Practice-1

Machine Learning Techniques

Karthik Thiagarajan

Q-1

- Logistic Regression
- Threshold for prediction is T
- Find the decision boundary

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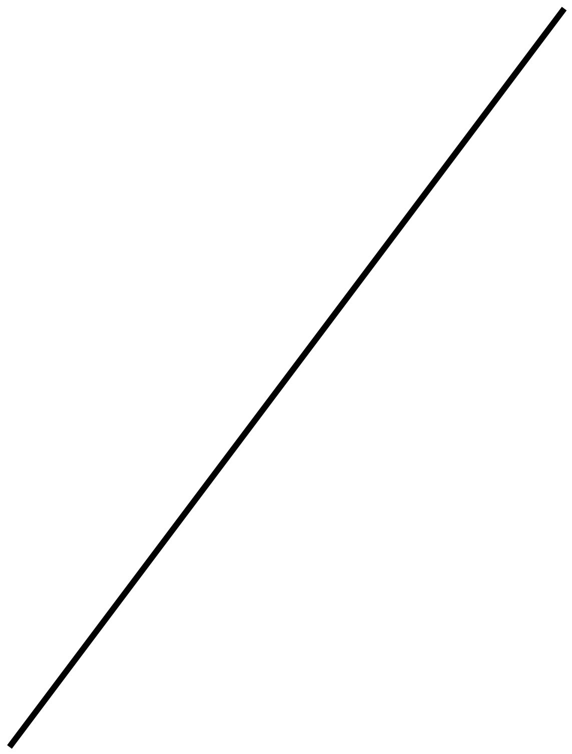

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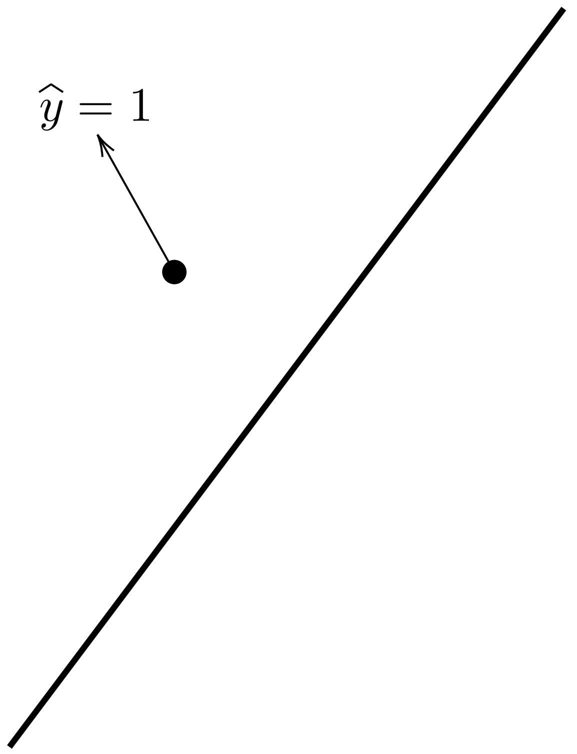

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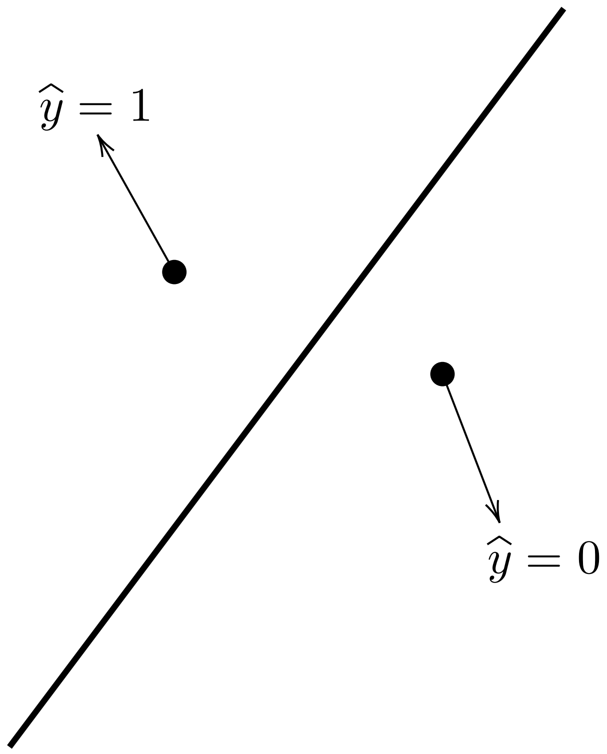

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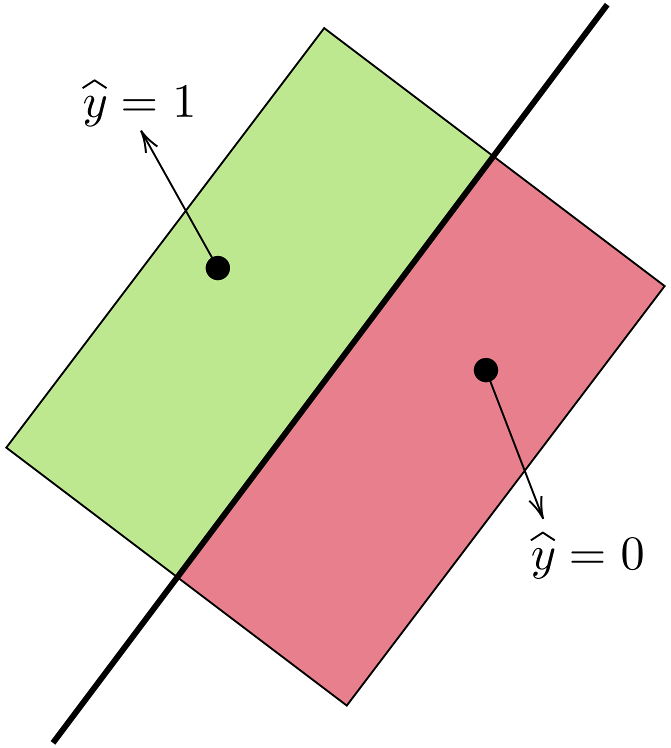

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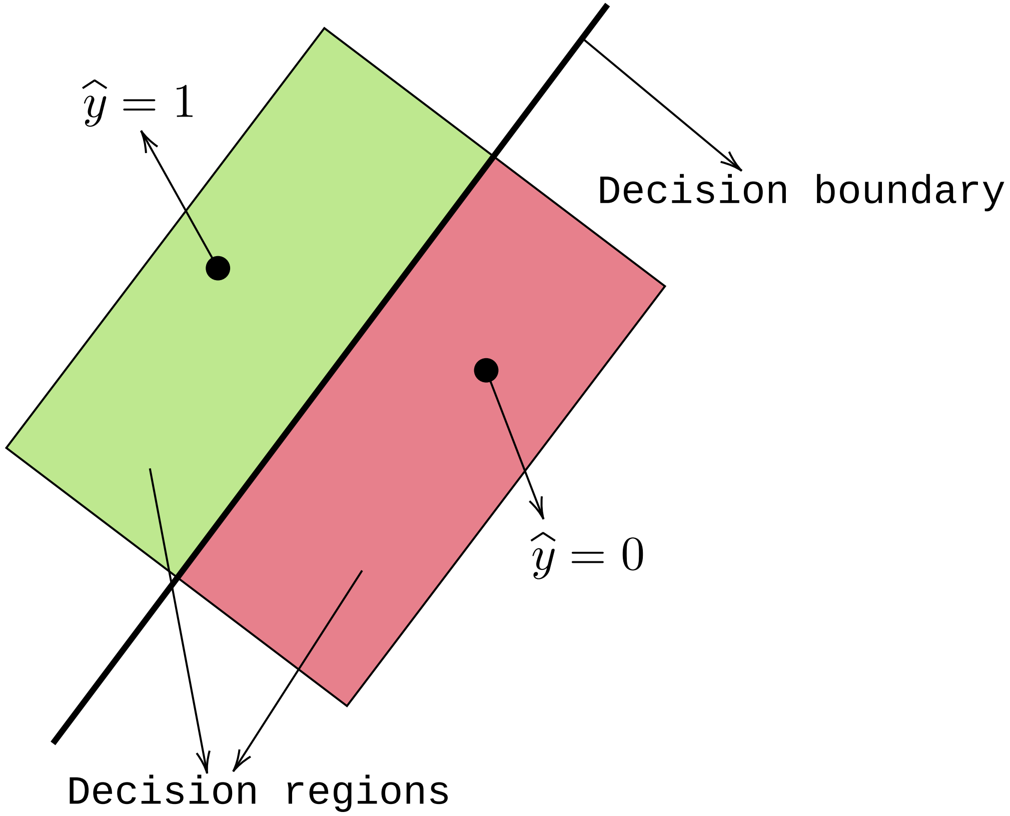

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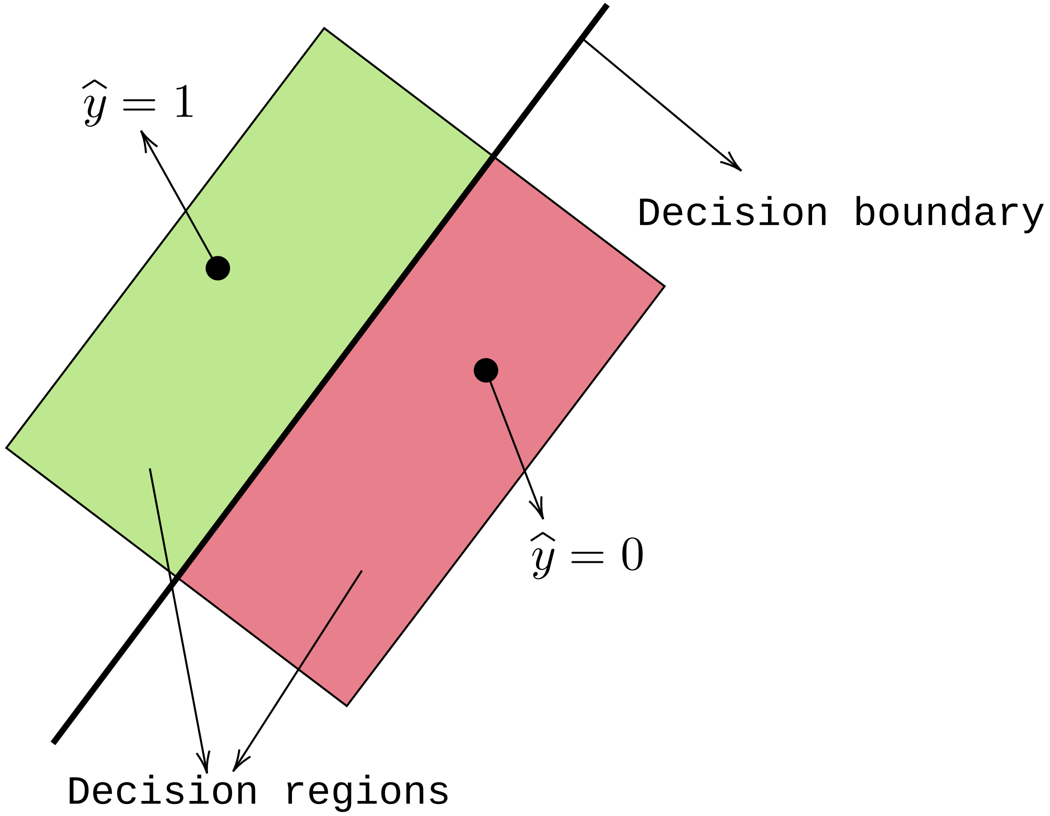

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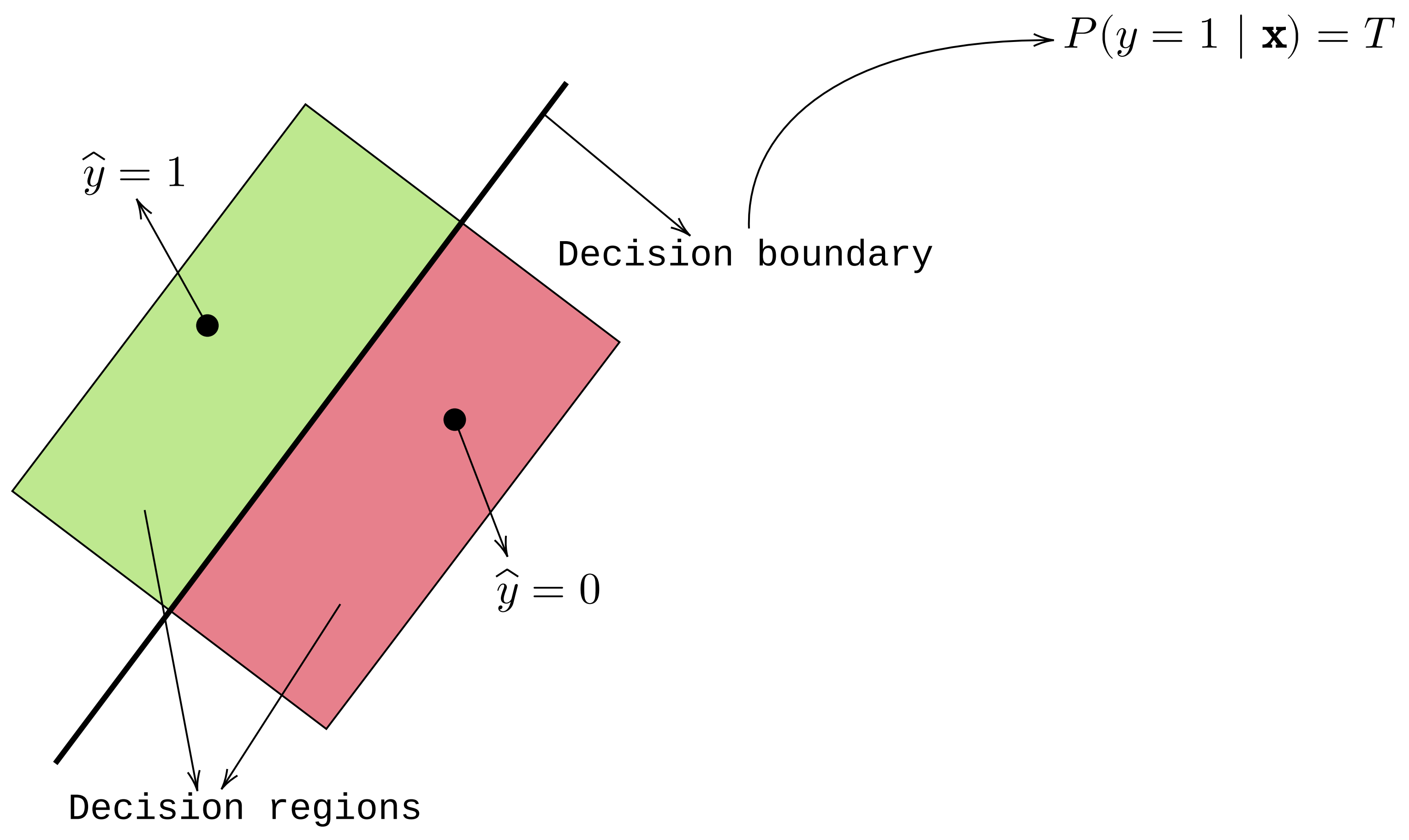

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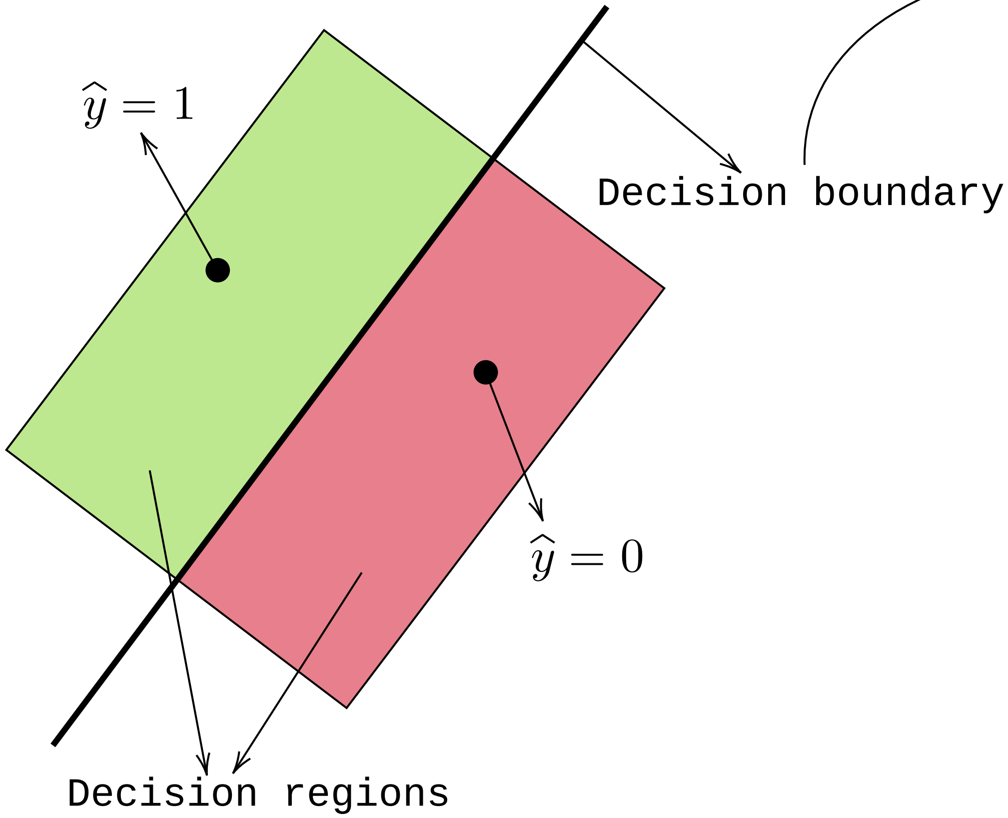
$P(y=1 \mid \mathbf{x}) = T$

$\sigma(\mathbf{w}^T\mathbf{x}) = T$



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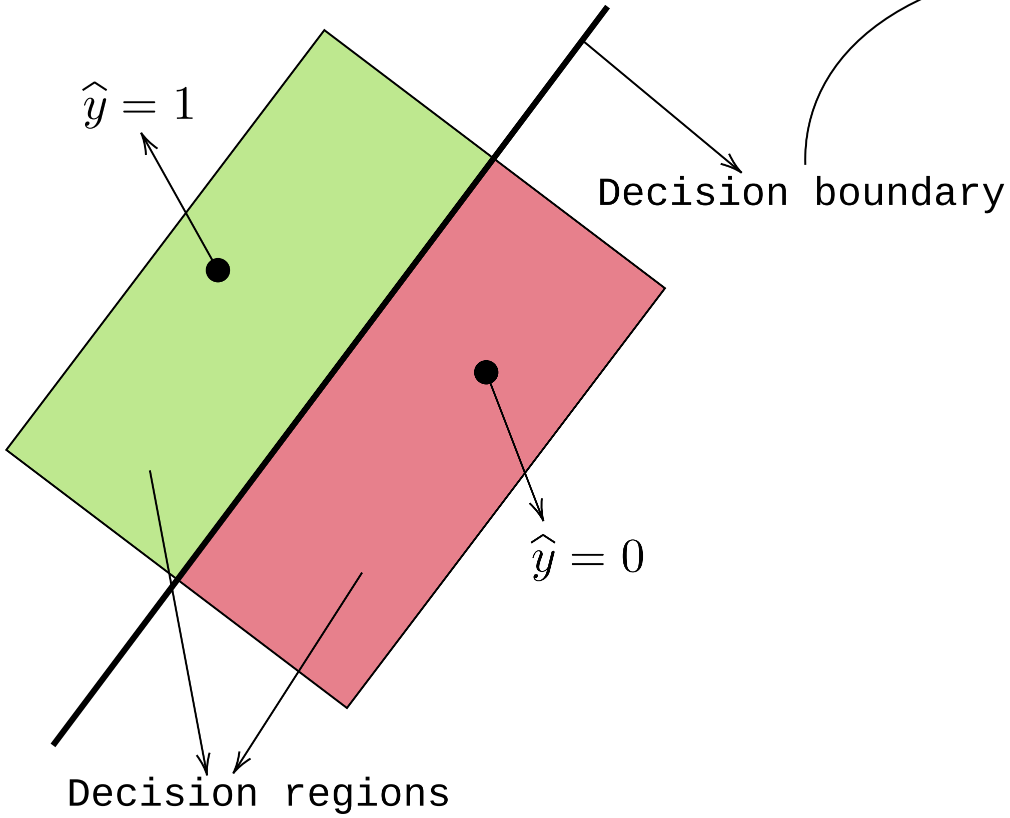

$$P(y=1 \mid \mathbf{x}) = T$$

$$\sigma(\mathbf{w}^T\mathbf{x}) = T$$

$$\frac{1}{1+e^{-\mathbf{w}^T\mathbf{x}}} = T$$

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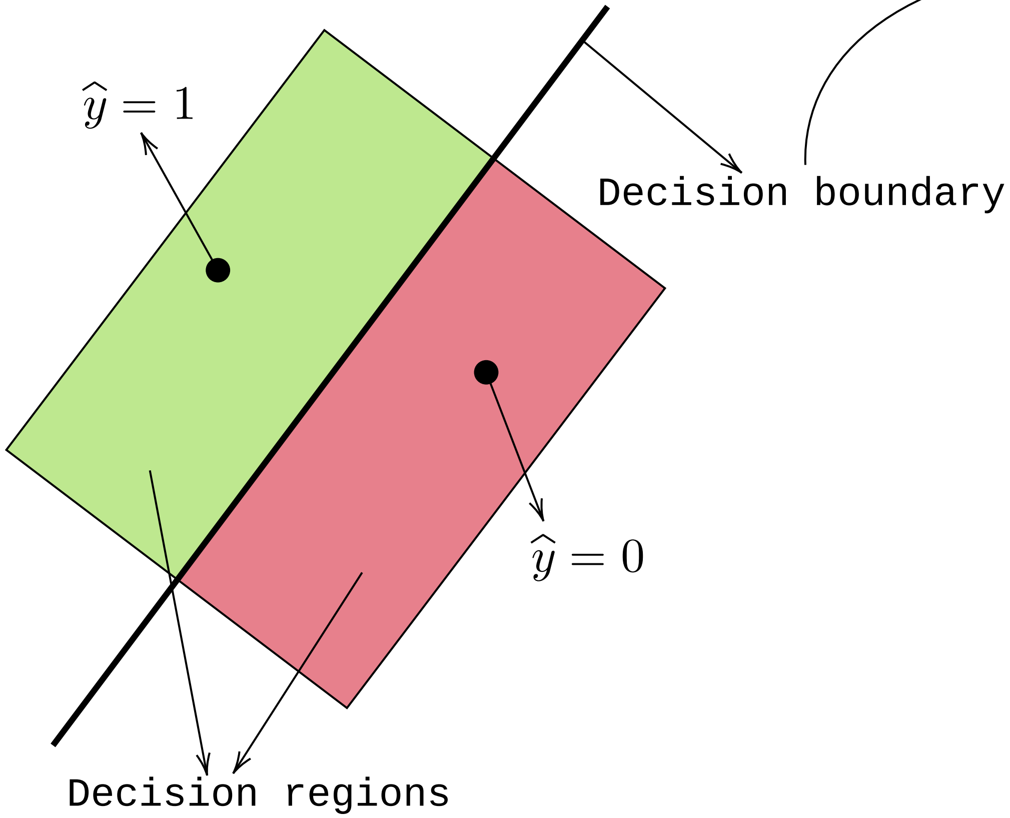

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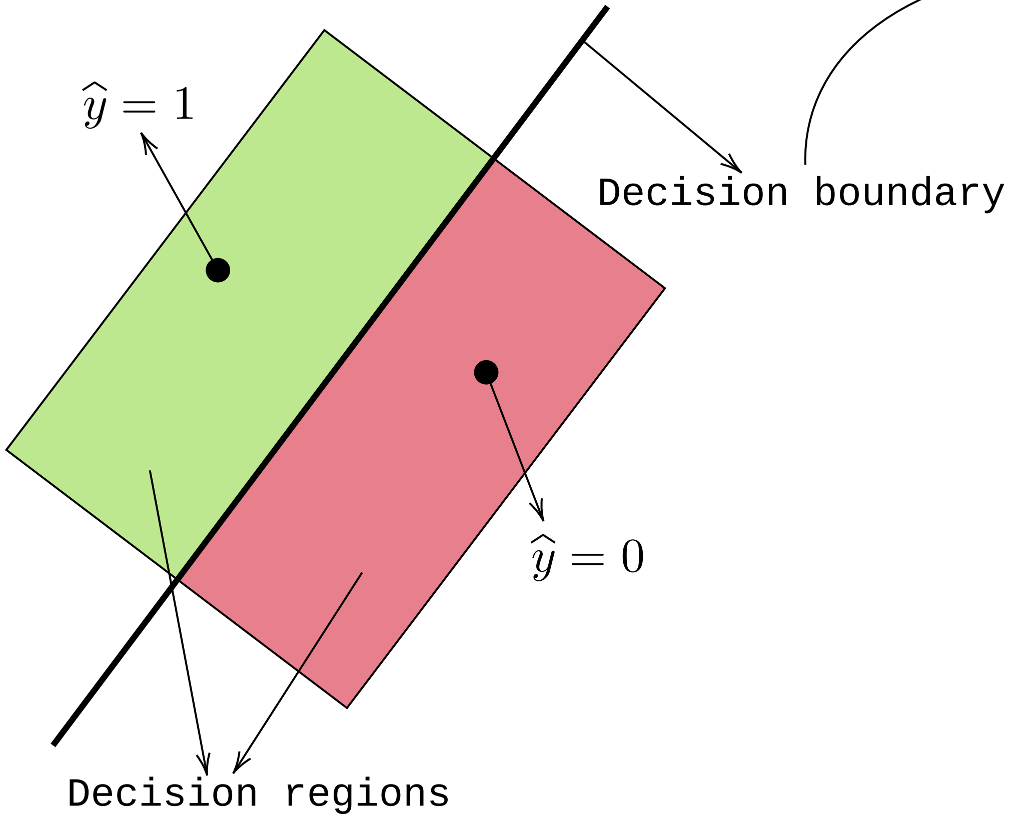

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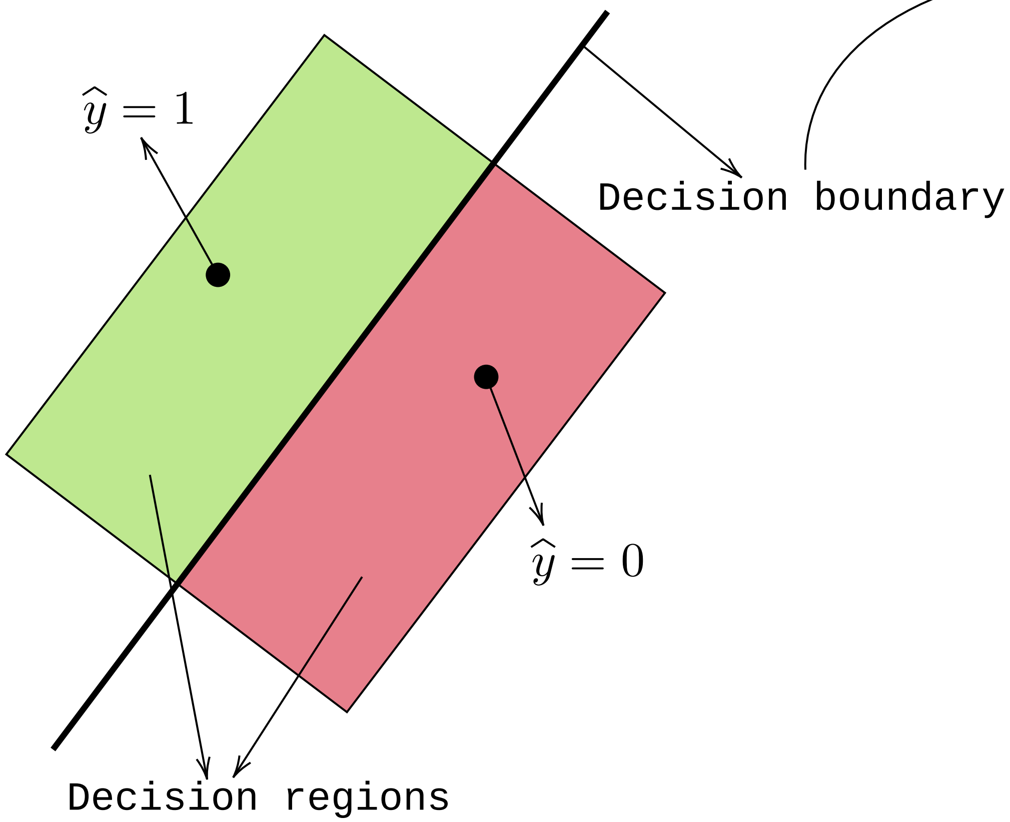

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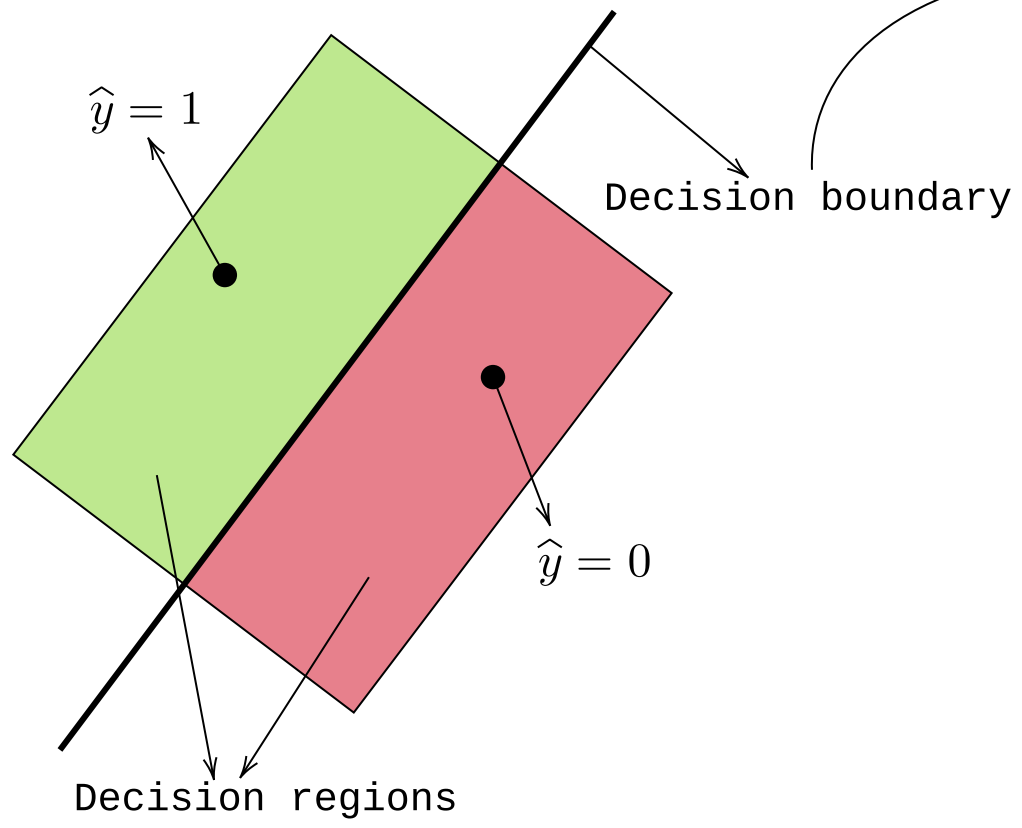

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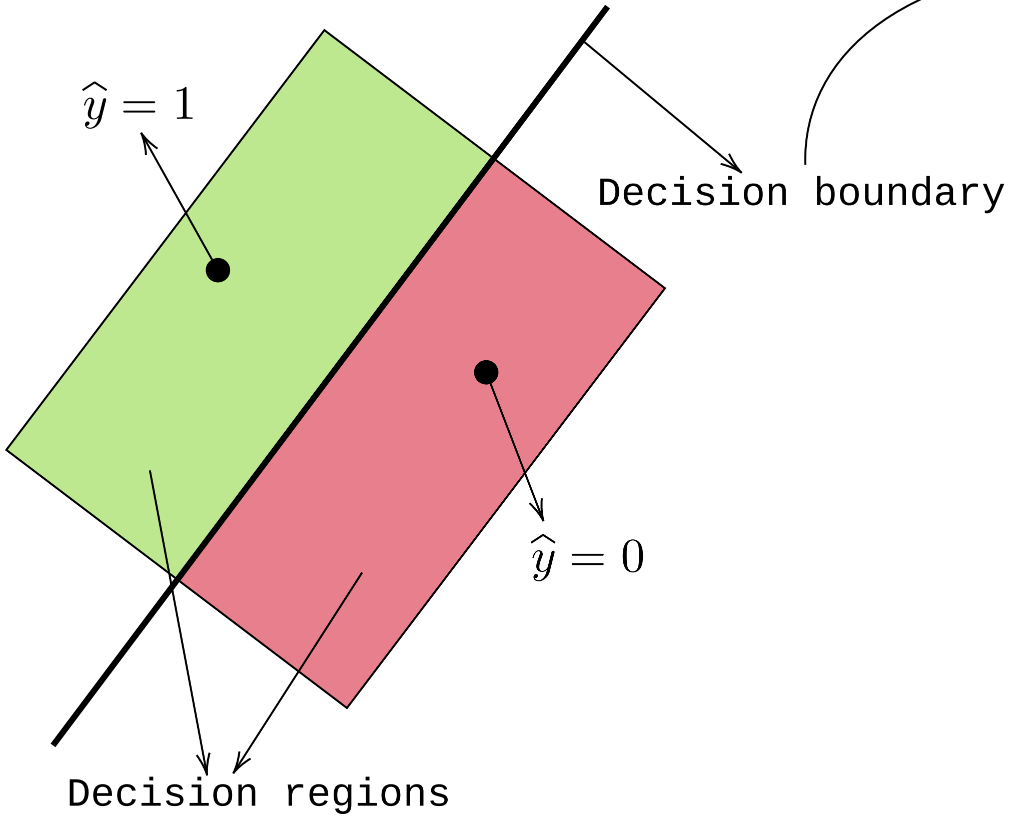
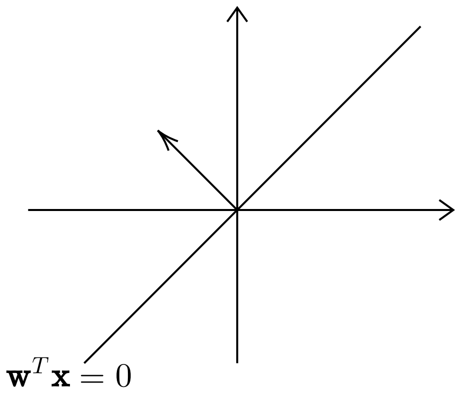


$T = 0.5$



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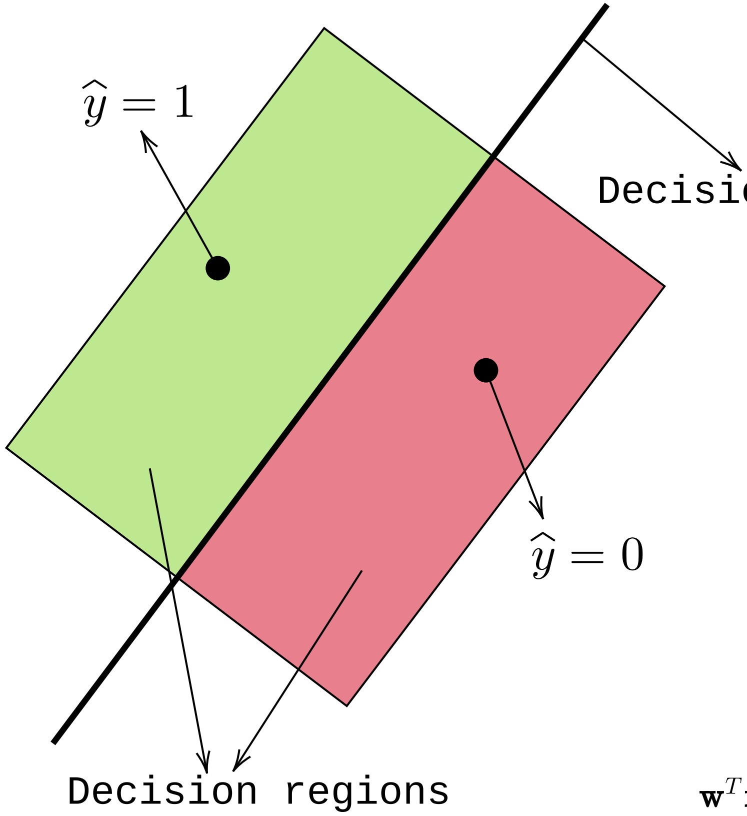

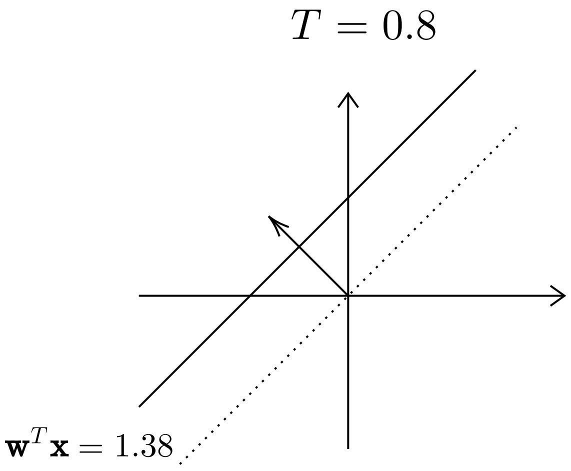

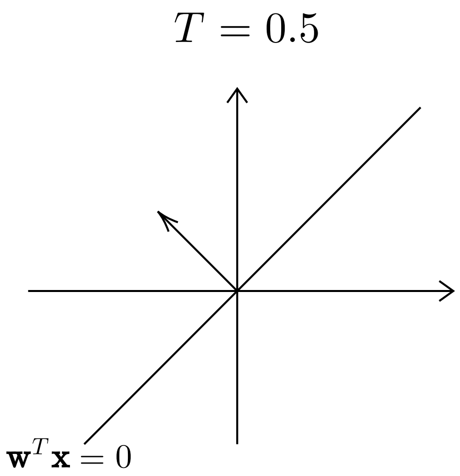

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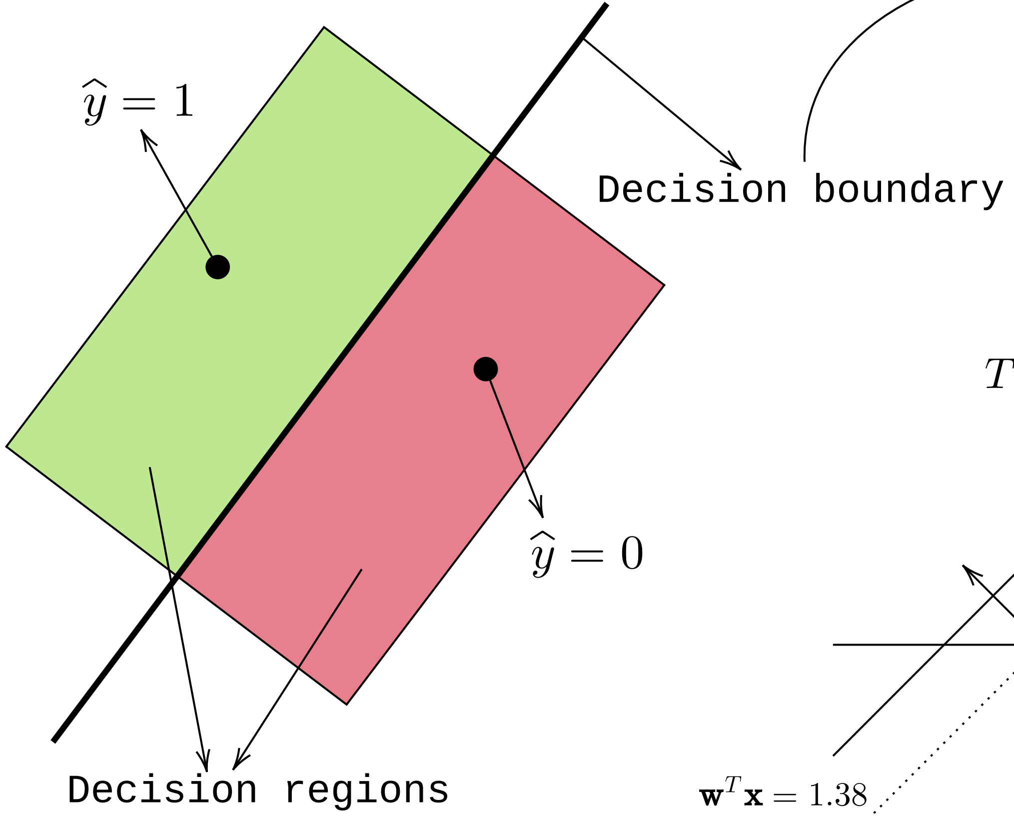

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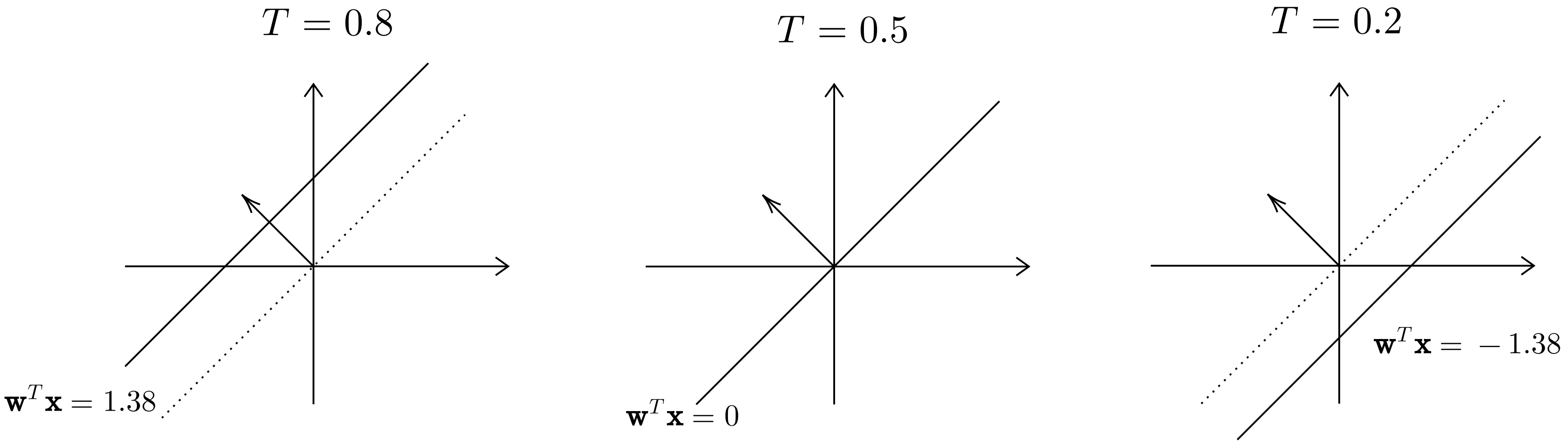

Q-2

- Soft-margin, Linear-SVM
- $\mathbf{w} = [0 \quad 1]^T$
- Hinge loss

$\mathbf{X}, \mathbf{y} =$

x_1	x_2	y
2	1	1
-2	1	1
-1	2	1
0	2	-1
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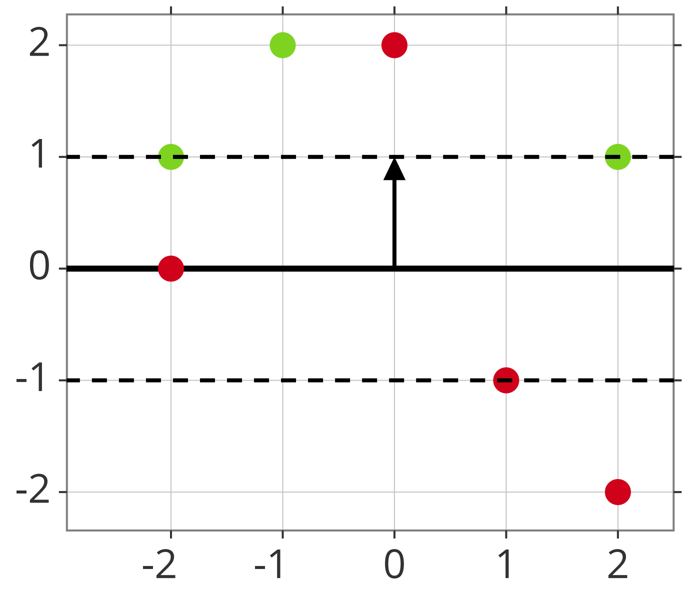

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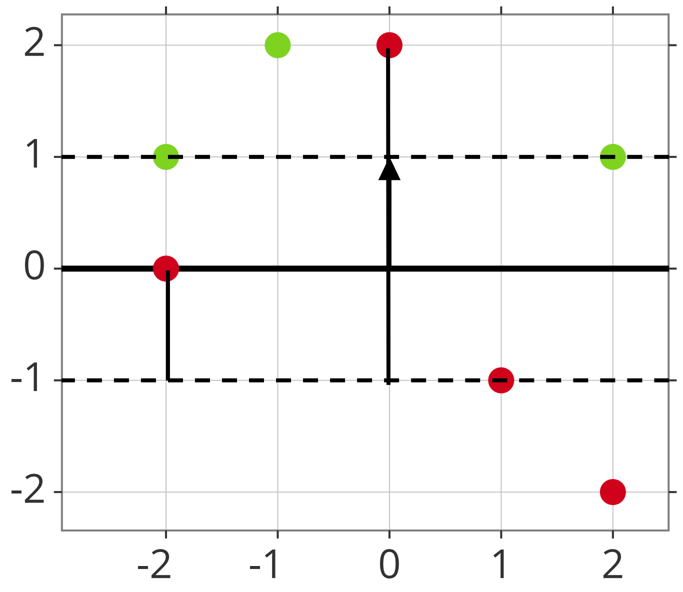

Q-3

Valid covariance matrix?

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 3 & 0 \\ 1 & 9 & 2 \end{bmatrix}$$

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$$\mathbf{C} = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T$$

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$$\mathbf{C} = \frac{1}{n} \cdot \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^T$$

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$$|\mathbf{C} - \lambda \mathbf{I}|$$

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$$= (1-\lambda)(1-\lambda)(-\lambda) - (1-\lambda)$$

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$$\lambda_1 = \frac{1+\sqrt{5}}{2},$$

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$$\begin{aligned} \mathbf{A} &= \mathbf{Q}\mathbf{D}\mathbf{Q}^T \\ &= \mathbf{Q}\mathbf{D}^{1/2}\mathbf{D}^{1/2}\mathbf{Q}^T \\ &= \mathbf{B}\mathbf{B}^T \end{aligned}$$

Q-4

Find the principal components

$$\mathbf{X} = \begin{bmatrix} 2 & -2 & 4 & -4 & 2 & -2 & 4 & -4 \\ 2 & -2 & 4 & -4 & -2 & 2 & -4 & 4 \end{bmatrix}$$

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$$= \frac{1}{8} \cdot \begin{bmatrix} 2 & -2 & 4 & -4 & 2 & -2 & 4 & -4 \\ 2 & -2 & 4 & -4 & -2 & 2 & -4 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -2 & -2 \\ 4 & 4 \\ -4 & -4 \\ 2 & -2 \\ -2 & 2 \\ 4 & -4 \\ -4 & 4 \end{bmatrix}$$

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$$\mathbf{X}^T = \begin{bmatrix} 2 & 2 \\ -2 & -2 \\ 4 & 4 \\ -4 & -4 \\ 2 & -2 \\ -2 & 2 \\ 4 & -4 \\ -4 & 4 \end{bmatrix}$$

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$$\mathbf{C} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

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$$\mathbf{w}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{w}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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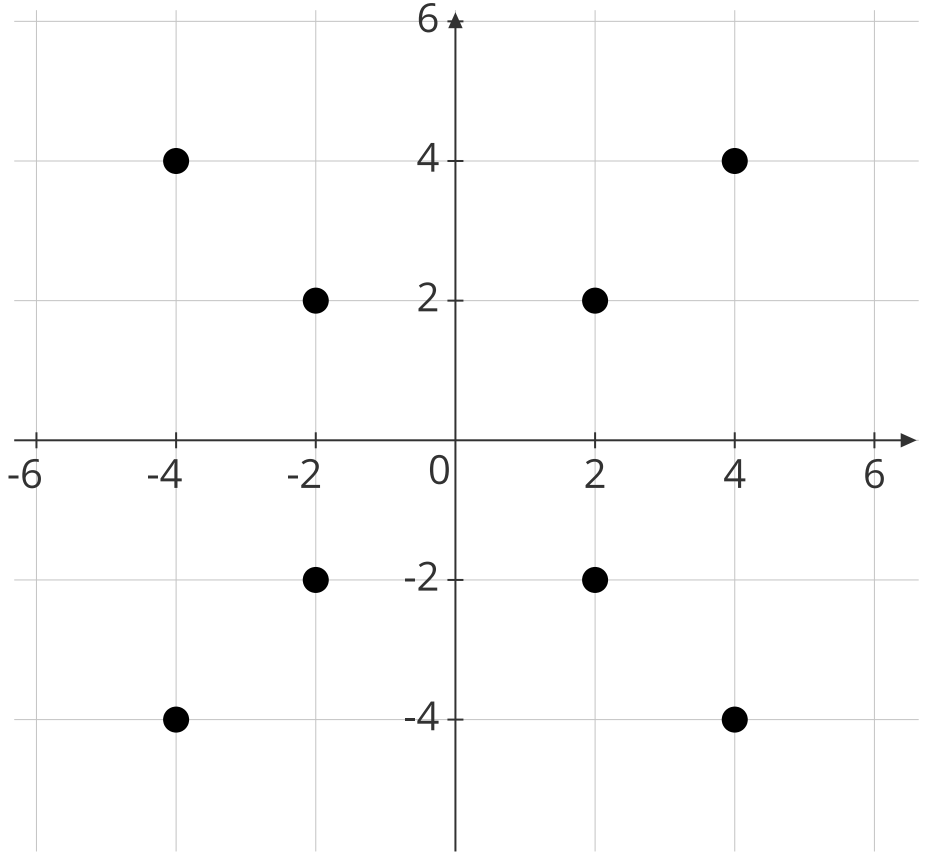

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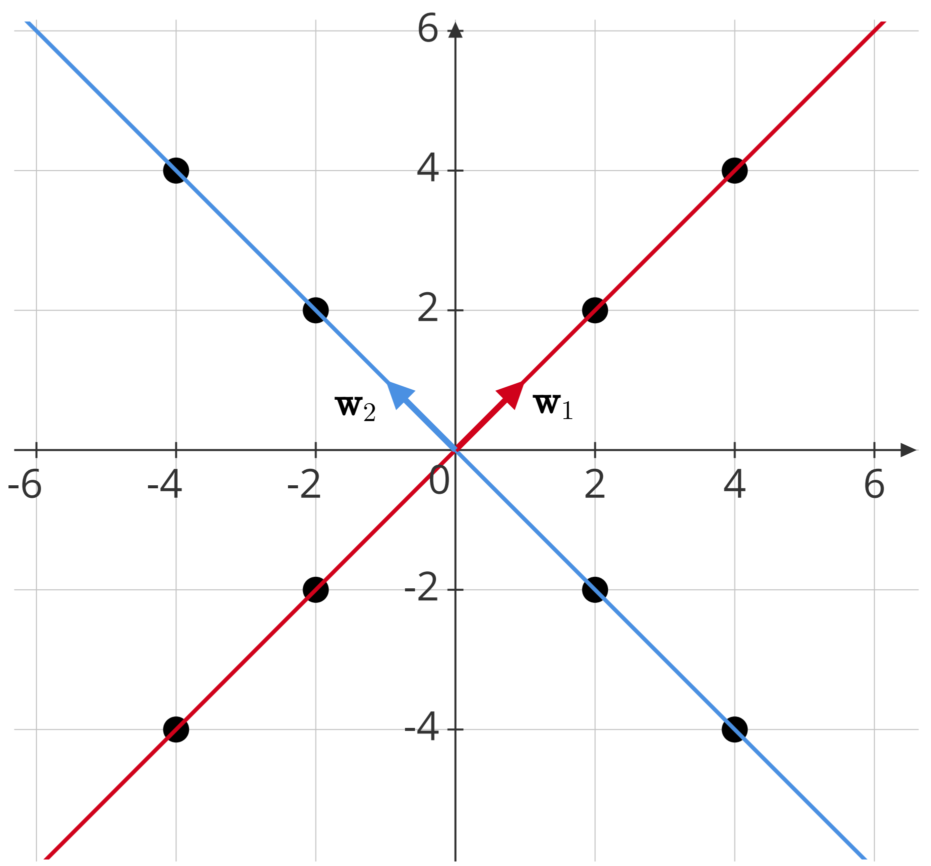

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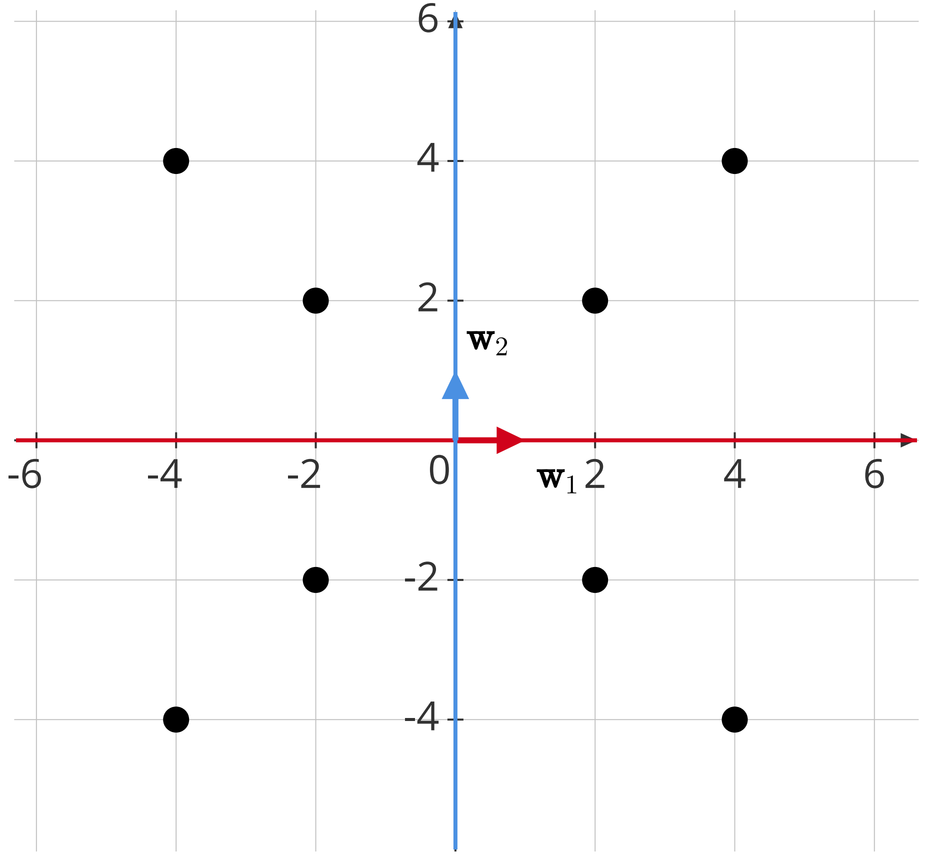

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Q-5

- $D = \{(c_1 \cdot \mathbf{u}, y_1),\ \cdots, (c_n \cdot \mathbf{u}, y_n)\}$
- $\mathbf{u} \in \mathbb{R}^d$ and $\mathbf{u} \neq 0$
- $c_i \neq 0$ for some i
- $\sum_{i=1}^{n} c_i \cdot y_i = 20$, $\sum_{i=1}^{n} c_i^2 = 100$
- $\mathbf{x}_{\texttt{test}} = 5 \cdot \mathbf{u}$

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$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

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$$\nabla L(\mathbf{w}, D)$$

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$$\nabla L(\mathbf{w}, D) = 2 \cdot \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)\mathbf{x}_i$$

$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

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$$\begin{aligned}\nabla L(\mathbf{w}, D) &= 2 \cdot \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)\mathbf{x}_i \\ &= 2 \cdot \sum_{i=1}^{n} [(\mathbf{w}^T \mathbf{u})c_i^2 - c_i y_i]\mathbf{u}\end{aligned}$$

$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

Q-5

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$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

Q-5

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$$
\begin{aligned}
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&= 2 \cdot \sum_{i=1}^{n} [(\mathbf{w}^T \mathbf{u})c_i^2 - c_i y_i]\mathbf{u} \\
&= 0
\end{aligned}
$$

$$\implies \mathbf{w}^T \mathbf{u}$$

$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

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$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

$$\begin{aligned}
\nabla L(\mathbf{w}, D) &= 2 \cdot \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)\mathbf{x}_i \\
&= 2 \cdot \sum_{i=1}^{n} [(\mathbf{w}^T \mathbf{u})c_i^2 - c_i y_i]\mathbf{u} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\implies \mathbf{w}^T \mathbf{u} &= \frac{\sum_{i=1}^{n} c_i y_i}{\sum_{i=1}^{n} c_i^2} \\
&= 0.2
\end{aligned}$$

Q-5

- $D = \{(c_1 \cdot \mathbf{u}, y_1),\ \cdots, (c_n \cdot \mathbf{u}, y_n)\}$
- $\mathbf{u} \in \mathbb{R}^d$ and $\mathbf{u} \neq 0$
- $c_i \neq 0$ for some i
- $\sum_{i=1}^{n} c_i \cdot y_i = 20,\ \sum_{i=1}^{n} c_i^2 = 100$
- $\mathbf{x}_{\texttt{test}} = 5 \cdot \mathbf{u}$

$$\begin{aligned} \nabla L(\mathbf{w}, D) &= 2 \cdot \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)\mathbf{x}_i \\ &= 2 \cdot \sum_{i=1}^{n} [(\mathbf{w}^T \mathbf{u})c_i^2 - c_i y_i]\mathbf{u} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \implies \mathbf{w}^T \mathbf{u} &= \frac{\sum_{i=1}^{n} c_i y_i}{\sum_{i=1}^{n} c_i^2} \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} \mathbf{w}^T \mathbf{x}_{\texttt{test}} &= \mathbf{w}^T (5 \cdot \mathbf{u}) \\ &= 5 \cdot \left(\mathbf{w}^T \mathbf{u}\right) \\ &= 1 \end{aligned}$$

$$L(\mathbf{w}, D) = \sum_{i=1}^{n} (\mathbf{w}^T \mathbf{x}_i - y_i)^2$$

Q-6

- Hard-margin, linear-SVM
- Add 10 new + data-points
  - correct side of margin
  - very far away from it
- Retrain the classifier
- Comment on decision boundary

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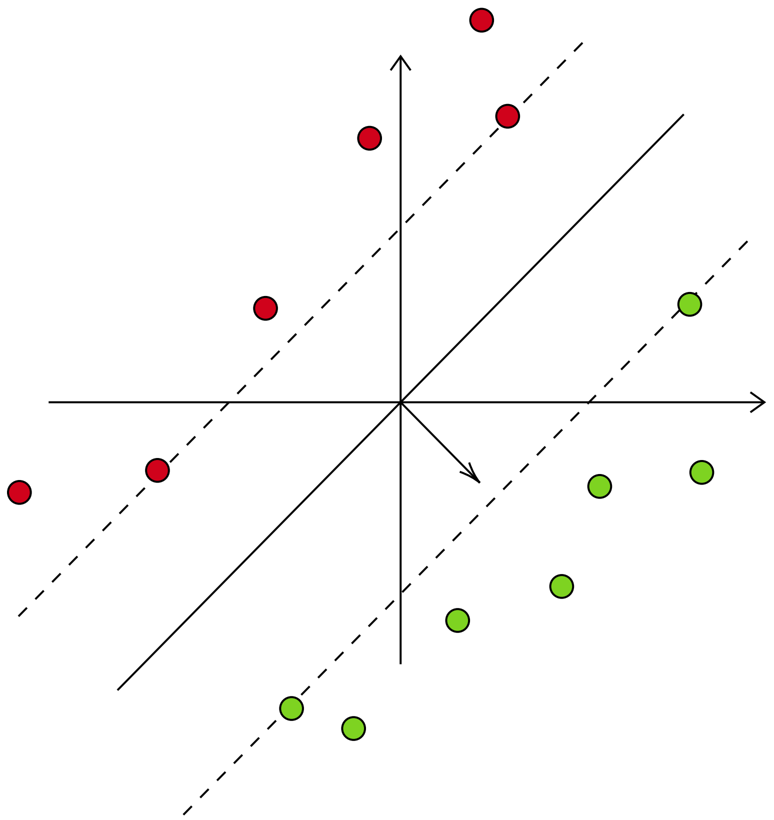

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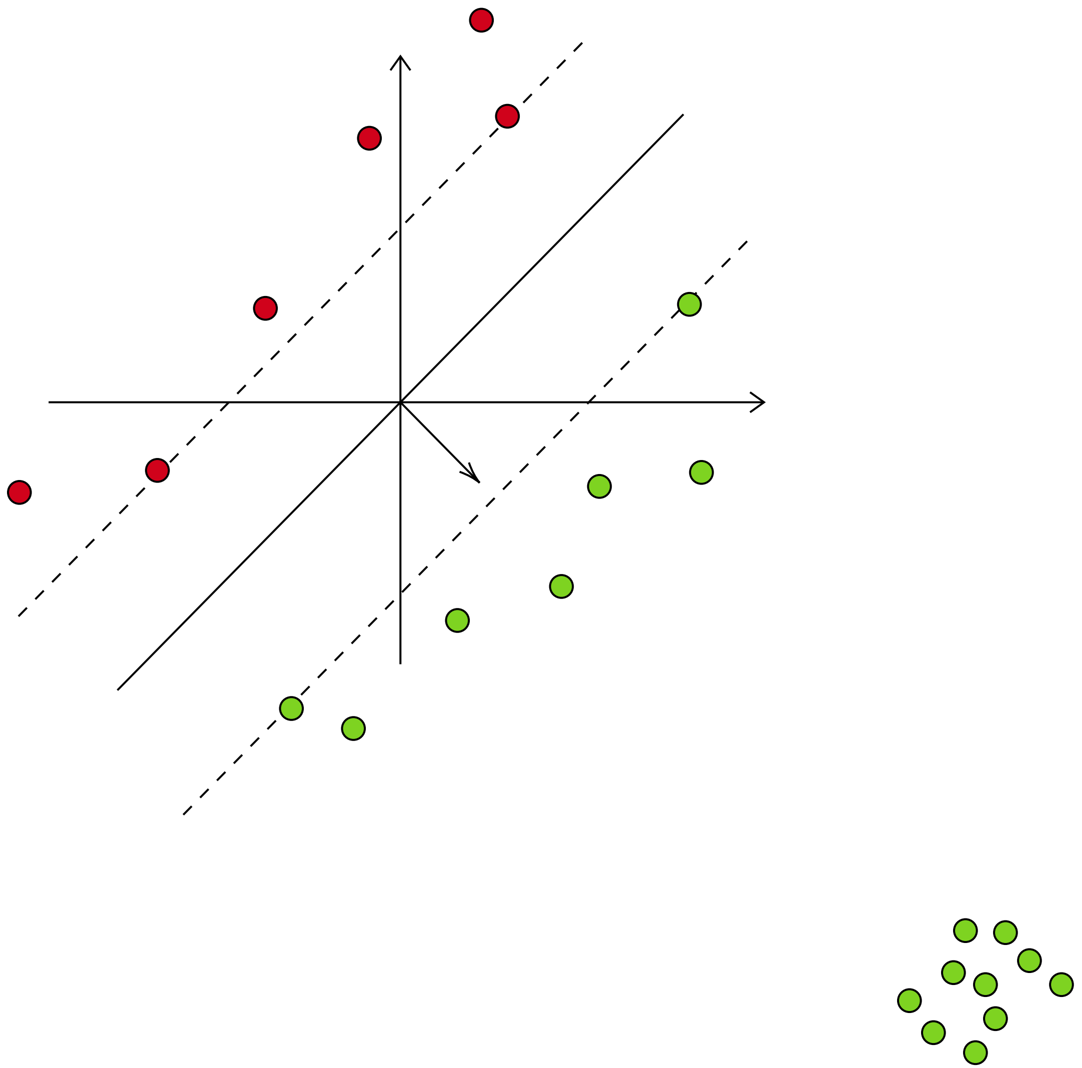

Q-7

- Hard-margin, linear-SVM
- Optimal weight vector $\mathbf{w}^* = [1 \quad 2 \quad 3]^T$
- Distance of the closest point from the boundary

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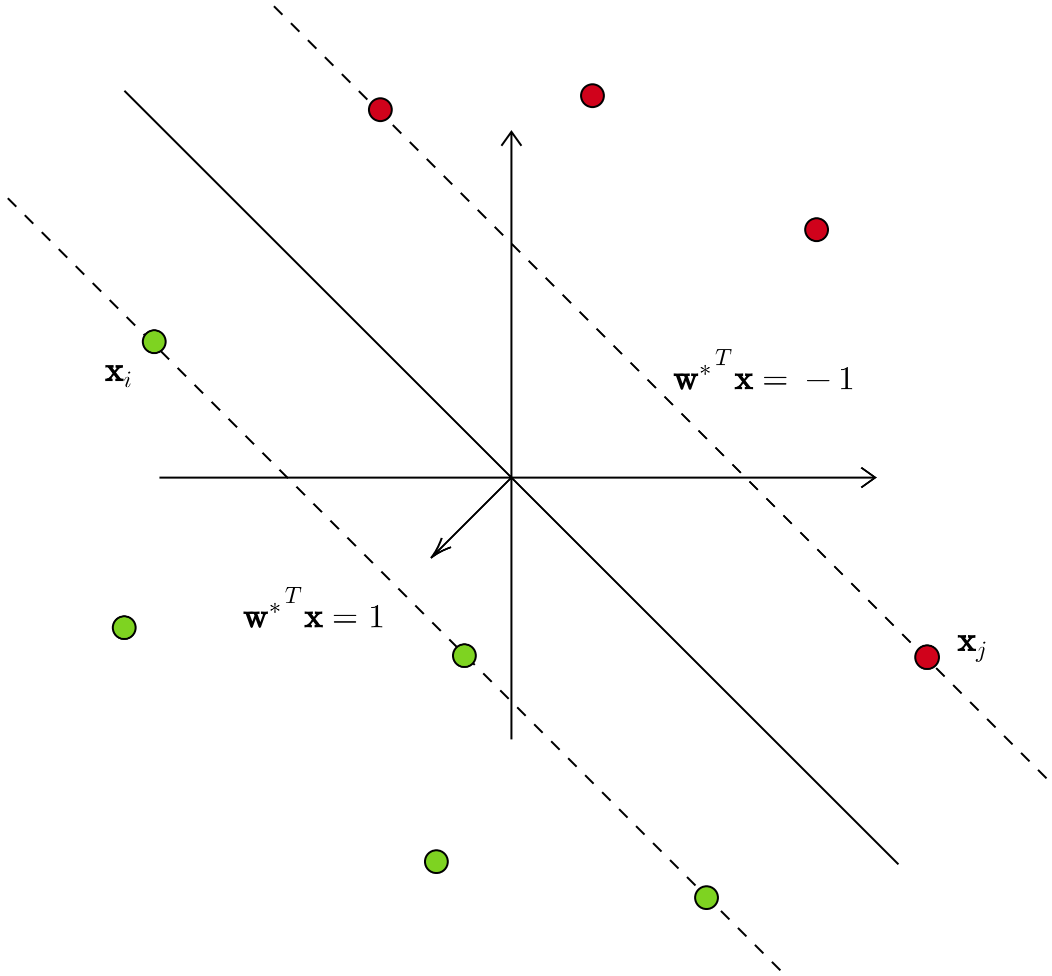

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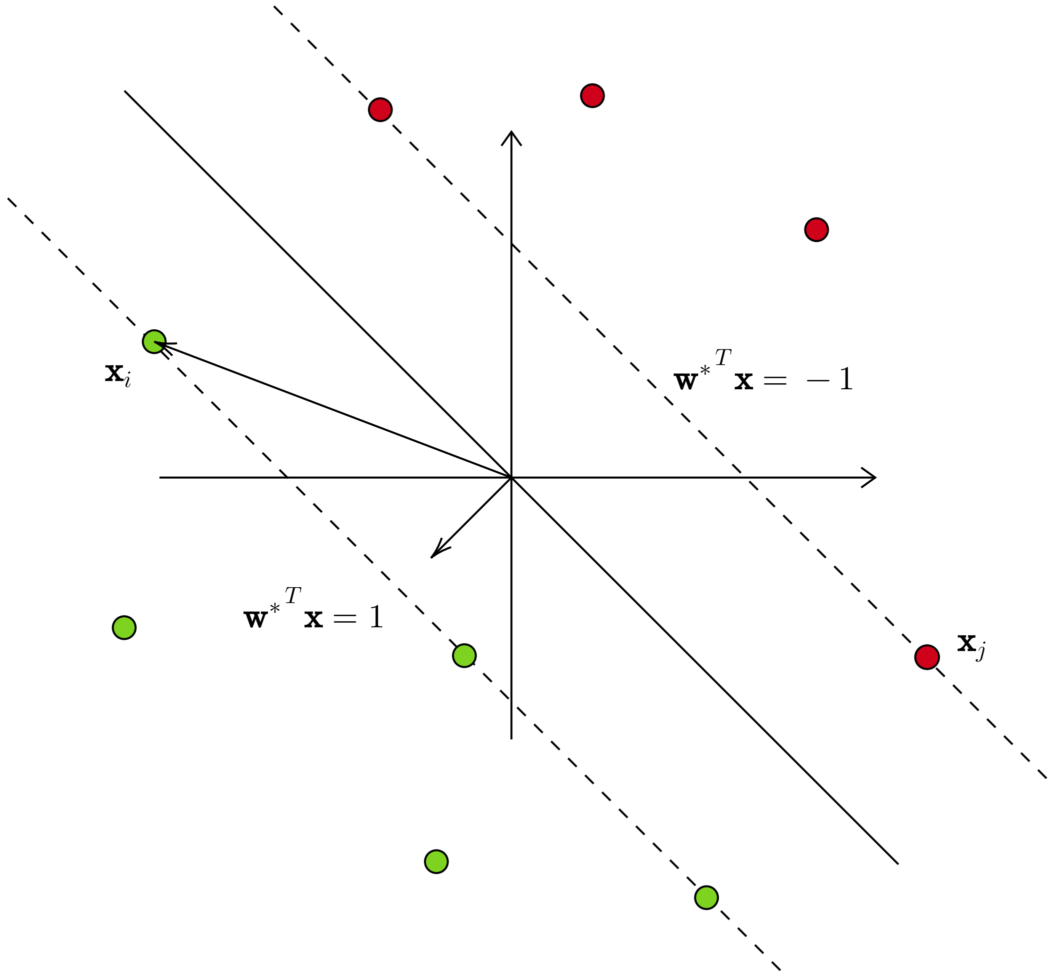

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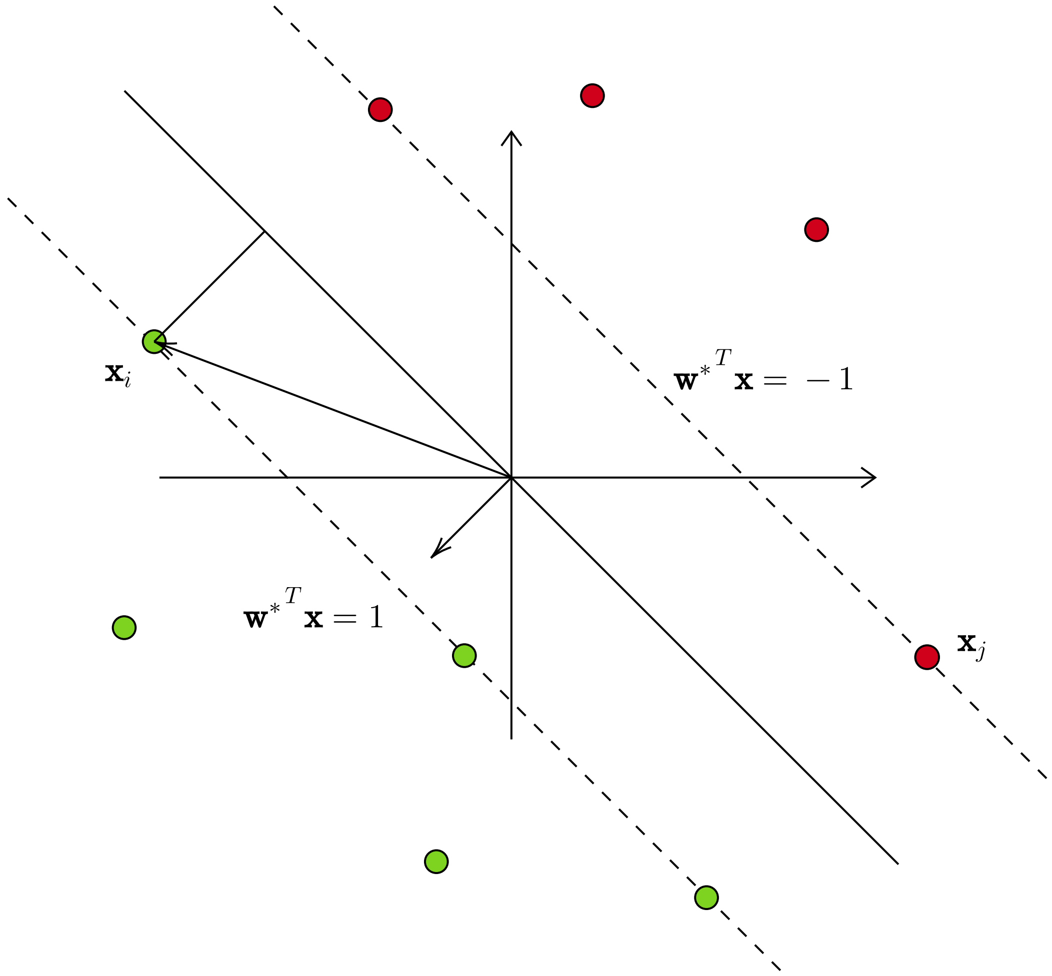

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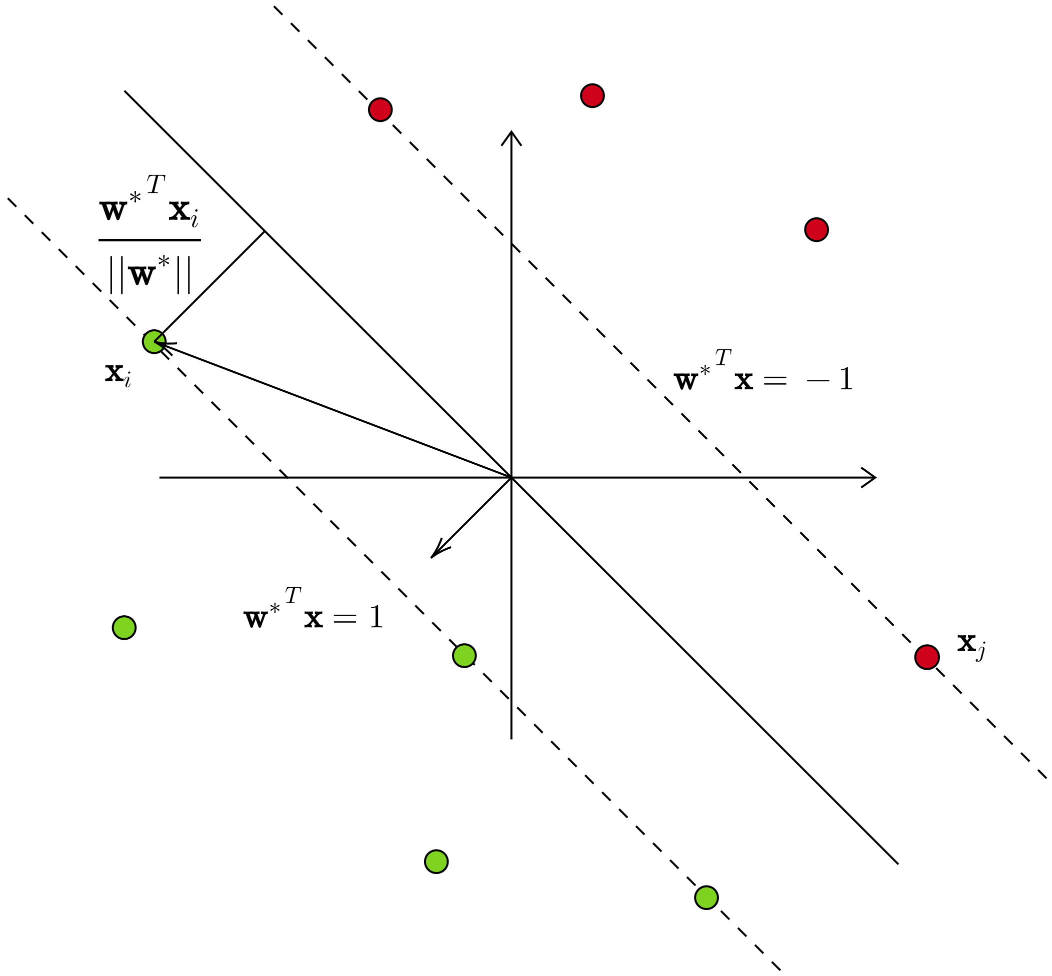

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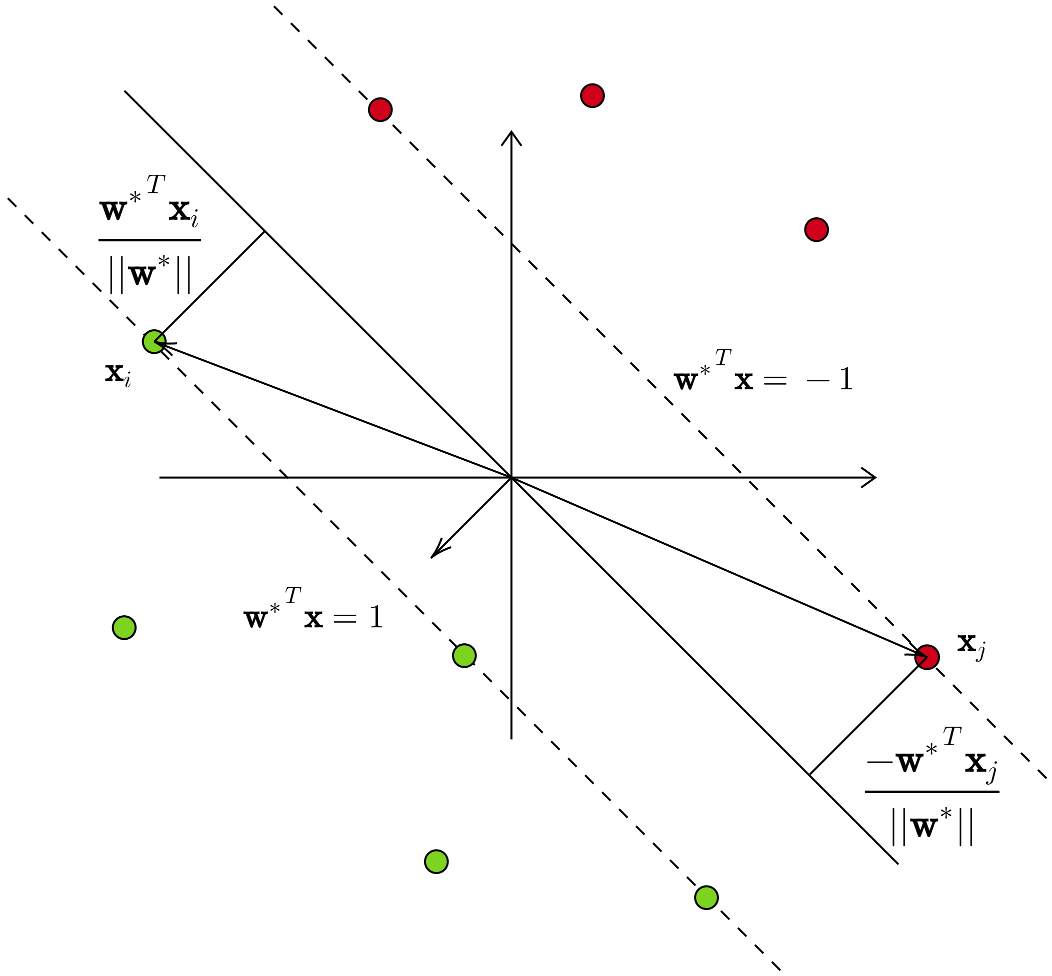

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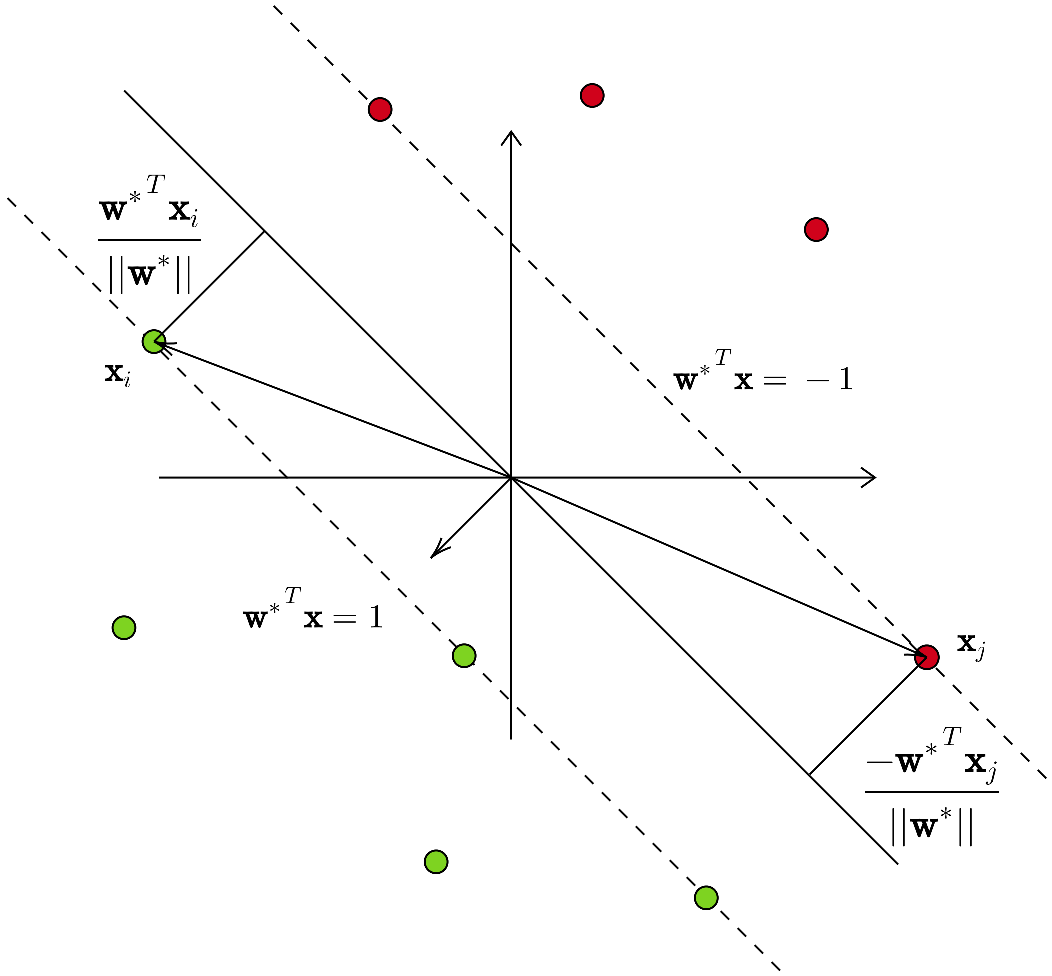

$$d =$$

Q-7

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- Optimal weight vector $\mathbf{w}^* = [1 \quad 2 \quad 3]^T$
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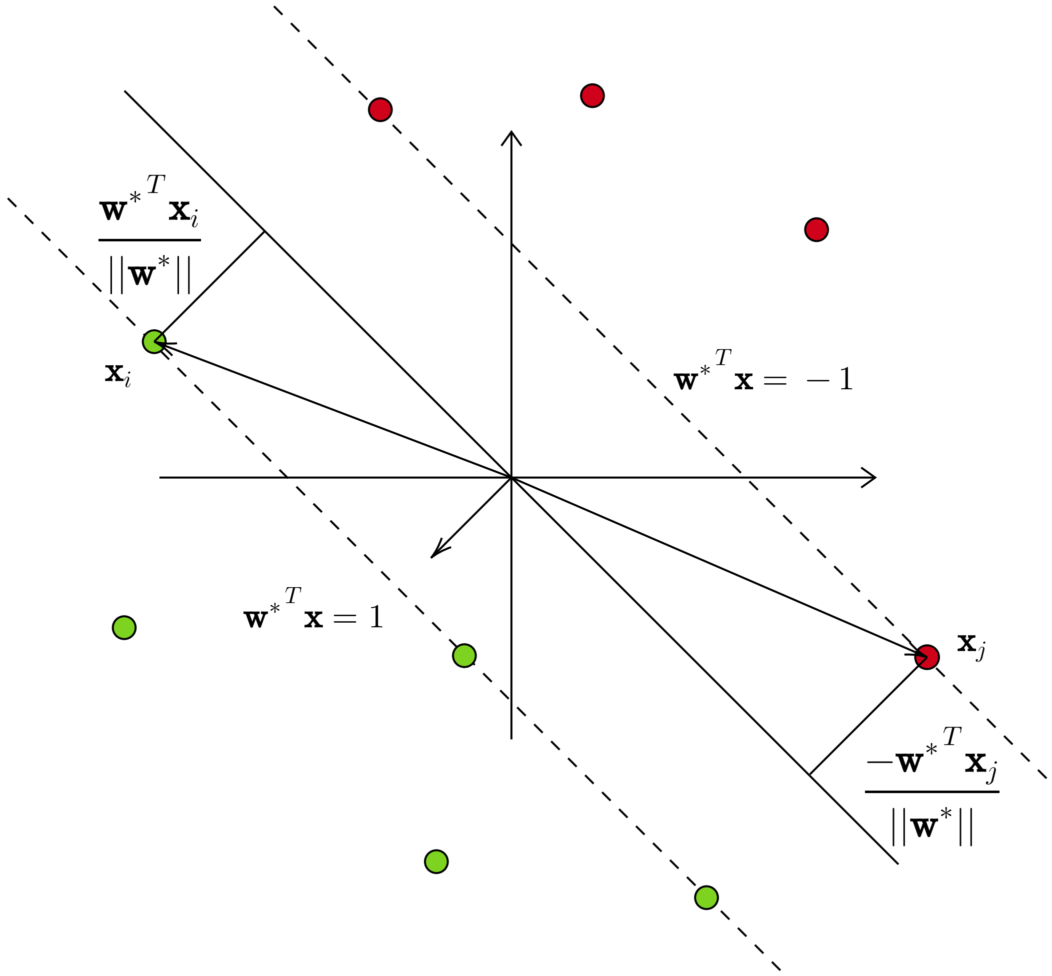

$$d = \frac{1}{||\mathbf{w}^*||} = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{14}}$$

Q-7

- Hard-margin, linear-SVM
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Geometric margin

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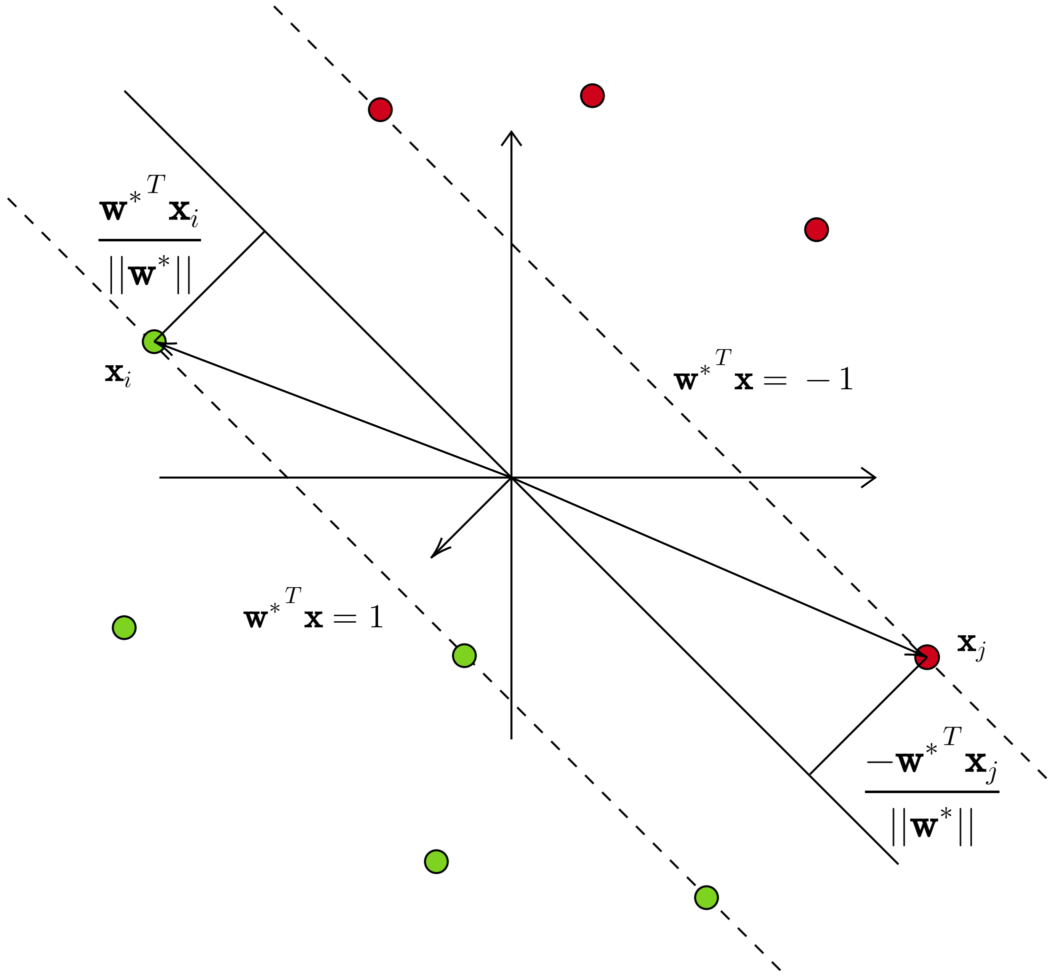

Q-8

- Weighted linear regression
- Weight for each data-point in the loss function
- $\mathbf{R} = \mathtt{diag}(r_1,\ \cdots .\, r_n)$

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$$\nabla L(\mathbf{w})$$

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$$\mathbf{XRX}^T \mathbf{w} = \mathbf{XRy}$$

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$$\mathbf{XRX}^T \mathbf{w} = \mathbf{XRy}$$

$$\mathbf{w} = \left(\mathbf{XRX}^T\right)^{-1} \mathbf{XRy}$$

Q-9

- Hard-margin, linear-SVM
- $\mathbf{w}^* = [2 \quad -1]^T$
- Unit square (0,0), (1, 0), (0, 1), (1, 1)
- Probability that a point picked at random from the unit square is is predicted as 1

Q-9

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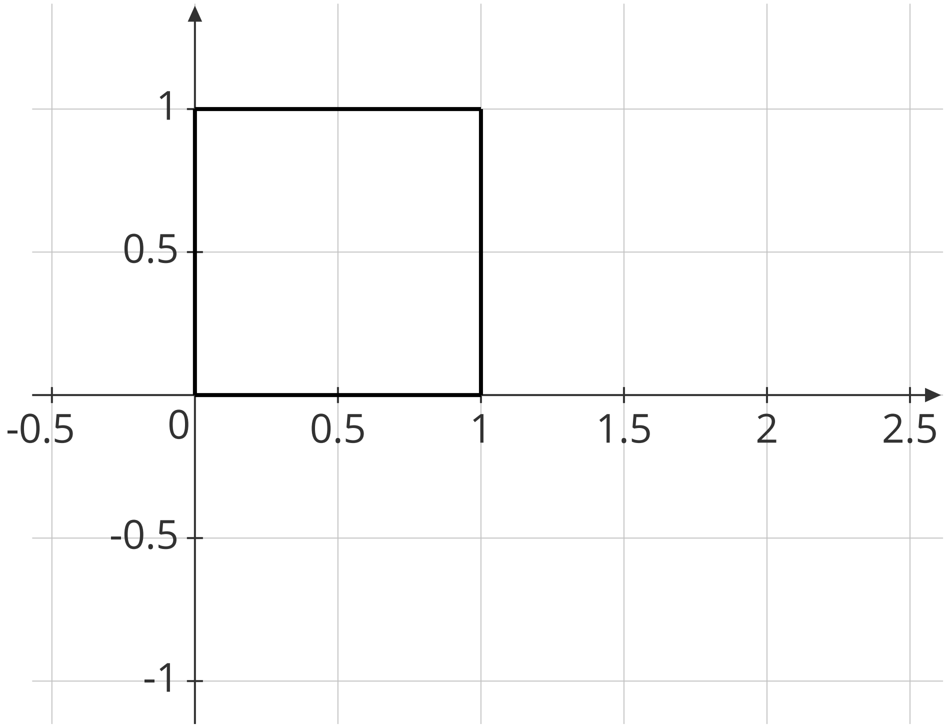

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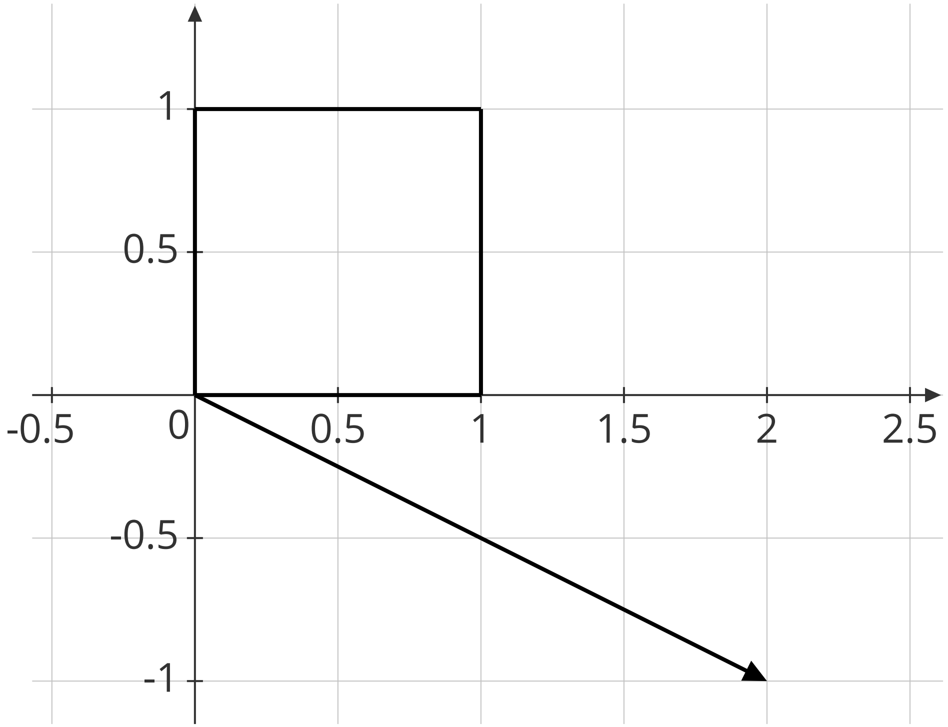

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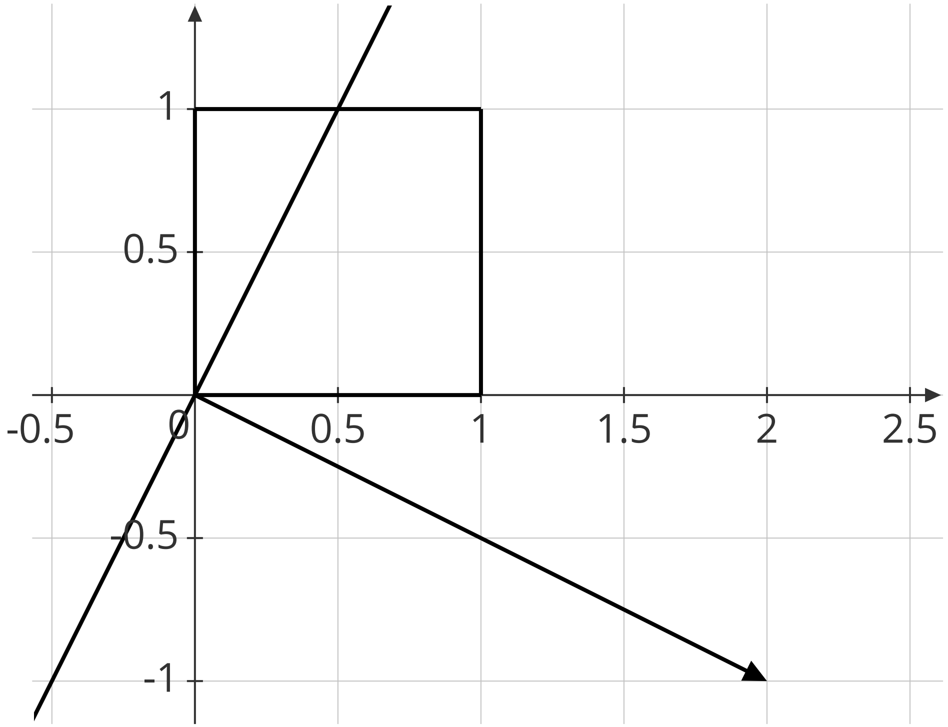

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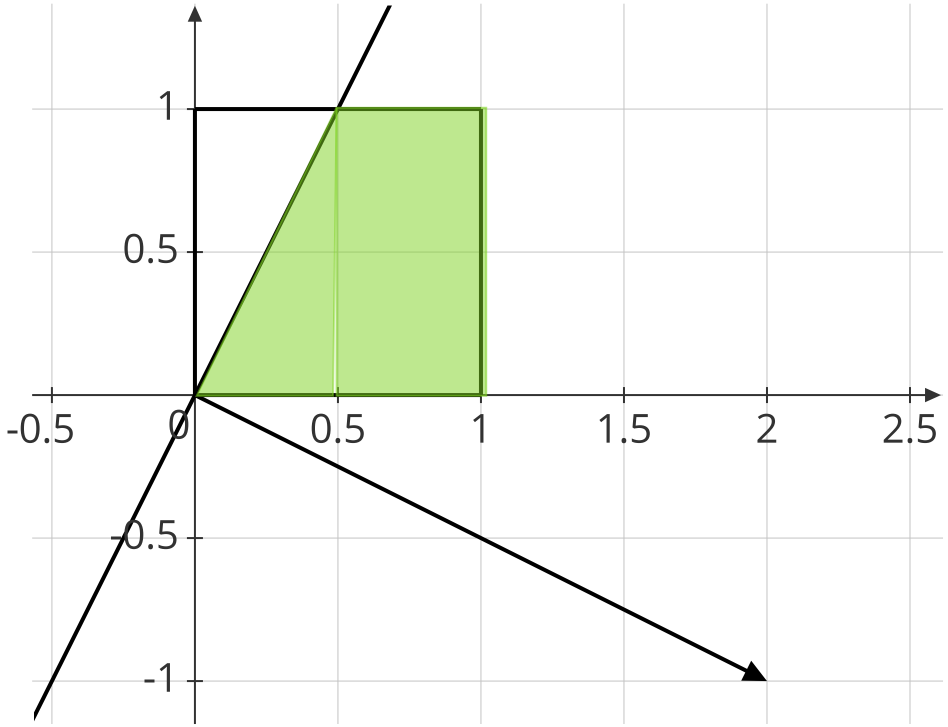

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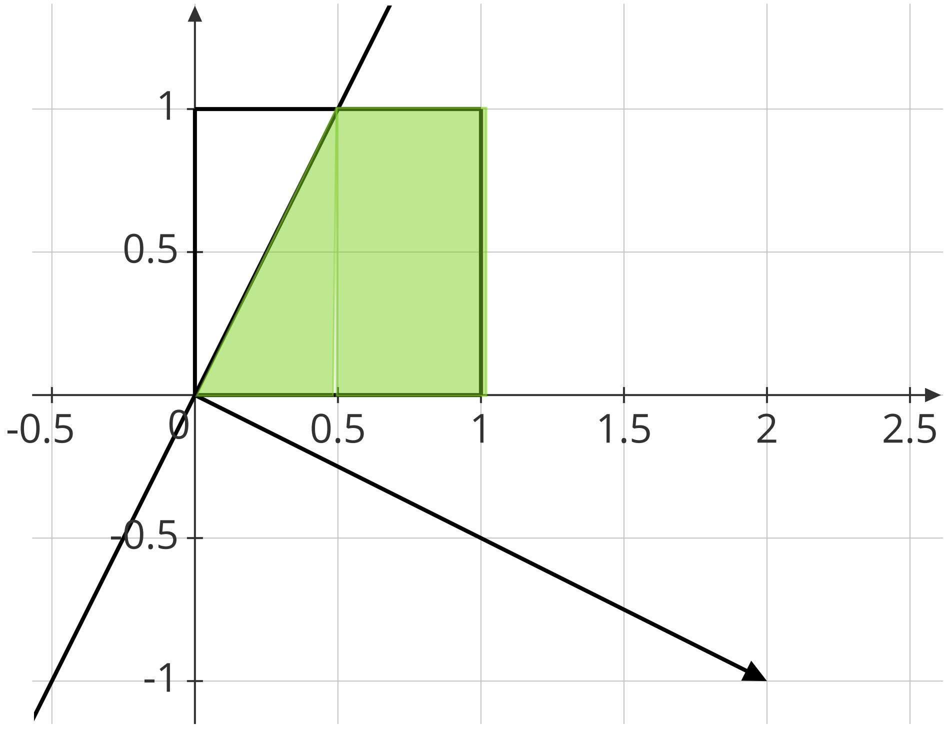

$$P(y = 1 \mid \mathbf{x}) = 1 - \frac{1}{2} \cdot 1 \cdot \frac{1}{2}$$

$$= 1 - 0.25$$

$$= 0.75$$

Q-10

Select all true statements regarding a hard-margin SVM:

(1) Every support vector lies on one of the two supporting hyperplanes.
(2) Every point on one of the two supporting hyperplanes is a support vector.

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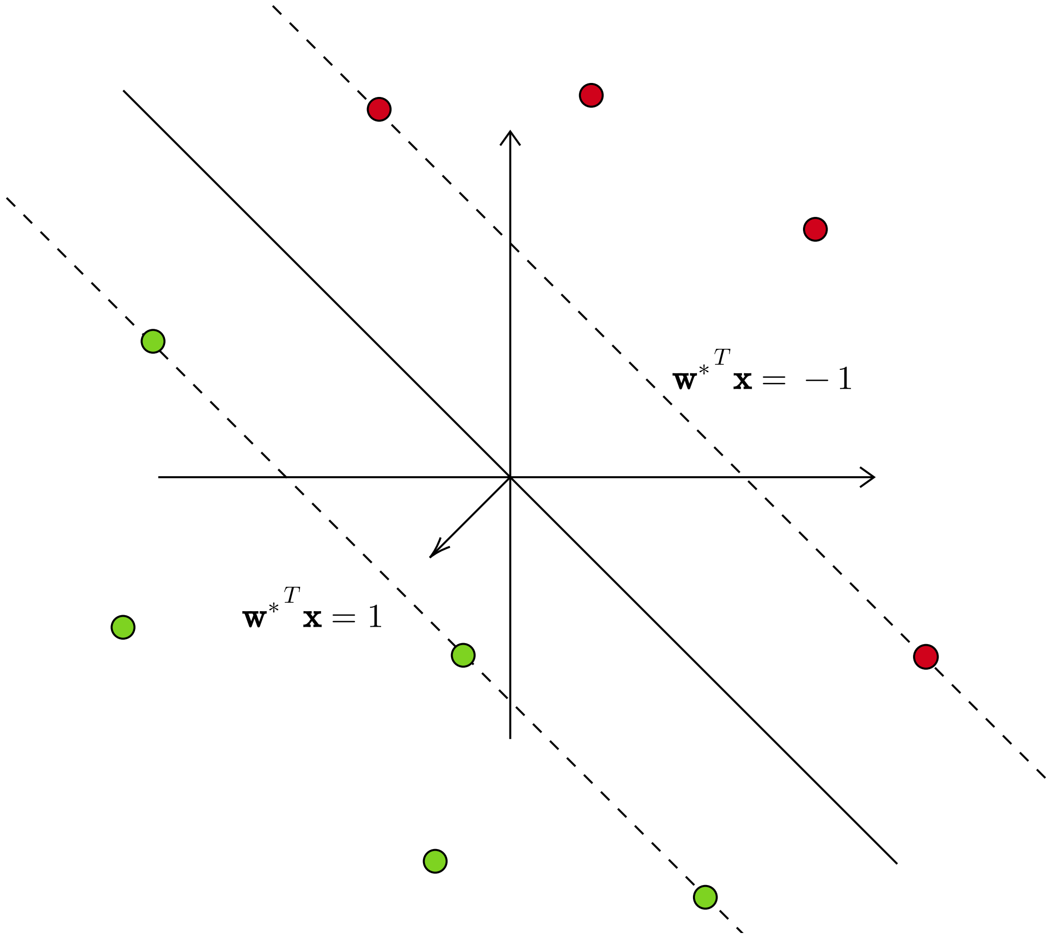

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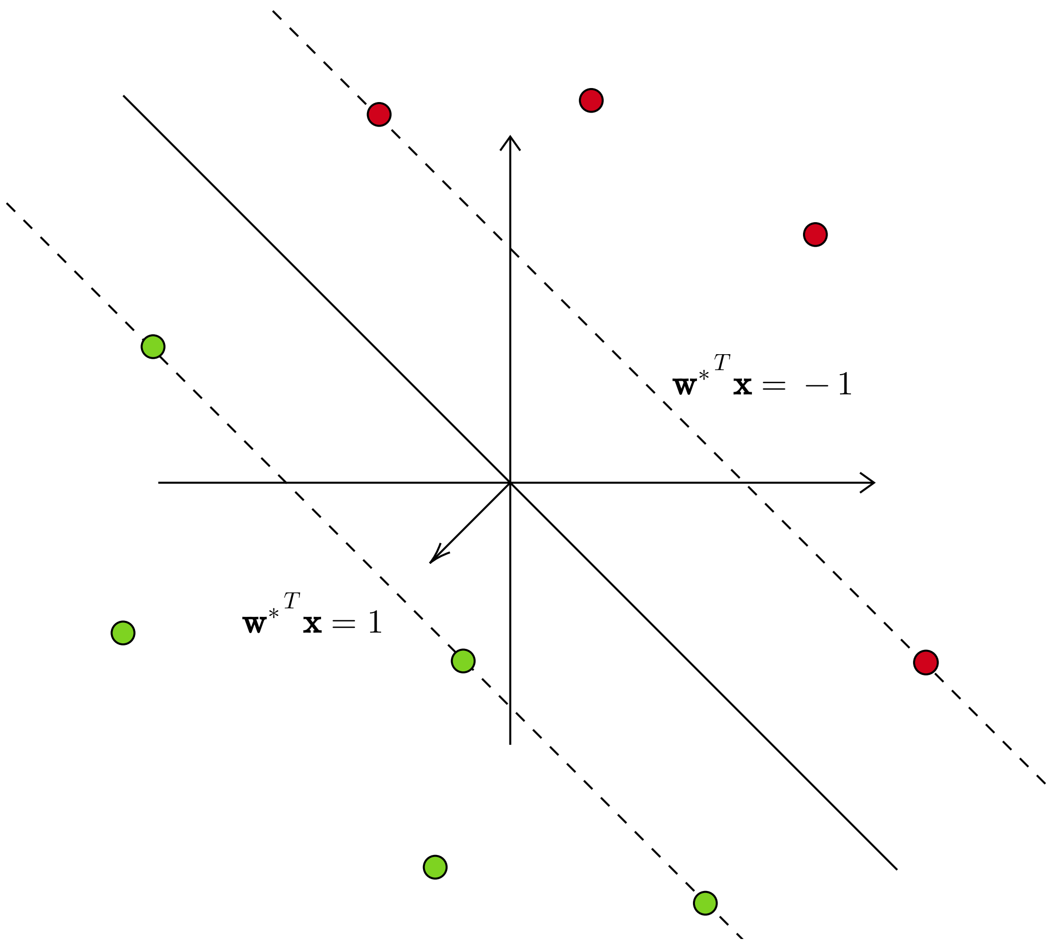

$$\left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i \geqslant 1 \Longrightarrow 1 - \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i \leqslant 0$$

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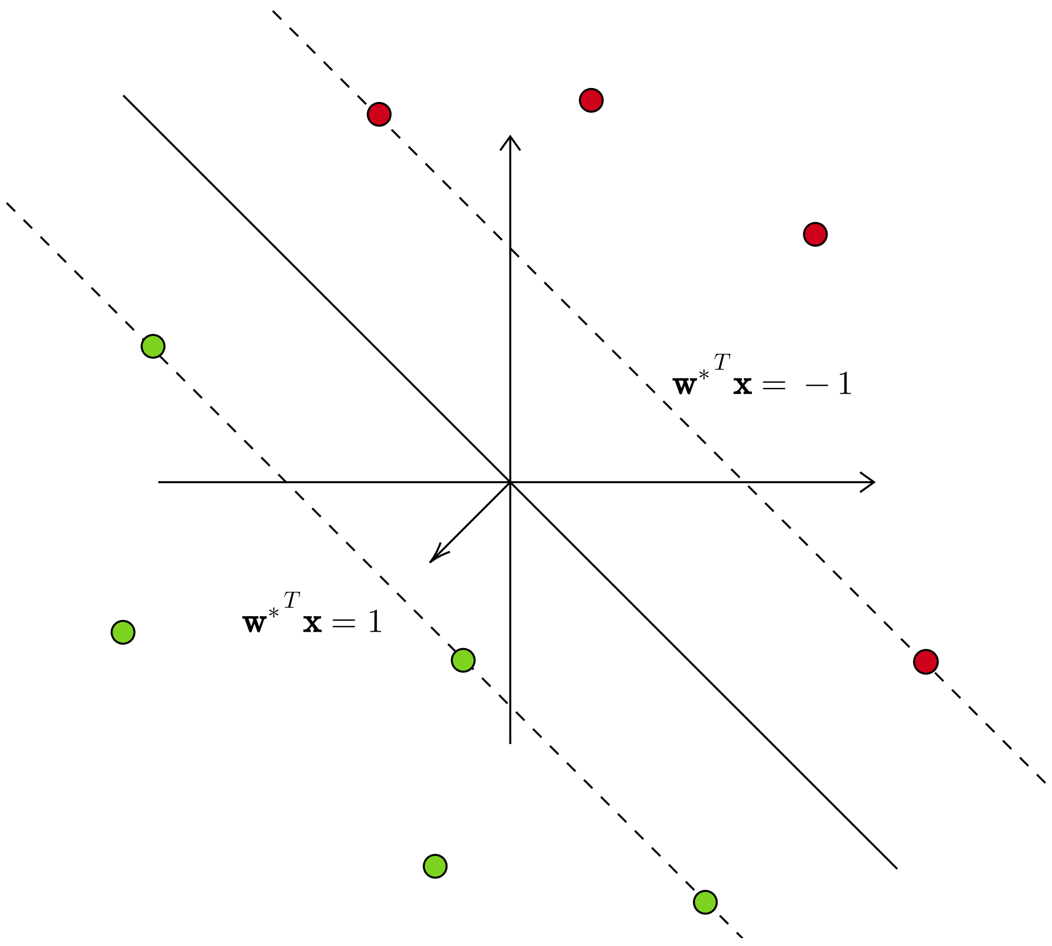

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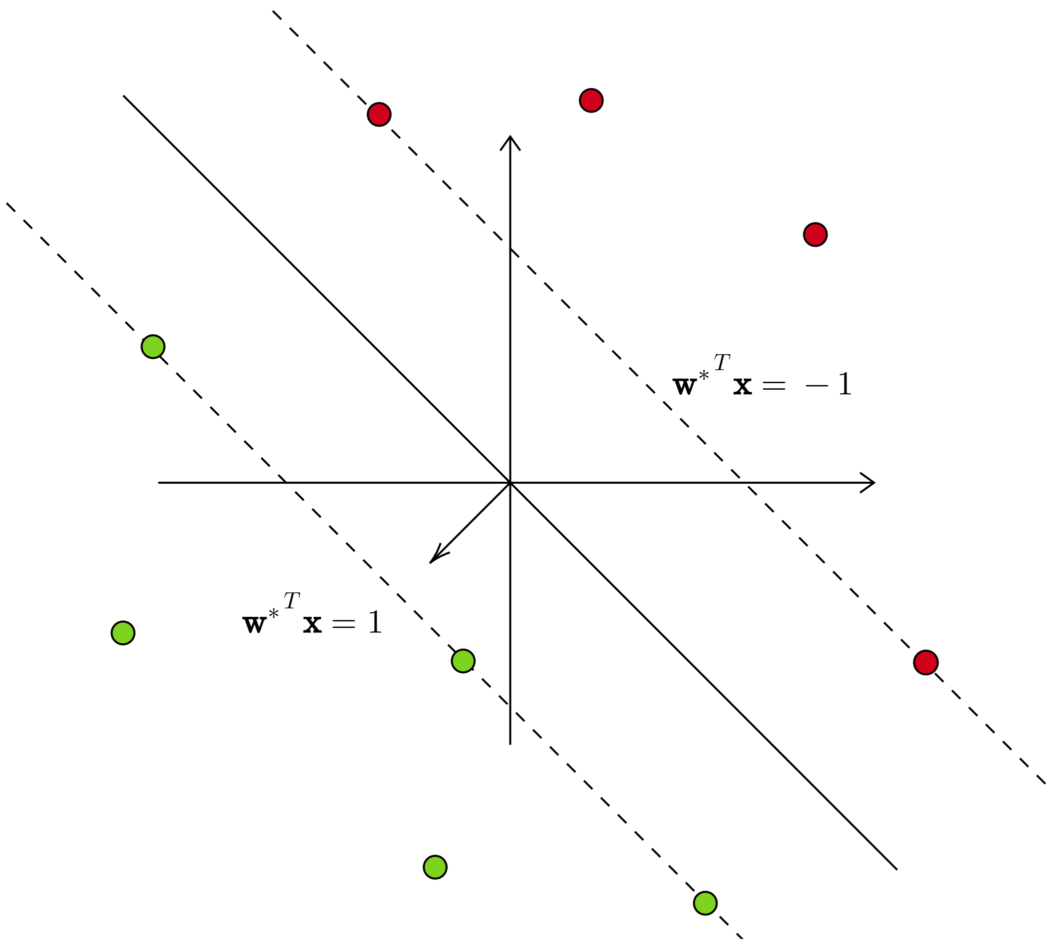

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Support vector $\implies$

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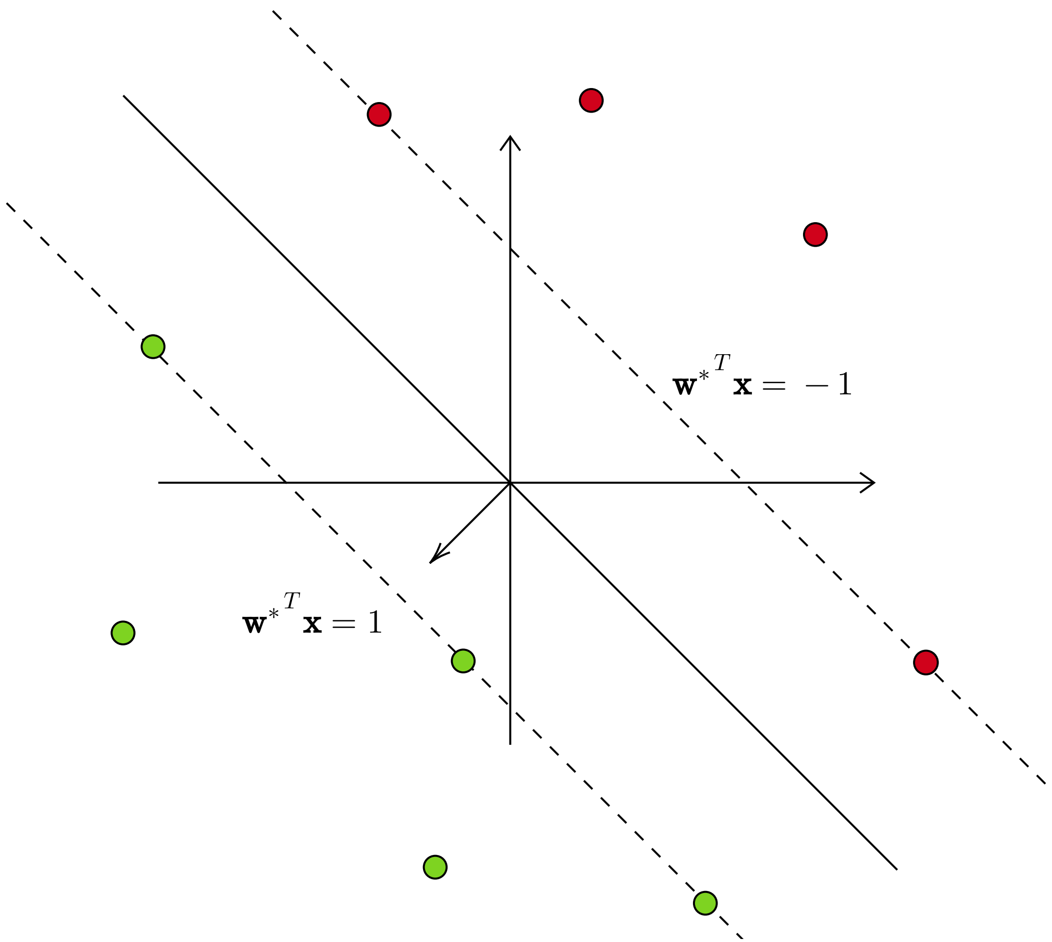

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Support vector $\implies$ $\alpha_i^* > 0$ $\implies$

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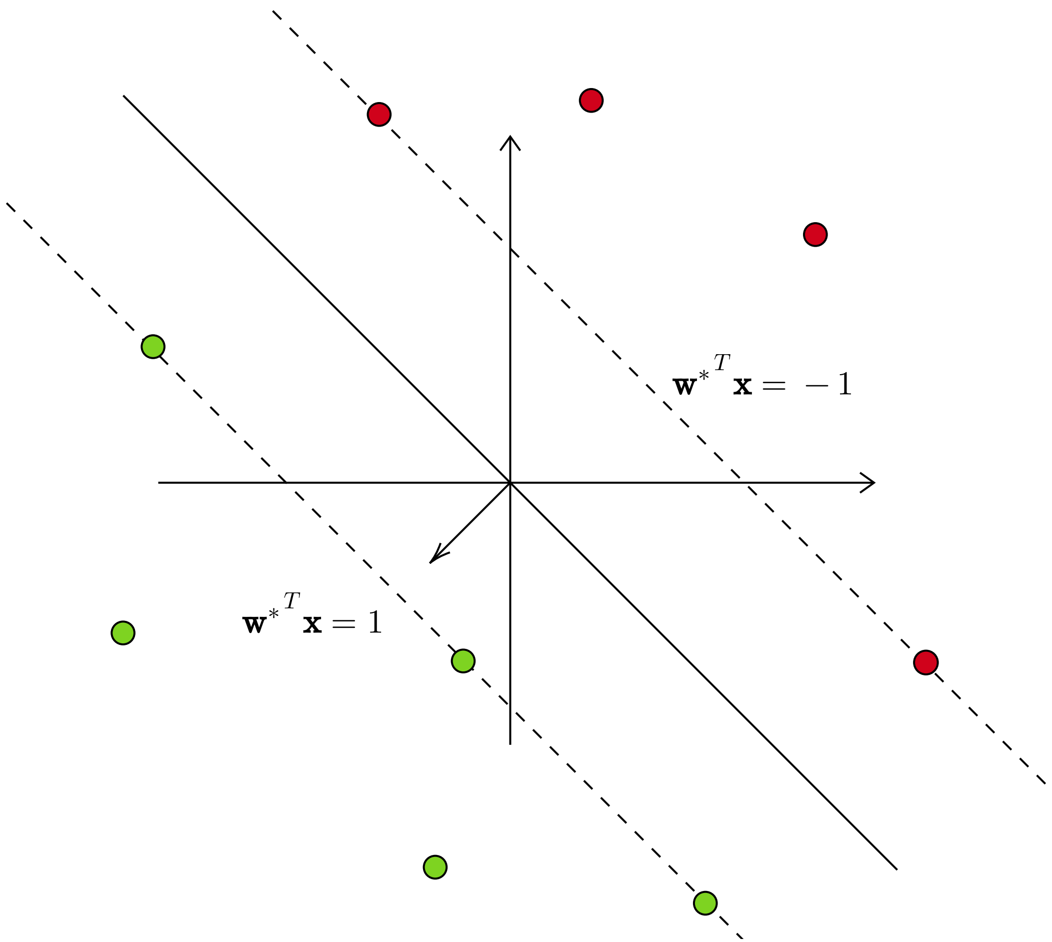

$$\left(\mathbf{w^*}^T\mathbf{x}_i\right)y_i \geqslant 1 \implies 1 - \left(\mathbf{w^*}^T\mathbf{x}_i\right)y_i \leqslant 0$$

$$\alpha_i^* \cdot \left[1 - \left(\mathbf{w^*}^T\mathbf{x}_i\right)y_i\right] = 0$$

Support vector $\implies \alpha_i^* > 0 \implies \left(\mathbf{w^*}^T\mathbf{x}_i\right)y_i = 1$

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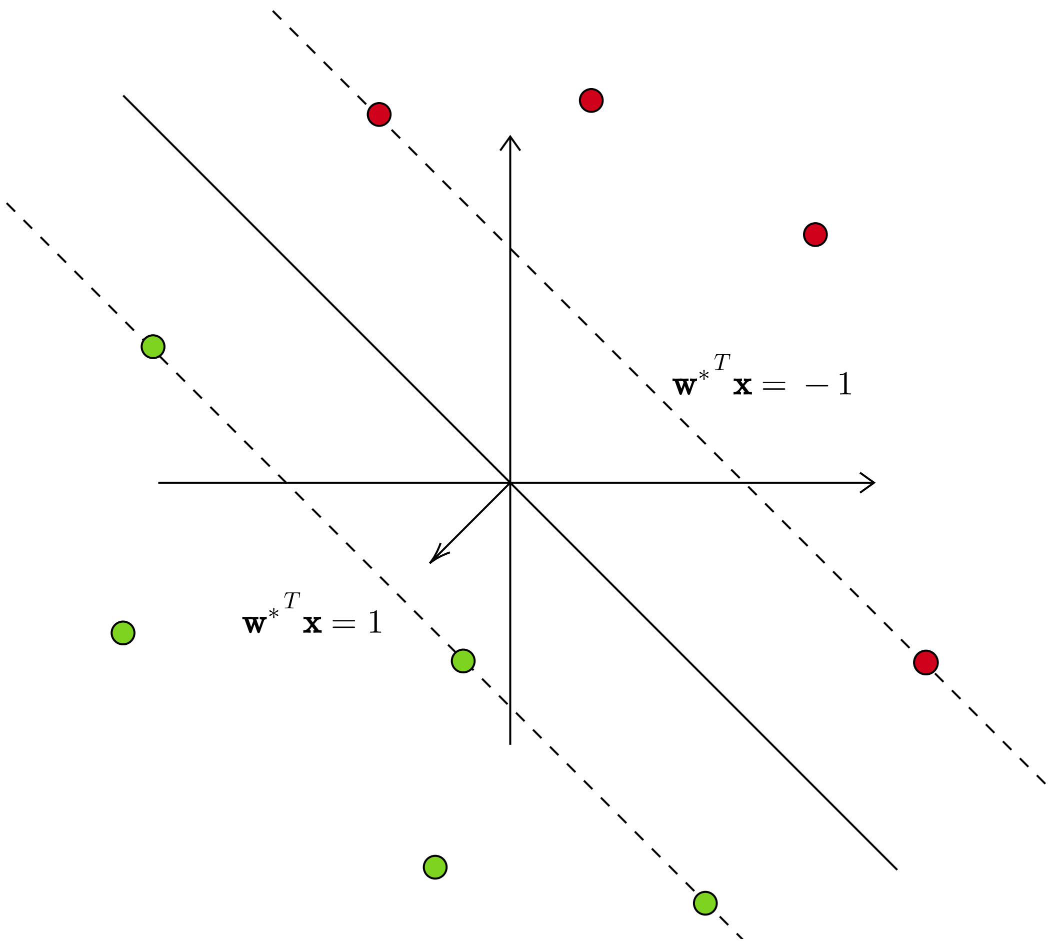

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$$\alpha_i^* \cdot \left[1 - \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i\right] = 0$$

Support vector $\Longrightarrow \alpha_i^* > 0 \Longrightarrow \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i = 1$

$$\left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i = 1$$

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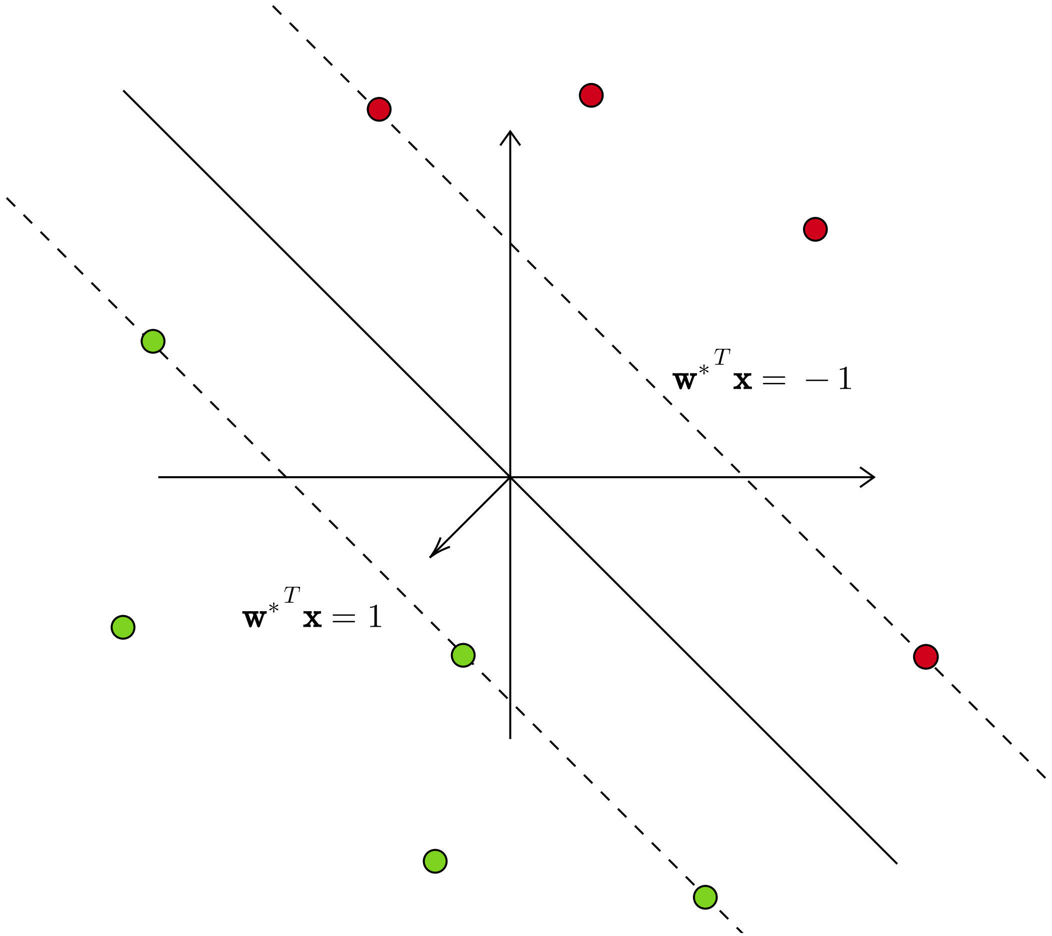

$$\left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i \geqslant 1 \Longrightarrow 1 - \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i \leqslant 0$$

$$\alpha_i^* \cdot \left[1 - \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i\right] = 0$$

Support vector $\Longrightarrow \alpha_i^* > 0 \Longrightarrow \left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i = 1$

$$\left(\mathbf{w}^{*T}\mathbf{x}_i\right)y_i = 1 \quad \not\Longrightarrow \quad \alpha_i^* > 0$$