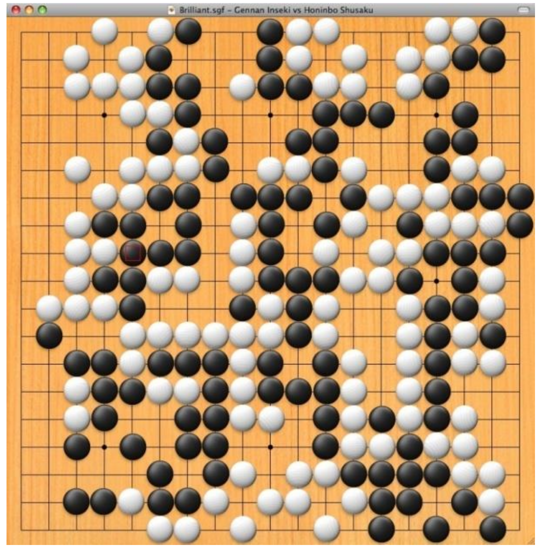
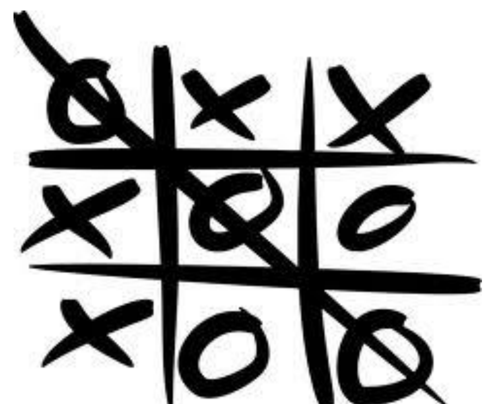
Artificial Intelligence: Adversarial Search

Motivation



GO

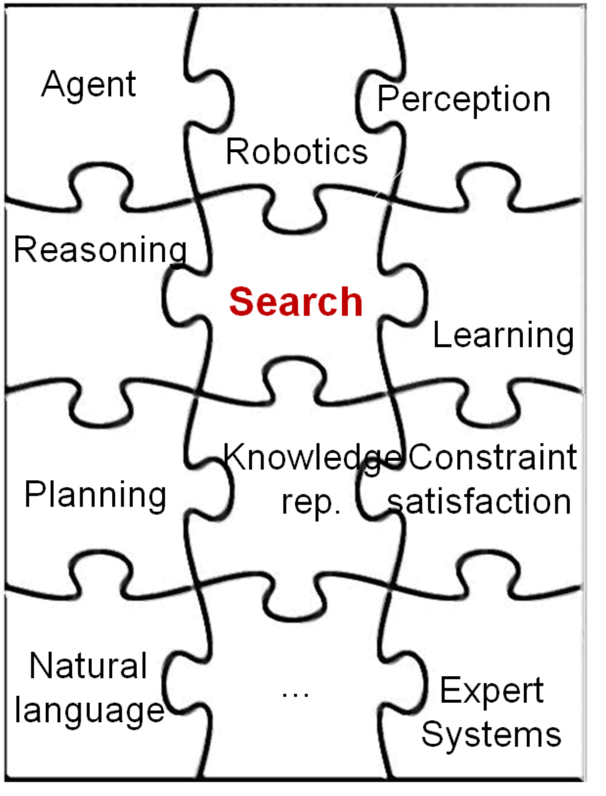
chess



tic-tac-toe

Today

- State Space Search for Game Playing
  - MiniMax
  - Alpha-beta pruning
  - Stochastic Games
- Where we are today



Adversarial Search

- Classical application for heuristic search
  - simple games: exhaustively searchable
  - complex games: only partial search possible
  - additional problem: playing against opponent
- Here, we look at 2-player adversarial games
  - win, lose, or tie

Types of Games

- **Perfect Information**
  - A game with the perfect information is that in which agents can look into the complete board. Agents have all the information about the game, and they can see each other moves also.
  - Examples: Chess, Checkers, Go, etc.
- **Imperfect Information**
  - Game state only *partially observable*, choices by opponent are not visible (hidden)
  - Example: Battleship, Stratego, many card games, etc.

Types of Games (II)

- **Deterministic games**
  - No games of chance (e.g., rolling dice)
  - Examples: Chess, Tic-Tac-Toe, Go, etc.
- **Non-deterministic games**
  - Games with unpredictable (random) events (involving chance or luck)
  - Example: Backgammon, Monopoly, Poker, etc.

Types of Games (III)

- **Zero-Sum Game**
  - If the total gains of one player are added up, and the total losses are subtracted, they will sum to zero (example: cutting a cake)
  - A gain by one player must be matched by a loss by the other player
  - One player tries to maximize a single value, the other player tries to minimize it
  - Examples: Checkers, Chess, etc.
- **Non-Zero-Sum Game**
  - Win-Win or Lose-Lose type games
  - Famous example: The Prisoner's Dilemma

https://en.wikipedia.org/wiki/Prisoner%27s_dilemma

Today

- State Spa rch for Game Playing

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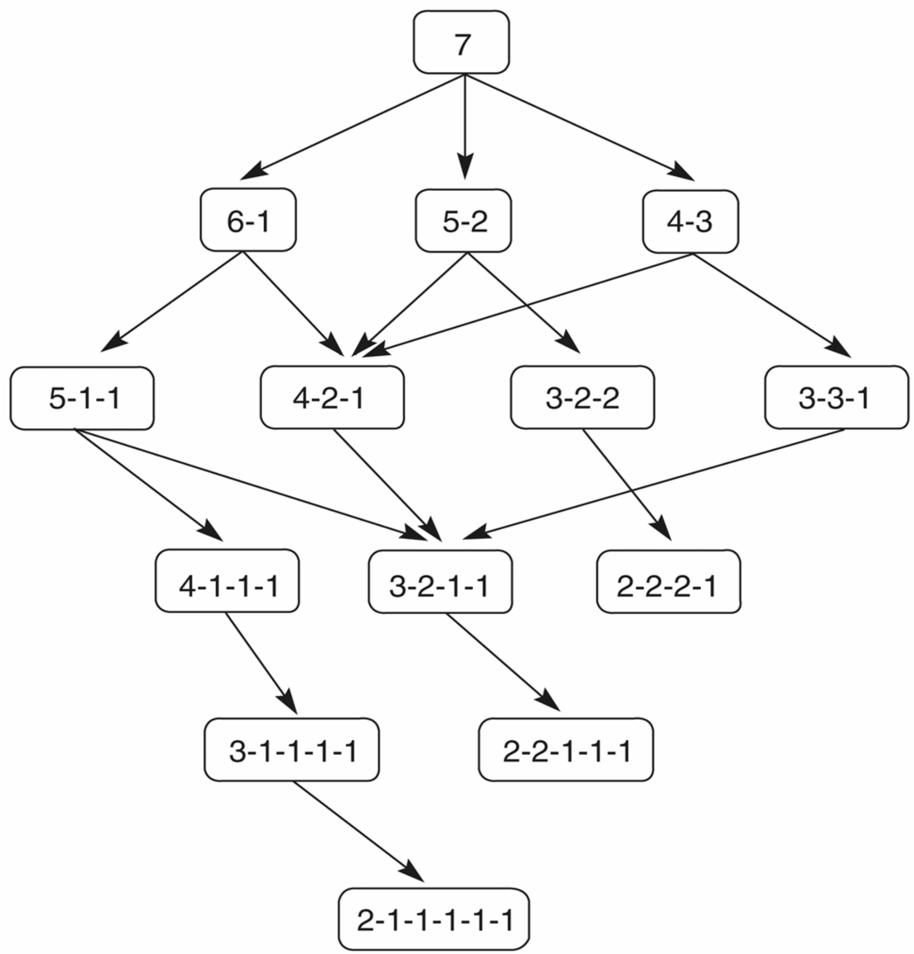
- Where we are today

Example: Game of Nim

- Rules
  - 2 players start with a pile of tokens
  - move: split (any) existing pile into two **non-empty differently-sized** piles
  - game ends when no pile can be unevenly split
  - player who cannot make his move loses

State Space of Game Nim



- start with one pile of tokens
- each step has to divide one pile of tokens into 2 non-empty piles of different size
- player without a move left loses game

source: *G. Luger (2005)*

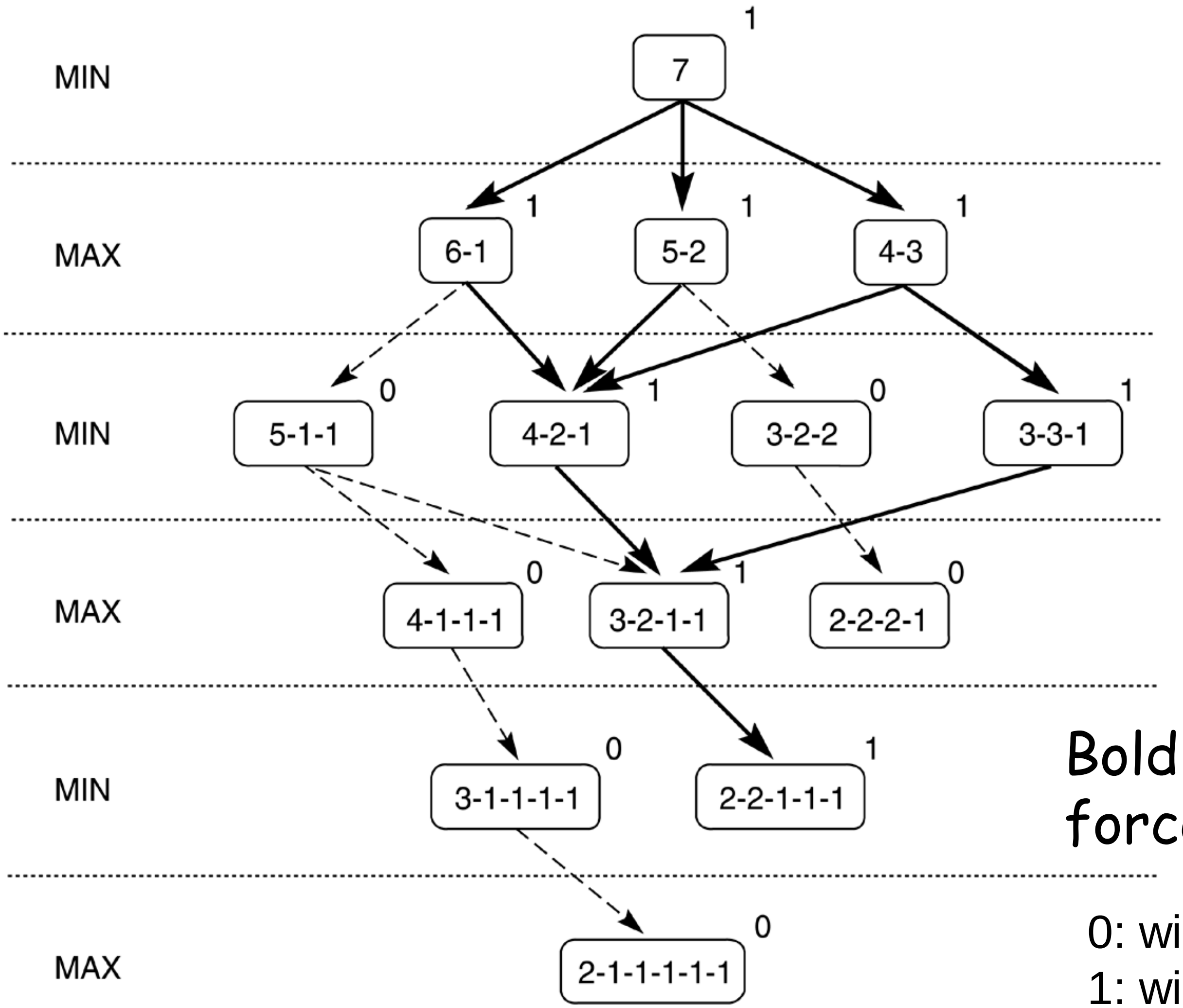
MiniMax Search

- Game between two opponents, MIN and MAX
  - MAX tries to win, and
  - MIN tries to minimize MAX's score
- Existing heuristic search methods do not work
  - would require a helpful opponent
  - Need to incorporate "hostile" moves into search strategy

Exhaustive MiniMax Search

- For small games where exhaustive search is feasible
- Procedure:
  1. build complete game tree
  2. label each level according to player's turn (MAX or MIN)
  3. label leaves with a utility function to determine the outcome of the game
     - e.g., (0, 1) or (-1, 0, 1)
  4. propagate this value up:
     - if parent=MAX, give it max value of children
     - if parent=MIN, give it min value of children
  5. Select best next move for player at root as the move leading to the child with the highest value (for MAX) or lowest values (for MIN)

Exhaustive MiniMax for Nim



Bold lines indicate forced win for MAX

0: win for MIN
1: win for MAX

source: G. Luger (2005)

n-ply MiniMax with Heuristic

- Exhaustive search for interesting games is rarely feasible
- Search only to predefined level
  - called n-ply look-ahead
  - n is number of levels
- No exhaustive search
  - nodes evaluated with heuristics and not win/loss
  - indicates best state that can be reached
  - horizon effect
- Games with opponent
  - simple strategy: try to maximize difference between players using a heuristic function *e(n)*

Heuristic Function for 2-player games

- simple strategy:
  - try to maximize difference between MAX's game and MIN's game
- typically called *e(n)*
- *e(n)* is a heuristic that estimates how favorable a node n is for MAX
  - $e(n) > 0$ --> n is favorable to MAX
  - $e(n) < 0$ --> n is favorable to MIN
  - $e(n) = 0$ --> n is neutral

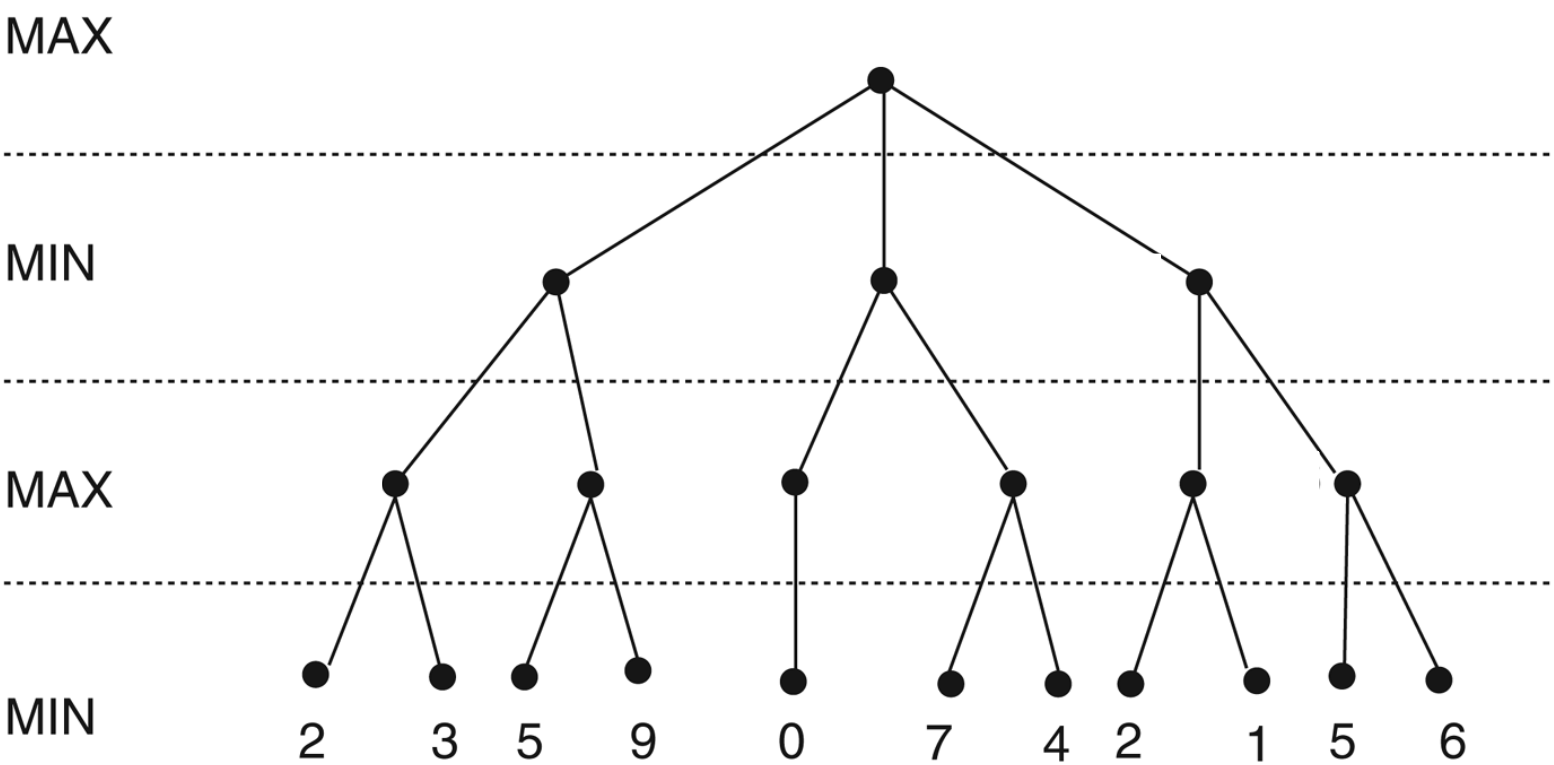
Choosing a Heuristic Function e(n)

- Usually e(n) is a weighted sum of various features:

$$e(n) = \sum w_i \, f_i(n)$$

- E.g. of features:
  - f_1 = number of pieces left on the game for MAX
  - f_2 = number of possible moves left for MAX
  - f_3 = -(number of pieces left on the game for MIN)
  - f_4 = -(number of possible moves left for MIN)
- E.g. of weights:
  - $w_1 = 0.5$ // f_1 is a very important feature
  - $w_2 = 0.2$ // f_2 is not very important
  - $w_3 = 0.2$ // f_3 is not very important
  - $w_4 = 0.1$ // f_4 is really not important

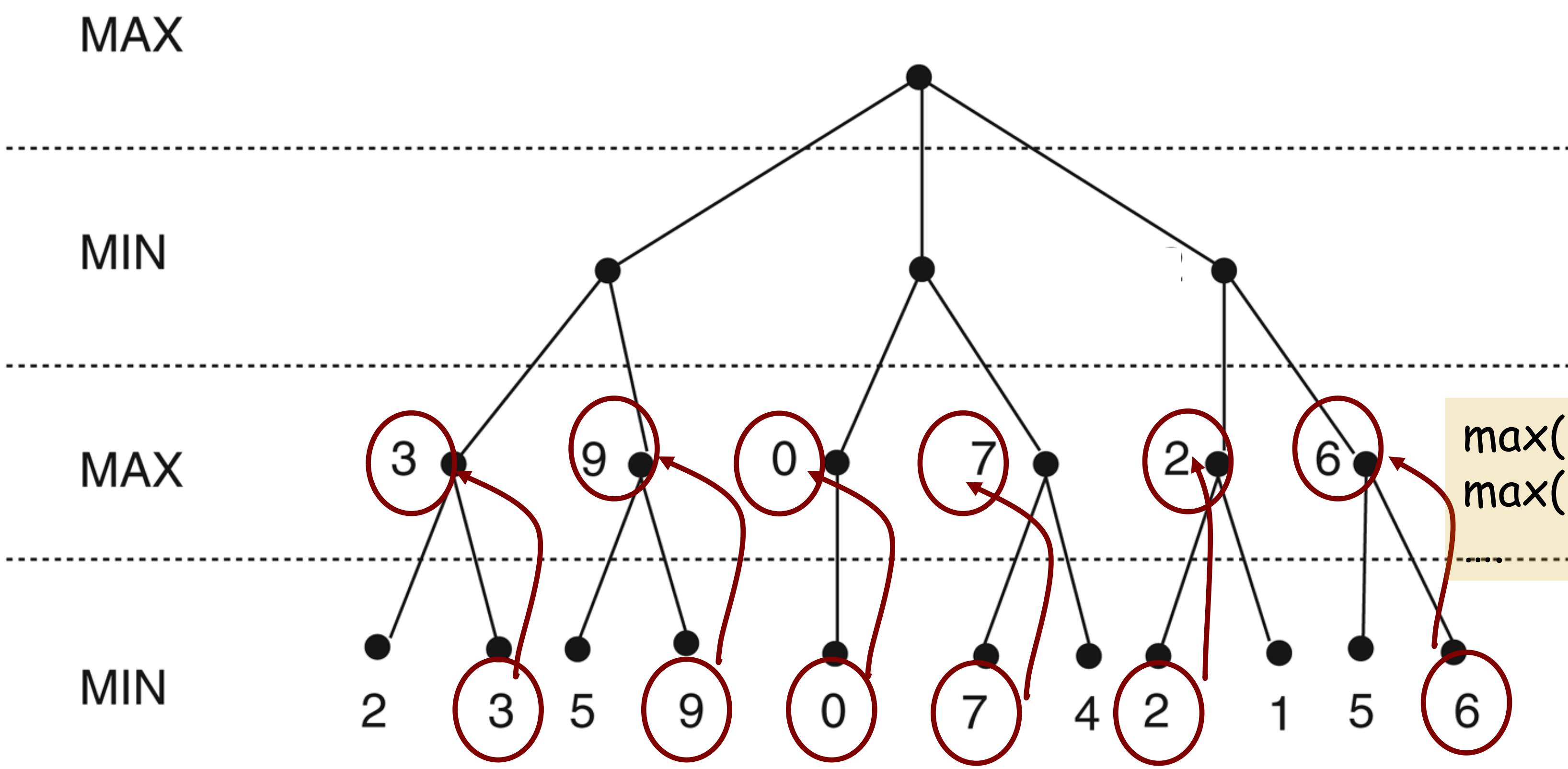
MiniMax with Fixed Ply Depth



Leaf nodes show the actual heuristic value *e(n)*

source: *G. Luger (2005)*

MiniMax with Fixed Ply Depth

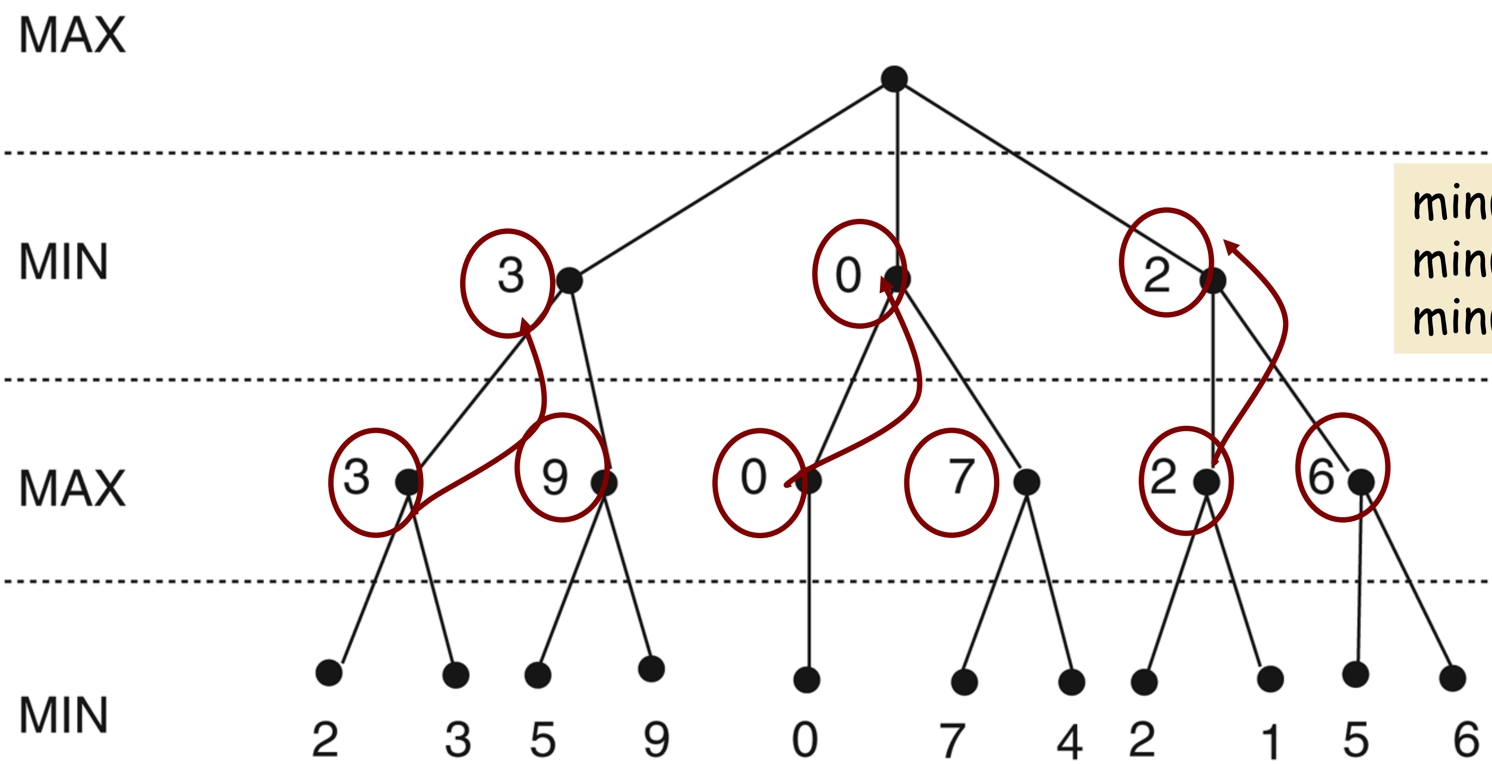


max(2,3) = 3
max(5,9) = 9
....

Leaf nodes show the actual heuristic value *e(n)*
Internal nodes show <u>back-up</u> heuristic value

source: G. Luger (2005)

MiniMax with Fixed Ply Depth



min(3,9) = 3
min(0,7) = 9
min(2,6) = 2

Leaf nodes show the actual heuristic value *e(n)*
Internal nodes show <u>back-up</u> heuristic value

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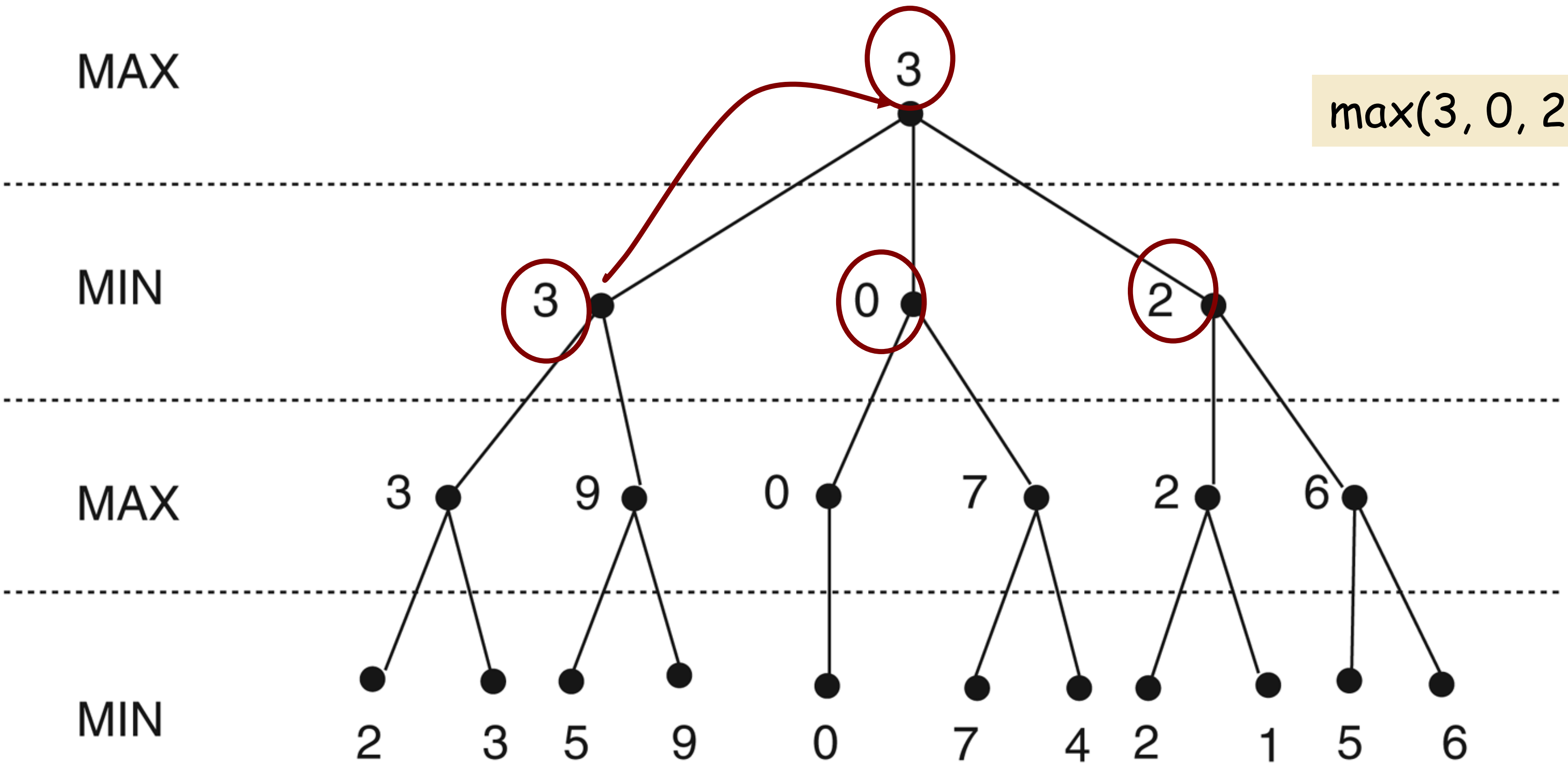
MiniMax with Fixed Ply Depth

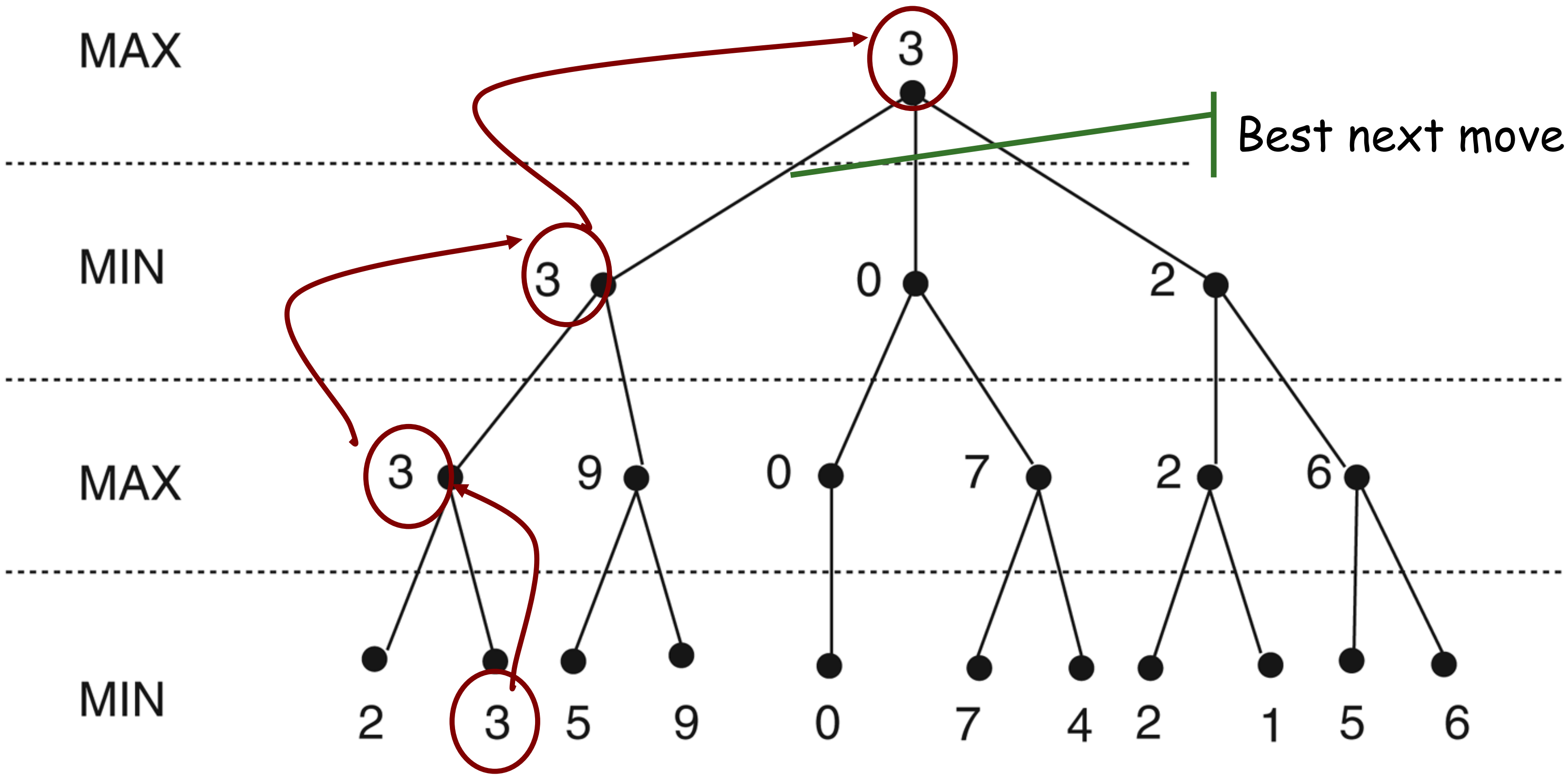


max(3, 0, 2) = 3

Leaf nodes show the actual heuristic value e(n)
Internal nodes show back-up heuristic value

source: G. Luger (2005)

MiniMax with Fixed Ply Depth



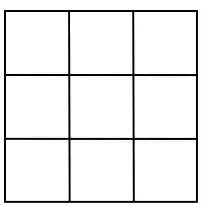
Leaf nodes show the actual heuristic value *e(n)*
Internal nodes show <u>back-up</u> heuristic value

source: G. Luger (2005)

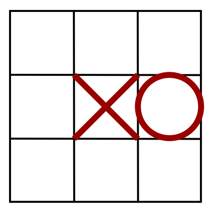
Example: *e(n)* for Tic-Tac-Toe

- Possible *e(n)*

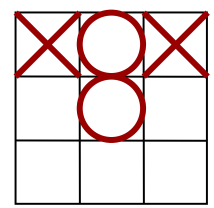
$$e(n) = \begin{cases} \text{number of rows, columns, and diagonals open for MAX} \\ \qquad \text{- number of rows, columns, and diagonals open for MIN} \\ + \quad \text{, if } n \text{ is a forced win for MAX} \\ - \quad \text{, if } n \text{ is a forced win for MIN} \end{cases}$$



$e(n) = 8 – 8 = 0$

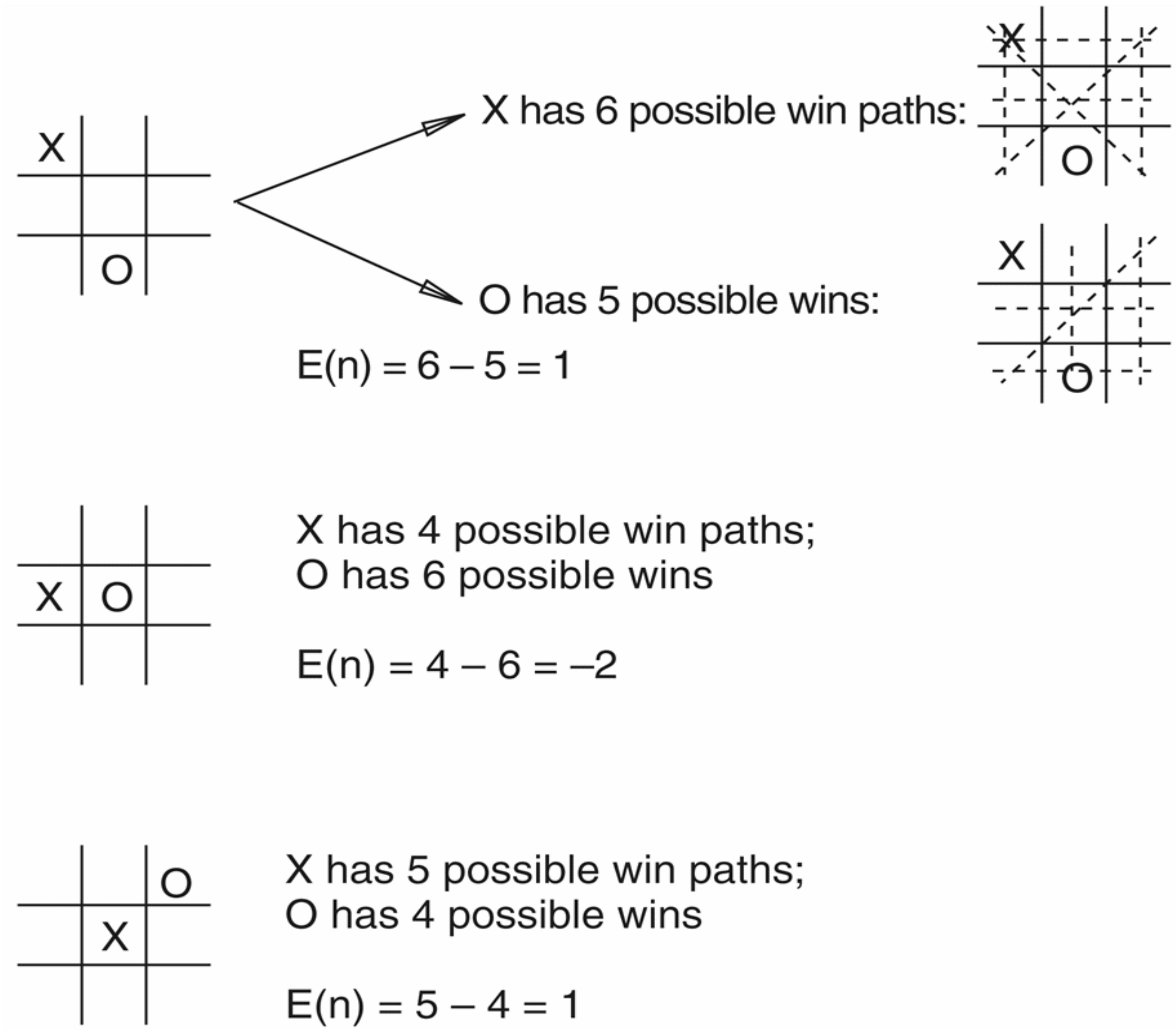


$e(n) = 6 – 4 = 2$



$e(n) = 3 – 3 = 0$

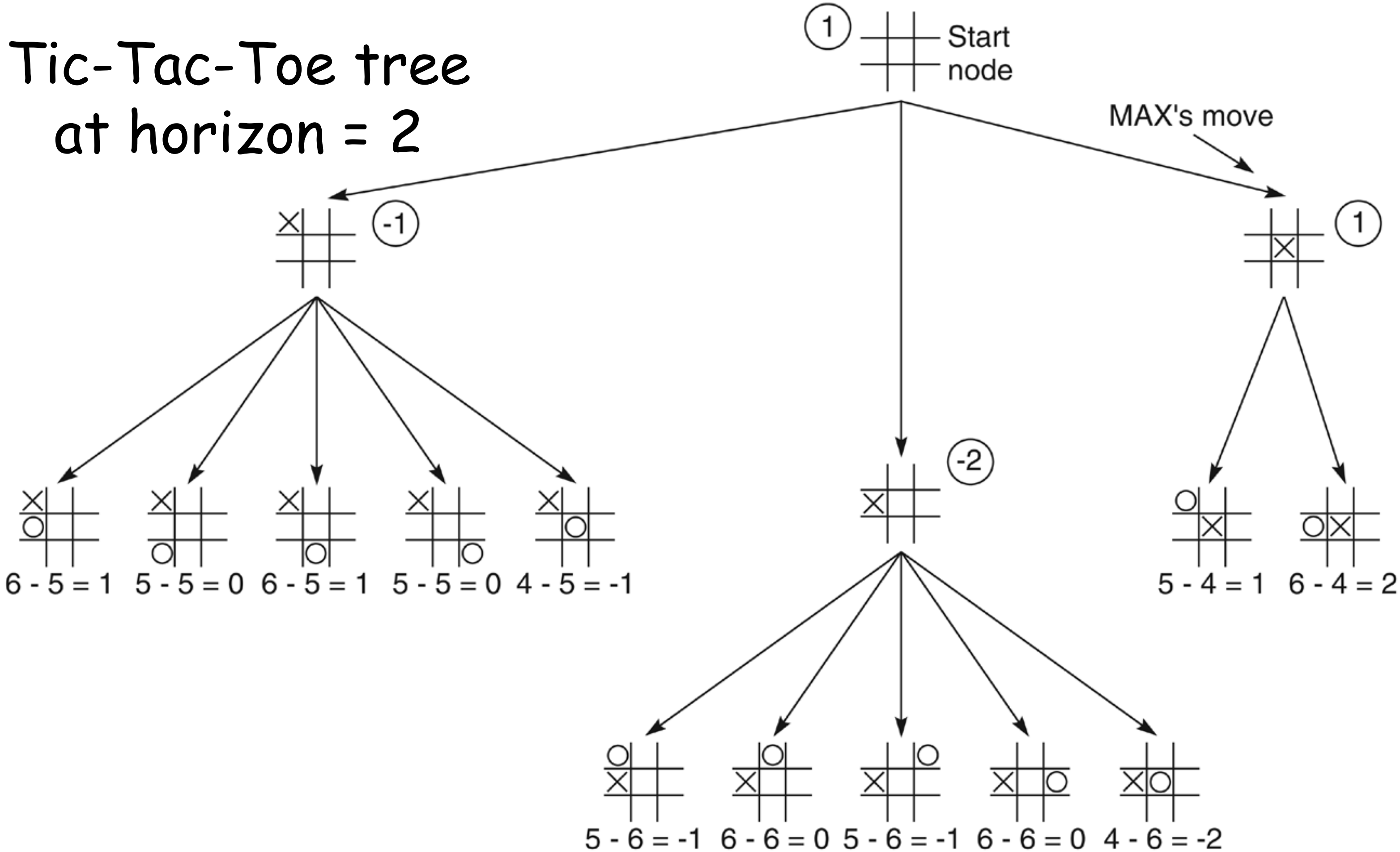
More examples…



source: G. Luger (2005)

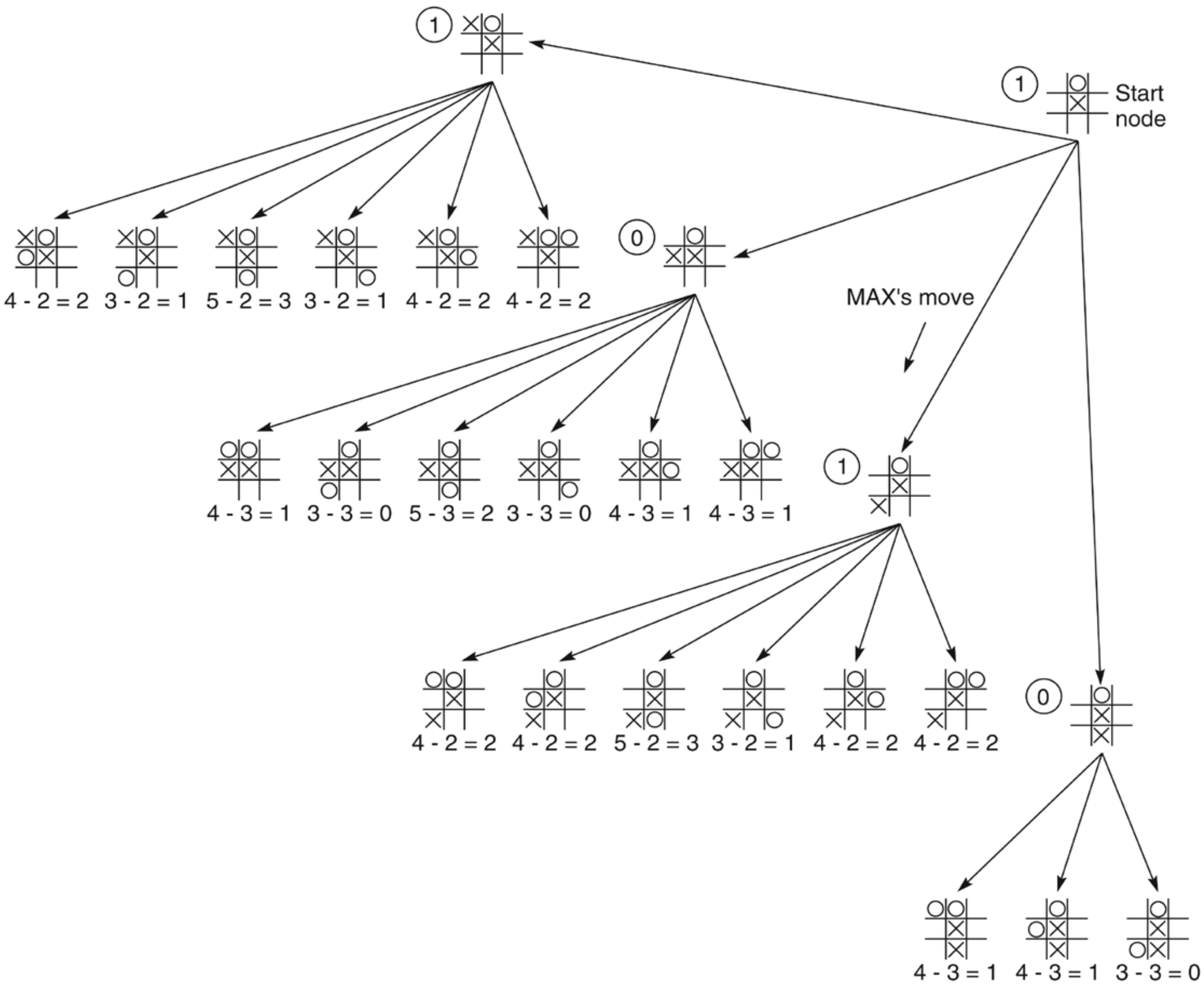
Two-ply MiniMax for Opening Move

Tic-Tac-Toe tree
at horizon = 2



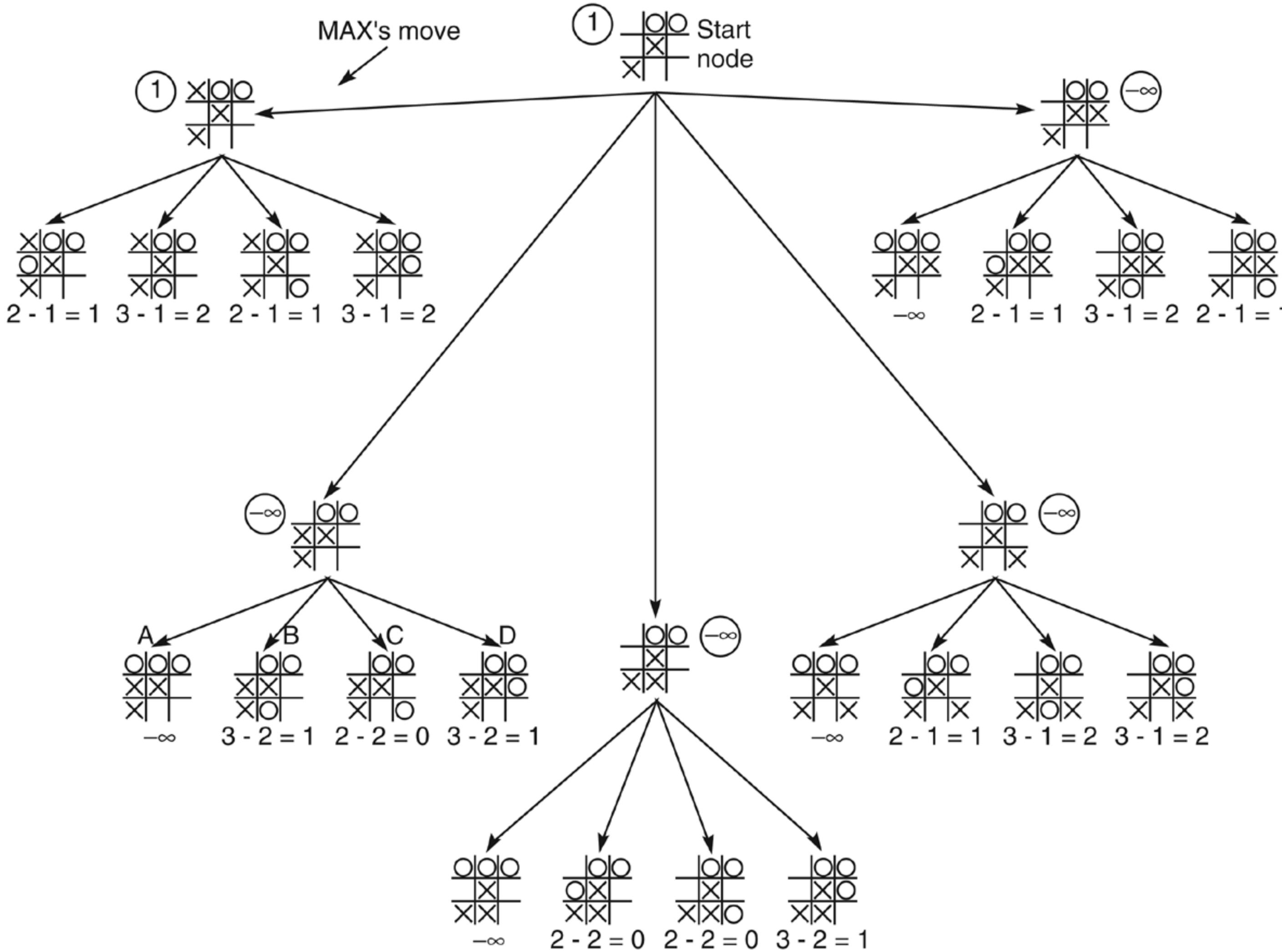
source: G. Luger (2005)

Two-ply MiniMax: MAX's possible 2nd moves



source: G. Luger (2005)

Two-ply minimax: MAX's move at end



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Alpha-Beta Pruning

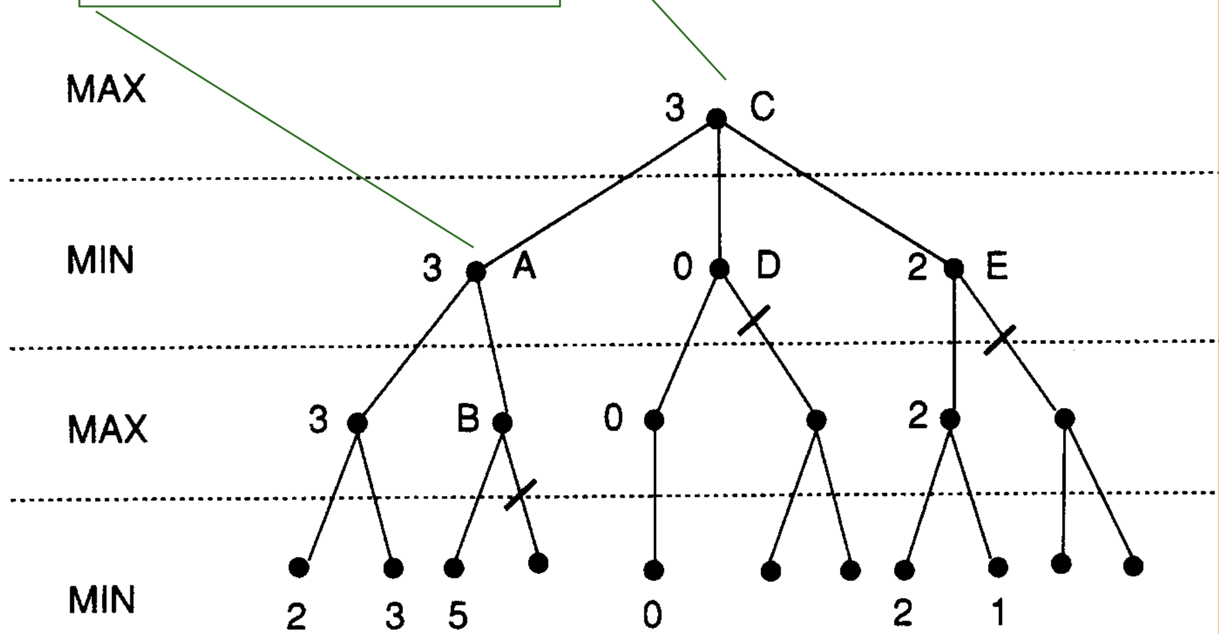
- Optimization over MiniMax, that:
  - ignores (cuts off, prunes) branches of the tree that cannot possibly lead to a better solution
  - reduces branching factor
  - allows deeper search with same effort

Alpha-Beta Pruning: Example 1

- With MiniMax, we look at all possible nodes at the *n*-ply depth
- With α-β pruning, we ignore branches that could not possibly contribute to the final decision

A=min(3, max(5,?))

C=max(3, min(0,?), min(2,?))



B will be >= 5

So we can ignore B's right branch, because A must be 3

D will be <= 0

But C will be >= 3

So we can ignore D's right branch

E will be <= 2.

So we can ignore E's right branch

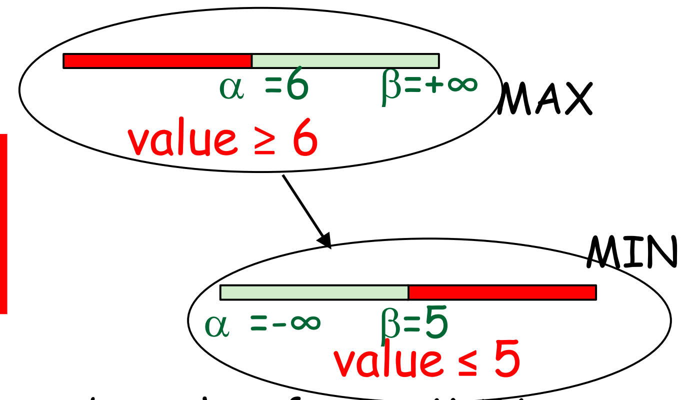
Because C will be 3.

source: G. Luger (2005)

Alpha-Beta Pruning Algorithm

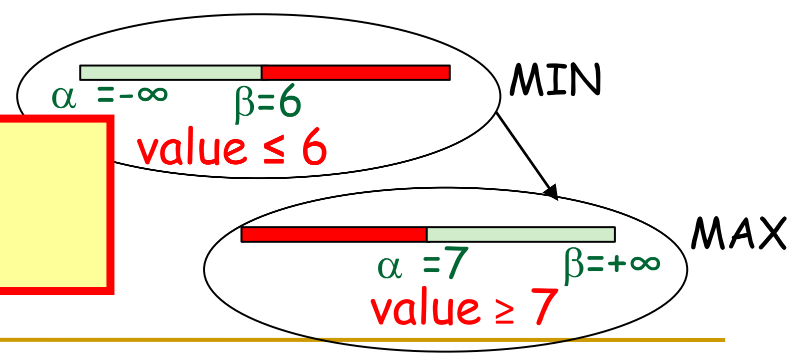
- α : lower bound on the final backed-up value.
- β : upper bound on the final backed-up value.
- Alpha pruning:
  - eg. if MAX node's α = 6, then the search can prune branches from a MIN descendant that has a β <= 6.
  - if child β <= ancestor α $\rightarrow$ prune

> incompatible…
> so stop searching the right branch;
> the value cannot come from there!



- Beta pruning:
  - eg. if a MIN node's β = 6, then the search can prune branches from a MAX descendant that has an α >= 6.
  - if ancestor β <= child α $\rightarrow$ prune

> incompatible…
> so stop searching the right branch;
> the value cannot come from there!



Alpha-Beta Pruning Algorithm

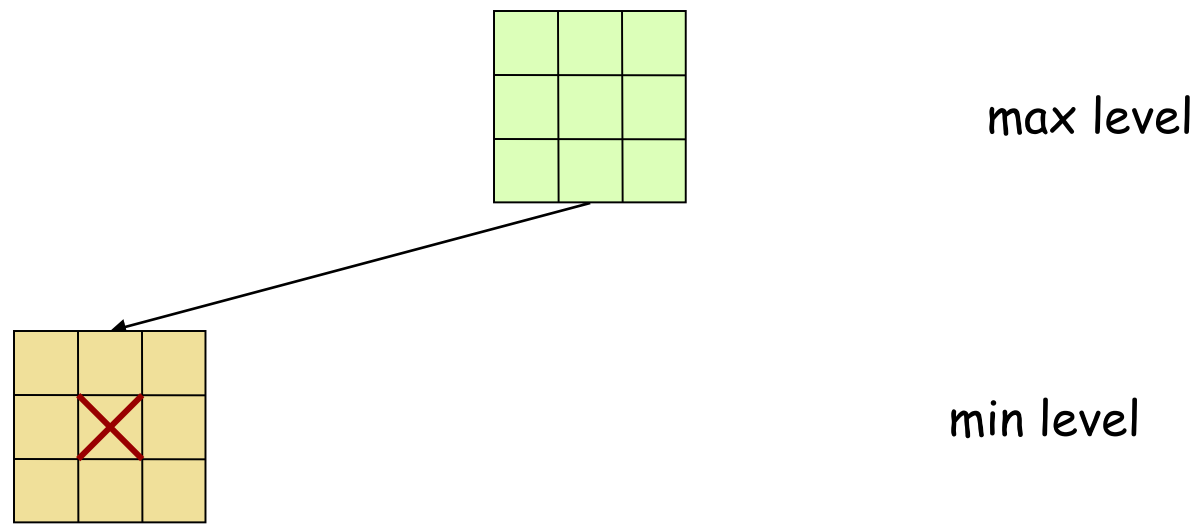
```
function alphabeta(node, depth, α, β, maximizingPlayer)
     if depth = 0 or node is a terminal node
         return the heuristic value of node
     if maximizingPlayer
         v := -∞
         for each child of node
             v := max(v, alphabeta(child, depth - 1, α, β, FALSE))
             α := max(α, v)
             if β ≤ α
                 break (* β cut-off *)
         return v
     else
         v := ∞
         for each child of node
             v := min(v, alphabeta(child, depth - 1, α, β, TRUE))
             β := min(β, v)
             if β ≤ α
                 break (* α cut-off *)
         return v
```

Initial call:

`alphabeta(origin, depth, -∞, +∞, TRUE)`

source: http://en.wikipedia.org/wiki/Alpha%E2%80%93beta_pruning

Example with tic-tac-toe



source: robotics.stanford.edu/~latombe/cs121/2003/home.htm

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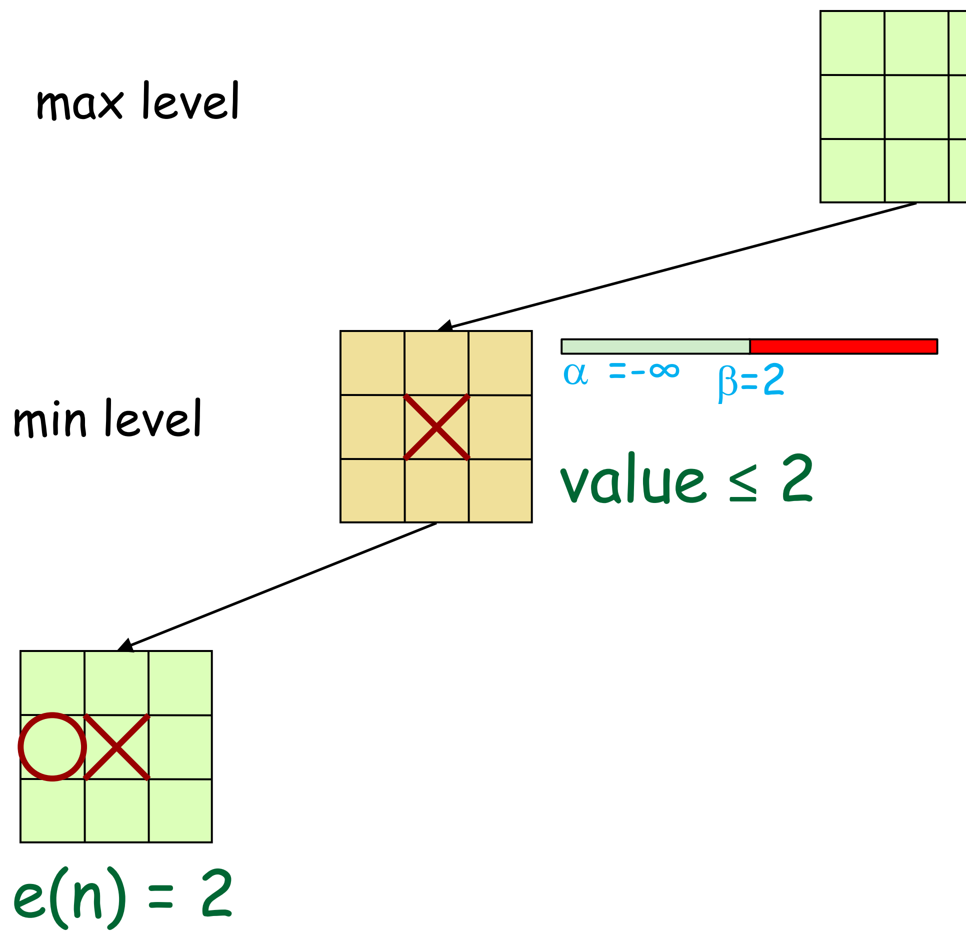

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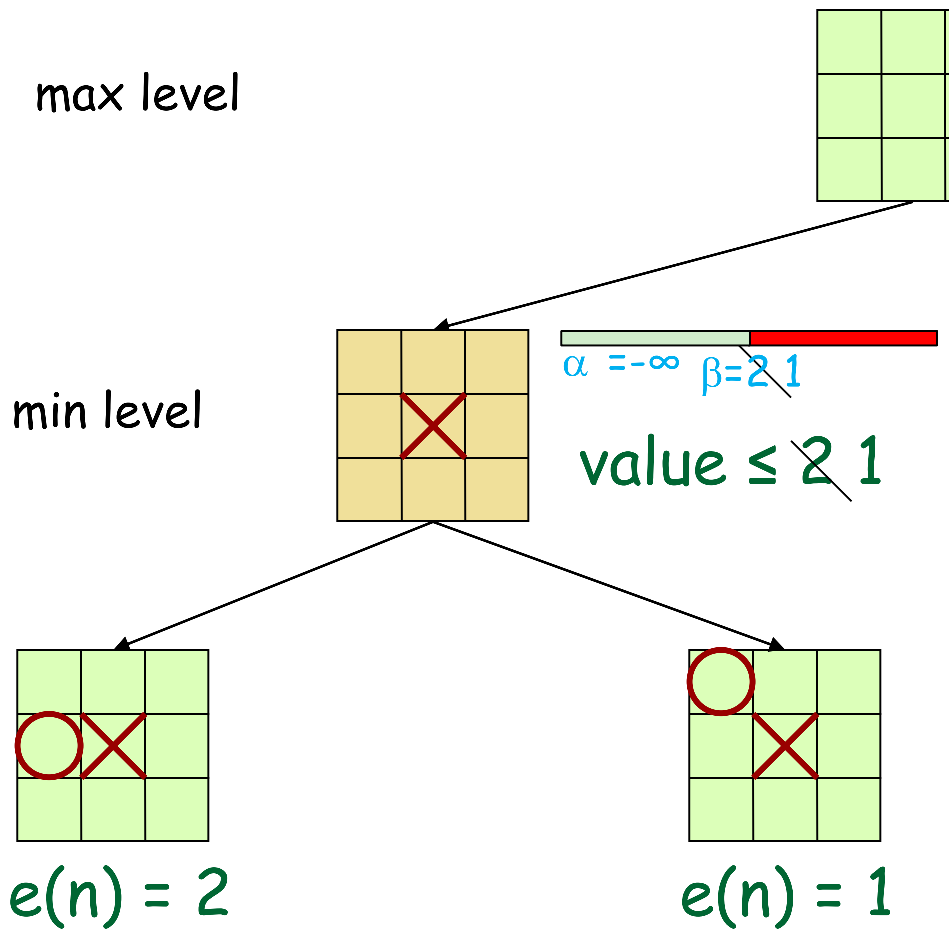

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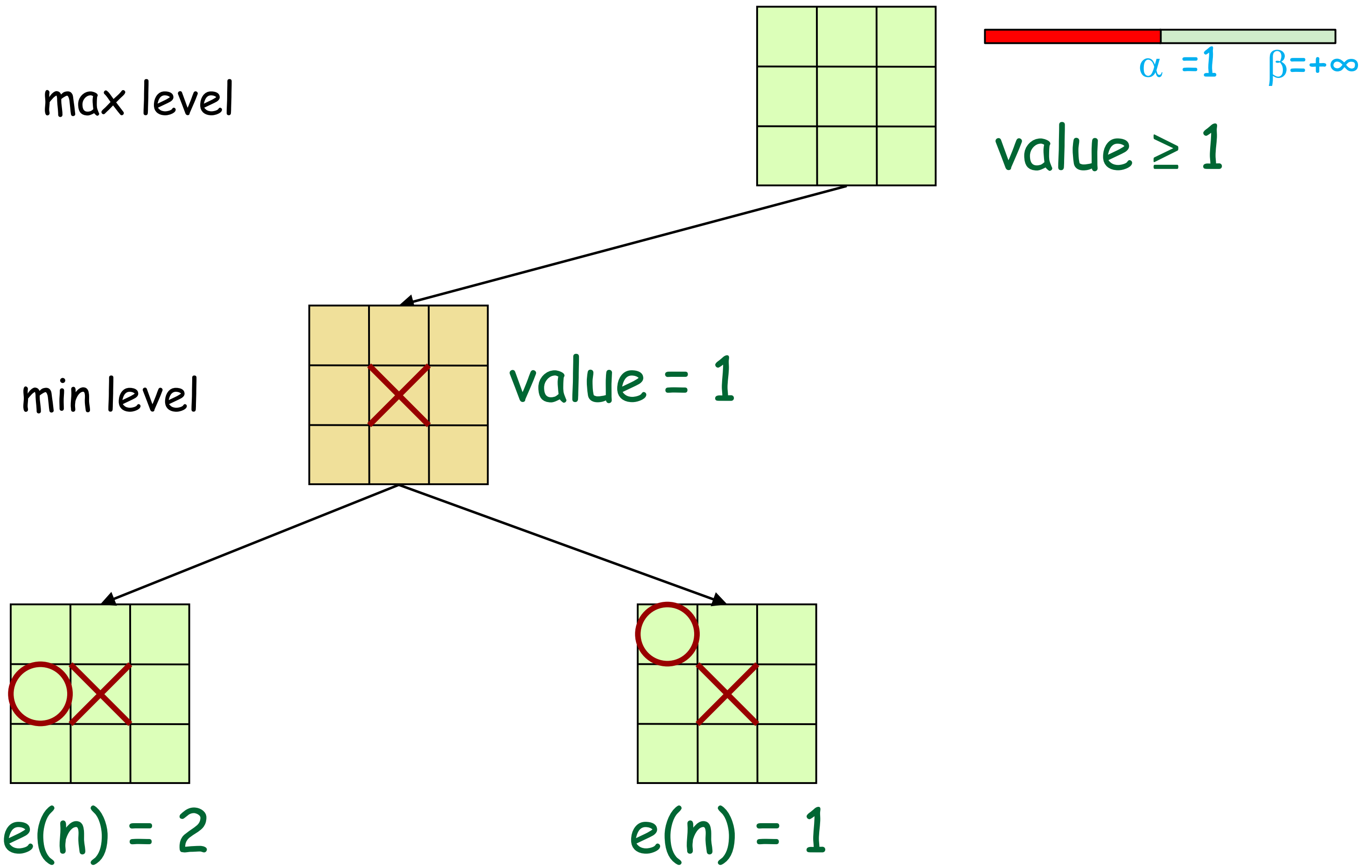

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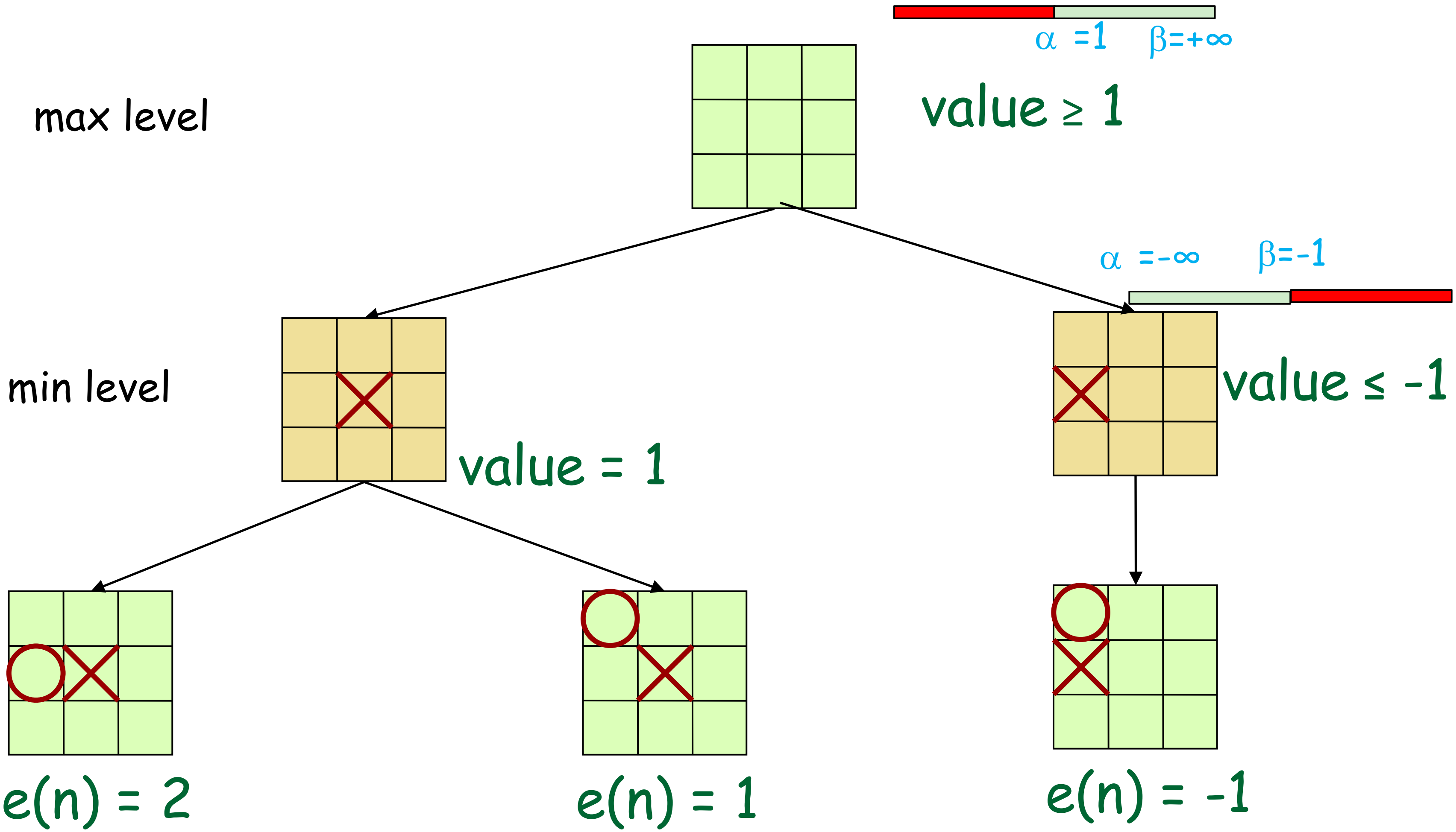

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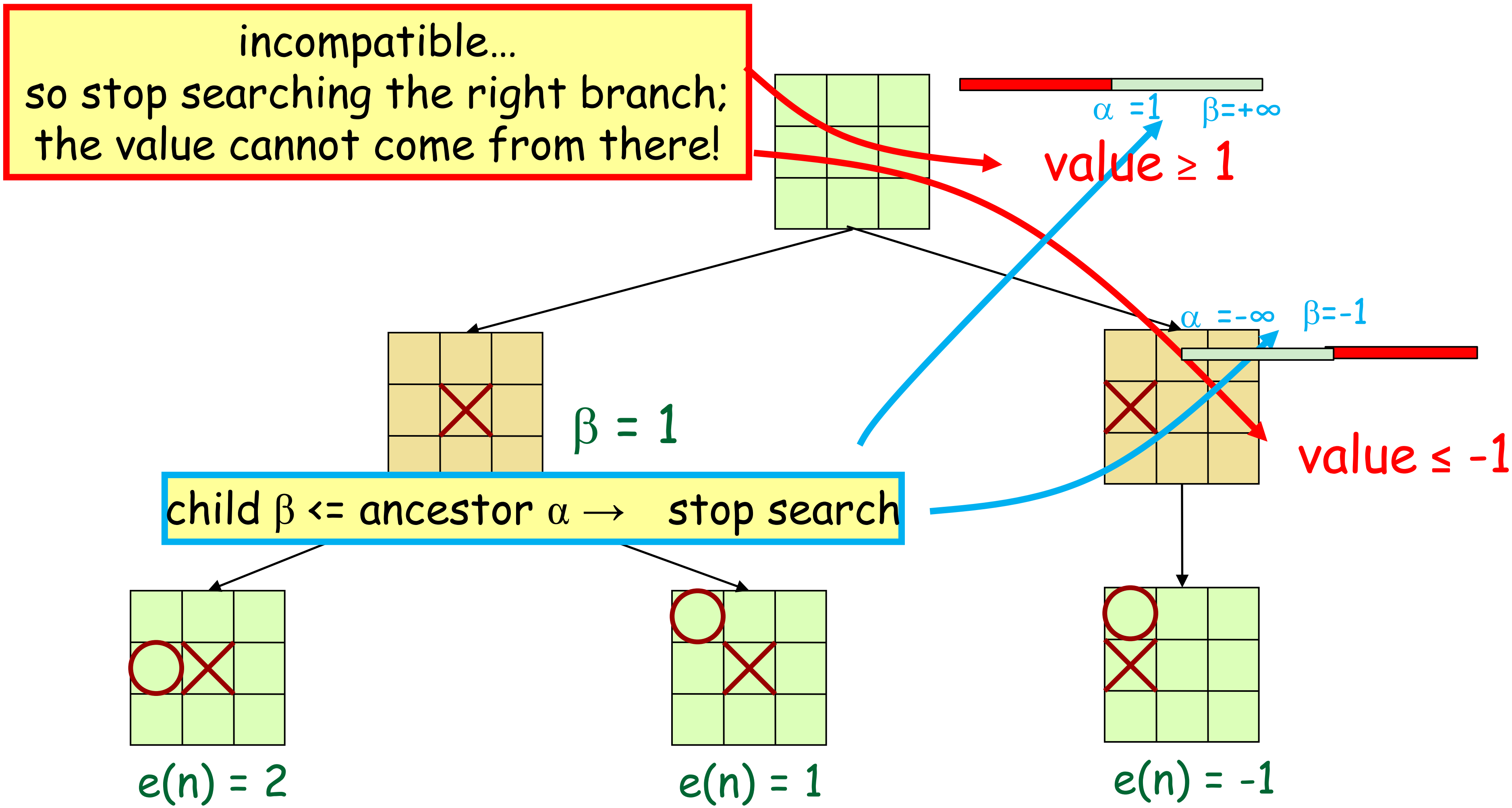

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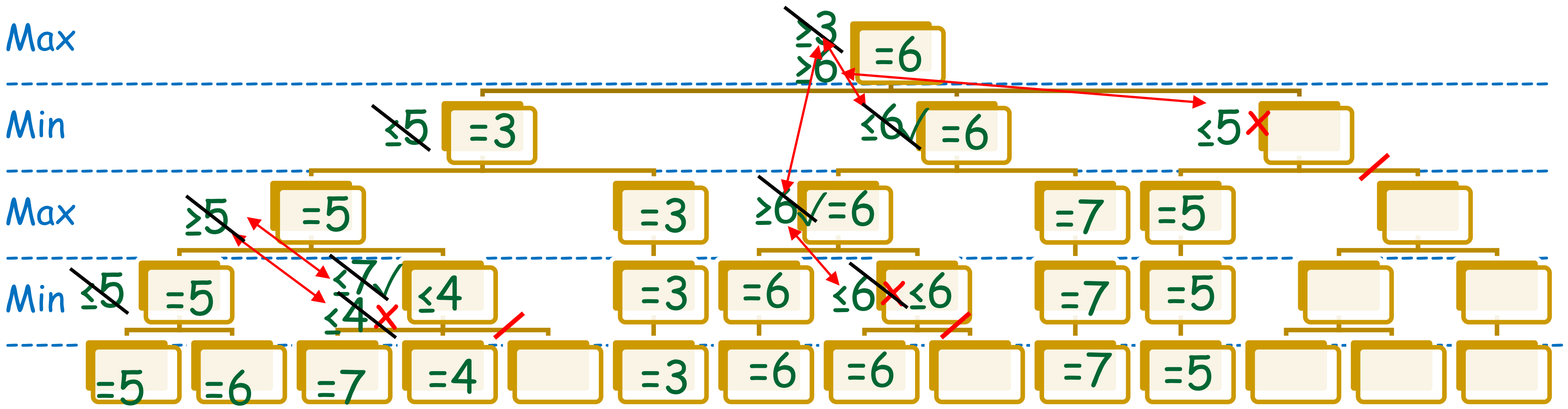
Alpha-Beta Pruning: Example 2



source: http://en.wikipedia.org/wiki/File:AB_pruning.svg

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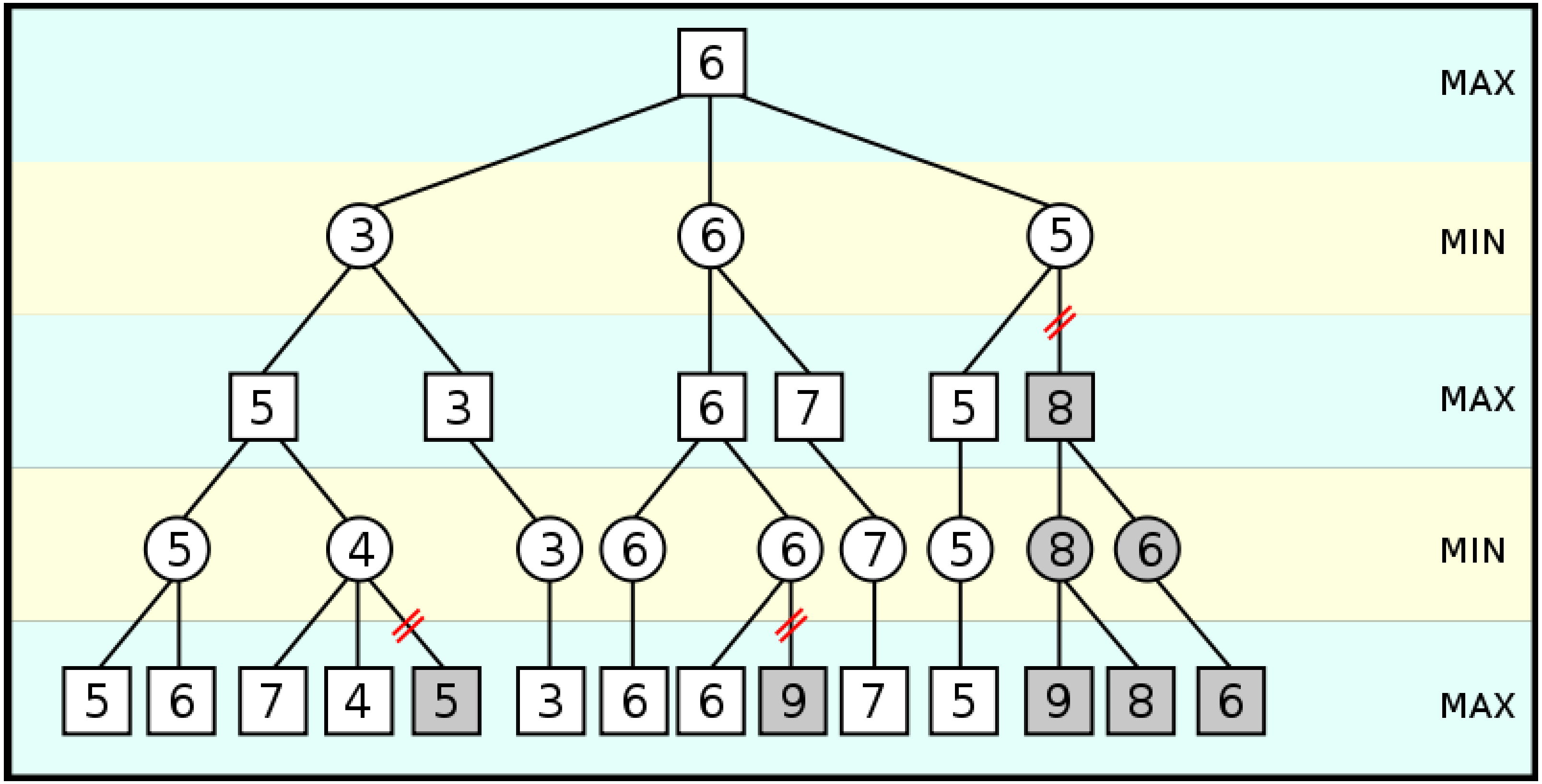

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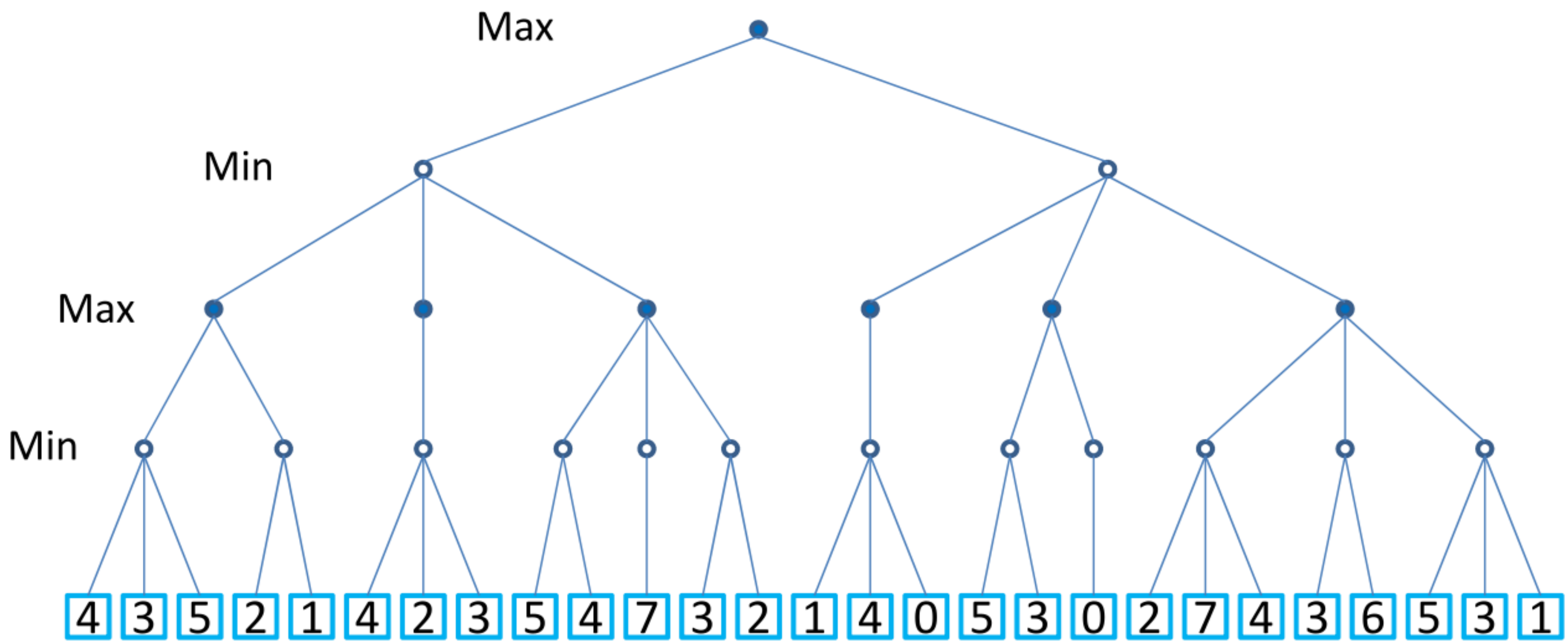
Alpha-Beta Pruning: Example 3



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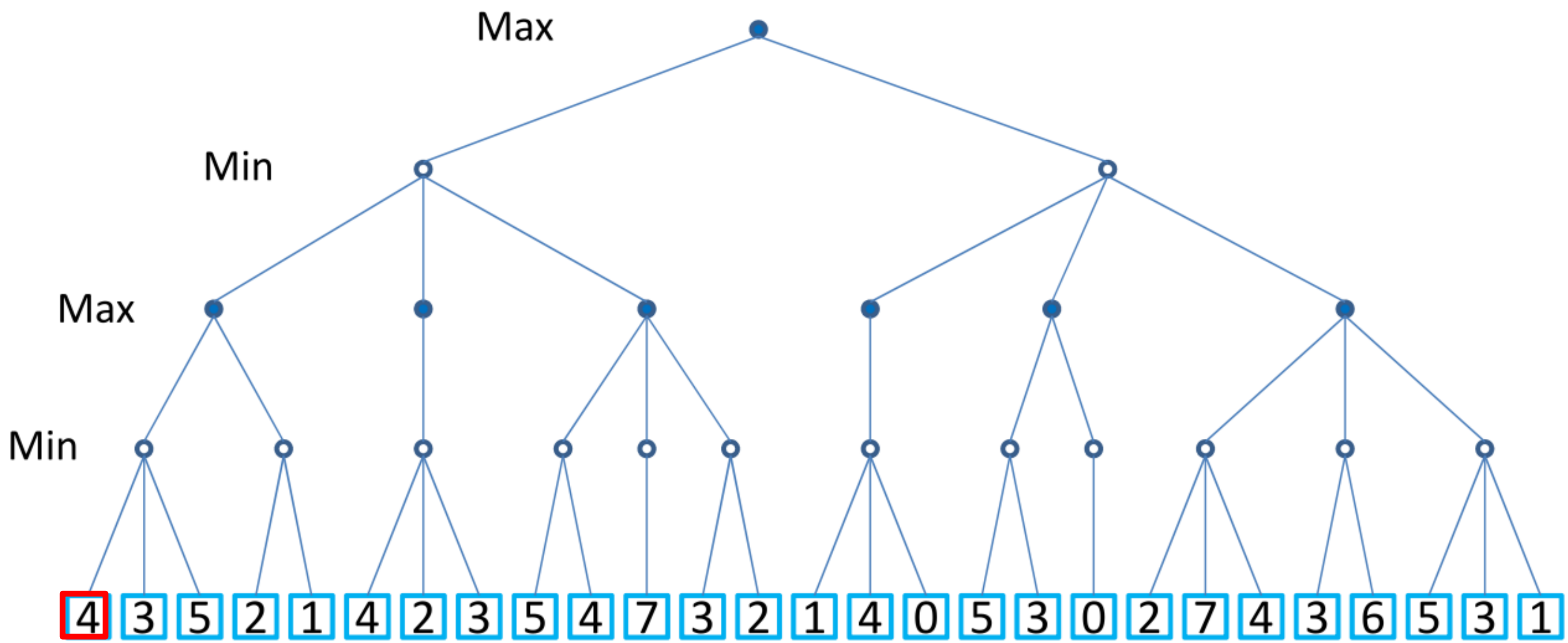

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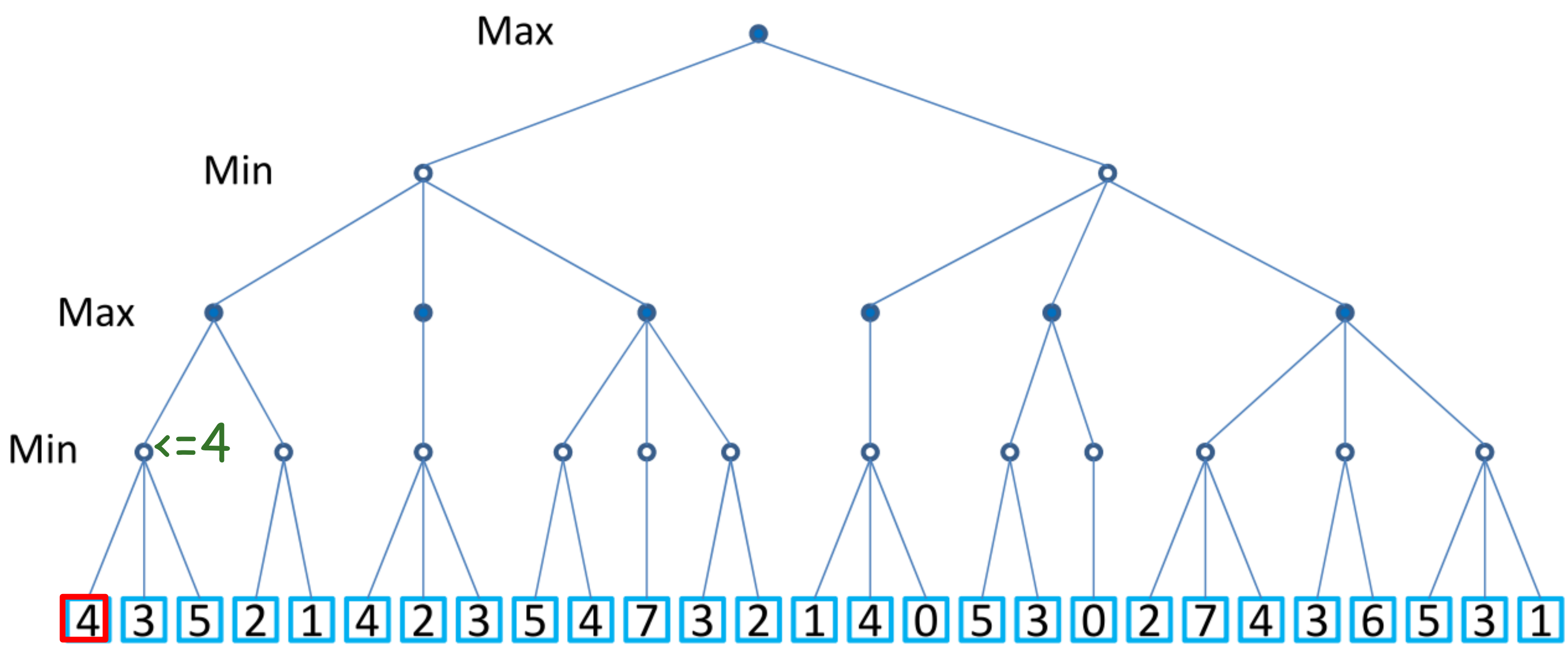

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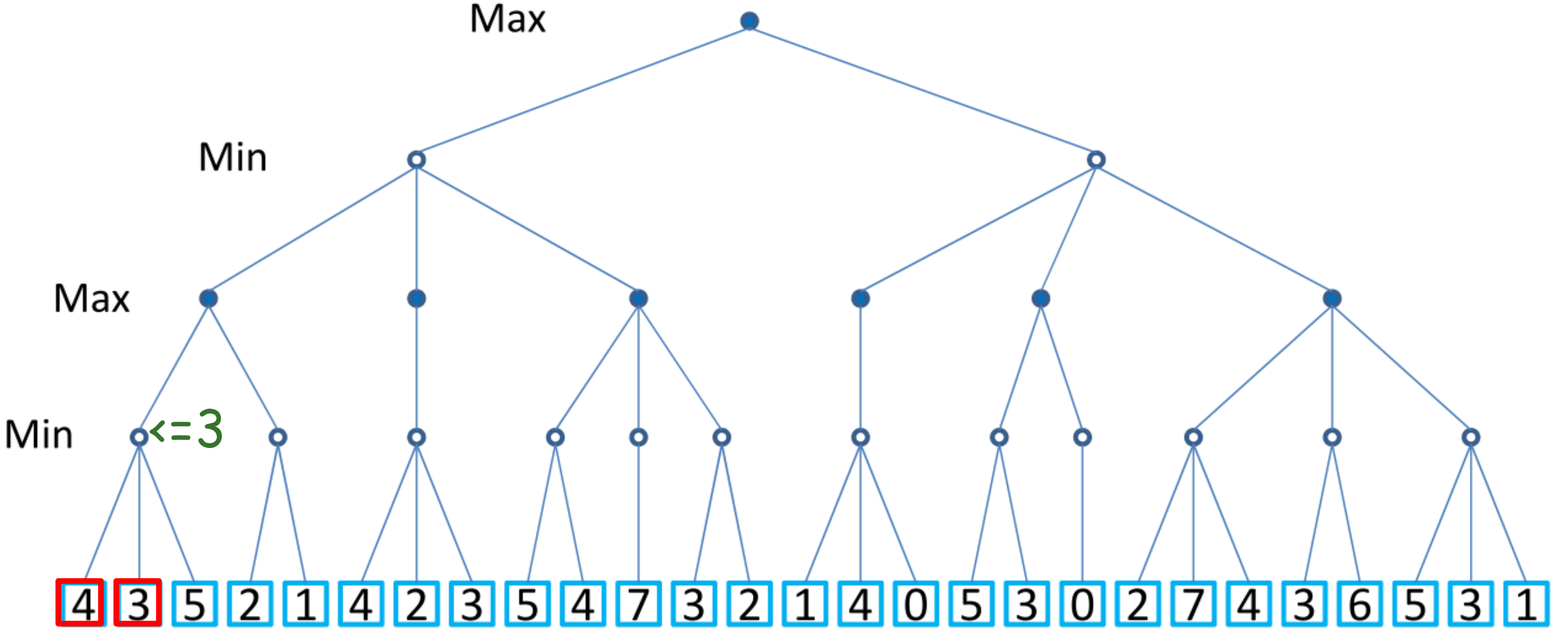

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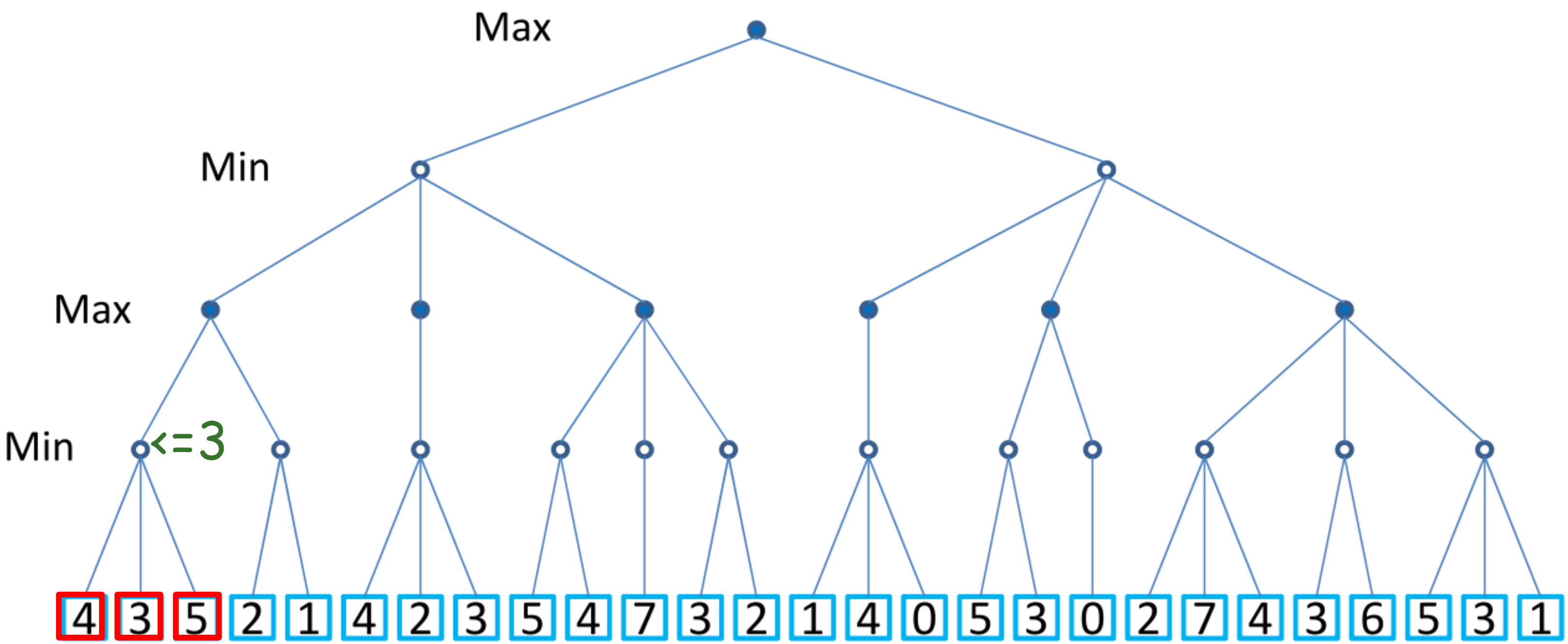

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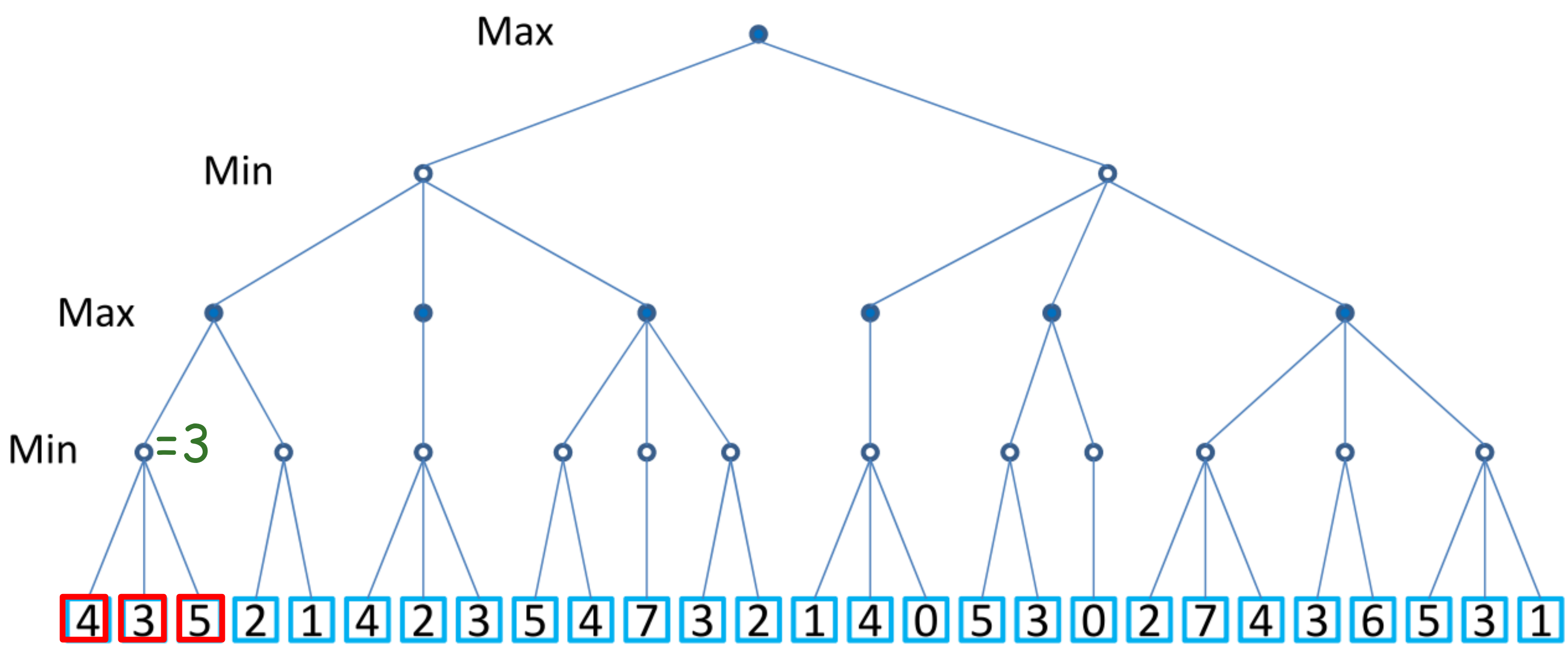

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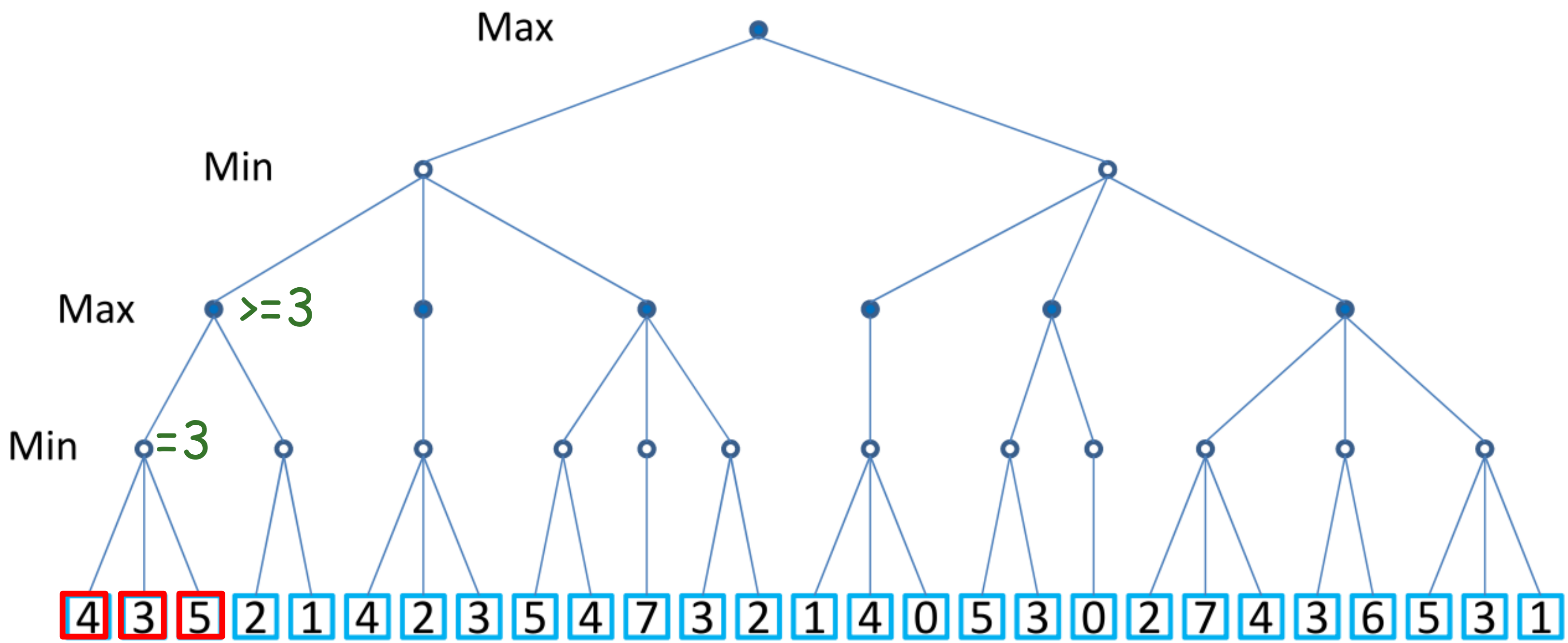

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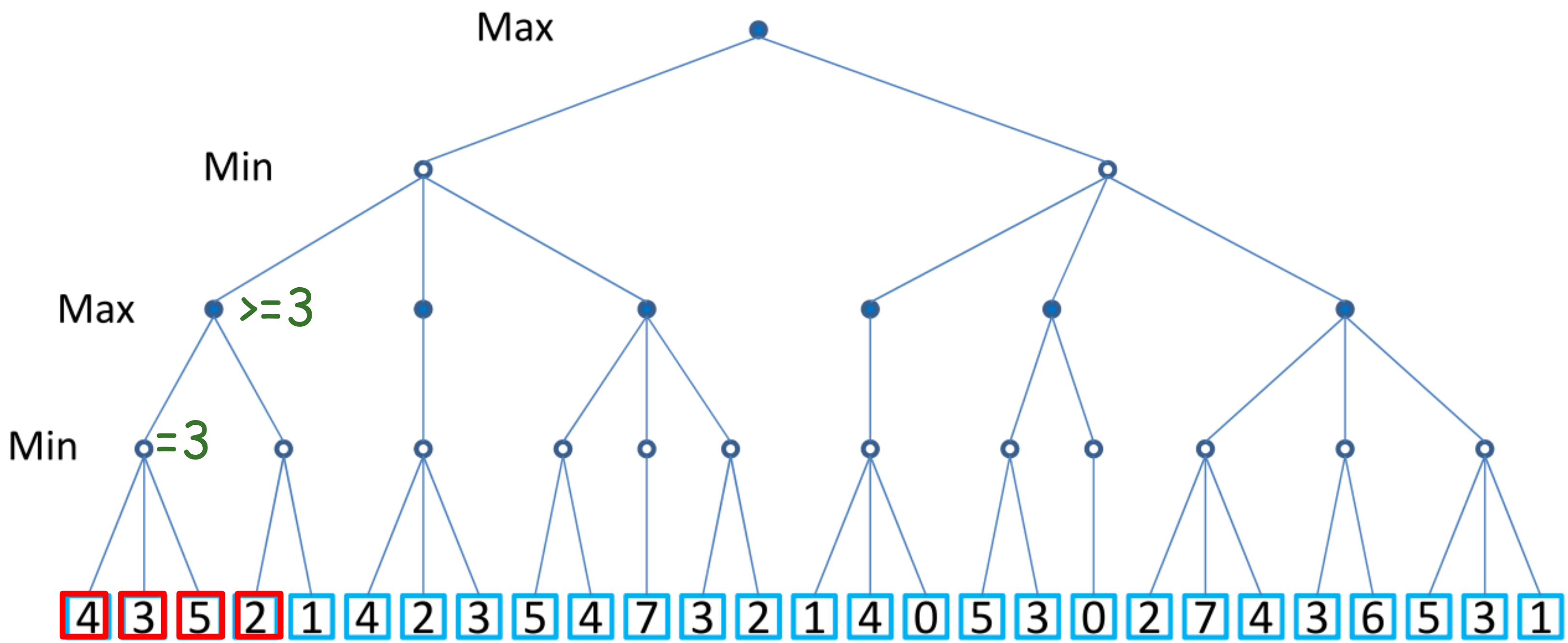

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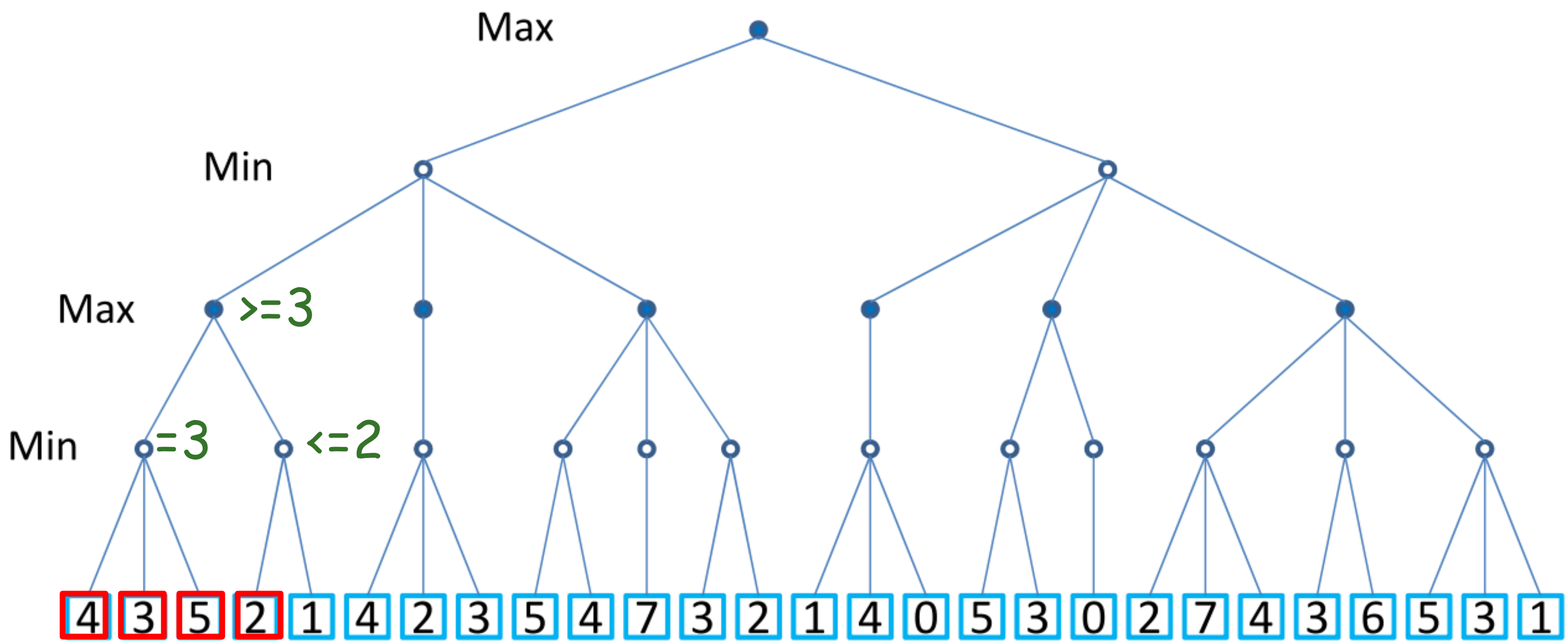

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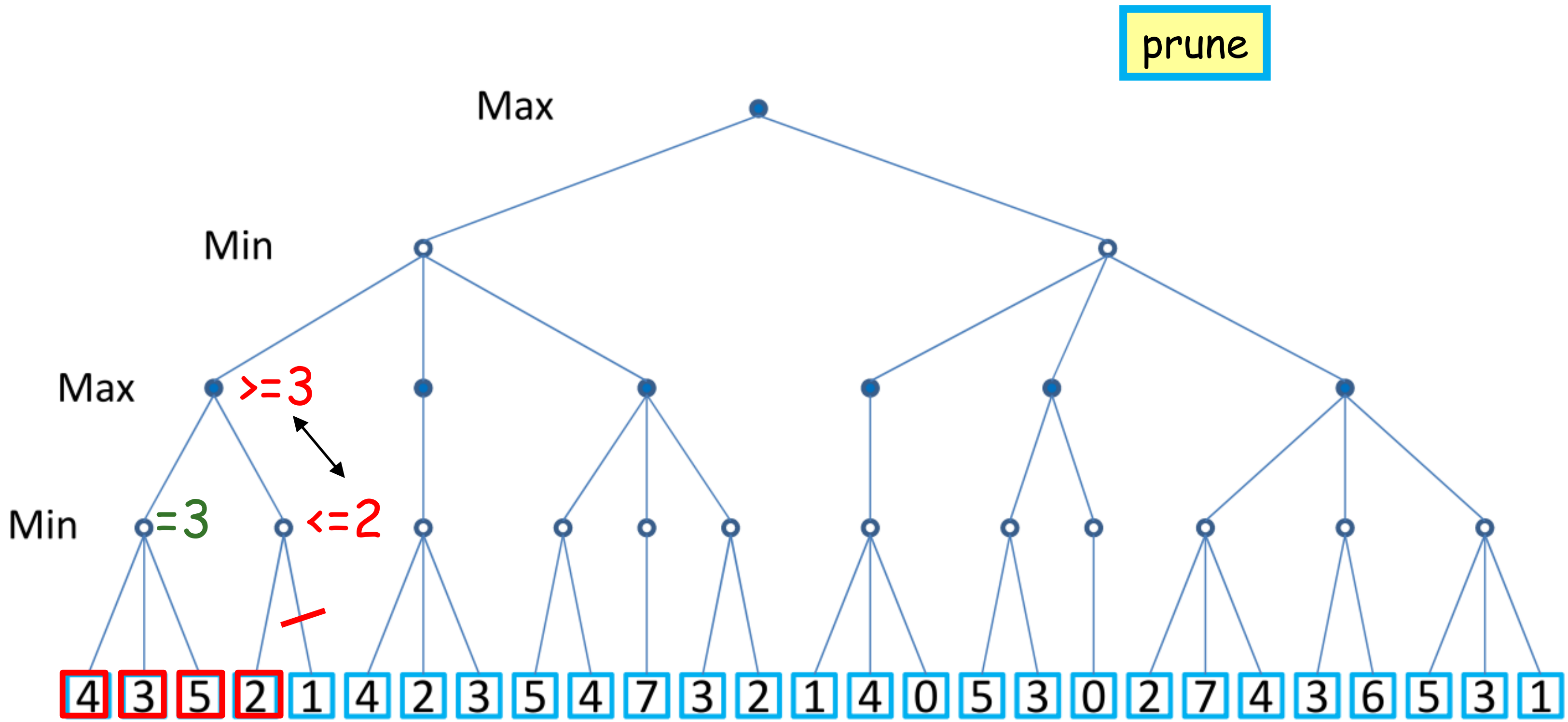

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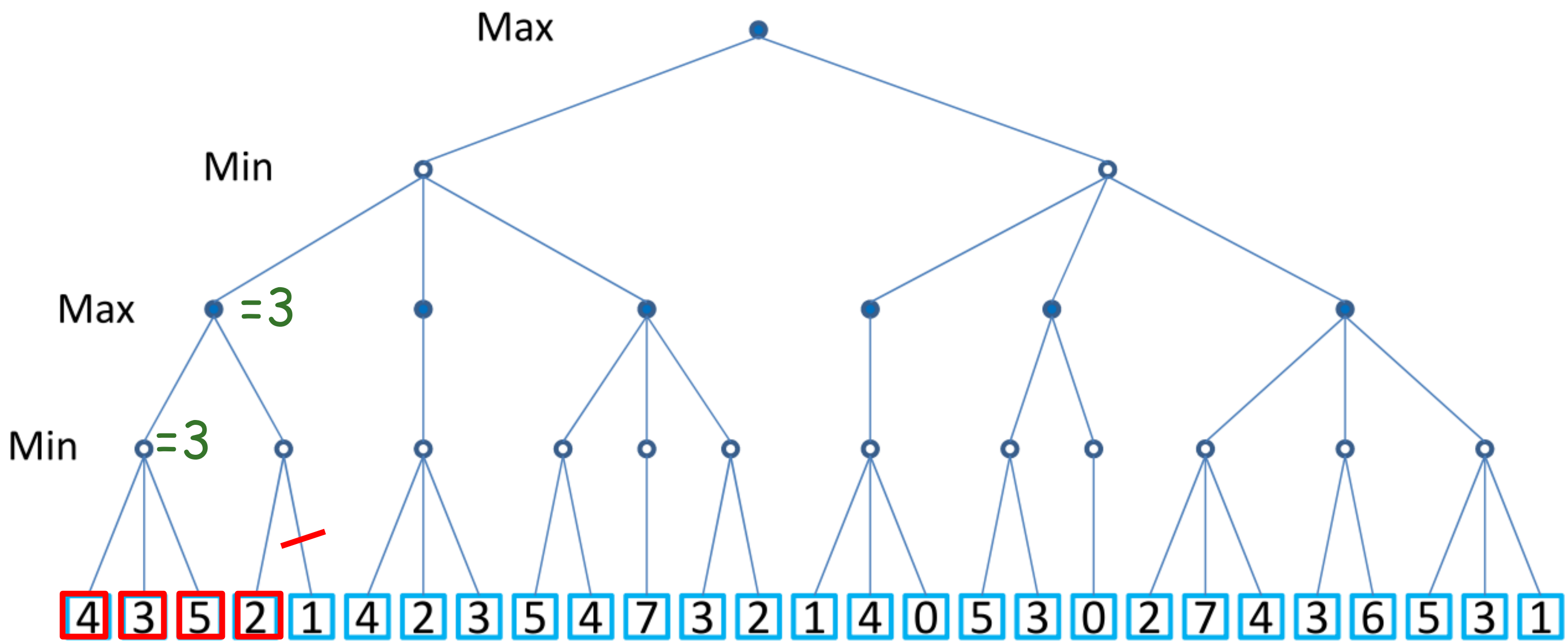

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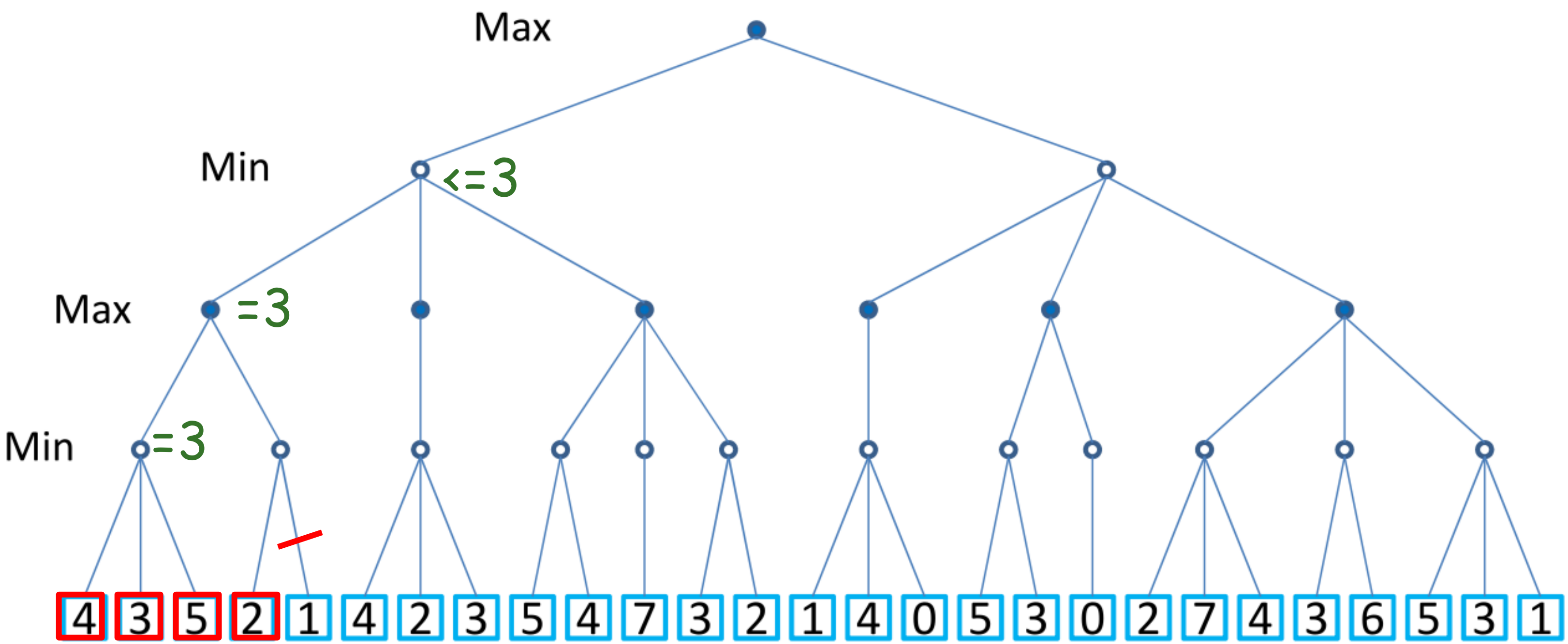

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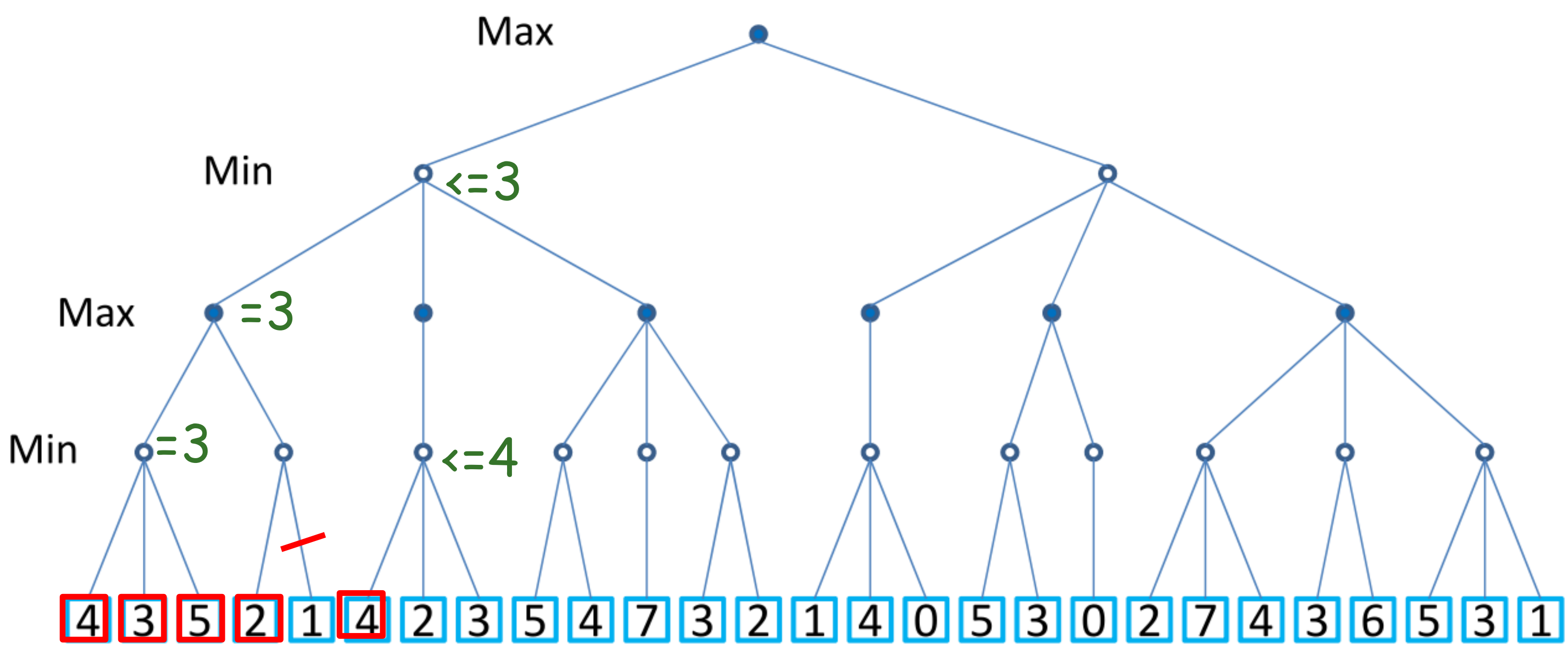

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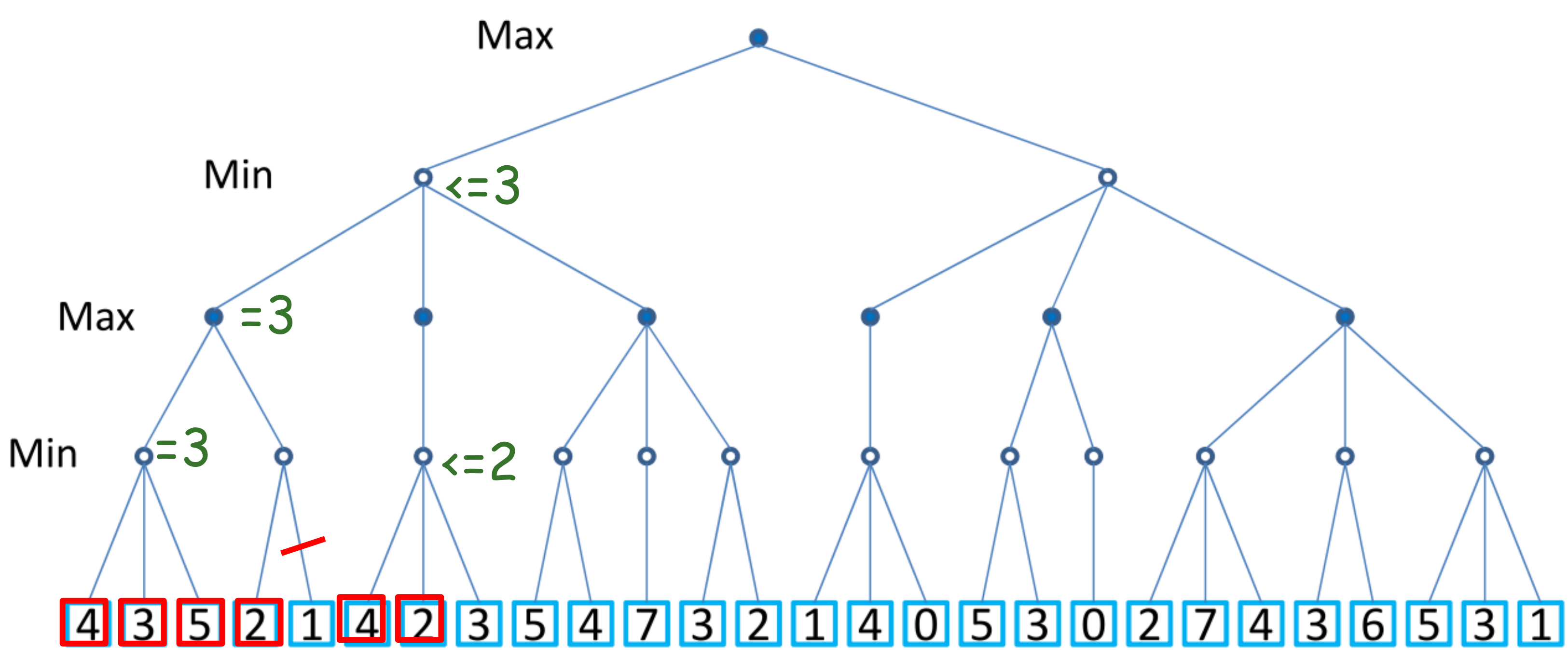

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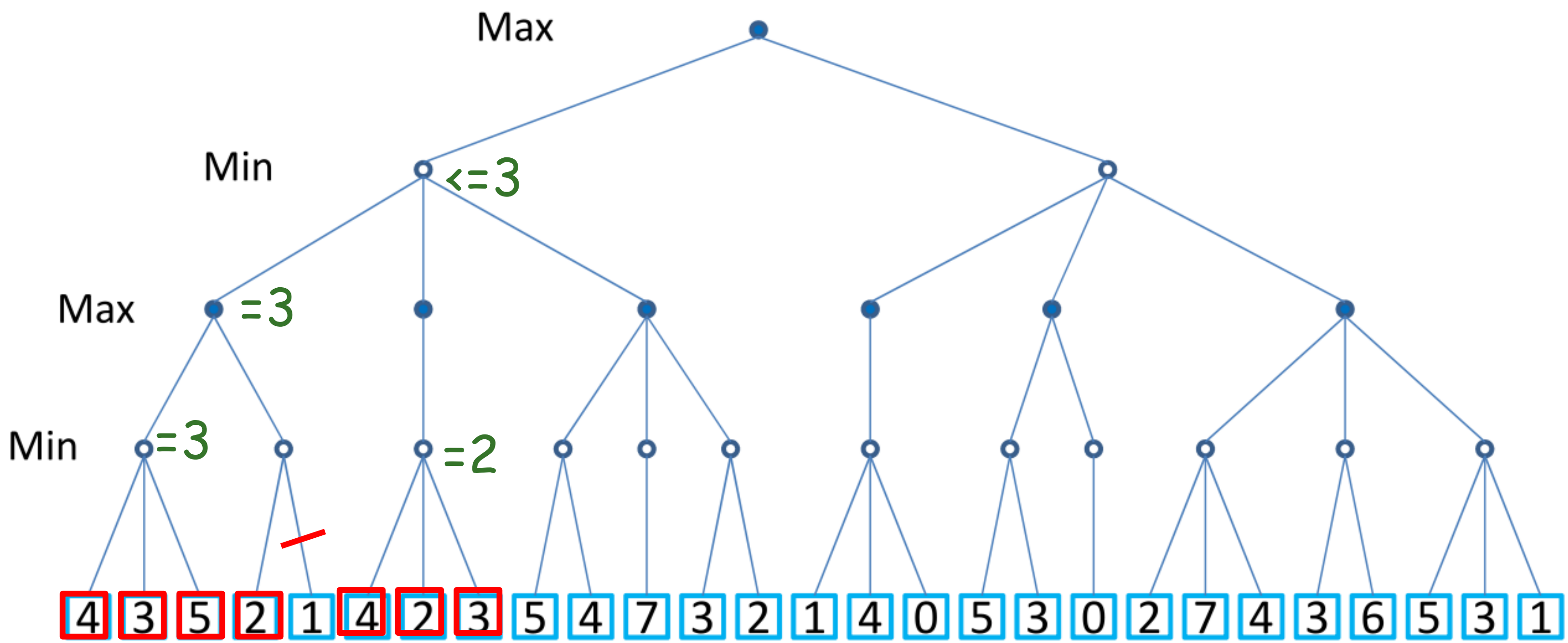

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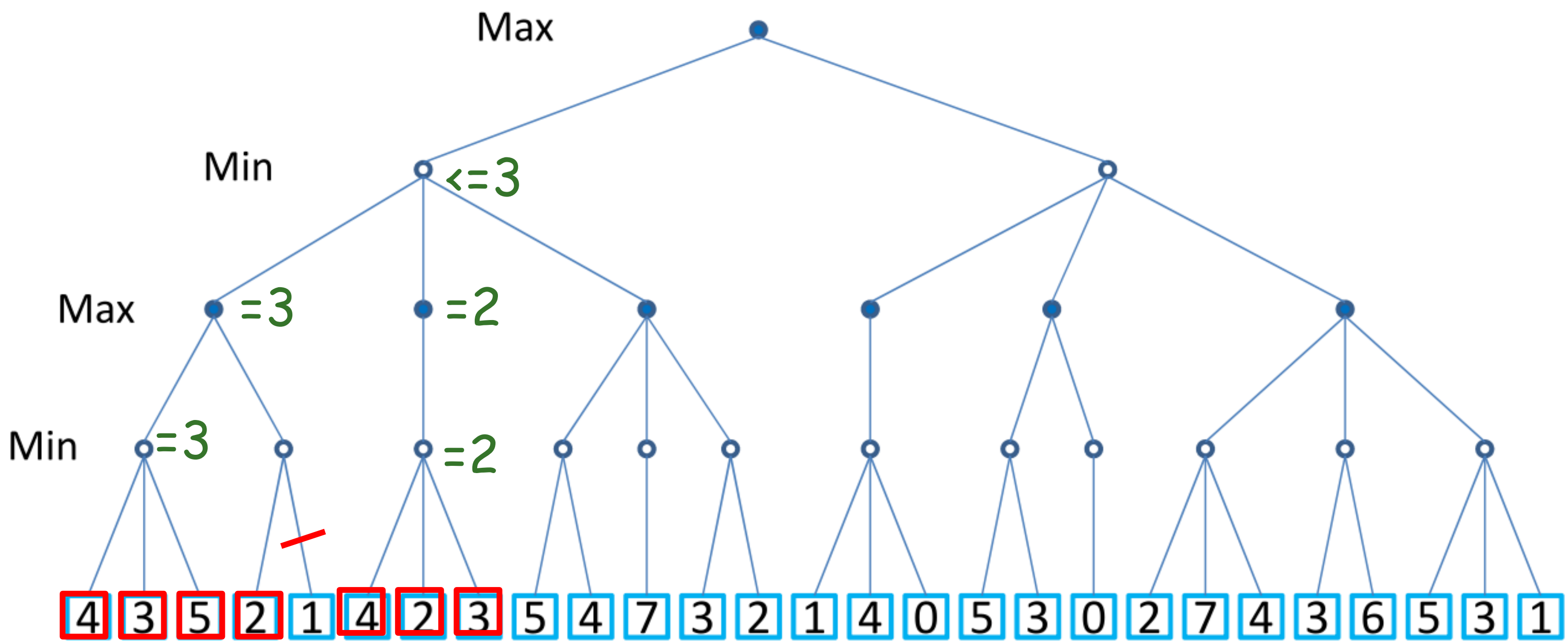

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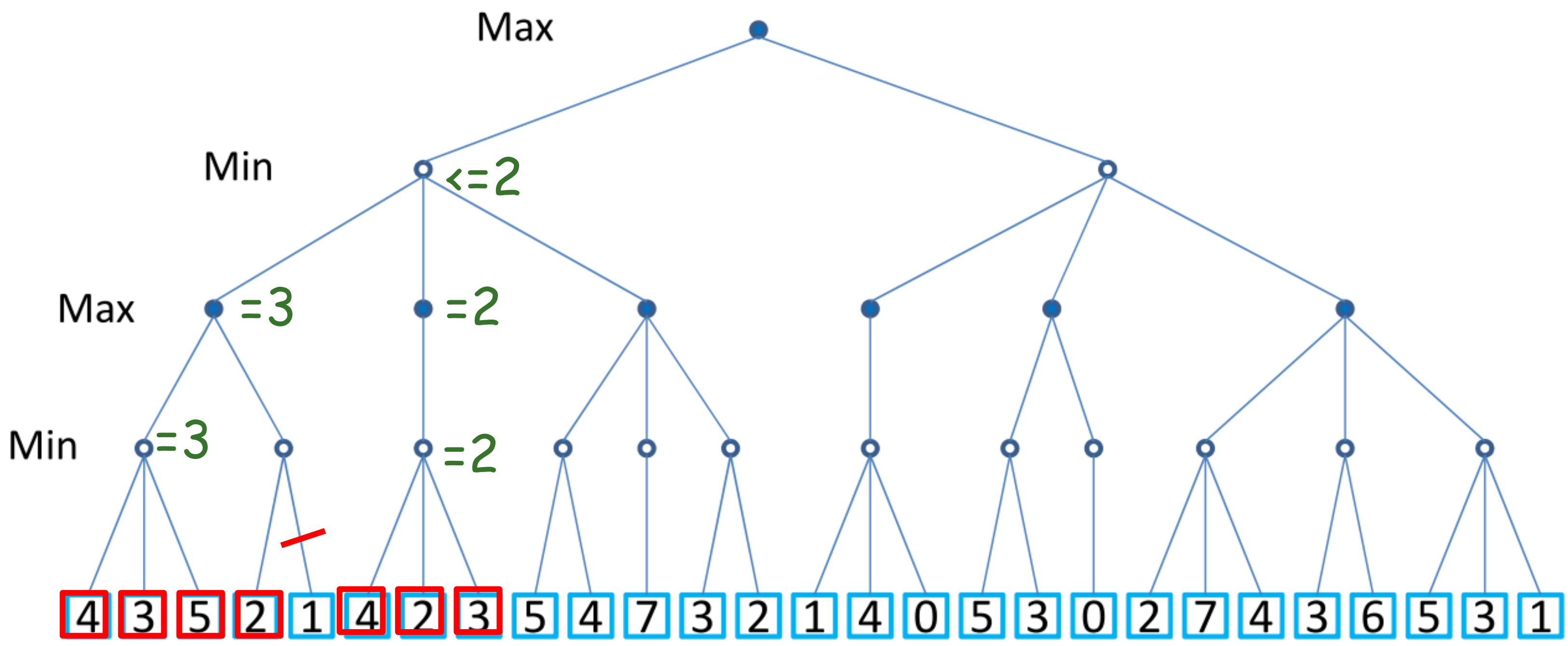

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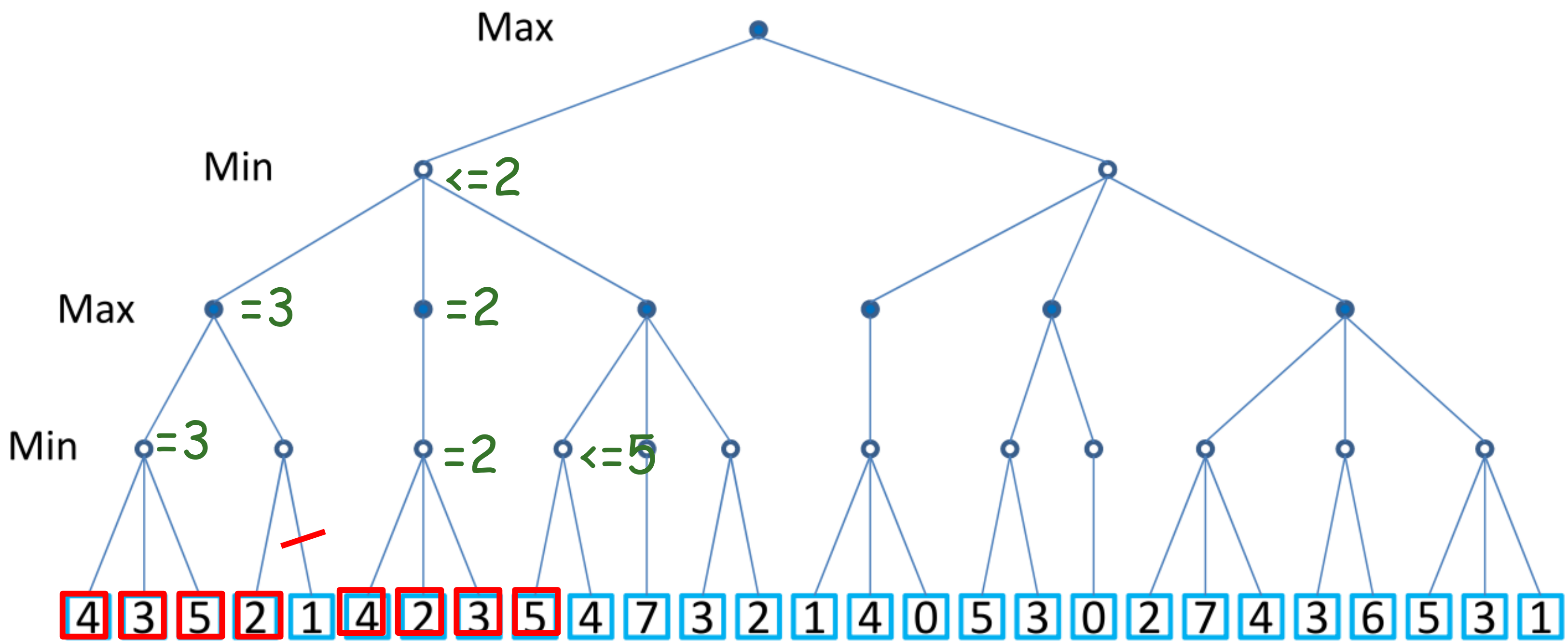

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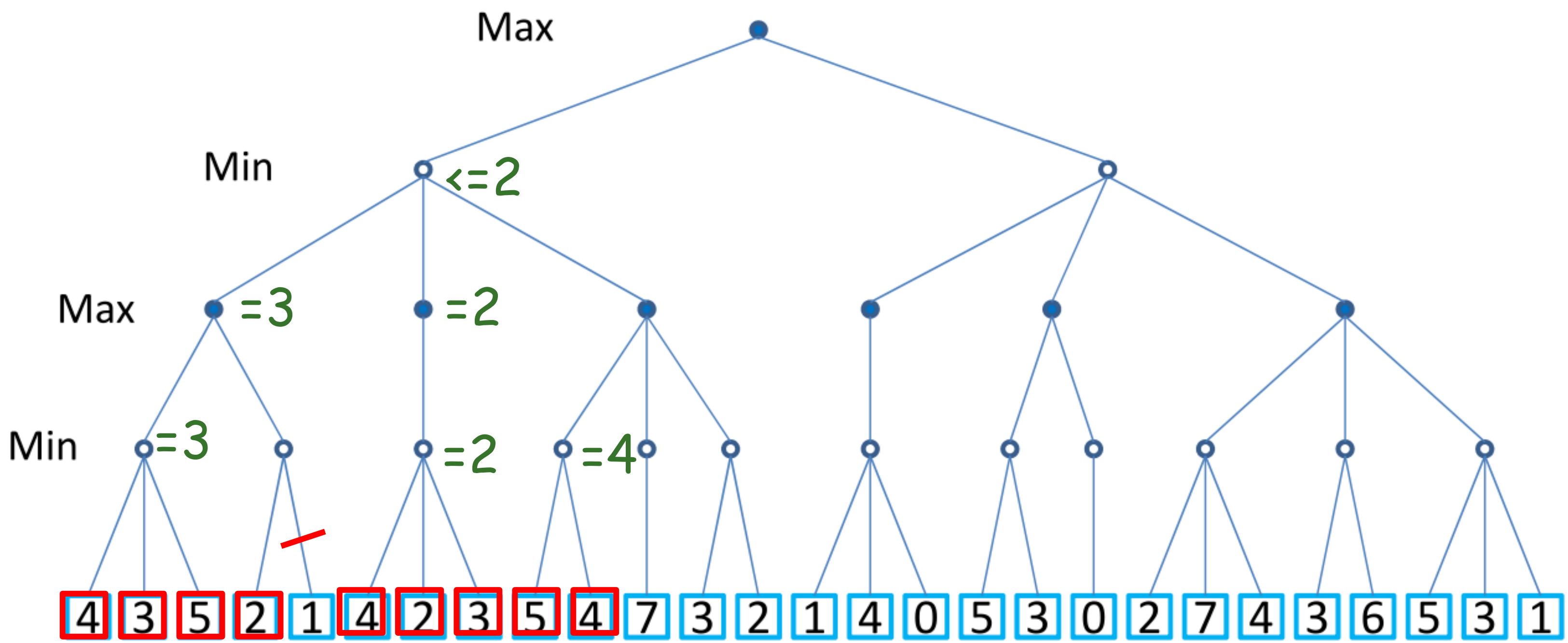
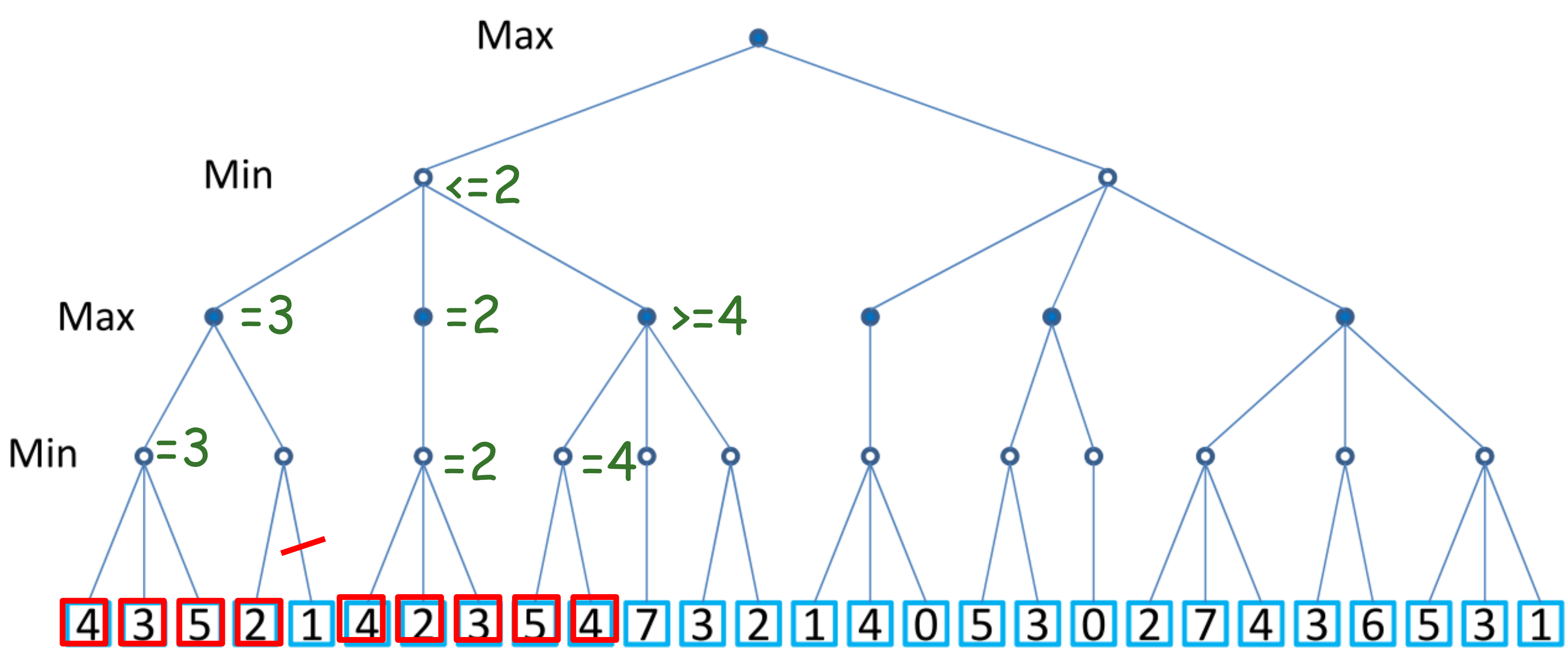
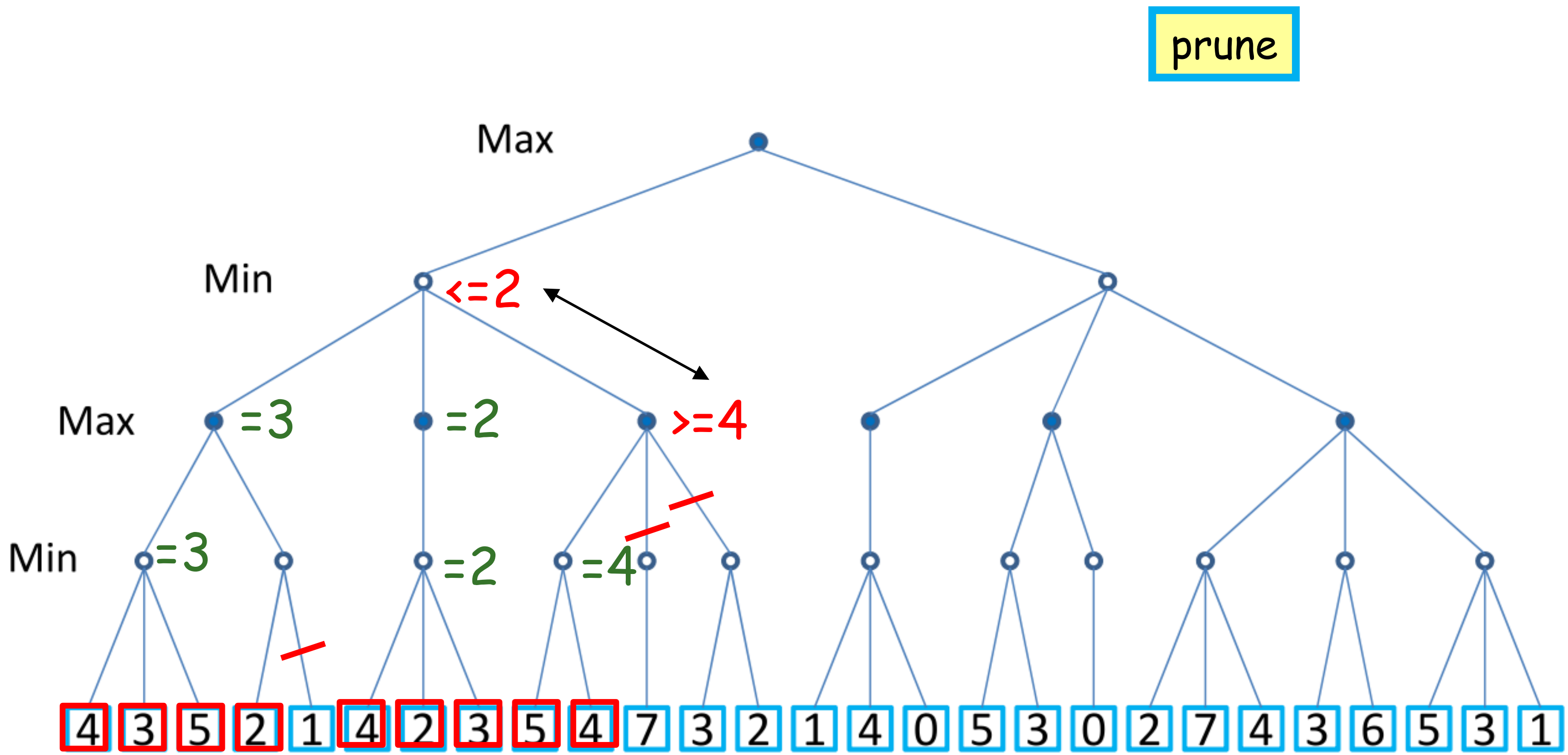


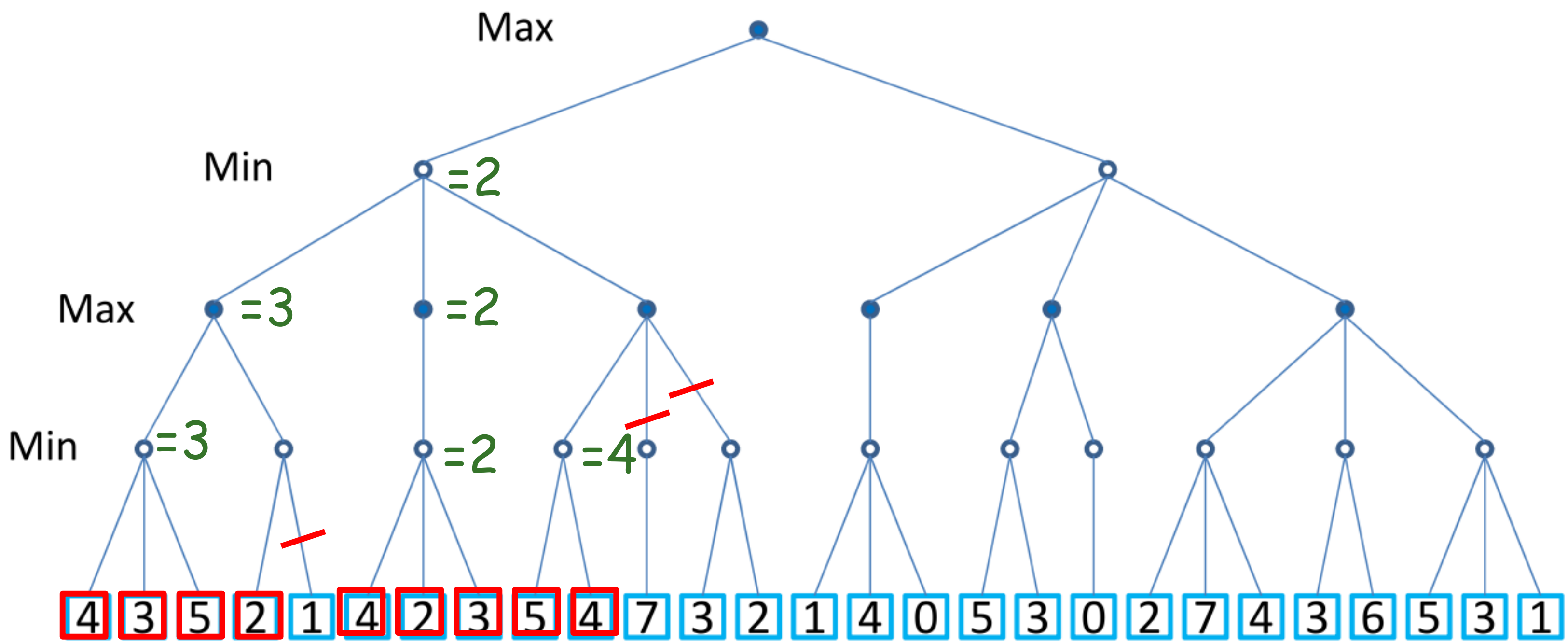
Alpha-Beta Pruning: Example 3



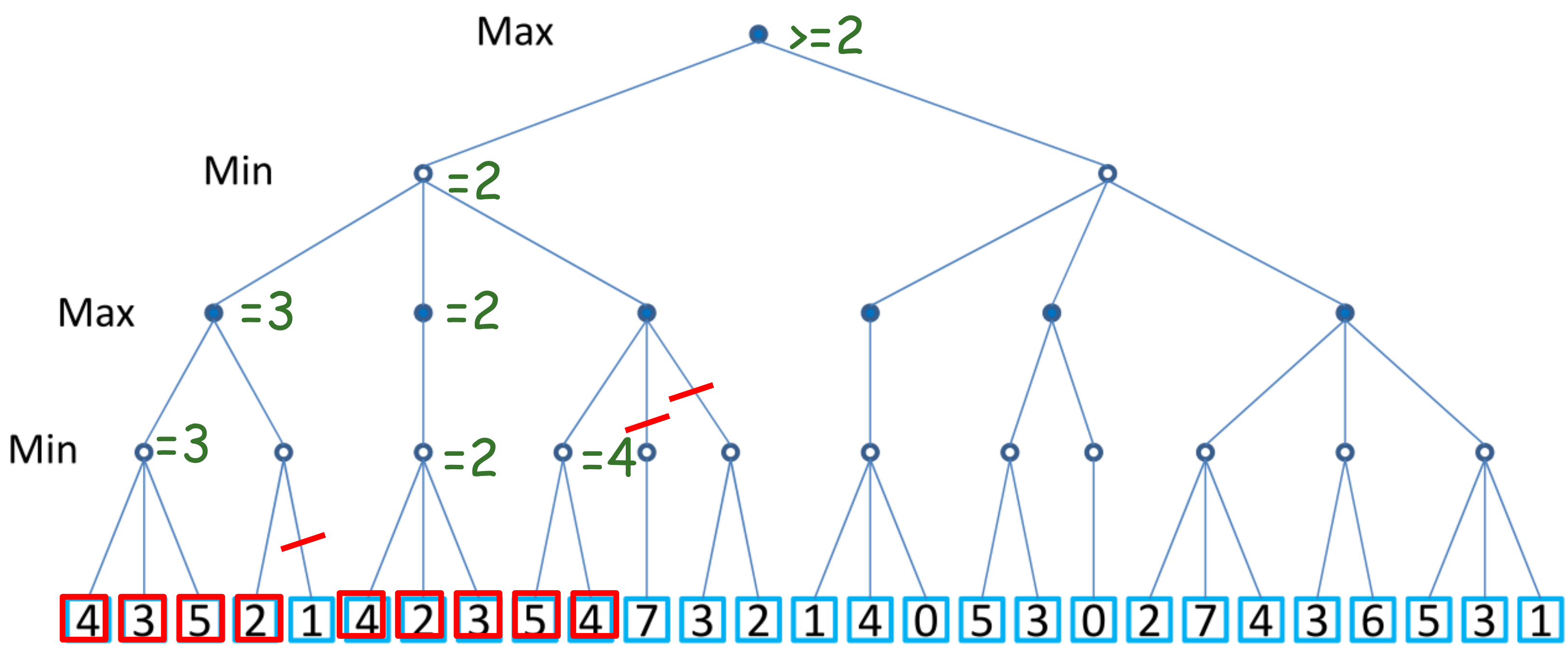
Alpha-Beta Pruning: Example 3



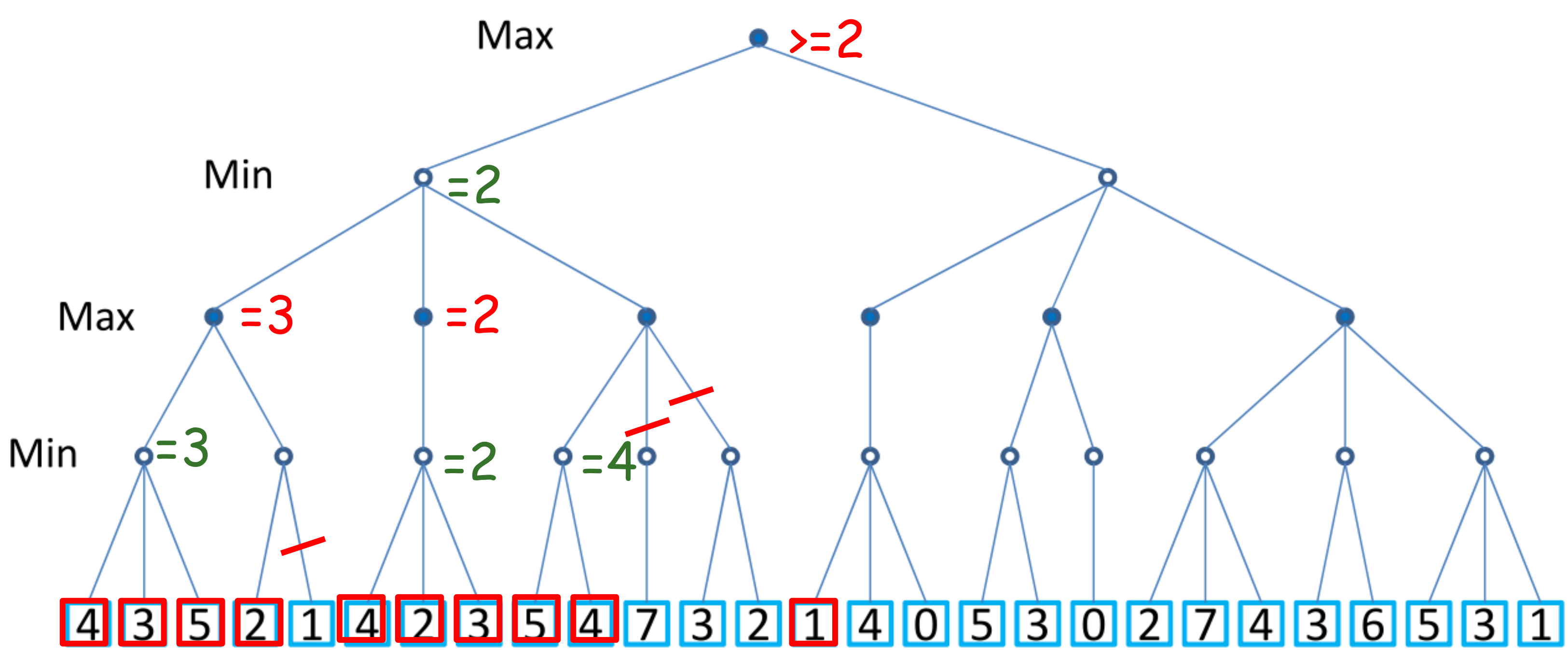
Alpha-Beta Pruning: Example 3



Alpha-Beta Pruning: Example 3



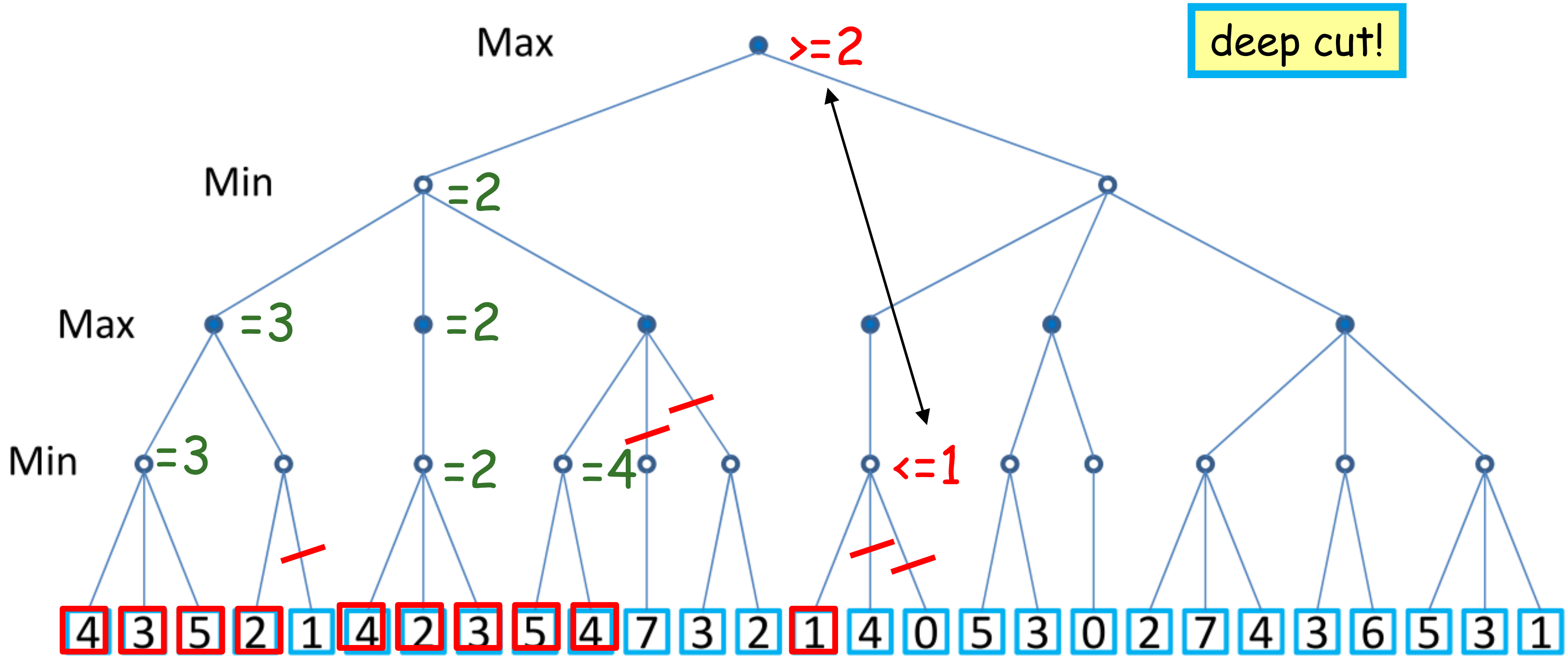
Alpha-Beta Pruning: Example 3



Alpha-Beta Pruning: Example 3

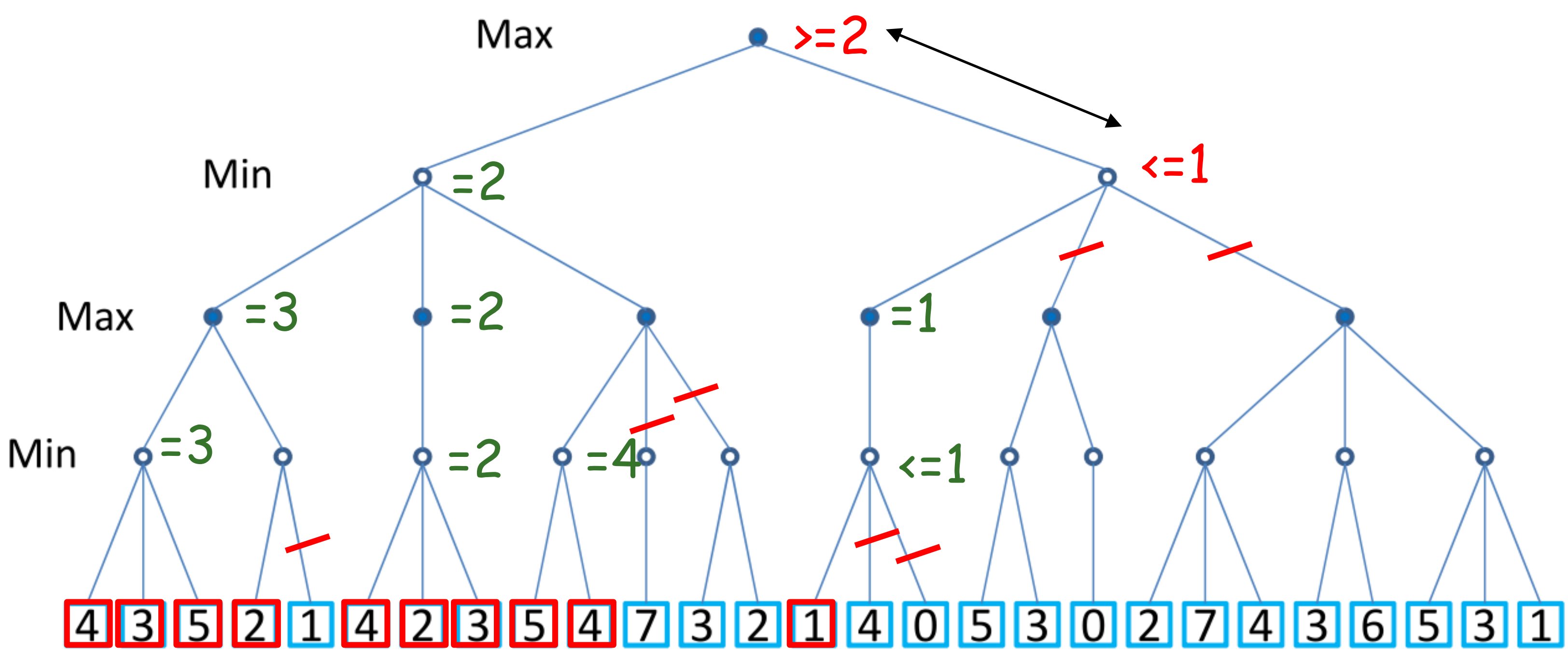
prune

deep cut!



Alpha-Beta Pruning: Example 3

prune



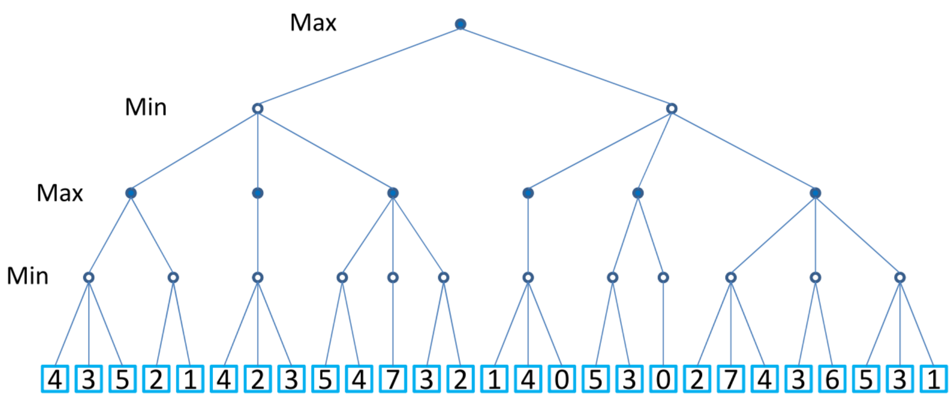
10 nodes explored out of 27

Efficiency of Alpha-Beta Pruning

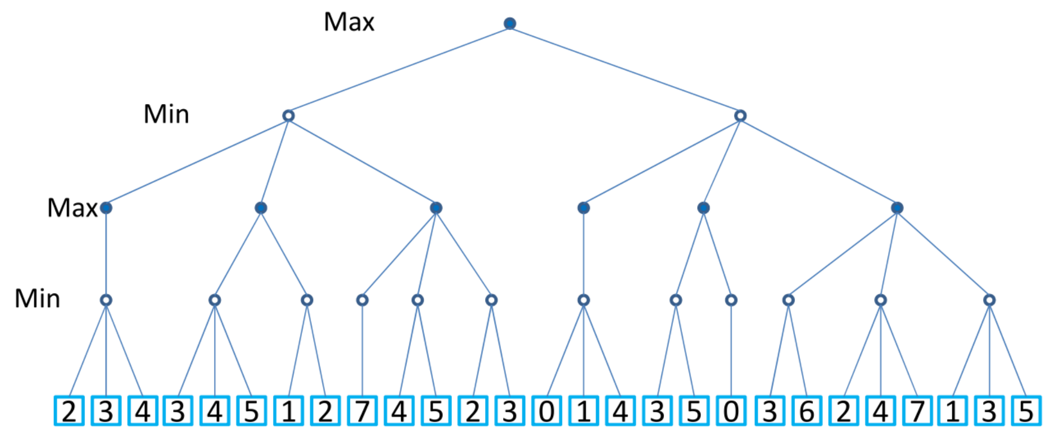
- Depends on the order of the siblings

- In worst case:
  - alpha-beta provides no pruning
- In best case:
  - branching factor is reduced to its square root

Alpha-Beta: Best ordering



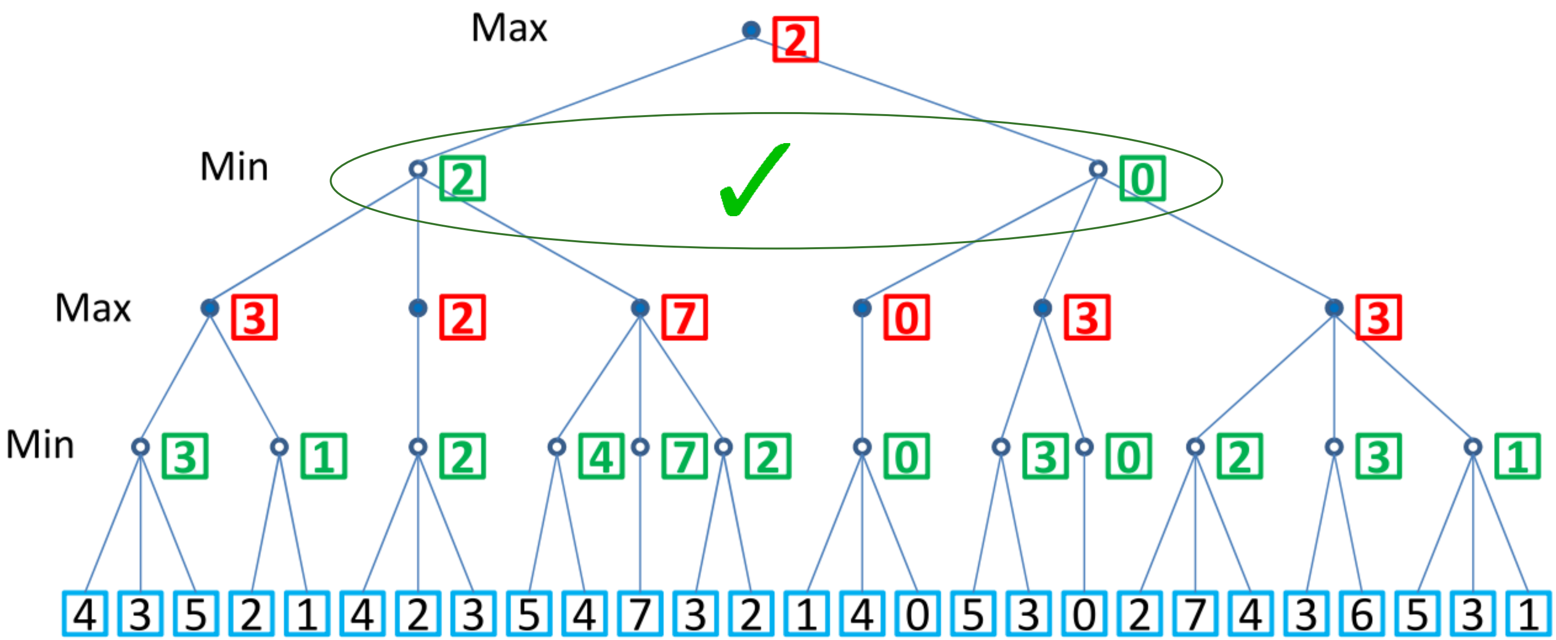
Original (arbitrary) game tree



Best ordering for alpha-beta

Alpha-Beta: Best ordering

- best ordering:
  1. children of MIN : smallest node first
  2. children of MAX: largest node first



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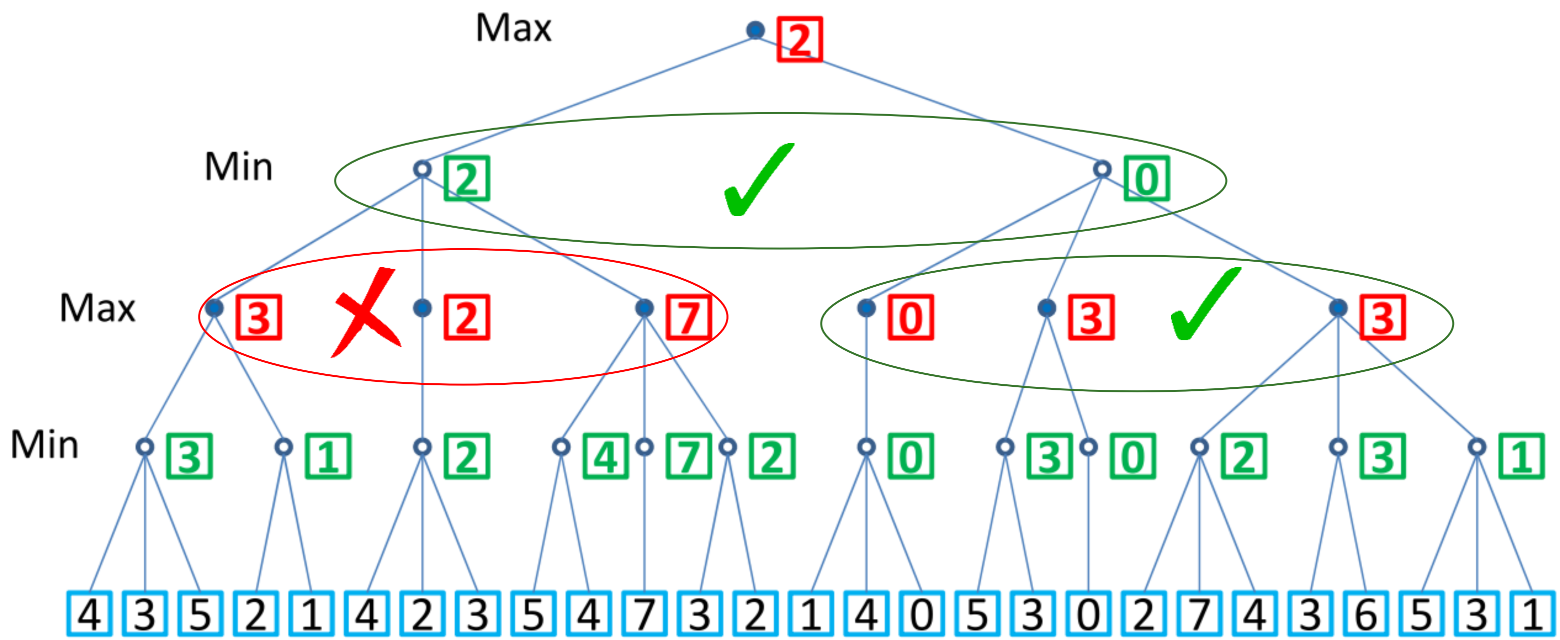

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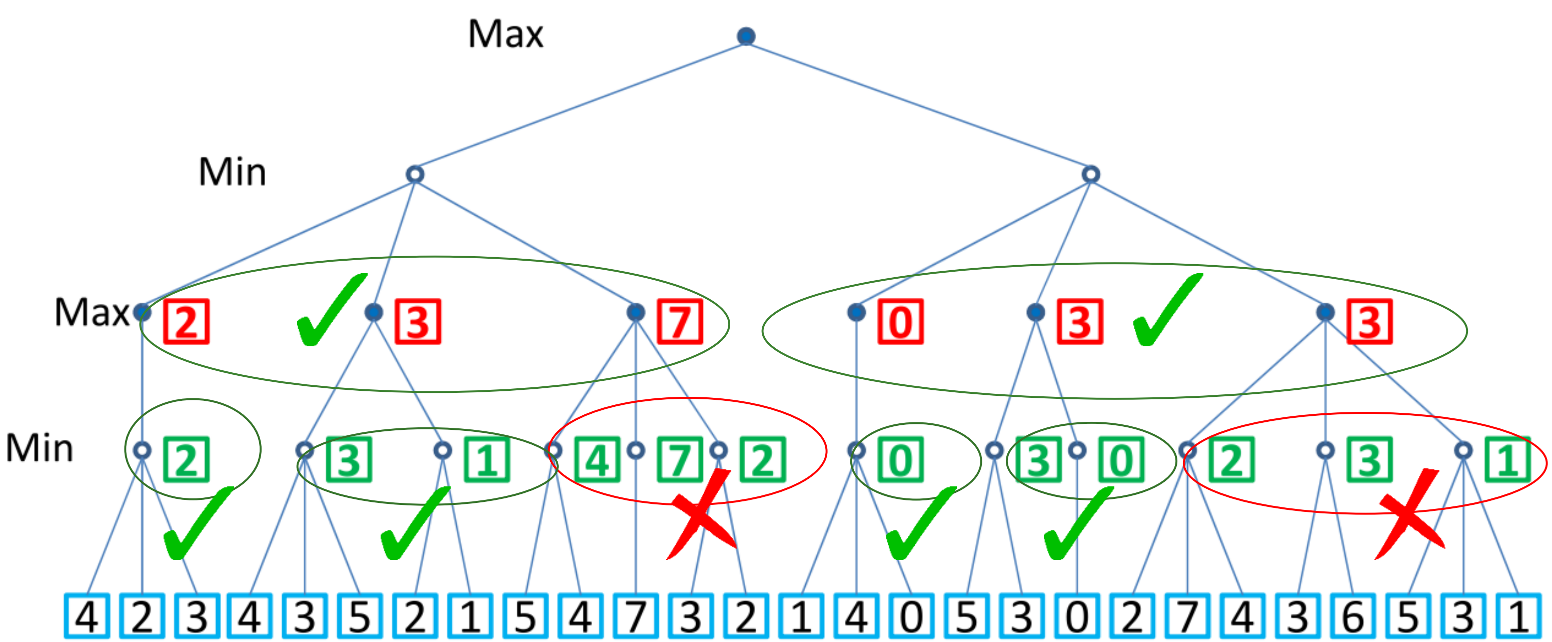
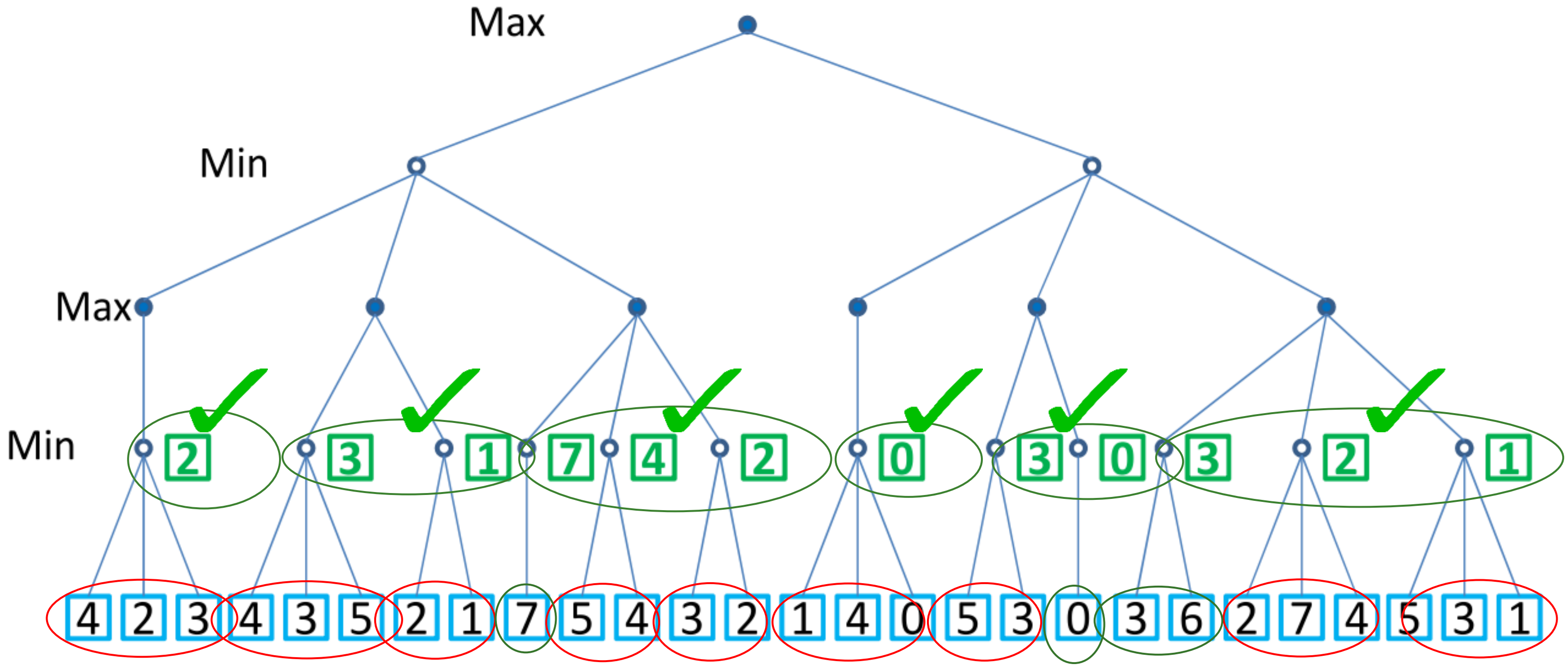
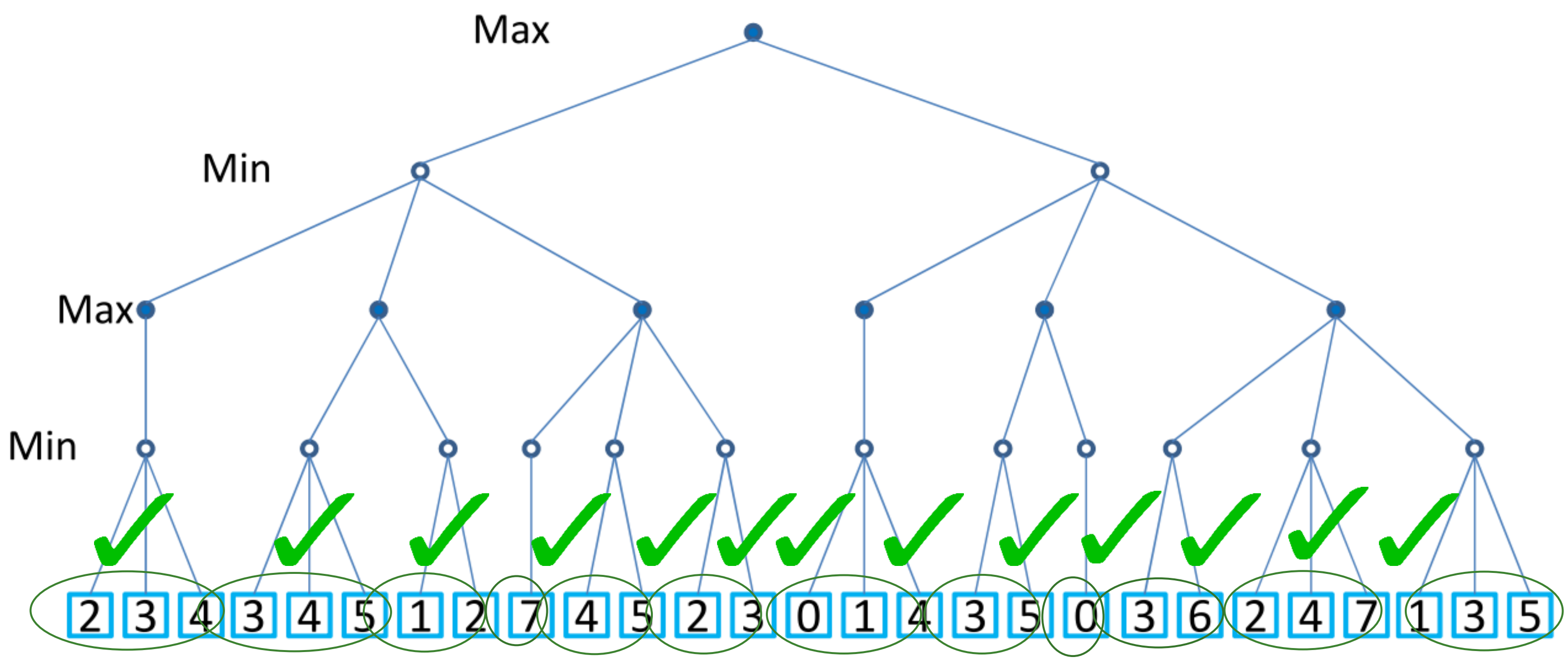


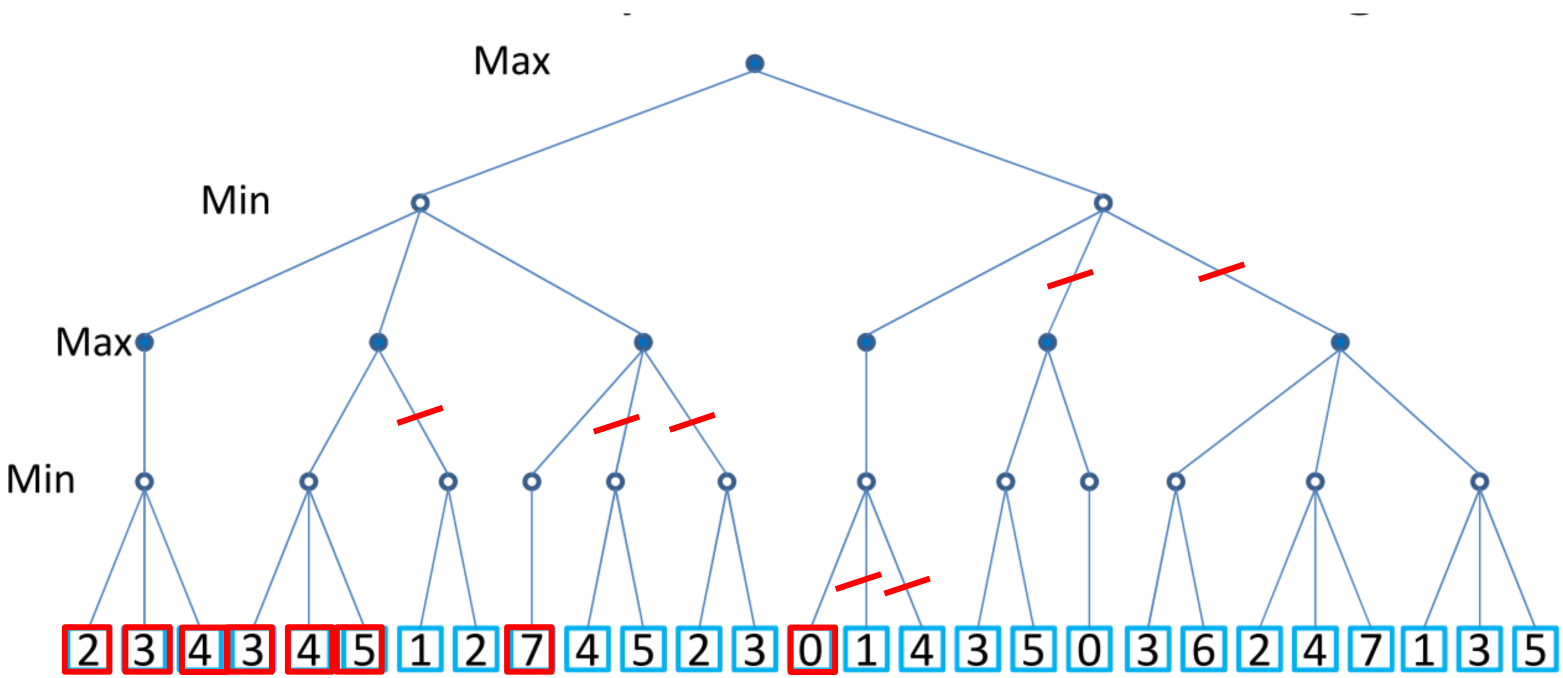
Alpha-Beta: Best ordering



Alpha-Beta: Best ordering



Alpha-Beta: Best ordering



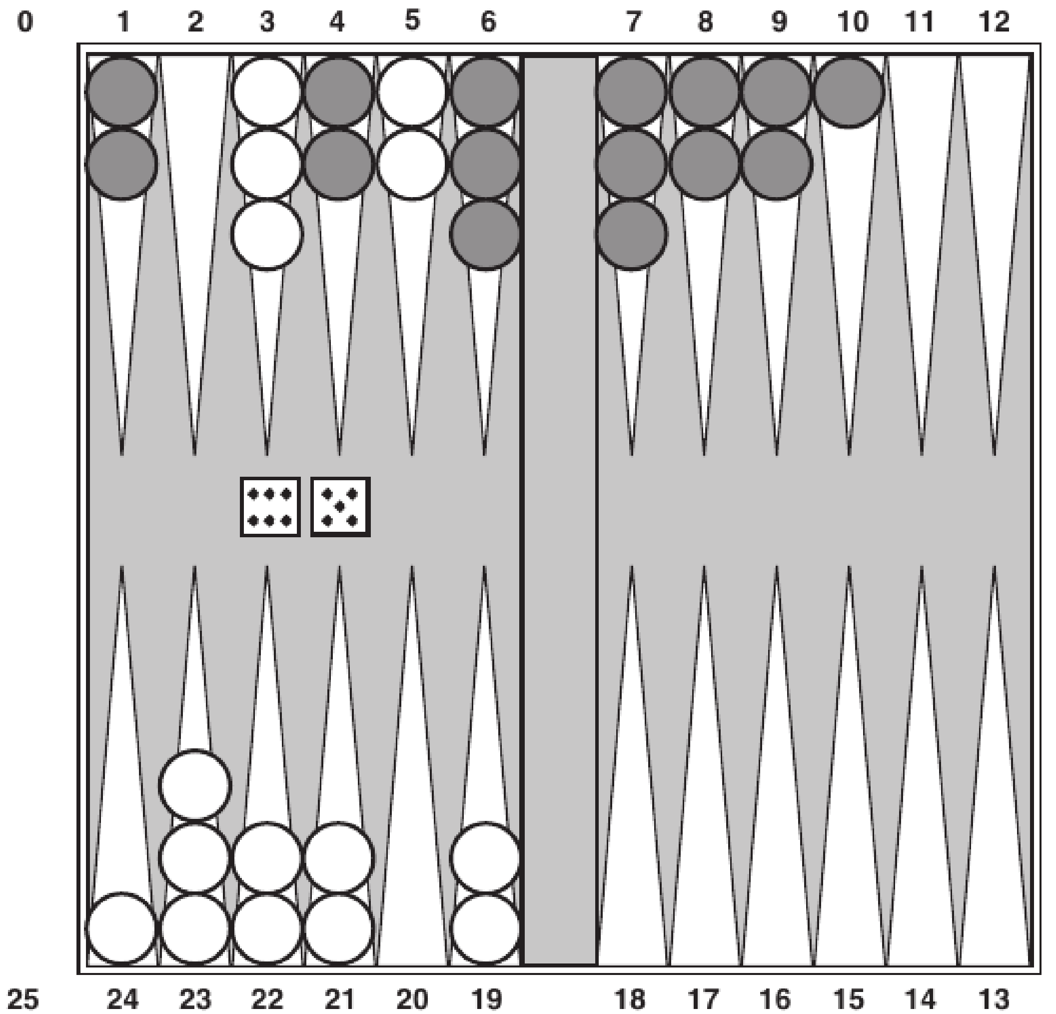
8 nodes explored out of 27

Today

- State Space Search for Game Playing
  - MiniMax
  - Alpha-beta pruning
  - Stochastic Games

- Where we are today

Backgammon

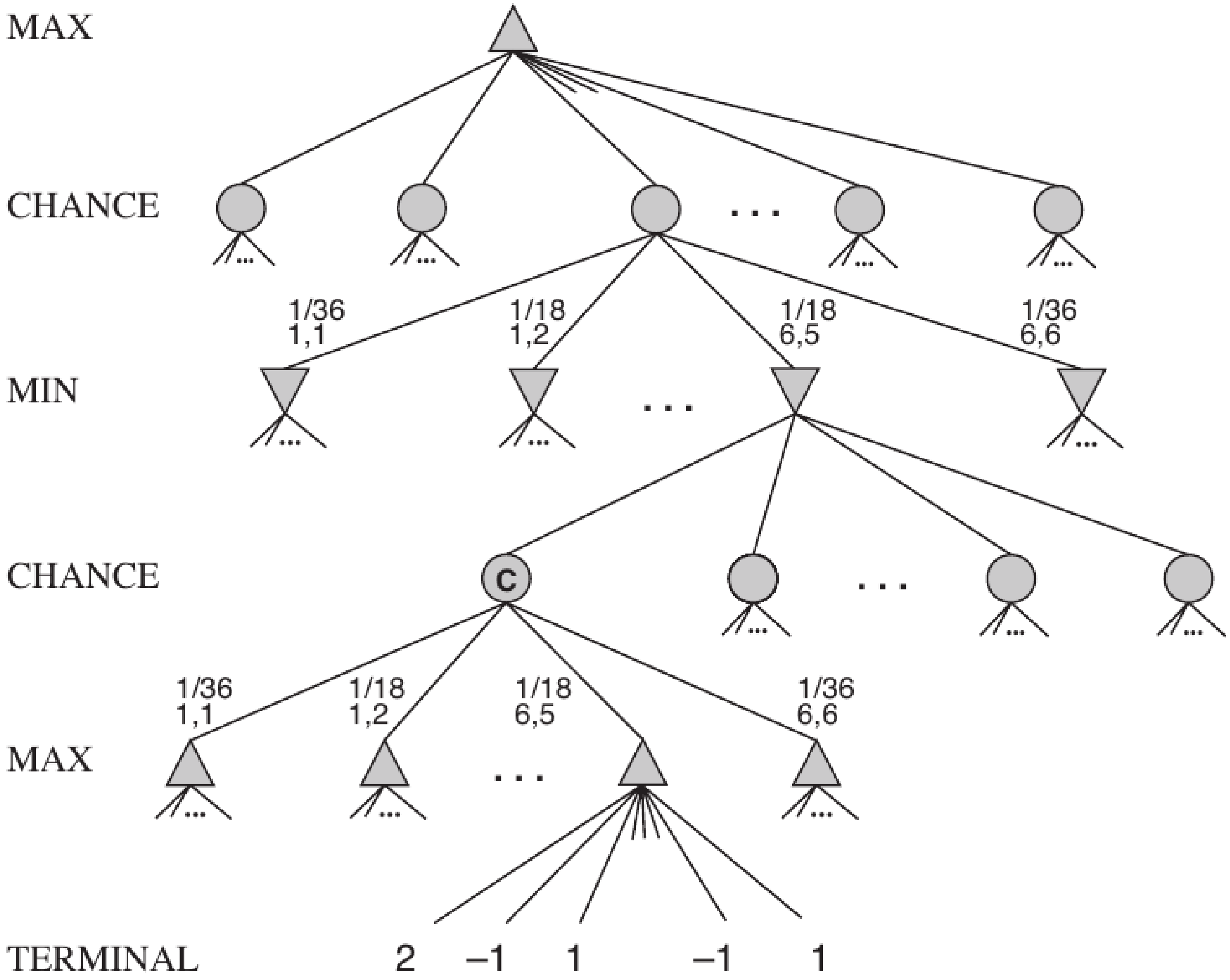


source: Russel & Norvig (2010)

Stochastic (Non-Deterministic) Games

- Search tree for games of chance
  - white can calculate its own legal moves
  - but it does not know what black will roll…
- Idea: add **chance nodes** to the search tree
  - branches indicate possible dice rolls
  - each branch labeled with the roll and its probability (e.g., 1/6 for a single dice roll)

Search Tree for Backgammon



Expectiminimax Algorithm

- Calculating Expectiminimax
  - Like MiniMax, but using the weighted sum for Chance nodes:

$$\sum P(r) Expectiminimax\left(Result(s,r)\right)$$

  - r is a possible dice roll (or other random event)
  - P(r) the probability of the event
  - Result(s, r) is the same state s with dice roll result r
  - Note: **very** expensive due to the high branching factor!
  - See https://en.wikipedia.org/wiki/Expectiminimax
    for the whole algorithm

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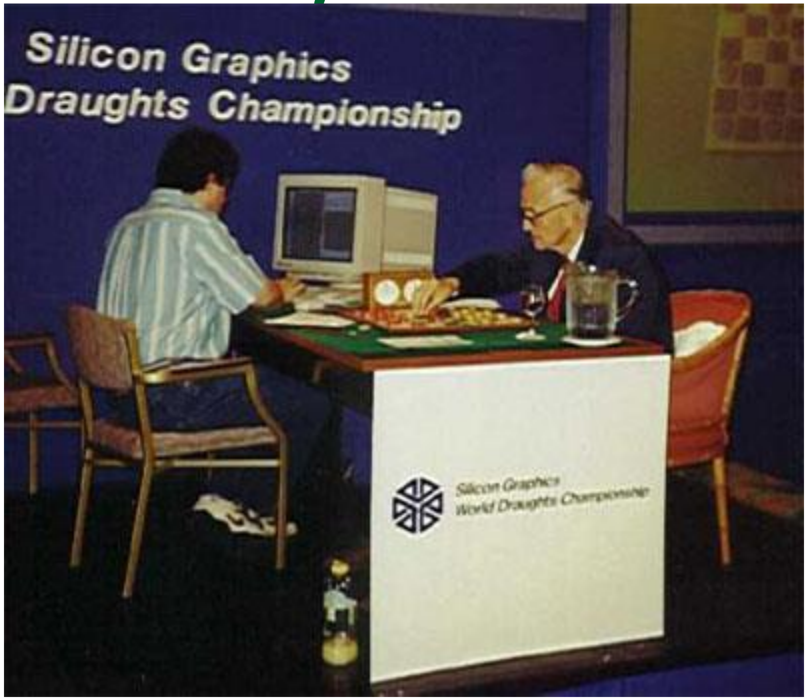
1992-1994 - Checkers: Tinsley vs. Chinook



Marion Tinsley
World champion
for over 40 years

VS

Chinook
Developed by
Jonathan Schaeffer,
professor at the U. of Alberta

1992: Tinsley beat Chinook in 4 games to 2, with 33 draws.
1994: 6 draws

In 2007, Schaeffer announced that checkers was solved,
and anyone playing against Chinook would only be able to draw, never win.

Play against Chinook: http://games.cs.ualberta.ca/cgi-bin/player.cgi?nodemo

1997 - Othello: Murakami vs. Logistello

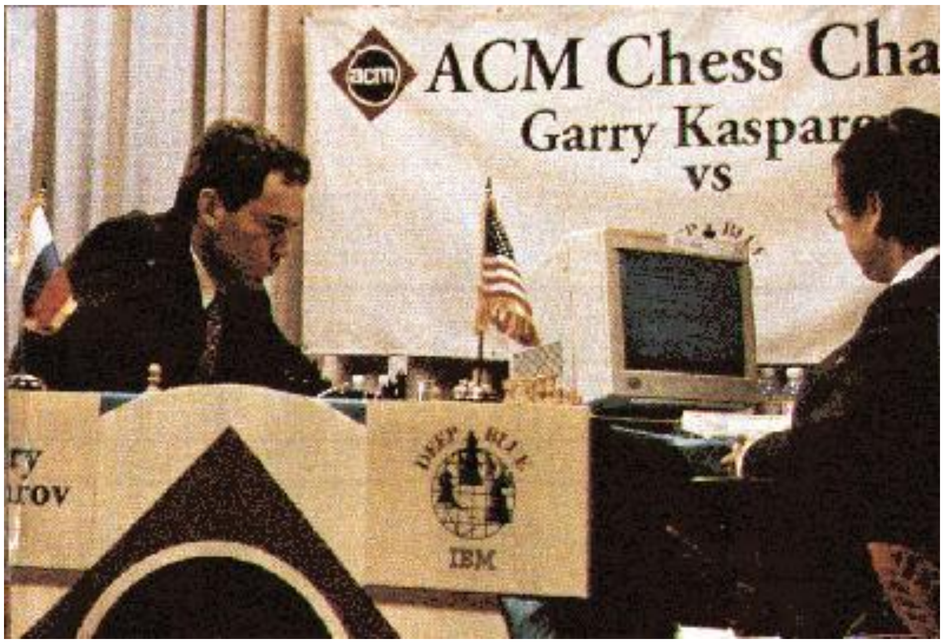
Takeshi Murakami
World Othello (aka Reversi) champion

VS

Logistello
developed by Michael Buro
runs on a standard PC
https://skatgame.net/mburo/log.html
(including source code)

Logistello beat Murakami by 6 games to 0

1997- Chess: Kasparov vs. Deep Blue



Garry Kasparov
50 billion neurons
2 positions/sec
VS
Deep Blue
32 RISC processors
+ 256 VLSI chess engines
200,000,000 pos/sec

Deep Blue wins by 3 wins, 1 loss, and 2 draws

2003 - Chess: Kasparov vs. Deep Junior

Garry Kasparov
still 50 billion neurons
still 2 positions/sec

VS

Deep Junior
8 CPU, 8 GB RAM, Win 2000
2,000,000 pos/sec
Available at $100

Match ends in a 3/3 tie!

2016 – Go: AlphaGo vs Lee Se-dol

- GO was always considered a much harder game to automate than chess because of its very high branching factor (35 for chess vs 250 for Go!)
- In 2016, AlphaGo beat Lee Sedol in a five-game match of GO.
- In 2017 AlphaGo beat Ke Jie, the world No.1 ranked player at the time

- uses a Monte Carlo tree search algorithm to find its moves based on knowledge previously "learned" by deep learning

https://www.theverge.com/2016/3/15/11213518/alphago-deepmind-go-match-5-result

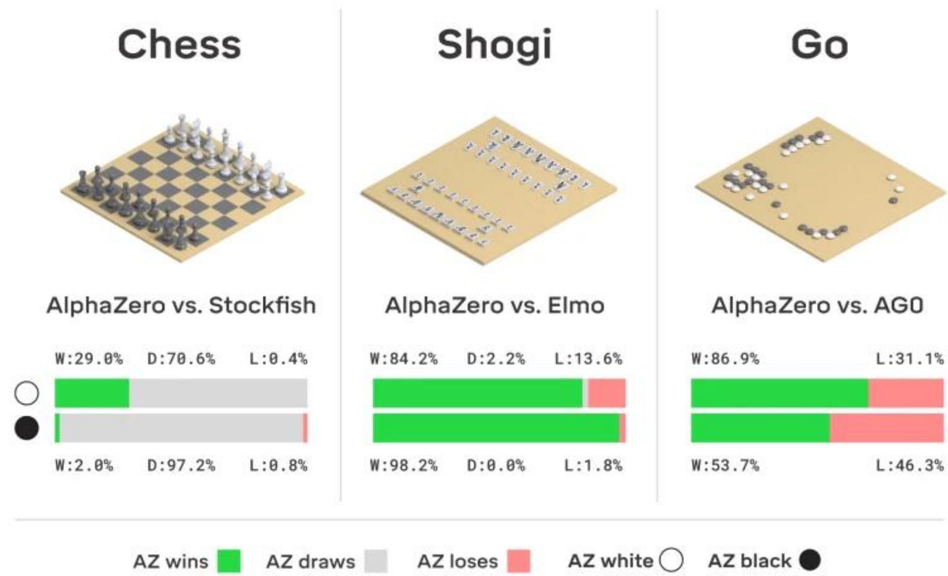
2017 – AlphaGo Zero & AlphaZero

AlphaGo Zero learned the Game by itself, without input of human games

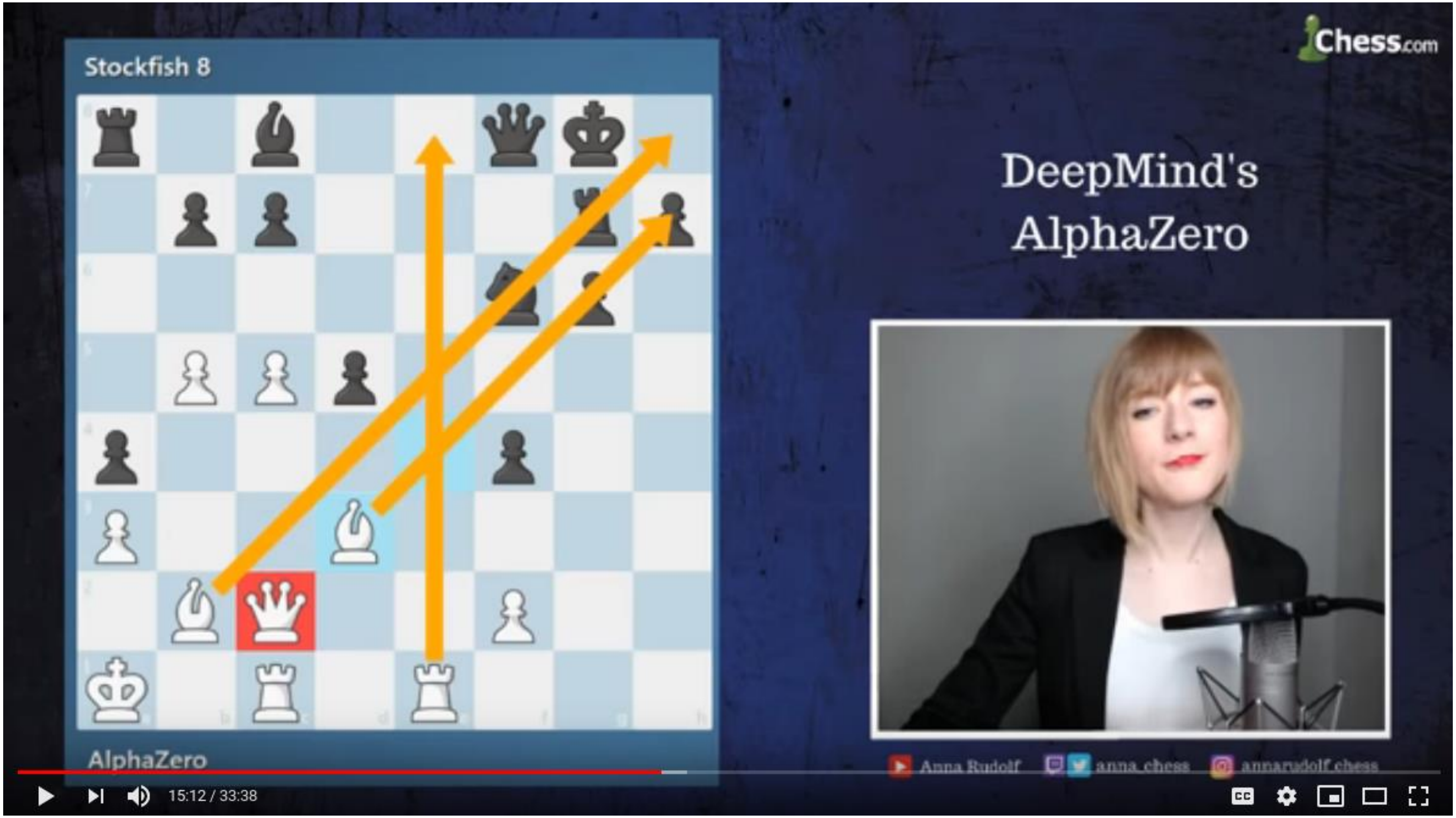
- Became better than all old versions after 40 days of training
- In the first three days, AlphaGo Zero played 4.9 million games against itself using reinforcement learning

AlphaZero can learn other games, like Chess and Shogi

- In 2018, it beat the then-best chess program, Stockfish 8 in a 100-game tournament
- Trained using 5,000 tensor processing units (TPUs), run on four TPUs and a 44-core CPU during matches



2018 - AlphaZero vs Stockfish 8



Game commentary: https://www.youtube.com/watch?v=nPexHaFL1uo

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