Network Dynamics and Learning Homework #3

Lal Akin

Politecnico di Torino
Turin, Italy
s300192@studenti.polito.it

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Introduction

This report is written to convey the details of the work done for the third homework of Network Dynamics and Learning lecture of 2023/2024 semester of Politecnico di Torino, Master's of Data Science and Engineering.

The main topics are: K-regular graphs, Preferential Attachment model, SIR models, Coloring problem, Anti-Coordination Games

Exercise 1

1.1

Epidemic models on a symmetric k-regular undirected graph.

In this question, firstly we have to construct a symmetric k-regular undirected graph to then run a SIR infection model. This graph contains 500 nodes and each node is connected to its two neighbors and since k is given as 4, two other closest nodes. At any given time each node can be in one of the 3 status. Susceptible nodes are open to infection, Infected nodes can get better and go into Recovered node status and Recovered nodes will be recovered until the end. We are given two probabilities, β which is the probability of infection, ρ which is the probability of recovery.

Parameters:

- $\beta = 0.3$
- $\rho = 0.7$

Incorporating these two probabilities into the model, we are asked to select 10 nodes out of the 500 randomly to be infected as the initial configuration, and run this epidemic model for 15 weeks, 100 iterations.

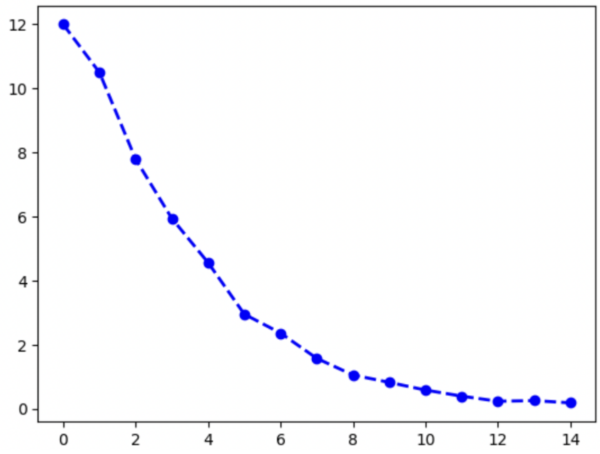

Figure 1: Number of individuals that became infected with each passing week on average

Also it is useful to check the counts of each state at this step, and it can be seen in Figure 2. It is evident that as the number of susceptible individuals stay the same, which also corresponds to a very high portion of the population, the infected individuals decrease and recovered individuals increase proportionally.

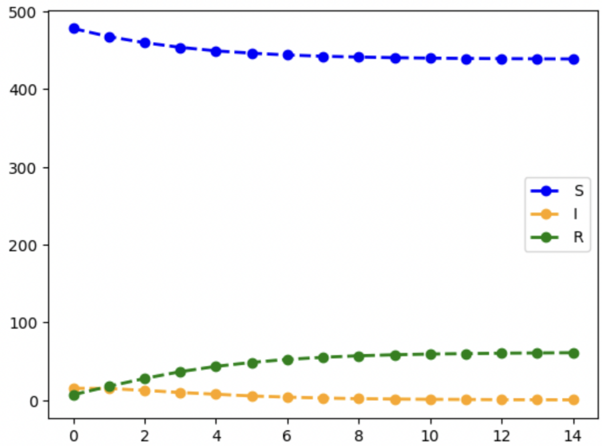

Figure 2: Evolution of each state

1.2

Preferential attachment model, SIR dynamics without vaccination.

For this step now we will draw our population graph with Preferential Attachment Model. The key point about this model is that, in this configuration if a node is attached to many others it will be more likely for this node to be added new links which is also intuitively true for epidemic models. We draw a large graph made up of 900 nodes.

Then for Problem 2, we draw a Preferential Attachment Model graph with 500 nodes with average degree as 6, and the same parameters as before.

The results of the simulation is as follows:

It can be seen that firstly there is a peak in the infection, and slowly the number falls as more and more people becomes recovered.

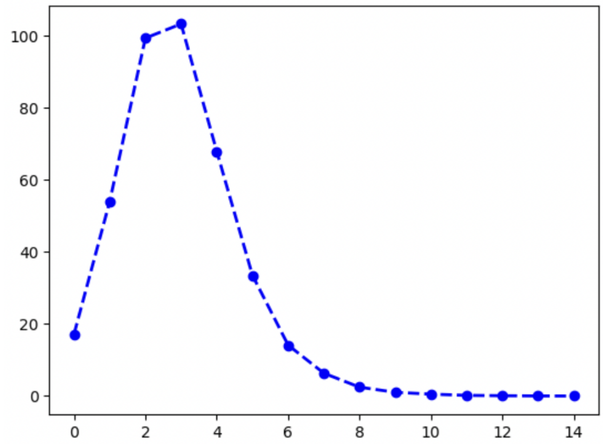

Figure 3: Number of individuals that became infected with each passing week on average

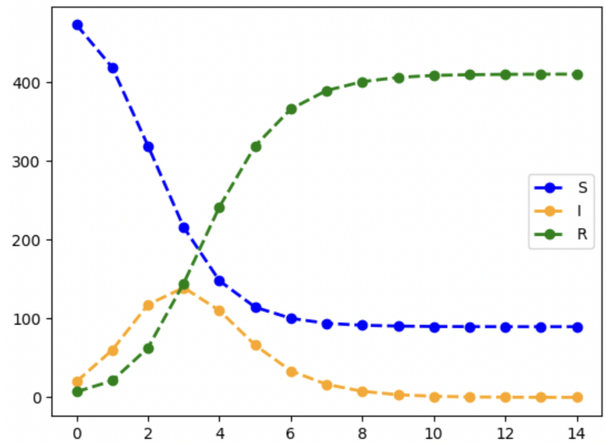

Figure 4: State Count per week

From Figure 4 we can see how the number of recovered individuals and the number of infected individuals follow reverse trends, and as the number of people who were susceptible, got infected, and then recovered increase, the number of people who are susceptible still falls.

Another point worth mentioning is how different the graph dynamics are, between the k-regular graph and the random graph with preferential attachment style. With the first model, it is visible that the first 10 randomly selected infectious nodes didn't have a substantial amount of links to all the other nodes, and therefore the number of infection didn't increase. But this isn't a realistic approach for simulating epidemic models.

In the second configuration, the preferential attachment model depicts the real world behaviour much better, since also in the real world if someone who is infected has many other people he or she comes into contact with, the number of new people exposed to the infection will grow. This is reason why the first model's states follow a somewhat linear trend, because there isn't enough interaction. Whereas the second model shows the state's increasing and decreasing non-linearly.

1.3

Preferential attachment model, SIR dynamics with vaccination.

We will implement a chance of slowing down the infection by vaccination. Another state will be added now which is Vaccinated. Once vaccinated, that individual will not be infected or will not be able to spread the disease if they were infected. We are given a percentage of the population to be vaccinated at the end of each week, for the 15 weeks timeline. According to these percentages, we are asked to select randomly which nodes to vaccinate. This epidemic model will be run once more on the graph that was constructed at Section 1.2.

Parameters:

- Number of nodes = 500
- k = 6
- $\beta = 0.3$
- $\rho = 0.7$

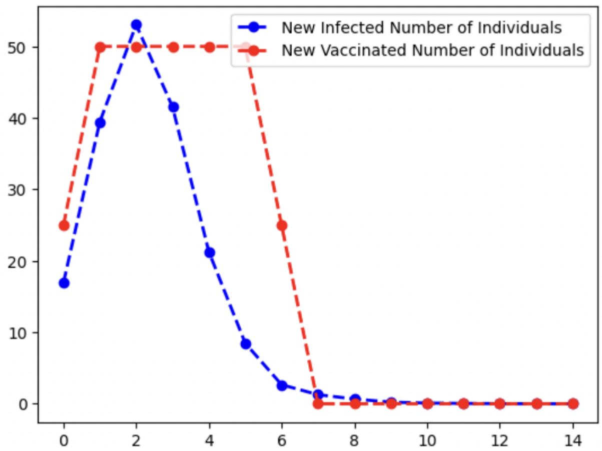

Figure 5: Weekly average number of new infected and vaccinated individuals

We can see from Figure 5 that there is a significant difference in the dynamics now. The average number of newly vaccinated individuals follow the trend given in the problem specification. The number of newly infected individuals quickly rise until the point in which the number of newly vaccinated individuals reaches about %15-20 of the population. Then it can be seen that the infection is slowed down.

From Figure 6 it is seen that after about 8 weeks, number of vaccinated people and susceptible people converge to the same point and stay at that number until the end of all the weeks. This is in accordance with the problem specifications, since after week 8, the number of newly vaccinated people will not change. After %60 of the population is vaccinated, there is a large portion of susceptible people, but the infection is not spread after that point because it has been contained by the vaccination.

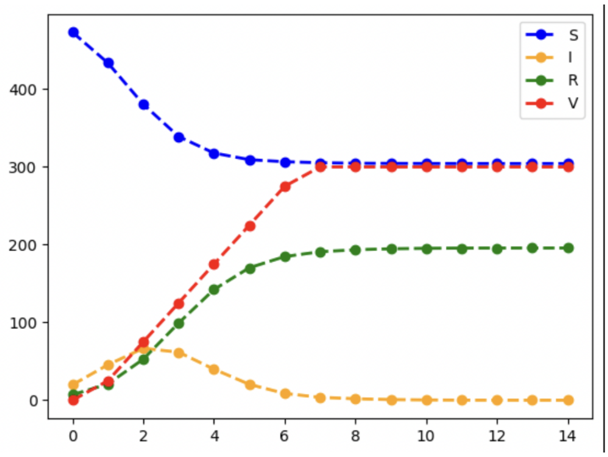

Figure 6: State Count per week

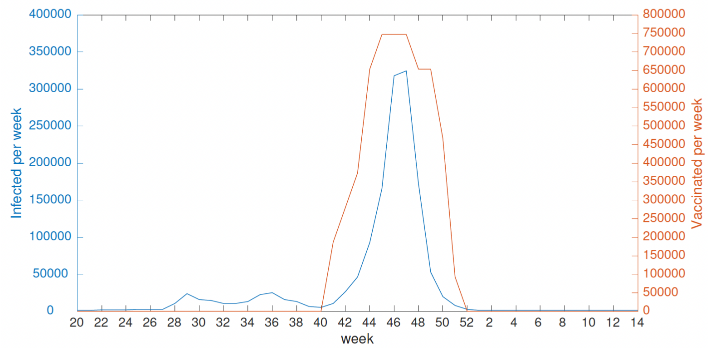

Figure 7: H1N1 2009 Pandemic weekly new cases of infection and vaccinated people

1.4

H1N1 2009 Pandemic in Sweden Simulation with Preferential Attachment Model

Finally, we will simulate the H1N1 Pandemic that was seen in Sweden during 2009. The government issued a vaccination against this during the 40'th week, and as it can be seen from Figure 7, the graph shows how the number of infections started to decrease as sixty percent of the population was vaccinated.

To simulate this pandemic, we will have a SIRV model, with 934 nodes. We have to find the best values for the other hyperparameters such as k, β and ρ. We will do this by calculating the RMSE (Root Mean Square Error) of each configuration in the grid search. Initial values for the hyperparameters are given and this will be simulated for N = 10 times.

$RMSE = \sqrt{\frac{1}{15}\sum(I(t) - I_0(t)^2}$

After the simulation of this question, we expect the graph of the simulation to be similar to Figure 7.

Best parameters are found to be k=10, $\beta = 0.09$, $\rho = 0.7$. Best Root Mean Square Error is 10.542928119518473.

Figure 8 shows the trend of simulation for the H1N1 scenario. It is not as similar to the real case trend, however some similarities can be seen. The number of newly infected

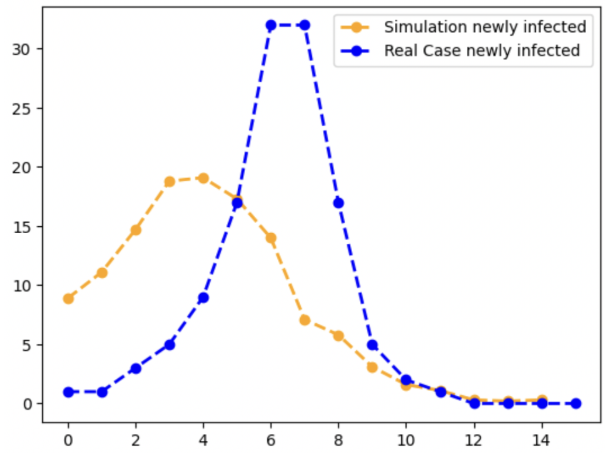

Figure 8: Weekly average number of new infected and vaccinated individuals

people rise at first, and then around the middle of the time frame of 15 weeks it peaks, followed by a fall reaching almost 0 by the end.

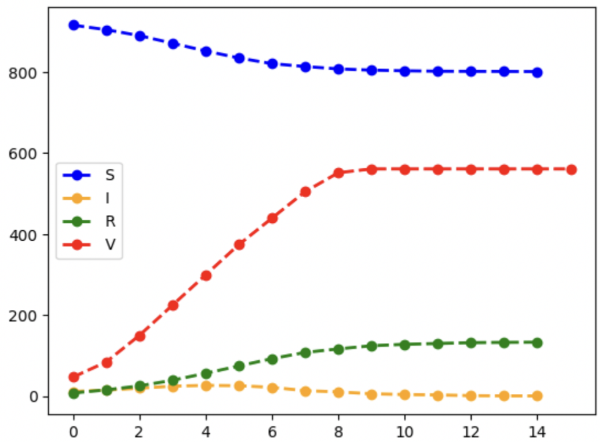

Figure 9: State Count per week

In Figure 9 each different state and their evolution throughout the weeks is shown. It is similar to the previous problem in Section 1.3. However the lines of Susceptible and Vaccinated states do not merge in this problem, instead there is a portion of the population left as un-vaccinated and susceptible since sufficient amount of people are now recovered or vaccinated.

Exercise 2

2.1

Coloring problem on a line graph

In this problem we have a line graph with 10 nodes, and we have to apply a coloring problem but this question can also be thought of a potential game. Every node has to pick the different color of their immediate neighbours. We have two states, "Red" and "Green" and we start with each node in "Red" state.

The new state of each node will be chosen with the probability function:

$P(Xi(t+1) = a|X(t), I(t) = i) = \frac{e^{\eta^{(t)} \sum W_{ij} c(a, Xj(t))}}{\sum e^{\eta^{(t)} \sum W_{ij} c(a, Xj(t))}}$

The cost function is:

$$c(s, Xj(t)) = \begin{cases} 1 & if Xj(t) = s \\ 0 & otherwise. \end{cases}$$

We start with $\eta^{(t)}$ which is the inverse of noise, as $\frac{t}{100}$.

Potential function is given by: $U(t) = \frac{1}{2} \sum W_{ij} c(X_i(t), X_j(t))$ We are trying to get the potential to converge to 0, which will mean that the goal of coloring algorithm has been reached.

Figure 10: Line Graph with all nodes in "Red" State

Figure 11: Output of Coloring Problem

From Figure 11 we can see how the nodes all assume different states then their immediate neighbours. The cost function aims to penalize nodes with the same state, therefore when this dynamics are run, the best configuration will be the one with all nodes assuming different states. The potential is 0 in this configuration and it is visible that zero potential is reached in Figure 12.

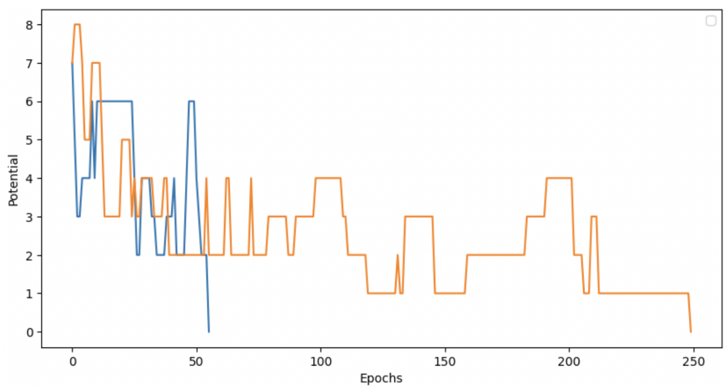

Figure 12: Potential over 2 different Epochs

2.2

Coloring algorithm for the problem of assigning Wifi-Channels to routers

Finally, we have a coloring algorithm again but this time the nodes symbolize different routers. A link between two routers mean they are able to interfere with each others Wi-fi channel and this is not a desirable scenario. Therefore the goal is to apply the coloring model in order to reach a scenario in which the routers interfere with each other the least. There are 8 possible states for the routers(nodes) which are different colors.

This will be a "Dissatisfaction Minimization" in a way, since the potential can be described as the dissatisfaction of the routers. By applying the coloring algorithm on this network we are basically applying a **Anti-Coordination Game**.

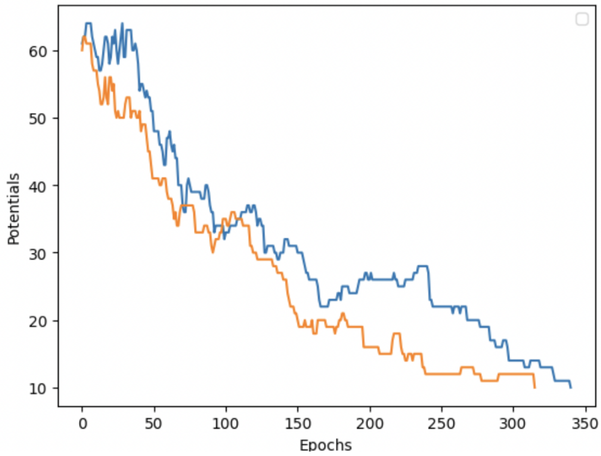

Figure 13: Evolutions of Potential

We considered 2 epochs, and after running the dynamics it is seen that near-zero potential is reached after sufficient iterations. The maximum number of iterations needed was around 700 iterations as it can be seen from Figure 13.

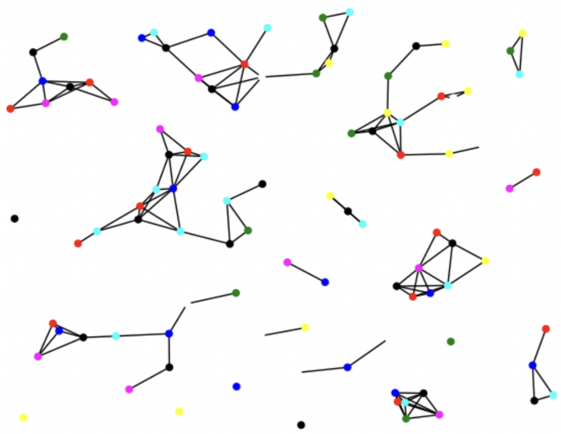

Figure 14: Routers with 8 different states (Wi-fi bands)

The optimal configuration for the routers is shown in Figure 14. The links signify routers that can interfere with each-other, but since the cost of this problem makes sure of routers using the same band or routers that are close to be assigned to different bands, we optimize the Wi-fi channel assignment in this configuration.