Newtons Laws. of Motion.

While studying Newtons Laws you must remember that you have started reading this topic in your school and this may be your Last time when you are studying as a part of your curriculum. Try to understand the significance of 1st Law of motion. It defins Inertial frame of reference. You must Know how to write free body diagram. In Circular motion be clear about the real forces & the term Centripetal force. In the end I have included one example from Kleppner. It is very very important example which shows that at certain times you have to use polar coordinats otherwise you may make mistake. Practise more problems from the book.

- Different Sources are used.
- Please Point- out if you find any mistake.
- Please add some more good exampls.

Anwesi

18 August, 2016.

Newtons Laws of Motion.

Newton's Laws of Motion are three Physical Laws that together laid the foundation of classical Mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces.

Some Imp Characteristics of Newtons Laws.

- They are applied to objects which are idealized as single point masses — deformation and rotation of the body are not considered — so a planet can be idealized as a particle.
- Euler in 1750 introduced generalization of Newton's Laws of motion for rigid bodies called Euler's laws of motion. later applied as well for deformable bodies.
- Newton's Laws hold only with respect to a certain set of frames of reference called Newtonian or Inertial frames of reference. Some authors interpret the first Law as defining what an inertial reference frame is

Newton's First Law.

The first law stats That if the net force (the vector sum of all forces acting on a object) is zero, then the velocity of the object is constant.

(Velocity is a vector quantity which express both the object's speed and the direction of motion).

This means

- An object that is at ~~s~~ rest will stay at rest unless a force acts upon it
- An object That is in motion will not change its velocity unless a force acts upon it.

This is known as Law of Inertia (either retain the state of rest if it is at rest or in state of motion if it is in motion)

Newton placed the first Law of motion to establish frames of reference for which other Laws are applicable. The first law of motion postulats the existence of atleast one frame of reference called a Newtonian or Inertial reference frame, relative to which the motion of a particle not subject to forces is a straight line at constant speed

So first Law can also be restated as :-

First Law – When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a net force.

Ex: Suppose I accelerate in front of a crowd. The crowd will see me accelerating and ask if I feel some net force. I will say yes. Now I will see the crowd accelerating back. If I ask the same question they say no. So Newton first Law is not valid. But that is not true because I am seeing from an accelerating frame. which is not an inertial frame.

So how can we define an ~~Fra~~ Inertial frame of ref.

Definition of Inertial frame of reference –

- A frame of reference in which Newton's first Law is valid is called an inertial frame of reference.
- If the net force acting on a body is zero, then it is possible to find a set of reference frames in which that body has no acceleration.
- An inertial frame is a frame of reference that describes time and space homogeneously, isotropically and in a time-independent manner.

How to test whether a particular frame of reference is an inertial frame?

Place a test body at rest in the frame & ascertain that no net force acts on it. ~~Similarly~~ If the body does not remain at rest, the frame is not inertial (Similarly for motion). A frame in which these tests are everywhere passed is an inertial frame.

- All inertial frames are in a state of constant, rectilinear motion with respect to one another. The observers in different inertial frames may find different velocity but if the net force = 0 ~~but~~ ~~no~~ in all frames. acceleration = 0. Both may conclude from first Law that no net force acts on the body.

- **Mathematical Expression of First Law.**

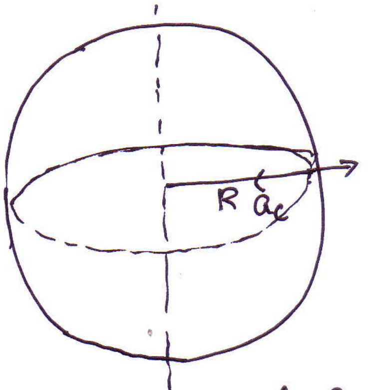
$$\sum F = 0 \quad \Rightarrow \quad \frac{d\bar{V}}{dt} = 0.$$

- An object that is at rest will stay at rest unless a force acts upon it
- An object that is in motion will not change its velocity unless a force acts upon it.

Is earth an Inertial frame.

Though earth is rotating but the Centripetal acceleration comes out to be very small, around 300 times smaller than Gravitational acceleration. So approx earth can be taken as inertial frames (Later we will study Foucault pendulum and Coriolis forces).

⇒ Let us calculate the Centripetal acc. a_c at the equator which is highest there.



Earth

$$a_c = -\omega^2 R\,\hat{r}$$

$$\omega = \frac{2\pi}{T}.$$

$$|\bar{a}_c| = \left(\frac{2\pi}{T}\right)^2 R$$

$$= \left(\frac{2\pi}{24\times3600\,\text{sec}}\right)^2 \times 6400\times10^3\,\text{m}.$$

$$|\bar{a}_c| = \underline{0.034}\ \text{m}/\text{sec}^2$$

a_c = Centripetal acceleration.

R = 6400 km

= 6400 × 1000 m.

Radius of earth

Acc due to Gravity

Comparison ⇒ $\dfrac{g}{a_c} = \dfrac{9.8}{0.034} \simeq 289.54 \simeq \underline{\underline{300}}.$

Newton's Second Law.

The Second Law states that the rate of change of momentum of a body is directly proportional to the force applied and this change in momentum takes place in the direction of applied force.

$$\boxed{\bar{F} = \frac{d\bar{P}}{dt} = \frac{d}{dt}(m\bar{v})}$$

Since Newton's second Law is only valid for Const mass system, so mass can be taken out and the equation can be written in terms of acceleration.

$$\bar{F} = m.\frac{d\bar{V}}{dt} = m\bar{a} \qquad \text{OR} \qquad \Sigma \bar{F} = m\bar{a}$$

F is the net force applied, m is the mass of the body and $\bar{a}$ is the acceleration of the body.

If there is a Constant Force applied to two bodies having masses m_1 and m_2 then it is found

$$\underline{F} = m_1 a_1 = m_2 a_2 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{a_2}{a_1}$$

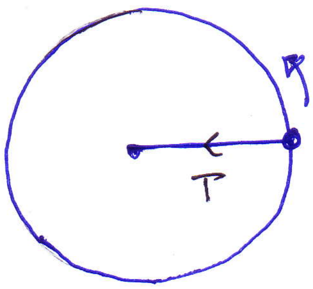
Mass is the quantitative measure of its inertia. It is the intrinsic property of the body.

In Component form one can write

$$\Sigma F_x = ma_x, \quad \Sigma F_y = ma_y, \quad \Sigma F_z = ma_z.$$

Gravitational Weight = mg = Weight, mass & Weight are different.

A body Moving in Uniform Circular Motion.



Suppose a body is moving in a circle with constant speed in the floor/Table attached by a rope. Here Velocity is not constant because its direction is changing. So it is accelerating. Its acceleration is directed towards the Center which is known by the name Centripetal acceleration. (a_c).
And there is a Constant physical Force towards the Centre which is tension here. [Try to avoid the term Centripetal force, why?] [You must be very clear about this term before using it]

Relativistic Case.

In special relativity ($v \to c$) Newton's Second Law does not hold in the Form $\bar{F} = m\bar{a}$ but it does if it is expressed as $\bar{F} = \frac{d\bar{P}}{dt}$.

So $\bar{F} = m\bar{a}$ is valid for inertial frame & $v \ll c$.

Newton's Third Law of Motion:

If body A exerts a force on body B (an "action"), then body B exerts a force on body A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies.

$$\bar{F}_{A \text{ on } B} = -\bar{F}_{B \text{ on } A}$$

* The two forces in an action-reaction pair act on different bodies.

\# A horse refuses to pull a cart. The horse reasons "according to Newton Third's Law whatever force I exert on the cart, the cart will exert an equal & opposite force on me, so the net force will be zero and I will have no chance of accelerating the cart." What is wrong with this statement? Show using the free body diagram.

Free Body Diagram & Equations of Motion.

1) Single body.

Resolve all the forces along x and y axis.

$\Sigma F_x = m a_x$

$\Sigma F_y = m a_y$.

2) Many bodies connected to each other.

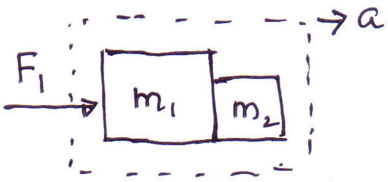
(a) Separate the bodies.

(b) Draw FBD (free body diagram) for each of them

(c) Resolve the forces along x & y axes.

(d) $\Sigma F_x = m a_x$ $\quad$ $\Sigma F_y = m a_y$ for each one of them.

3) If there is a system of bodies & they don't move relative to each other then the whole system of bodies can be treated as one body. The action reaction pair will cancel. However one can separate them & solve independently as well.



$\Sigma F_x = m a_x$.

$F_1 = (m_1 + m_2) a \Rightarrow a = \dfrac{F_1}{m_1 + m_2}$

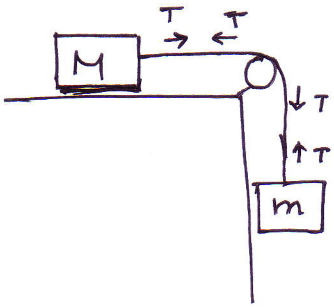
(4) Bodies connected by strings & Pulleys.

(a) String ⇒ ~~inex~~ inextensible, acc ⇒ same for all bodies.

(b) string ⇒ massless, tension ⇒ same everywhere

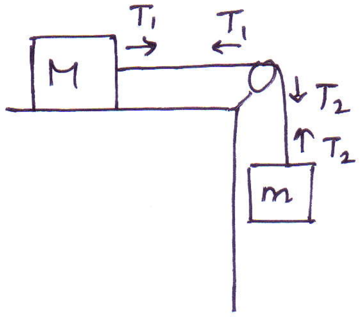
string ⇒ Mass, tension ⇒ diff at diff points.

(c) Pulley ⇒ massless & frictionless, tension ⇒ same on both sids of Pulley.



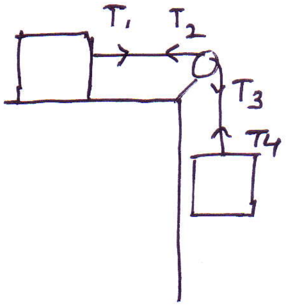
String & Pulley ⇒ masslees & no friction anywhere

(i)



String ⇒ massless
pulley ⇒ not massless
not frictionless

(ii)



string & Pulley ⇒ Not Masslees
Not frictionless.

(iii)

(5) 3 types of forces in Mechanics.

Forces

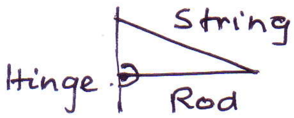
→ **Field Forces** — (a) Gravitational force
(b) Electrostatic force

(Contact is not necessary).

→ **Contact Forces** — Normal Reaction.
— Frictional Force

- equal & opposite forces on each other
- In general There are perpendicular to each other

→ **Hinge Force**



Find F_x, F_y

(6) Circular Motion - Concept of Centrifugal force, Centripetal force, Centripetal acceleration.

Whenever a body is moving in a circle, a net resultant force should act on that body to change its direction at every point and give rise to circular motion. This net resultant force may rise because of vector sum of real forces like gravitational force, frictional force, tension etc. Now this net resultant force towards the Centre is known by the name Centripetal force. So centripetal force is not the name of any real force but it is the name given for the resultant force towards the Centre. That is why according to many it is a misnomer. One must try to understand that. One should never write Centripetal force in the FBD. But this net resultant force gives rise to a resultant acceleration which is directed towards Centre known by the name centripetal acceleration.

Now many a times a problem can be solved using a rotating frame which is a noninertial frame. In solving a question in non-inertial frame we have to apply a pseudo force to apply Newton's Laws. Pseudo force is applied opposite to the direction in which acceleration is taking place. given by

$$\bar{F}_p = -m\bar{a}$$

m = mass of the body

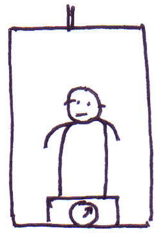
a = acceleration of the frame of reference.

Only in noninertial frame show centrifugal force in FBD

A passenger P of mass m is riding in an elevator while standing on a platform scale (which is essentially a calibrated spring scale that reads the upward force F_{ps} exerted on the passenger by scale) What does the scale read when the elevator cab is

(a) descending with constant velocity v

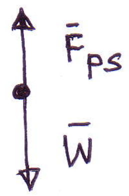
(b) ascending with an acceleration of a.



Step 1: Choose the inertial reference frame, that is the building in which the elevator is located. because an accelerating elevator is not an inertial frame. Both g and a are measured by an observer in this inertial frame.

Step 2: Choose a coord. syst. ⇒ Y axis is vertical & +ve

Step 3. Draw the free body diagram of passenger.



Step 4. Equations of motion.

$$\Sigma F_y = F_{ps} - W = m a_y.$$

$$F_{ps} = m a_y + W = m a_y + mg.$$

(a) Const velocity case, $a_y = 0$ $\quad F_{ps} = mg.$

The case is same as it is at rest

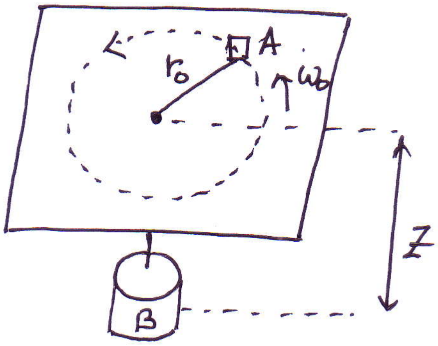
(b) $F_{ps} = ma + mg = m(a+g)$

Scale reading increases – If it accelerates down its reading decreases. Think & Conclude !!

Problems of Circular Motion - Use of Radial Polar Coordinats.

In certain problems of Circular motion use of ~~radial~~ Polar Coordinats is necessary. Let us see where ?

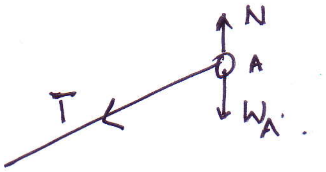
Ex 2.7 of KPK.

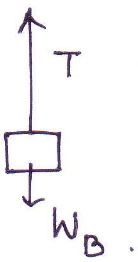


A horizontal frictionless table has a small hole in its Centre. Block A on the table is connected to Block B hanging beneath by a string of negligible mass which passes through the hole. Initially, B is held stationary and A rotats at Constant radius r_0 with steady angular Velocity ω_0. If B is released at $t = 0$, what is its acceleration immediately afterward ?

⇒ Let us first try it using simple method that we have been doing.

FBD





For body A since it is moving in a steady circle with Constant Circular motion we can write the eqs of motion as follows.

$$T = m_A \omega_0^2 r_0 \quad (\text{before } t=0)$$

AT $t=0$, body B is released so one can write in the limit $t \to 0$

$$W_B - m_A \omega_0^2 r_0 = m_B a_B \; ?$$

$$a_B = \frac{m_B g - m_A \omega_0^2 r_0}{m_B}.$$

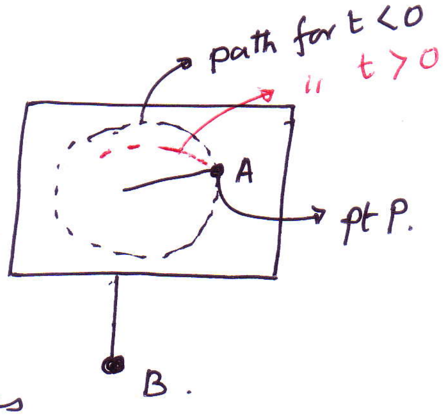
But this answer is Wrong X Why ?

Think about it & Write the answer below.

When to use the polar Coordinats.

Most of the problems that you have done in your intermediat relats to bodies moving in Uniform Circular motion or circular motion. But if the body is moving in a spiral or in a curve, at that time it is necessary to use the expressions of polar coordinats.

In the Last problem before $t=0$, the body was moving in a circular motion. At $t=0$, the body A was at point P. At $t=0$ the body B was released.



The moment it was released, now the motion of A was no longer circular but some kind of spiral motion. Then the net radial acceleration That is acceleration towards the centre is not $\omega^2 r$ or $\frac{v^2}{r}$.

Please note radial acc is $(\ddot{r} - r\dot{\theta}^2)$ not only $r\dot{\theta}^2$

Though Velocity at A at time $t=0$ is ω_0, radius is r_0 but the radial acceleration is not $\omega_0^2 r_0$ because acceleration is the rate of change of velocity and not velocity. [You remember in SHM when velocity is minimum, acceleration is max].

So in all these cases where the particle is not exactly in circular motion one must use the expressions of polar Coordinats.

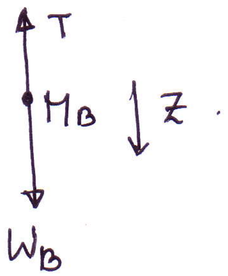
⇒ Sol

Using polar coord r, θ for A
Single linear coord z for B.

The equations of motion are.

$$-T = M_A(\ddot{r} - r\dot{\theta}^2) \quad \text{radial}$$

$$0 = M_A(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \quad \text{Tangential}$$



$$W_B - T = M_B\ddot{z} \quad \text{vertical.}$$

Constraint: length of string is const.

$$r + z = l. \quad \text{Diff twice.}$$

$$\ddot{r} = -\ddot{z}$$

As A moves inward B falls.

Combining all the equations.

$$\ddot{z} = \frac{W_B - M_A r\dot{\theta}^2}{M_A + M_B}.$$

Imp: Acceleration can change instantaneously, Velocity and position cannot. Thus immediately B is released, $r = r_0$ and $\dot{\theta} = \omega_0$. Hence.

$$\ddot{z}(0) = \frac{W_B - M_A r_0 \omega_0^2}{M_A + M_B}.$$

$\ddot{z}(0)$ can be +ve, −ve or zero depending on the value of the numerator. If ω_0 is large enough, block B will begin to rise after release.