

Preface

Chance – or what appears to us as such – is ubiquitous. Not only in the games of chance such as lottery or roulette where risk is played with, but also in substantial parts of everyday life. Every time an insurance company uses the claim frequencies to calculate the future premium, or a fund manager the most recent stock charts to rearrange his portfolio, every time cars are jamming at a traffic node or data packages at an internet router, every time an infection spreads out or a bacterium turns into a resistant mutant, every time pollutant concentrations are measured or political decisions are based on opinion polls – in all such cases a considerable amount of randomness comes into play, and there is a need to analyse the random situation and to reach at rational conclusions in spite of the inherent uncertainty. Precisely this is the objective of the field of stochastics, the ‘mathematics of chance’. Stochastics is therefore a highly applied science, which tries to solve concrete demands of many disciplines. At the same time, it is genuine mathematics – with sound systematics, clear-cut concepts, deep theorems and sometimes surprising cross-connections. This interplay between applicability on the one side and mathematical precision and elegance on the other makes up the specific appeal of stochastics, and a variety of natural questions determines its lively and broad development.

This book offers an introduction to the typical way of thinking, as well as the basic methods and results of stochastics. It grew out of a two-semester course which I gave repeatedly at the University of Munich. It is addressed to students of mathematics in the second year, and also to scientists and computer scientists who intend not only to apply stochastics, but also to understand its mathematical side. The two parts of stochastics – probability theory and statistics – are presented in two separate parts of the book because of their own scopes and methods, but are united under the same cover on purpose. For, the statistics is built on the concepts and methods of probability theory, whereas the latter needs the former as a bridge to reality. In the choice of the material I confined myself deliberately to the central subjects belonging to the standard curriculum of the corresponding mathematical courses. (It is thus unavoidable that some readers will miss their favourite subjects, e.g., the resampling methods of statistics.) The standard themes, however, are discussed with all necessary details. Rather than starting with discrete models I preferred to present (and motivate) the general measure theoretic framework right from the beginning, and some theoretical issues are also treated later as the case arises. In general, however, the measure theoretic apparatus is confined to what is absolutely necessary, and the emphasis is on the development of a stochastic intuition.

This text comprises a little more material than can be presented in a four-hour course over two semesters. The reader may thus want to make a selection. Several possibilities present themselves. For a first overview, the reader may concentrate on concepts, theorems, and examples and skip all proofs. In particular, this is a practicable route for non-mathematicians. A deeper understanding, of course, requires the study of a representative selection of proofs. On the other hand, a reader mainly interested in the theory and familiar with some applications may skip a portion of examples. For a short route through Part I leading directly to statistics, one can restrict oneself to the essentials of the first chapters up to Section 3.4, as well as Sections 4.1 and 4.3, and 5.1 and 5.2. The core of Part II consists of Sections 7.1–7.5, 8.1–8.2, 9.2, Chapter 10, as well as 11.2 and 12.1. Depending on the specific interests, it will facilitate orientation to browse through some portions of the text and return to them later when needed. A list of notational conventions can be found on page 395.

At the end of each chapter there is a collection of problems offering applications, additions, or supplements to the text. Their difficulty varies, but is not indicated because the reader should follow only his or her interests. The main point is to try for oneself. Nevertheless, this second English edition now provides draft solutions of selected problems, marked with ^S. These should be used for self-testing, rather than lulling the reader's willingness to tackle the problems independently.

As every textbook, this one grew out of more sources than can possibly be identified. Much inspiration came from the classical German texts of Ulrich Krengel [38] and Klaus Krickeberg and Herbert Ziezold [39], which had strong influence on the introductory courses in stochastics all over Germany. I also got many impulses from my Munich stochastics colleagues Peter Gänßler and Helmut Pruscha as well as all those responsible for the problem classes of my lectures during more than two decades: Peter Imkeller, Andreas Schief, Franz Strobl, Karin Münch-Berndl, Klaus Ziegler, Bernhard Emmer, and Stefan Adams. I am very grateful to all of them.

The English translation of the German original would not have appeared without the assistance of two further colleagues: Marcel Ortgiese accepted to lay the foundation by preparing an initial English version, so that I could concentrate on details and cosmetic changes. Ellen Baake took pleasure in giving a final polish to the English and suggesting numerous clarifications. I gratefully acknowledge their help.

Munich, June 2012

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