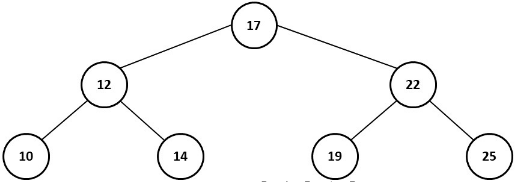
# Binary Search and Binary Search Tree



- Consider the data set:

| 10 | 12 | 14 | 17 | 19 | 22 | 25 |
|---|---|---|---|---|---|---|
| LL | L | LR | M | RL | R | RR |

- Search order is:
  - First: M
  - Second: L or R
  - Third:
    - For L: LL or LR
    - For R: RL or RR
  - Recur ...
- Put as a tree:

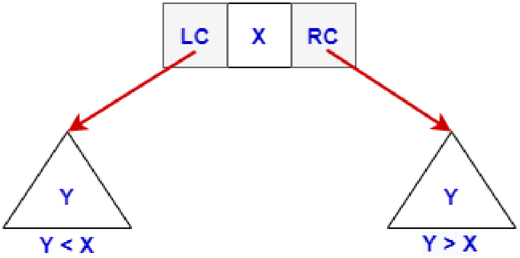


# Binary Search Tree





- **Binary Search Tree (BST)**: Is a tree in which all the nodes hold the following:
  - The value of each node in the left sub-tree is less than the value of its root
  - The value of each node in the right sub-tree is greater than the value of its root

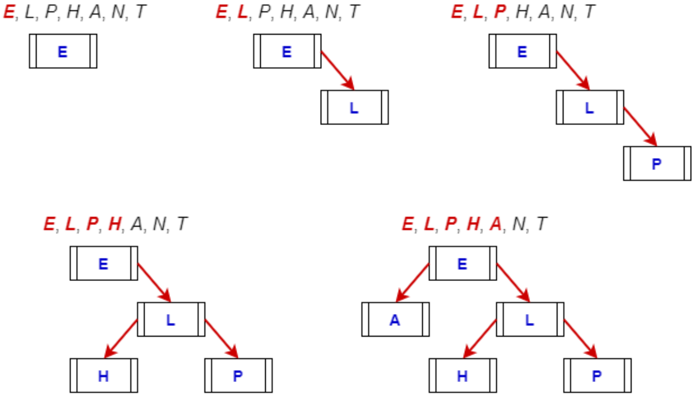



- **Structure of BST node**: Each node consists of an element (**X**), and a link to the left child or the left subtree (**LC**), and a link to the right child or the right subtree (**RC**)

# Binary Search Tree (2)



- **Example**: Obtain the BST by inserting the following values- *E, L, P, H, A, N, T*.

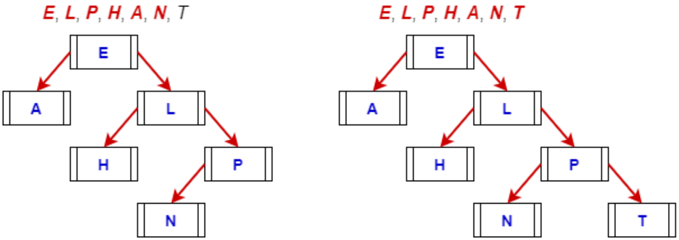


# Binary Search Tree (3)



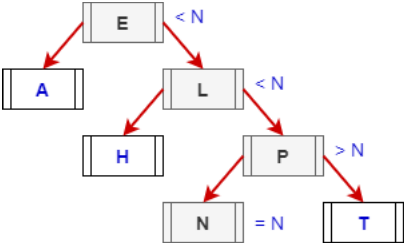


- **Example**: Obtain the BST by inserting the following values- *E, L, P, H, A, N, T*.



# Searching a key in BST

```
search(root, key)
1. Compare the key with the element  at root.
    1.1. If the key is equal to root’s element then
        1.1.1 Element found and return
    1.2. else if the key is lesser than the root’s element
        1.2.1 search(root.lc) #search on the left subtree
    1.3  else: #if the key is greater than the root’s element
        1.3.1 search(root.rc) #search on the right subtree
```



# Searching a key in BST (2)



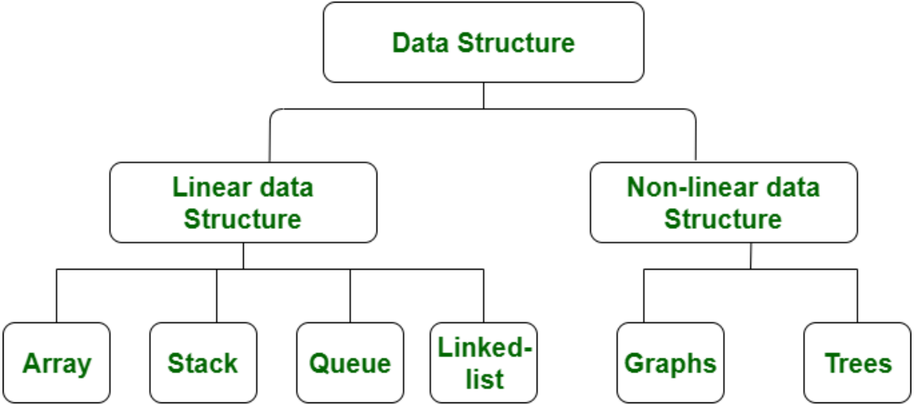


- Searching a key in a BST is $O(h)$, where $h$ is the height of the key
- **Worst Case**
  - The BST is a skewed binary search tree (all the nodes except the leaf would have only one child)
  - This can happen if keys are inserted in sorted order
  - Height ($h$) of the BST having $n$ elements becomes $n - 1$
  - Time complexity of search in BST becomes $O(n)$
- **Best Case**
  - The BST is a balanced binary search tree
  - This is possible if
    - If keys are inserted in purely randomized order, Or
    - If the tree is explicitly balanced after every insertion
  - Height ($h$) of the binary search tree becomes $\lg n$
  - Time complexity of search in BST becomes $O(\lg n)$

# Comparison of Linear and Non-Linear Data Structures

# Linear and Non-Linear Data Structures






**Source**: Difference between Linear and Non-linear Data Structures (11-Aug-2021)

# Comparison of Linear and Non-Linear Data Structures



| Linear Data Structure | Non-Linear Data Structure |
|---|---|
| • Data elements are *arranged* in a linear order where each and every elements are attached to its previous and next adjacent | • Data elements are *arranged* in hierarchical or networked manner |
| • Single *level* is involved | • Multiple *level* are involved |
| • *Implementation* is easy in comparison to non-linear data structure | • *Implementation* is complex in comparison to linear data structure |
| • Data elements can be *traversed* in one way only | • Data elements can be *traversed* in multiple ways. Various traversals may be defined to linearize the data: Depth-First, Breadth-First, Inorder, Prepoder, Postorder, etc. |
| • *Examples*: array, stack, queue, linked list, and their variants | • *Examples*: trees, graphs, skip list, hash map, and several variants |

# Complexity of Common Data Structure Operations



| | Data Structure | Time Complexity | | | | | | | | Space Complexity |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Average | | | | Worst | | | | Worst |
| | | Access | Search | Insertion | Deletion | Access | Search | Insertion | Deletion | |
| **Linear Data Structures** | Array | Θ(1) | Θ(n) | Θ(n) | Θ(n) | O(1) | O(n) | O(n) | O(n) | O(n) |
| | Stack | Θ(n) | Θ(n) | Θ(1) | Θ(1) | O(n) | O(n) | O(1) | O(1) | O(n) |
| | Queue | Θ(n) | Θ(n) | Θ(1) | Θ(1) | O(n) | O(n) | O(1) | O(1) | O(n) |
| | Singly-Linked List | Θ(n) | Θ(n) | Θ(1) | Θ(1) | O(n) | O(n) | O(1) | O(1) | O(n) |
| | Doubly-Linked List | Θ(n) | Θ(n) | Θ(1) | Θ(1) | O(n) | O(n) | O(1) | O(1) | O(n) |
| **Non-Linear Data Structures** | Skip List | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(n) | O(n) | O(n) | O(n) | O(n log(n)) |
| | Hash Table | N/A | Θ(1) | Θ(1) | Θ(1) | N/A | O(n) | O(n) | O(n) | O(n) |
| | Binary Search Tree | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(n) | O(n) | O(n) | O(n) | O(n) |
| | Cartesian Tree | N/A | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | N/A | O(n) | O(n) | O(n) | O(n) |
| | B-Tree | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(n) |
| | Red-Black Tree | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(n) |
| | Splay Tree | N/A | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | N/A | O(log(n)) | O(log(n)) | O(log(n)) | O(n) |
| | AVL Tree | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(log(n)) | O(n) |
| | KD Tree | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | Θ(log(n)) | O(n) | O(n) | O(n) | O(n) | O(n) |

**Source**: Know Thy Complexities! (06-Apr-2021)

# Module Summary



- Introduced Non-linear Data Structures - graph, tree, hash table
- Studied Binary Search Tree as an adaptation of binary search
- Compared Linear and Non-Linear Data Structures

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