

## Homework #1 (Theory of Computation)

Instructor: Dr. Lalatendu Behera

Total: 75 Points

**Assignment Policy:** Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- Please include your name and roll number with the submission. You need to submit the soft copy of the assignment in the google classroom and the hard copy in the class.
- This assignment is due on 7<sup>th</sup> October 2022 at 10:30 AM. Late assignments would be penalized by deducting (10% number of days lateness) % of the marks. Any form of copying will incur zero marks.
- The Institute Academic Code of Conduct will be strictly enforced.

**Problem 1**

(25 points)

Write a regular expression for the language given below.

$$L = \{0x \in \{0,1\}^* \mid 0x \text{ contains an equal number of occurrences of the substrings } 01 \text{ and } 10\}$$

Construct an  $\epsilon$ -NFA using the methods discussed in the class from the above regular expression. Then find an equivalent NFA from the  $\epsilon$ -NFA. Construct a DFA from the NFA. Finally, apply the DFA minimization algorithm to find the final DFA of the language  $L$ .

**Problem 2**

(5 points)

Let  $L$  be any language over the alphabet  $\Sigma$ . Prove that  $L = L^+$  iff  $LL \subseteq L$ .

**Problem 3**

(20 points)

Construct DFAs recognizing the following languages. In all parts, the alphabet is  $\{0,1\}$

- $\{w \mid w \text{ begins with a } 1 \text{ and ends with a } 0\}$
- $\{w \mid w \text{ contains at least three } 1\text{s}\}$
- $\{w \mid w \text{ contains the substring } 0101\}$
- $\{w \mid w \text{ has length at least } 3 \text{ and its third symbol is a } 0\}$
- $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$
- $\{w \mid w \text{ does not contain the substring } 110\}$
- $\{w \mid \text{the length of } w \text{ is at most } 5\}$
- $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$
- $\{w \mid \text{every odd position of } w \text{ is a } 1\}$
- $\{w \mid w \text{ contains at least two } 0\text{'s and at most one } 1\}$
- $\{\epsilon, 0\}$
- $\{w \mid w \text{ contains an even number of } 0\text{s, or contains exactly two } 1\text{s}\}$
- The empty set
- All strings except the empty string
- $\{w \mid w \text{ is any string not in } 0^*1^*\}$

**Problem 4**

(25 points)

Prove that the following languages are not regular using pumping lemma and the closure properties of regular languages. If you think the language is regular, then construct a DFA for the same.

- a)  $\{0^n 1^m 0^n \mid m, n \geq 0\}$
- b)  $\{0^m 1^n \mid m \neq n\}$
- c)  $\{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$
- d)  $\{wxw \mid w, x \in \{0, 1\}^*\}$
- e)  $\{0^n 1^n 01^{n+1} \mid n \geq 0\}$