Exercise: Removing the insignificant knots

- Exercise 1. Define a function `f` in R:

```
f0 <- function(x){
 ans <- x*sin(20*x)
 ans[x<0] <- 0
 return(ans)
}

f <- function(x){ f0(2*(x-0.5))}
curve(f,0,1)
```

It is expected that we can approximate `f` well using a cubic spline (order 4) with knots `(1:13)/14`.

  (a) Generate data

```
n <- 1000
x <- seq(0,1,length=n)
y <- f(x) + rnorm(n, sd=0.02)
```

and fit a cubic spline to the data with knots `(1:13)/14` using truncated power basis functions. Remove the insignificant knots using backward elimination. How many knots are left in the model?

  (b) Let `knots0` be the remaining knots from Part (a). Generate data

```
n <- 1000
x <- seq(0,1,length=n)
y <- f(x)
```

Fit a cubic spline to the data with knots `=knots0` using B-spline basis functions on $[0, 1]$. Find the ISE. Denote this ISE by ISE.SP. Recall that we can also approximate `f` well on $[0, 0.5]$ using a polynomial of high degree. Is it possible to use a polynomial of degree 10 to approximate `f` so that the ISE is smaller or equal to ISE.SP?