RSE4502 Manipulators
Lab 2: Spatial Transformation

Objective

The objective of this lab exercise is to familiarize learners with spatial transformations using MATLAB. Learners will learn how to perform various spatial transformations such as translation, rotation, and spatial transform.

Contents

Prerequisites

Basic understanding of MATLB commands.

Understanding of spatial transformations (translation, rotation, scaling).

Familiarity with matrices and vectors.

Tools

- MATLAB visualization function: visualizeFrames.m from LMS.
- MATLAB RST Toolboxes.

Exercise 1: Rotational Transformations

1. Using the Z–Y–X (ψ, θ, ϕ) Euler angle convention, write a MATLAB program to calculate the rotation matrix ${}^{A}_{B}R$ when the user enters the Euler angles (ψ, θ, ϕ). Test for two examples:
   1.1. $\psi = 10^{o}, \theta = 20^{o}, \phi = 30^{0}$
   1.2. $\psi = 30^{o}, \theta = 90^{o}, \phi = -55^{0}$

   For case (1.1), demonstrate the six constraints for unitary orthonormal rotation matrices (i.e., there are nine numbers in a 3x3 matrix, but only three are independent). Also, demonstrate the beautiful property, ${}^{B}_{A}R = {}^{A}_{B}R^{-1} = {}^{A}_{B}R^{T}$, for case (1.1).

2. Write a MATLAB program to calculate the Euler angles (ψ, θ, ϕ) when the user enters the rotation matrix ${}^{A}_{B}R$ (the inverse problem). Calculate the two possible solutions. Demonstrate this inverse solution for the two cases from part (1). Use a circular check to verify your results (i.e., enter Euler angles in code from part (1); take the resulting rotation matrix ${}^{A}_{B}R$, and use this as the input to code from part (2); you get two sets of answers—one should be the original user input, and the second can be verified by once again using the code in part (1).

3. For a simple rotation of $, \theta$ about the Y axis only, for $, \theta$ =20° and ${}^{B}P = [1, 0, 1]^{T}$, calculate ${}^{A}P$; demonstrate with a sketch that your results are correct.

Exercise 2: Spatial Transformations

1. Write a MATLAB program to calculate the homogeneous transformation matrix ${}^{A}_{B}T$ when the user enters Z –Y –X Euler angles (ψ, θ, ϕ) and the position vector PBA. Test for two examples:
   1.1. $\psi = 10^o, \theta = 20^o, \phi = 30^0,\ {}^{A}P_{ORGB} = [1, 2, 3]^T$
   1.2. $\psi = 0^o, \theta = 20^o, \phi = 0^0,\ {}^{A}P_{ORGB} = [3, 0, 1]^T$

2. For $\psi = 0^o, \theta = 20^o, \phi = 0^0,\ {}^{A}P_{ORGB} = [3, 0, 1]^T$, as in test case above, suppose we have a vector represented in the B frame ${}^{B}P_1 = [1, 0, 1]^T$, use MATLAB to calculate ${}^{A}P_1$; demonstrate with a sketch that your results are correct. Also, using the same numbers demonstrate all three interpretations of the homogeneous transformation matrix—this part (2) assignment is the second interpretation, transform mapping.

3. Write a MATLAB program to calculate the inverse homogeneous transformation matrix ${}^{A}_{B}T^{-1} = {}^{B}_{A}T$, using the symbolic formula. Compare your result with a numerical MATLAB function (e.g., inv). Demonstrate that both methods yield correct results (i.e., ${}^{A}_{B}T^{-1}\,{}^{A}_{B}T = {}^{B}_{A}T^{-1}\,{}^{B}_{A}T = I_{4\times 4}$). Demonstrate this for examples (i) and (ii) from part (1) above.

4. Define ${}^{A}_{B}T$to be the result from Ex2 part (1.1) and ${}^{B}_{C}T$ to be the result from part Ex2 (1.2). Calculate ${}^{A}_{C}T$, and show the relationship via a transform graph.

5. Given ${}^{A}_{C}T$ and ${}^{B}_{C}T$ from part 4 above —assume you don't know ${}^{A}_{B}T$, calculate it using the loop equation. Compare your result with the answer you know from Ex2 part 1.2.

Demonstration Requirements

At the conclusion of the lab exercise on spatial transformations using MATLAB, learners are expected to demonstrate their understanding and proficiency in the following areas:

Transform Creation: Learners should create codes that accurately represent spatial transformations, including translation, and rotation.

Parameterization: Learners should appropriately parameterize their codes by assigning variables to translation vectors, and rotation angles, based on the given scenarios or input requirements.

Execution: Learners should execute their matlab codes and observe the resulting transformations to confirm the correctness of their implementation.

Observation and Analysis: After execution, learners should analyze the transformed outputs, including the positions and orientations of vectors, to ensure the successful application of spatial transformations.

Comparison: Learners should compare and contrast the effects of different spatial transformations (translation, rotation) on objects' positions and orientations, highlighting the differences between each transformation.

Presentation: During the demonstration, learners should articulate their findings, discussing their approach to setting up matlab codes, selecting appropriate parameters, and interpreting the observed transformations.

Question and Answer Session: learners should be prepared to answer questions from the instructor or peers, addressing inquiries related to their implementation choices, observations, and understanding of spatial transformations within the context of spatial transformation.