Discrete Math Final Exam

1. Let k, n, r be positive integers. Show that the number of integer solutions to the equation

$$x_1 + x_2 + \dots + x_n = r$$

such that $0 \le x_i \le k$ for each i = 1, 2, ..., n is given by

$$\sum_{i=0}^{n} (-1)^{n} \binom{n}{i} \binom{r - (k+1)i + n - 1}{n - 1}.$$

2. Solve

$$a_n - 6a_{n-1} + 9a_{n-2} = 0$$

given $a_0 = 2$ and $a_1 = 3$.