Lab Report №5 "Typical Dynamic Links"

Report

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1 Introduction

1.1 Objective

This work aims to investigate the following topics:

- Typical dynamic links.
- Frequency characteristics (amplitude-frequency response, phase-frequency response, frequency gain).

1.2 Reproducing the Results

All results can be reproduced using the repository.

1.3 Notations

Here and below, $\theta(t)$ denotes the Heaviside step function.

2 Experiment Execution

2.1 Brushed DC motor 2.0.

2.1.1 Theory

Given the equations of a separately excited DC motor:

$$J\dot{w} = M, M = k_m I, I = \frac{U + \varepsilon}{R}, \varepsilon = \varepsilon_i + \varepsilon_s, \varepsilon_i = -k_e w, \varepsilon_s = -L\dot{I}.$$

By simple transformations, we obtain:

$$\begin{bmatrix} \dot{w} \\ \dot{M} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{J} \\ -\frac{k_m k_e}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} w \\ M \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_m}{L} \end{bmatrix} U \tag{1}$$

$$w = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ M \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} U \tag{2}$$

From which we obtain the transfer function (TF):

$$W(s) = \frac{k_m}{JLs^2 + JRs + k_e k_m}$$

Then the differential equation (DE):

$$JL\ddot{w} + JR\dot{w} + k_e k_m w = k_m U$$
$$\ddot{w} + \frac{R}{L}\dot{w} + \frac{k_e k_m}{JL} w = \frac{k_m}{JL} U$$
$$\ddot{w} + 3.908\dot{w} + 48.81w = 134.2U$$

$$3.908^2 - 4 * 48.81 < 0 \Rightarrow \text{second-order oscillatory link.}$$

For frequency characteristics, we substitute $s = iw$ into TF:

$$W(w) = \frac{k_m}{-JLw^2 + JRiw + k_e k_m}$$

Applying the inverse Laplace transform for W, we get:

$$w_{i.r.} = \frac{2k_m e^{-\frac{Rt}{2L}} \sin\left(\frac{t\sqrt{\frac{-\frac{R^2}{L} + \frac{4k_e k_m}{J}}{L}}}{2}\right)\theta(t)}{JL\sqrt{\frac{-\frac{R^2}{L} + \frac{4k_e k_m}{J}}{L}}}$$
$$w_{i.r.} = 20e^{-1.95t}\sin(6.7t)\theta(t)$$

Applying the inverse Laplace transform for W/s, we get:

$$w_{s.r.} = k_m \left(\left(-\frac{e^{-\frac{Rt}{2L}} \cos\left(\frac{t\sqrt{-\frac{JR^2 - 4Lk_e k_m}{J}}}{2L}\right)}{k_e k_m} - \frac{Re^{-\frac{Rt}{2L}} \sin\left(\frac{t\sqrt{\frac{-\frac{R^2}{L} + \frac{4k_e k_m}{J}}{L}}}{2}\right)}{Lk_e k_m\sqrt{\frac{-\frac{R^2}{L} + \frac{4k_e k_m}{J}}{L}}} \right)\theta(t) + \frac{\theta(t)}{k_e k_m} \right)$$

]

$$w_{s.r.} = 0.36\left(-2.2e^{-1.95t}\sin\left(6.7t\right) - 7.56e^{-1.95t}\cos\left(6.7t\right)\right)\theta\left(t\right) + 2.75\theta\left(t\right)$$

2.1.2 Results

The results of the assignment are presented in the graph (Fig. ??). As can be seen, the practical result coincided with the theoretical one.

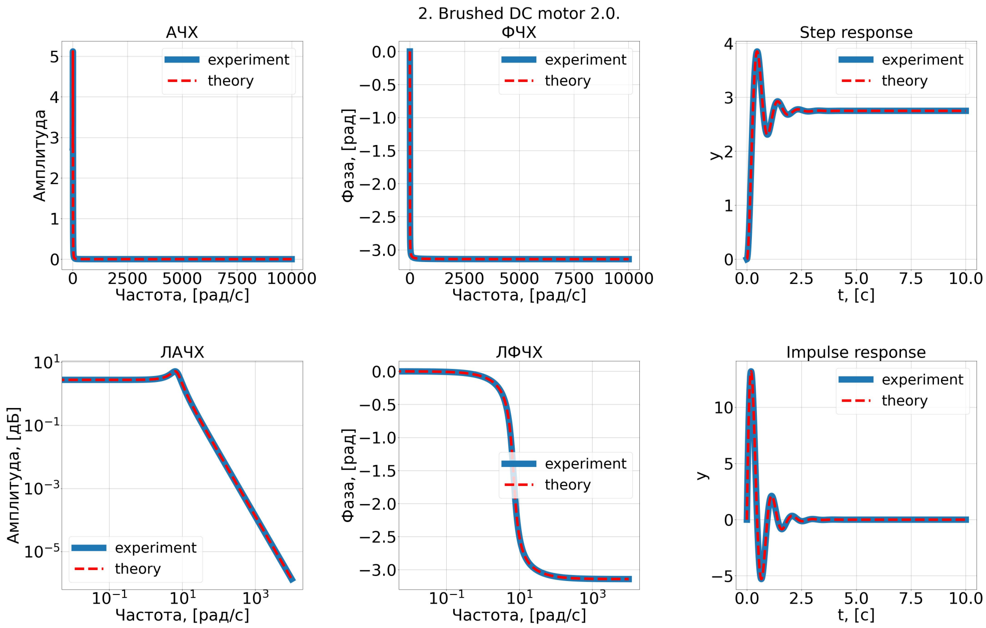

Рис. 1: The results of the assignment.

3 Conclusion

In this work, the following issues were investigated:

- Typical dynamic links.
- Frequency characteristics (amplitude-frequency response, phase-frequency response, frequency gain).

3.1 Conclusions

1. Typical dynamic links were studied using examples of different systems. The theoretical results matched the practical ones.