



Module 16

Partha Pratim  
Das

Week Recap

Objectives &  
Outline

Relational  
Algebra

Select

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Module Summary

# Database Management Systems

## Module 16: Formal Relational Query Languages/1

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Module Summary

- SQL Examples have been practiced for basic query structures
- Nested Subquery in SQL
- Data Modification
- SQL expressions for Join and Views
- Transactions
- Integrity Constraints
- More data types in SQL
- Authorization in SQL
- Functions and Procedures in SQL
- Triggers



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Module Summary

- To understand formal query language through relational algebra



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Module Summary

- Relational Algebra



# Formal Relational Query Language

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Module Summary

- Relational Algebra
  - Procedural and Algebra based
- Tuple Relational Calculus
  - Non-Procedural and Predicate Calculus based
- Domain Relational Calculus
  - Non-Procedural and Predicate Calculus based



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Module Summary

# Relational Algebra



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Module Summary

- Created by Edgar F Codd at IBM in 1970
- Procedural language
- Six basic operators
  - select:  $\sigma$
  - project:  $\Pi$
  - union:  $\cup$
  - set difference:  $-$
  - Cartesian product:  $\times$
  - rename:  $\rho$
- The operators take one or two relations as inputs and produce a new relation as a result



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Module Summary

- Notation:  $\sigma_p(r)$
- $p$  is called the **selection predicate**
- Defined as:

$$\sigma_p(r) = \{t | t \in r \text{ and } p(t)\}$$

where  $p$  is a formula in propositional calculus consisting of **terms** connected by :  $\wedge$  (**and**),  $\vee$  (**or**),  $\neg$  (**not**)

Each **terms** is one of:

$$< \text{attribute} > \text{ op } < \text{attribute} > \text{ or } < \text{constant} >$$

where op is one of:  $=, \neq, >, \geq, <, \leq$

- Example of selection:

$$\sigma_{\text{dept\_name} = 'Physics'}(\text{instructor})$$

| A        | B        | C  | D  |
|----------|----------|----|----|
| $\alpha$ | $\alpha$ | 1  | 7  |
| $\alpha$ | $\beta$  | 5  | 7  |
| $\beta$  | $\beta$  | 12 | 3  |
| $\beta$  | $\beta$  | 23 | 10 |

| A        | B        | C  | D  |
|----------|----------|----|----|
| $\alpha$ | $\alpha$ | 1  | 7  |
| $\beta$  | $\beta$  | 23 | 10 |

$$\sigma_{A=B \wedge D > 5}(r)$$





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Module Summary

- Notation:  $\Pi_{A_1, A_2, \dots, A_k}(r)$   
where  $A_1, A_2$  are attribute names and  $r$  is a relation
- The result is defined as the relation of  $k$  columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the *dept\_name* attribute of *instructor*

$$\Pi_{ID, name, salary}(instructor)$$

| A        | B  | C |
|----------|----|---|
| $\alpha$ | 10 | 1 |
| $\alpha$ | 20 | 1 |
| $\beta$  | 30 | 1 |
| $\beta$  | 40 | 2 |

| A        | C |
|----------|---|
| $\alpha$ | 1 |
| $\alpha$ | 1 |
| $\beta$  | 1 |
| $\beta$  | 2 |

=

| A        | C |
|----------|---|
| $\alpha$ | 1 |
| $\beta$  | 1 |
| $\beta$  | 2 |



# Union Operation

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Module Summary

- Notation:  $r \cup s$
- Defined as:  $r \cup s = \{t \mid t \in r \text{ or } t \in s\}$
- For  $r \cup s$  to be valid.
  - a)  $r, s$  must have the *same arity* (same number of attributes)
  - b) The attribute domains must be *compatible* (example: 2nd column of  $r$  deals with the same type of values as does the 2nd column of  $s$ )
  - c) Example: to find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$  | 1 |

$r$

| A        | B |
|----------|---|
| $\alpha$ | 2 |
| $\beta$  | 3 |

$s$

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$  | 1 |
| $\beta$  | 3 |

$r \cup s$

$$\Pi_{course\_id}(\sigma_{semester="Fall" \wedge year=2009}(section)) \cup \Pi_{course\_id}(\sigma_{semester="Spring" \wedge year=2010}(section))$$



# Difference Operation

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Module Summary

- Notation  $r - s$
- Defined as:  $r - s = \{t | t \in r \text{ and } t \notin s\}$
- Set differences must be taken between **compatible** relations
  - $r$  and  $s$  must have the **same** arity
  - attribute domains of  $r$  and  $s$  must be compatible
- Example: to find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\Pi_{course\_id}(\sigma_{semester="Fall" \wedge year=2009}(section)) -$$

$$\Pi_{course\_id}(\sigma_{semester="Spring" \wedge year=2010}(section))$$

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$  | 1 |

$r$

| A        | B |
|----------|---|
| $\alpha$ | 2 |
| $\beta$  | 3 |

$s$

  

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\beta$  | 1 |

$r - s$



# Intersection Operation

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Module Summary

- Notation:  $r \cap s$
- Defined as:

$$r \cap s = \{t \mid t \in r \text{ and } t \in s\}$$

- Assume:
  - $r, s$  have the *same arity*
  - attributes of  $r$  and  $s$  are compatible
- Note:  $r \cap s = r - (r - s)$

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\alpha$ | 2 |
| $\beta$  | 1 |

$r$

| A        | B |
|----------|---|
| $\alpha$ | 2 |
| $\beta$  | 3 |

$s$

| A        | B |
|----------|---|
| $\alpha$ | 2 |

$r \cap s$



# Cartesian-Product Operation

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Module Summary

- Notation  $r \times s$
- Defined as:

$$r \times s = \{t \mid t \in r \text{ and } q \in s\}$$

- Assume that attributes of  $r(R)$  and  $s(S)$  are disjoint.  
(That is,  $R \cap S = \phi$ )
- If attributes of  $r(R)$  and  $s(S)$  are not disjoint, then renaming must be used

| A        | B |
|----------|---|
| $\alpha$ | 1 |
| $\beta$  | 2 |

 $r$ 

| C        | D  | E |
|----------|----|---|
| $\alpha$ | 10 | a |
| $\beta$  | 10 | a |
| $\beta$  | 20 | b |
| $\gamma$ | 10 | b |

 $s$ 

| A        | B | C        | D  | E |
|----------|---|----------|----|---|
| $\alpha$ | 1 | $\alpha$ | 10 | a |
| $\alpha$ | 1 | $\beta$  | 10 | a |
| $\alpha$ | 1 | $\beta$  | 20 | b |
| $\alpha$ | 1 | $\gamma$ | 10 | b |
| $\beta$  | 2 | $\alpha$ | 10 | a |
| $\beta$  | 2 | $\beta$  | 10 | a |
| $\beta$  | 2 | $\beta$  | 20 | b |
| $\beta$  | 2 | $\gamma$ | 10 | b |

 $r \times s$



# Rename Operation

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Module Summary

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.
- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_X(E)$$

returns the expression  $E$  under the name  $X$

- If a relational-algebra expression  $E$  has arity  $n$ , then

$$\rho_X(A_1, A_2, \dots, A_n)(E)$$

returns the result of expression  $E$  under the name  $X$ , and with the attributes renamed to

$$A_1, A_2, \dots, A_n$$

.



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Module Summary

- The division operation is applied to two relations
- $R(Z) \div S(X)$ , where  $X$  subset  $Z$ . Let  $Y = Z - X$  (and hence  $Z = X \cup Y$ ); that is, let  $Y$  be the set of attributes of  $R$  that are not attributes of  $S$
- The result of DIVISION is a relation  $T(Y)$  that includes a tuple  $t$  if tuples  $t_R$  appear in  $R$  with  $t_R[Y] = t$ , and with
  - $t_R[X] = t_s$  for every tuple  $t_s$  in  $S$ .
- For a tuple  $t$  to appear in the result  $T$  of the DIVISION, the values in  $t$  must appear in  $R$  in combination with every tuple in  $S$
- Division is a derived operation and can be expressed in terms of other operations
- $r \div s \equiv \Pi_{R-S}(r) - \Pi_{R-S}((\Pi_{R-S}(r) \times s) - \Pi_{R-S,S}(r))$



# Division Examples

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**Division**

Module Summary

• R

| Lecturer | Module    |
|----------|-----------|
| Brown    | Compilers |
| Brown    | Databases |
| Green    | Prolog    |
| Green    | Databases |
| Lewis    | Prolog    |
| Smith    | Databases |

S

| Subject |
|---------|
| Prolog  |

R | S

| Lecturer       |
|----------------|
| Green<br>Lewis |





# Division Examples (2)

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Module Summary

• R

| Lecturer | Module    |
|----------|-----------|
| Brown    | Compilers |
| Brown    | Databases |
| Green    | Prolog    |
| Green    | Databases |
| Lewis    | Prolog    |
| Smith    | Databases |

S

| Subject   |
|-----------|
| Databases |
| Prolog    |

R | S

| Lecturer |
|----------|
| Green    |



## Division Examples (3)

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Module Summary

**A**

| <i>sno</i> | <i>pno</i> |
|------------|------------|
| s1         | p1         |
| s1         | p2         |
| s1         | p3         |
| s1         | p4         |
| s2         | p1         |
| s2         | p2         |
| s3         | p2         |
| s4         | p2         |
| s4         | p4         |

**B1**

|            |
|------------|
| <i>pno</i> |
| p2         |

**B2**

|            |
|------------|
| <i>pno</i> |
| p2         |
| p4         |

**B3**

|            |
|------------|
| <i>pno</i> |
| p1         |
| p2         |
| p4         |

**A/B1**

|            |
|------------|
| <i>sno</i> |
| s1         |
| s2         |
| s3         |
| s4         |

**A/B2**

|            |
|------------|
| <i>sno</i> |
| s1         |
| s4         |

**A/B3**

|            |
|------------|
| <i>sno</i> |
| s1         |



## Division Example (4)

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Module Summary

- Relations  $r, s$ :

| A          | B |
|------------|---|
| $\alpha$   | 1 |
| $\alpha$   | 2 |
| $\alpha$   | 3 |
| $\beta$    | 1 |
| $\gamma$   | 1 |
| $\delta$   | 1 |
| $\delta$   | 3 |
| $\delta$   | 4 |
| $\epsilon$ | 6 |
| $\epsilon$ | 1 |
| $\beta$    | 2 |

$r$

| B |
|---|
| 1 |
| 2 |

$s$

| A        |
|----------|
| $\alpha$ |
| $\beta$  |

- $r \div s$ :

*e.g. A is customer name  
B is branch-name  
1 and 2 here show two specific branch-names  
(Find customers who have an account in all  
branches of the bank)*



## Division Example (5)

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Module Summary

- Relations  $r, s$ :

| $A$      | $B$ | $C$      | $D$ | $E$ |
|----------|-----|----------|-----|-----|
| $\alpha$ | $a$ | $\alpha$ | $a$ | $1$ |
| $\alpha$ | $a$ | $\gamma$ | $a$ | $1$ |
| $\alpha$ | $a$ | $\gamma$ | $b$ | $1$ |
| $\beta$  | $a$ | $\gamma$ | $a$ | $1$ |
| $\beta$  | $a$ | $\gamma$ | $b$ | $3$ |
| $\gamma$ | $a$ | $\gamma$ | $a$ | $1$ |
| $\gamma$ | $a$ | $\gamma$ | $b$ | $1$ |
| $\gamma$ | $a$ | $\beta$  | $b$ | $1$ |

$r$

| $A$      | $B$ | $C$      |
|----------|-----|----------|
| $\alpha$ | $a$ | $\gamma$ |
| $\gamma$ | $a$ | $\gamma$ |

- $r \div s$ :

| $D$ | $E$ |
|-----|-----|
| $a$ | $1$ |
| $b$ | $1$ |

$s$

*e.g. Students who have taken both "a" and "b" courses, with instructor "1"*

*(Find students who have taken all courses given by instructor 1)*



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Module Summary

- Discussed relational algebra with examples

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