Time-Space Maps

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July 3, 2012

1 Introduction

Understanding the travel time relationships in a transportation system can be crucial to evaluating its performance. Time–space maps try to show an efficient and easy to understand portrayal of a transportation network.

1.1 Definition

Time space maps are a generalization of the concept of functional spaces where the physical distance corresponds to the travel time between locations.

Time–space mapping is a method that obtains a spatial configuration of cities (or more generally points), so that the Euclidean distances between points consist with the given time distances.

1.2 Terminology

Functional spaces – continuous regions generated based on proximity relationships that generalize the concept of physical distance (these concepts are generally based on psychological and cognitive distances like transportation flows, social differences or communication flows)

Multidimensional scaling (MDS) – takes the shortest paths matrix as input and generates another matrix containing the coordinates of points in time space

Distance cartogram – map which displays travel times between cities

1.3 Isochronic Maps

Are projections of a topographic or other geographical maps displaying accessibility to and/or from a specific central point in travel time. Isochronic maps can also be used to display other types of data like showing accessibility of number of jobs, potential employees, amenities within a specific time.

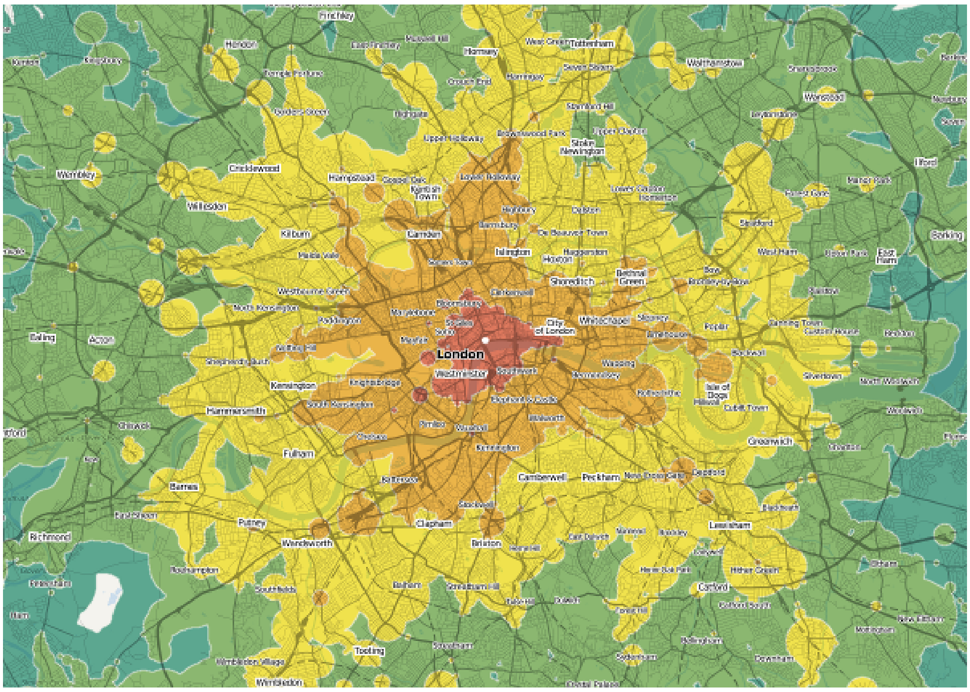


Figure 1: Isochronic map of London

1.4 Tempographic Maps

These are maps where a transformation of distance to time happens causing a change of the physical aspect of a country.

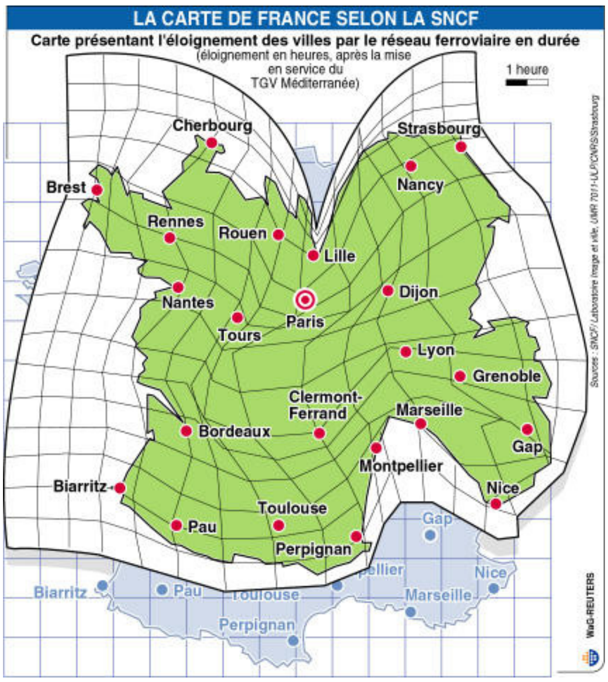


Figure 2: Tempographic map of France

2 Problem Analysis

2.1 Literature study & reverse engineering

Attempts to generate time-space maps back up to the 1960s. Bunge (1962) portrays two different time maps for Seattle. The first one uses irregular isochrones (lines of equal travel time) to show travel times from a specific origin. In the second one the map is distorted in such manner that the isochrones from the origin become concentric circles.

Ewing and Wolfe (1977) interpolate time–space locations of the transportation network as well as locations outside the network. After obtaining the set of displacement vectors they superimpose a uniform grid on the vectors and obtain a warped grid based on the influence of the vectors. They employ an inverse distance weighting method to interpolate timespace locations of lattice points of the grid. Marchand (1973) uses for the first time multidimensional scaling.

In the book Distance and Space, Gatrell (1983) summarizes time–space concepts and methodologies. He also presents an approach known as Q-analysis (Johnson, 1981). This approach focuses on set relations. The set is represented as a graph based on the connectivity of the nodes of a network to other nodes, degree of incidence and travel times along the connections. However, this generates an abstract graph that is not directly comparable with geographic space.

The goal of this study is to analyze and construct a time–space map from a set of travel–time relations on a specific country.

2.2 Project Description

For this study, we are going to focus solely on the Netherlands' railroad transportation system, there will be no display of travel times to other countries. Judging by the fact that in this time-space map lakes and other types of landform are irrelevant and may add some confusion to our end result, only cities, borders and islands will be shown. The map will have have one central point set at Utrecht and all the cities will be displayed as a circle with a fixed radius. The travel time will be shown as concentric circles around the central point set at an $\frac{1}{2}$ hour gap.

2.3 Issues

In timespace transformations, the metric nature of time–space is an issue since it is difficult to represent non-metric spaces as a two-dimensional map or analyze the space using spatial analytical techniques. The travel times in the origindestination matrix obtained from a transportation network may violate one of the metric space axioms, namely, symmetry (distances between two locations are the same regardless of direction).

The idea of rescaling the physical domain into the temporal domain is old, one of the earliest examples of time-scaled maps can be found in Tobler(1961) and Bunge (1962) where they present first examples of small-area scaling. Several works about time scaling follow, most notable ones are Shimizu (1993) and Spiekermann and Wegener (1994). In the latter they construct a two-dimensional time-scaled map of Europe's railroad system using an MDS algorithm.

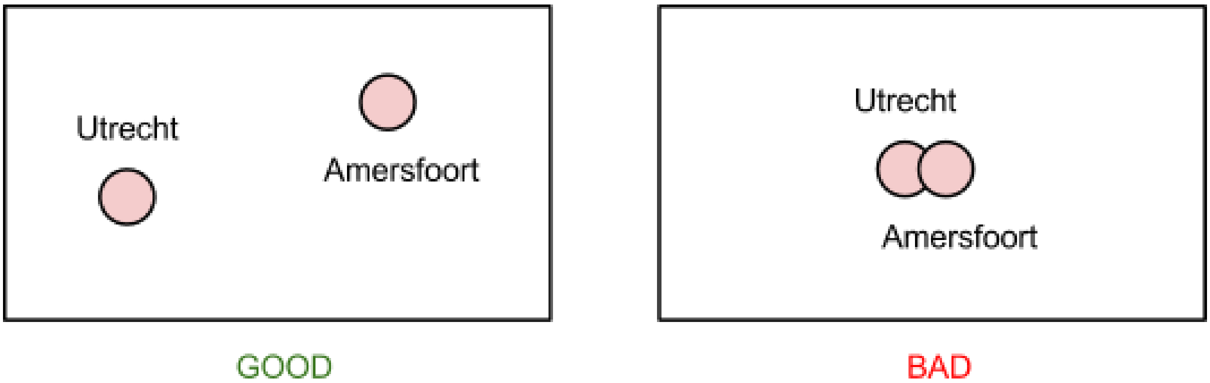
Inconclusive results regarding the nature of time–spaces and their structure probably result from the state of key transformation techniques such as multidimensional scaling (MDS) and map comparison techniques such as bidimensional regression: these techniques were not well developed or widely available until recently. Also, previous time–space mapping studies could not benefit from powerful geographic information system (GIS) software that allow management and processing of large and detailed geographic databases as well as tools for spatial analysis of these data and their product.

2.4 Statement of the criteria

Before describing an algorithm for our time–space map it is important to ask ourselves what accounts for a good time-space map. Strangely this question is not very popular in the literature and does not have a clear answer. Therefore, we present a set of criteria that would characterize a good time–space map. By measuring how well our algorithm satisfies this criteria we have a rough estimation on how well it is performing.

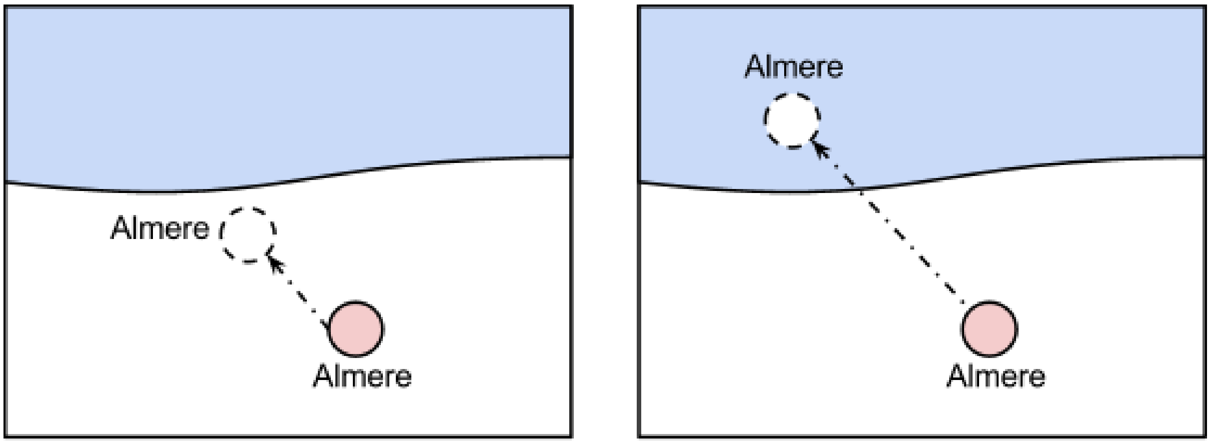
(i) cities must not be placed too close to each other (or overlap)

During the transformation cities can move closer to the central location, others can move further away. This leads to the possibility that certain cities may need to be placed very close to one another, which would make the map illegible.



(ii) cities must not be placed on water

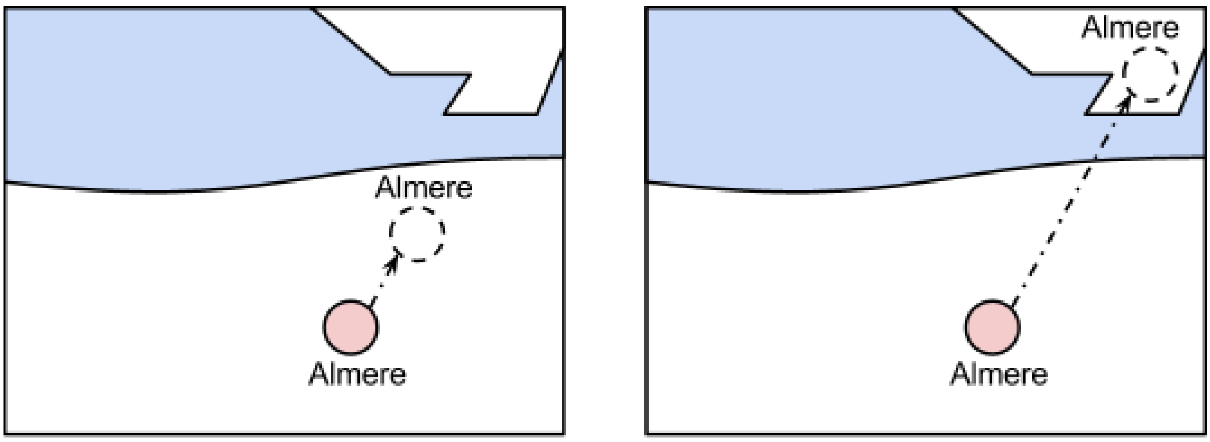
As cities move on the map to match the time distance, some bad travel connections could result in pushing a city far from its initial position. It is evidently imposed that whatever transformation is applied, the city cannot be positioned on water.



GOOD BAD

(iii) cities must not be placed on another island

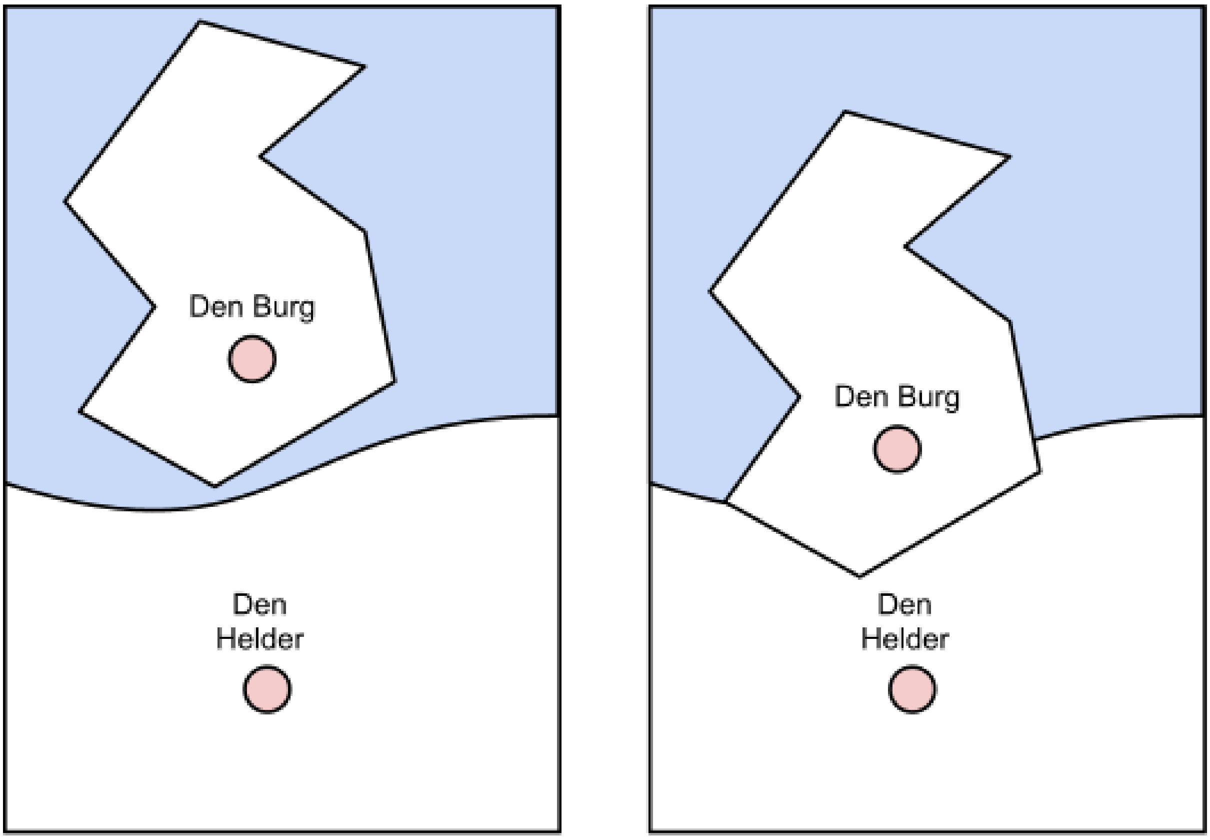
An obvious problem that can also occur after distortion is modifying the landform on which the city resides. For this reason, it is mandatory that after the distortion they do not travel outside their initial land boundaries. For the rest of the paper, when we define an island as a land areas surrounded by water. Continents are land areas having significant variation in nature of lands, climate, cultures, etc. For our purposes, when we refer to an island, we may also refer to a continent.



GOOD BAD

(iv) islands must not intersect with other forms of land

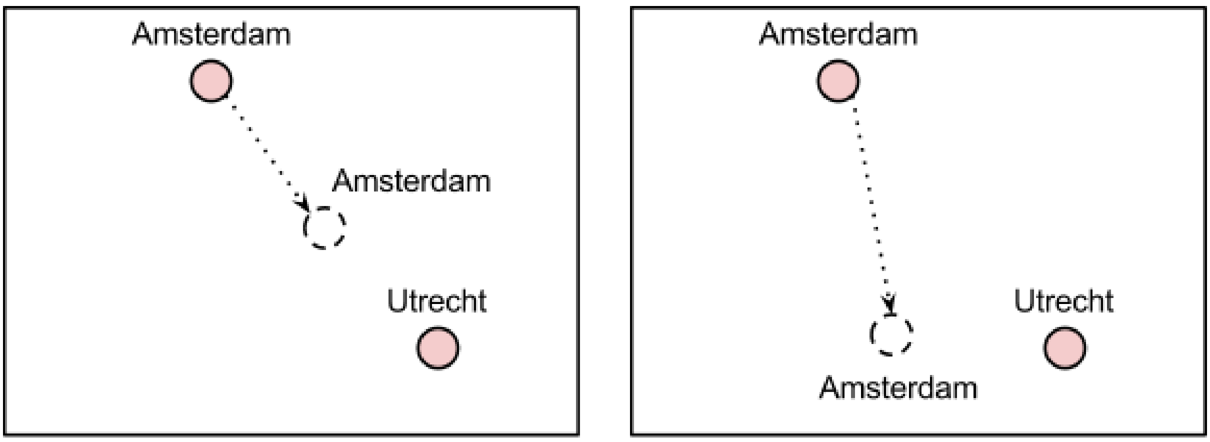
Another corner case we need to take into account is the possibility of an island to intersect with another island (or continent for that matter). This is definitely something that should be avoided, even if it introduces distortion.



GOOD BAD

(v) keep city orientation

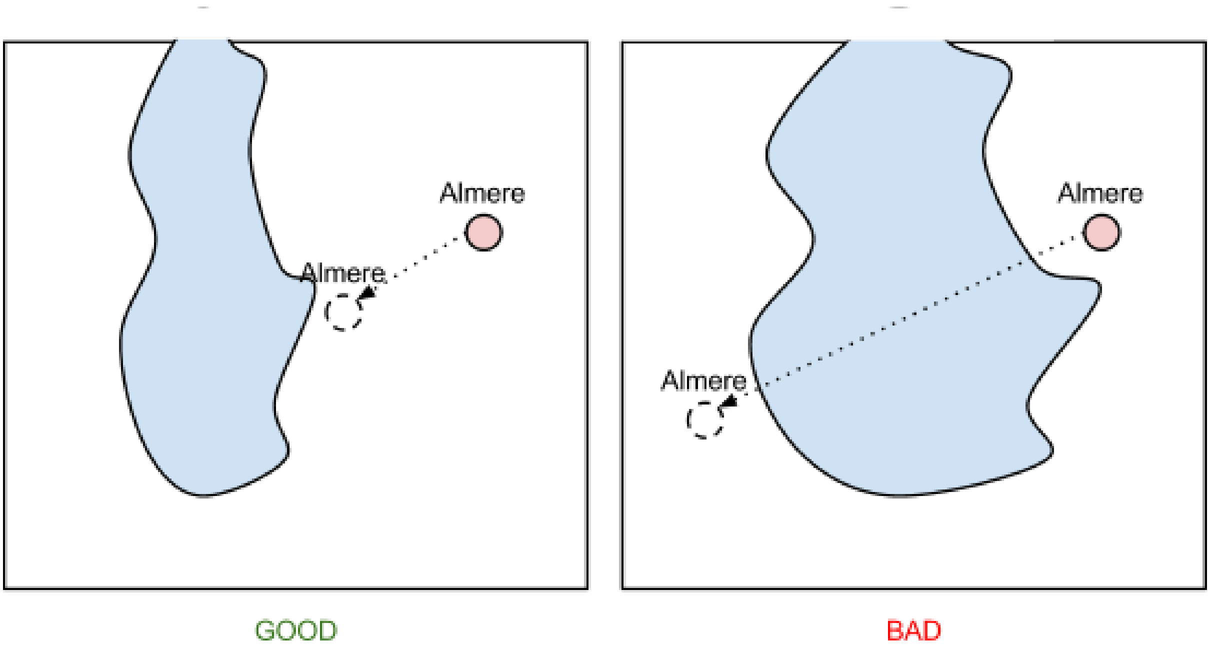
The accuracy of the map should not be altered in a way that would affect the reader's ability to recognize cities by their location on the land forms relative to the selected central point. Enforcing a limit on the orientation the city will have after distortion will avoid this situation.



GOOD BAD

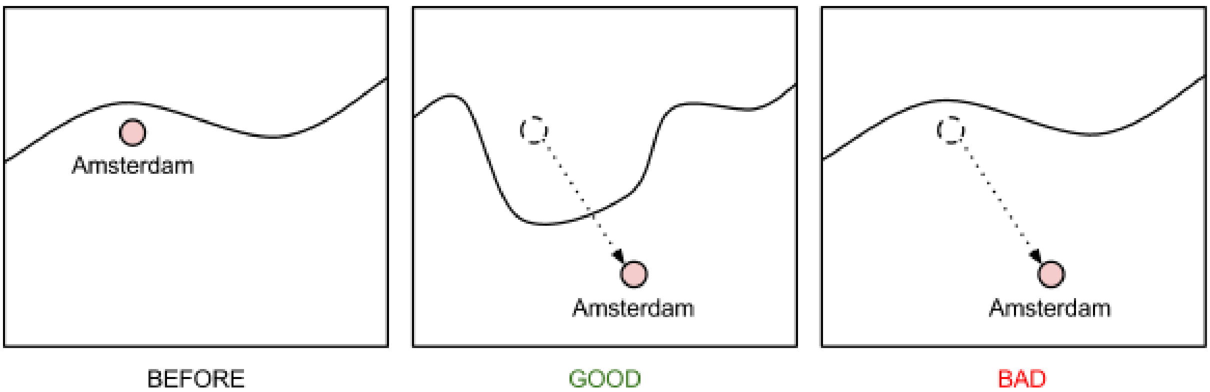
(vi) cities are not allowed to travel through a bay

The most common technique of identifying a city is to memorize its place relative to a bigger land form. As our time-space map includes only water body elements, the readability of the map can be further improved by restricting any movement that would traverse a bay. The actual identification of bays and validity of this criterion will be further explained in the "Quantification of the criteria" section of this paper.



(vii) border should follow city movement

Altering the position of the cities to emphasize the areas where the cities have a significant better travel connection is not enough to make an easily readable map. Although it would involve a great distortion of the initial map, moving the border in the direction the cities are travelling is a mandatory rule that needs to be applied in order to make a good time-space map. This immediately allows the reader to see the weak spots of the transportation network.



2.5 Dependency of the criteria

It is important to analyse the criteria stated above in order to gain a better understanding of the relationships between them. Some may be very similar, others may depend on another or there may be conflicting.

Criterion (iii) is very similar to (vi), since both of them constrain cities of travelling through water. Note that satisfying criterion (vi) also means satisfying (iii). Criterion (ii) is also similar to these, because it forbids placing a city on water. All these three could be carefully formulated into a single criteria, but for the sake of clarity and a better quantification, they are presented distinctly in this paper.

Criterion (i) and (v) impose restrictions on where we can place a city, while (iv) and (vii) account for border transformations. (iv) and (vii) may come to a conflict because of good connections to island cities. It is often possible that the border is dramatically altered so that the island overlaps the continent.

Notice how these three types of criteria all conflict to some extent with each other, since imposing some very strict restrictions on one of them leads to higher degrees of distortion in the others. For example, imposing criterion (i) may render satisfying (v) completely impossible, since two cities may want to overlap, and we need to respect the time scale. We have found that for every one of these criteria there is a corner case in which achieving a smaller distortion in one would render a higher distortion at least one of the others. A good solution would need to find the best trade-offs in satisfying all of them.

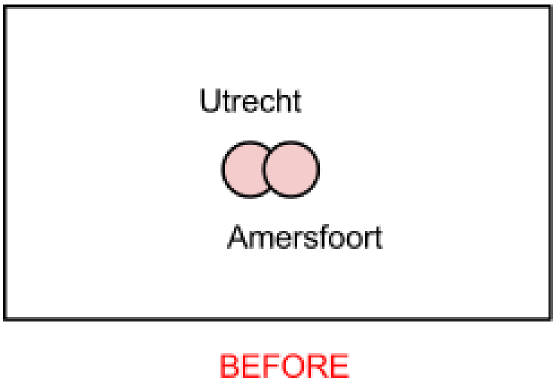
2.6 Quantification of the criteria

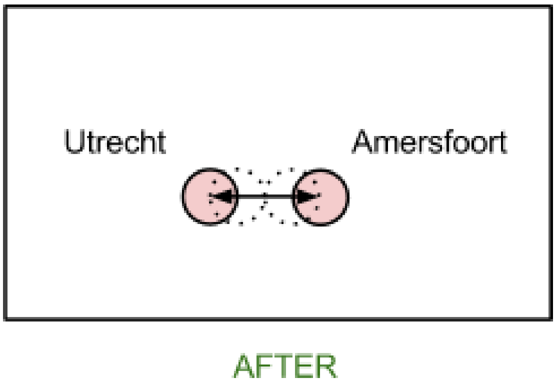
Now that we have determined our criteria, we can describe proper formal requirements. In general we will have two types of criteria, binary constraints, implying that some condition that the output of our algorithm must satisfy, and in quantitative manner, for which we would define a general quality function. This approach suggests the intent of minimizing or maximizing a specific value for a better quality output. A distortion function usually involves minimizing a value, while a quality function must be maximized. Since defining a distortion function f is analogous to defining a quality function as $1/f$, we will use the two interchangeably.

(i) cities must not be placed too close to each other (or overlap)

In order to properly define "closeness" in this context, it is important to realize that this criterion depends strictly on the scale of the map we are rendering, since its main role is purely esthetic (readability).

A reasonable quantification would be to enforce the distance between two cities to be at least two times the city point diameter. This would be what we call a hard or binary constraint.





(ii) cities must not be placed on water

This is another simple binary constraint. When moving a city to match the time travel times, it should never end up in the sea or ocean. A more general approach to this issue is that the land in its immediate vicinity should move along with it.

(iii) cities must not be placed on another island

Very similar with the criterion above, this enforces that a city must end up on the same island after the transformation. However, the same island after the transformation construct deserves a more formal explanation. While we can easily formalize same island as the existence of a land path between two points, the implication of the transformation itself makes this definition vague.

Assuming we are to place 4 anchors evenly distributed around a city point, they will always end on the land or on the coastline. Complying to this criterion would mean checking that after the transformation there is a continuous land path between the city center and all of the 4 control points.

(iv) islands must not intersect with other forms of land

This binary criterion states that none of the islands after the transformation are allowed to overlap. A basic requirement for most map transformations.

(v) keep city orientation

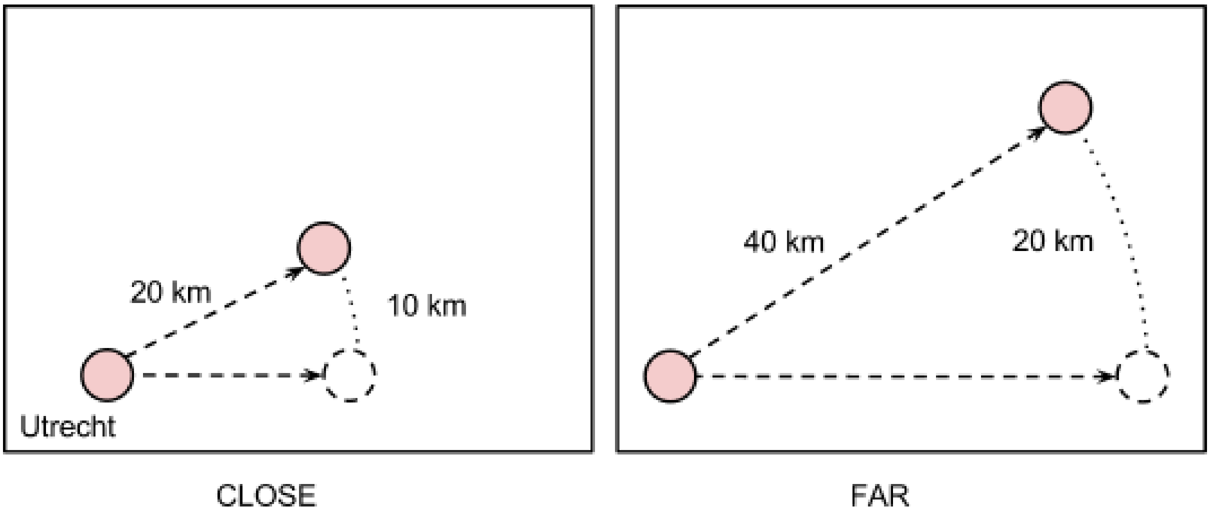
The map transformation should change the distances between the central point and different cities, but not the angle between them. We define the angle of two cities as the angle made by the straight line which joins them and the West-East axis. Due to the imposed restrictions (for example due to the fact that cities are not allowed to overlap), we cannot guarantee that this angle would always be the same. Therefore we need to describe a quantitative way of measuring the distortion introduced.

A first naive approach would be to define the distortion as the difference in the angles from before and after the transformation, but as we can see in the figure below. this is not a very good one, since $30°$ angle of distortion accounts for moving a city 10 or 20 km depending on how far it is. If we were to create a time scaled map of Europe, moving Amersfoort (a town very close to Utrecht) a few kilometers south would produce the same distortion as placing Warsaw next to Milan for example.

A better approach would be to measure the distortion in kilometers or miles. However 20 kilometers' worth of distortion would give $45°$ of freedom for placing Amersfoort and $2 - 3°$ of freedom for Groningen.

The best approach would be to define inversely proportional relationship between the angle and distance. If we note the angle measured in degrees(°) with a and distance measured in km with d, the most simple example would be the product da. Variants such as $d \log a$ could be used to decrease the importance of the angle in the product. The convolution $d * a$ is another option which probably deserves more investigation. For our purposes we found that the product d a works well enough so we will use that to measure the distortion introduced by variations in the original angle. A table with possible variation of the same distortion error is presented below.

distance(d)	10km	30km	50km	100km	150km
angle(a)	90°	30°	18°	9°	6°

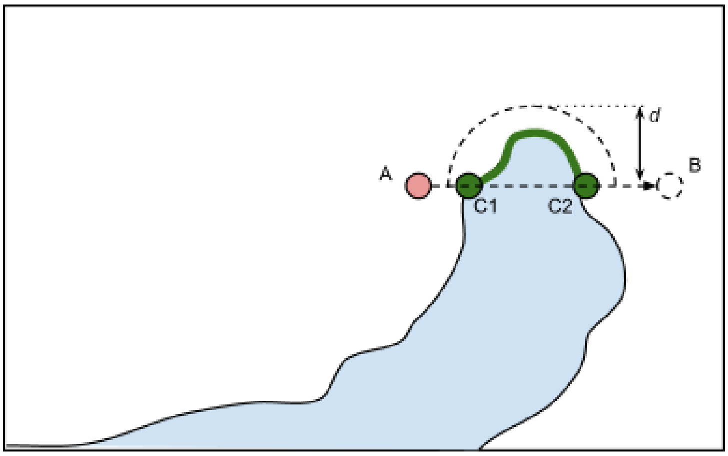


(vi) cities are not allowed to travel through a bay

Before we formalize this criterion we need to define a bay. A bay is a large body of water formed by an inlet of land. A bay can be a gulf, an inner sea, a lake, etc. Let us assume we want to move a city from A to B during a transformation algorithm (see figure below). We name as control segments all the intersections of segment $[AB]$ with water before and after applying the algorithm. Notice that A and B are on land (or on the coastline) so at the very least the union of all control segments before the transformation is included in (AB). Same thing applies to all control segments from the map generated by the transformation.

We define mapping A to B as "acceptable" with respect to d (where d is a real number) if and only if for all control segments C_iC_j we have a land path from C_i to C_j within the area defined by the circle with diameter d and center in the midpoint $[C_iC_j]$. The land path here is respective to the map the control segment belongs to (either the original map or the transformed map).

We say that the transformation of the map is "acceptable" with respect to d if all of its mappings are "acceptable" with respect to d. We can now introduce a distortion function to this criterion as the minimum real number d (measured in km) with respect to which the transformation is acceptable. Our algorithm should seek to minimize this number.



Acceptable mapping from A to B with respect to d

(vii) border should follow city movement

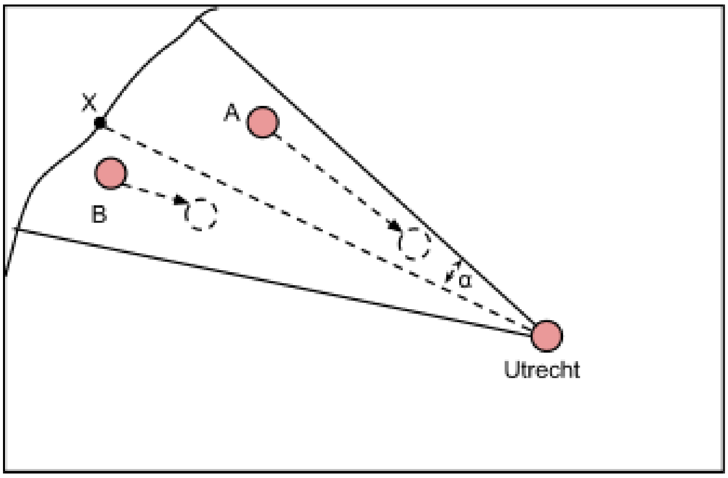
It should be clear by now that the border should be deformed to accommodate for changes in the surroundings. It is not realistic that all the cities nearby are pulled towards the center (assuming they have really good connections) but the coast remains far apart. It is even unrealistic to assume the speed with which we can travel towards the border is the same as the speed with which we travel towards the cities on the way there. Generally speaking there are so many unknowns in this equation that it is questionable whether we should even define a distortion function for this criterion. Still this is one of the most problematic aspects of the project and having some (even highly) rough estimations on how an algorithm is fairing is crucial to assess and improve it.

We start by selecting a point X on the border and assume our algorithm has generated a time t to get from the center(U) to X. We will define a distortion function $d(a)$ which returns a distortion based on an angle parameter a. First of all, we select an angle of $d(a)$ such that UX is its bisection (see figure below) and gather all the cities which are not farther away (in distance) than X.

For each of the cities mentioned, we calculate the total time it would take to reach X through that city, assuming constant speed. For example estimated time for getting to X through A would be $t(UA)(1 + \frac{d(AX)}{d(UA)})$.

For this time we set a weight of $\frac{1}{d(AX)}$ and then calculate the weighted average for all the cities within the aforementioned angle. This step ensures preference for the closest cities, favoring a decent connection to a close city (B for example) over an excellent one to a farther city (in this case A).

Finally we set $d(a)$ as the absolute value of the difference between the reported time t and the calculated weighted average. Although arguably an algorithm in its own right (albeit simple and naive) this distortion function keeps the main algorithm in check (at least to some extent) with one of the most basic approaches of solving this problem.



Border Movement

3 Algorithm design

3.1 Specification of the input and output

Building a map that outlines the transportation system of a country involves knowing some initial variables. As our focus was directed towards building a time space map of the Netherlands we will receive as input the following data:

(i) Map of the Netherlands

This will be represented as a list of cyclic points which build into polygons. A polygon is closed (we have an island) when the next point read is equal to the first one. The list may contain multiple islands.

Example:

```
(28,19), (81,7), (137,20), (173,49), (129,111), (94,71), (62,82),
(65,124), (11,124), (28,19), (202,37), (200,57), (225,62), (223,21),
(202,37)
```

(ii) List of cities

The list will be encoded as a JSON array with each element representing a city. Each component of the array will include the following details: name, position, best travel time to central city indicated in minutes. The first city in the list is the central city.

Example:

```
[ { "name":"Utrecht", "position": [23,23], "time": 0 },
  { "name":"Amersfoort", "position": [18,20], "time": 15 },
... ]
```

For convenience, after the algorithm is applied, the returned result will be in the same format as above but containing the deformed border of the map of the Netherlands and the list of cities with their new position on the map.

3.2 Algorithmic problem statement

Consider building a time space map that depicts the weaknesses and strengths of the Netherlands' train transportation system to and from a central city. The word "central" should not be used to refer to its position relative to the center of the map but rather as the center of importance to the map reader. As an initial phase it is necessary to calculate the transformation of a map from distance space to travel time space with respect to a central city U. To ensure high readability the distortion of the cities' angle with U should be minimized and the borders should scale relatively to the city movement vectors, restricting cities to not cross a geographical bay. A natural condition should also restrict movement so that islands don't intersect and cities do not travel to invalid areas (on water, another island or overlapping other cities).

3.3 Algorithm development

The basic algorithm is composed of several steps which are systematically described in Figure 3. The first values that need to be calculated are the city movement vectors, but these are highly dependent on the selected time-space scaling (so we choose this to minimize the MSE of the distances between old and new positions).

We then perform a triangulation of the distance map (see Figure 5), assuming all edges are elastic springs of spring constant k. When moving a city from their old position to their new one, it may happen that it wants to traverse an edge (another spring). In order to make sure the surface of the map is liquidly rescaled, we replace the spring with two springs tied in series with an equivalent spring constant ($\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$). We then attach the two springs to the city, so the city will have two sets of springs in parallel (see Figure 6). This adjustment ensures that the map is influenced by rebellious cities which move beyond the mainland.

It is clear however that such a modification creates the order in which to move cities very important. In order to minimize the distortion we pick cities to be moved over a small distance first.

The main issue remains simulating the physical phenomenon of chained tensions caused in the strings. In general such force-based approaches are iterated until the tensions generated on subsequent levels fall below a certain threshold ε, but the specifics of the implementation are not described in this paper.

The algorithm should satisfy all of the criteria mentioned above, but the first: minimum distance between the cities. In order to cater for this constraint we need to check violations and assign repulsing forces for the pairs of cities which do not satisfy it. Forces propagated through the network need to be recalculated through a similar algorithm with the one above.

Input: map $M = [(a_0, b_o), (a_1, b_1), \ldots, (a_n, b_n)]$
cities $C = [\{"name" : "Leiden", "position" : [23, 23], "time" : 20\}, \ldots]$
n border points, m city points

Output: map $M' = [(a'_0, b'_o), (a_1,' b'_1), \ldots, (a'_n, b'_n)]$
cities $C' = [\{"name" : "Leiden", "position" : [18, 20]\}, \ldots]$

Step 1. Compute islands
a) iterate over border points until we close a cycle (result is an island) and continue
b) for each city calculate which island it belongs to (point in polygon algorithm)

Step 2. Calculate optimal time-space scale (minimize MSE)
$\min\ E[(x_i - \alpha x'_i)^2 + (y_i - \alpha y'_i)^2] =$
$\min\ E[\alpha^2(x'^2_i + y'^2_i) - 2\alpha(x_i x'_i + y_i y'_i)]$
(α is the time-space conversion factor from time domain to space domain)

Step 3. Calculate city movement vectors (see Figure 4)
$V_i = [(x_i, y_i), (\alpha x'_i, \alpha y'_i)]$

Step 4. Create triangulation
* use a polar ray scanning approach (see Figure 5)
* triangulate islands and water bays

Step 5. Simulate tensions in springs
* sort cities by length of movement vector (ascending)
* pick first city and move it to final location
* the city gains more springs as it traverses triangles (Figure 6)
* calculate tensions in adjacent springs (breadth-first search)
* stop when tensions are lower than a predefined threshold ε
* fix city in place and repeat for all cities

Step. 6 Resolve collisions (nodes which break the minimum distance constraint)
* calculate distance required in order to solve collision
* apply half of it for each node as a repulsing force
* recalculate changes in the network
* if a city has been moved in the same iteration more then K times,
accept this error as a distance constraint violation

Figure 3: Pseudocode

3.4 Efficiency analysis

We will try to give some rough complexity estimations for the general algorithm presented above in the worst case scenario. Calculating which island each city belongs to takes $O(nm)$ time since the number of islands have an upper bound of $O(n)$. The optimal time-scale map is a mathematical optimization problem whose running time is linear, as well as calculating the

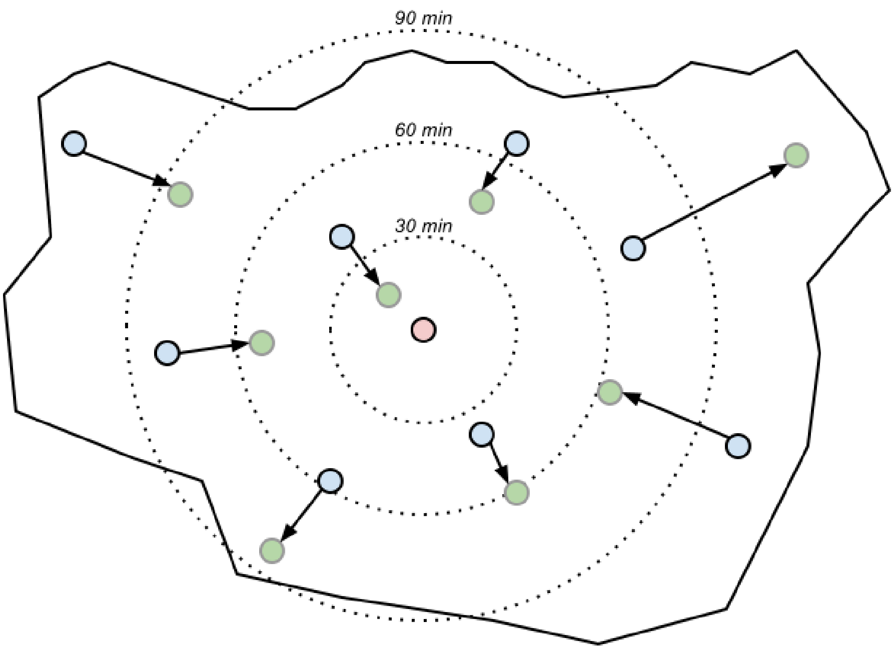


Figure 4: City movement vectors

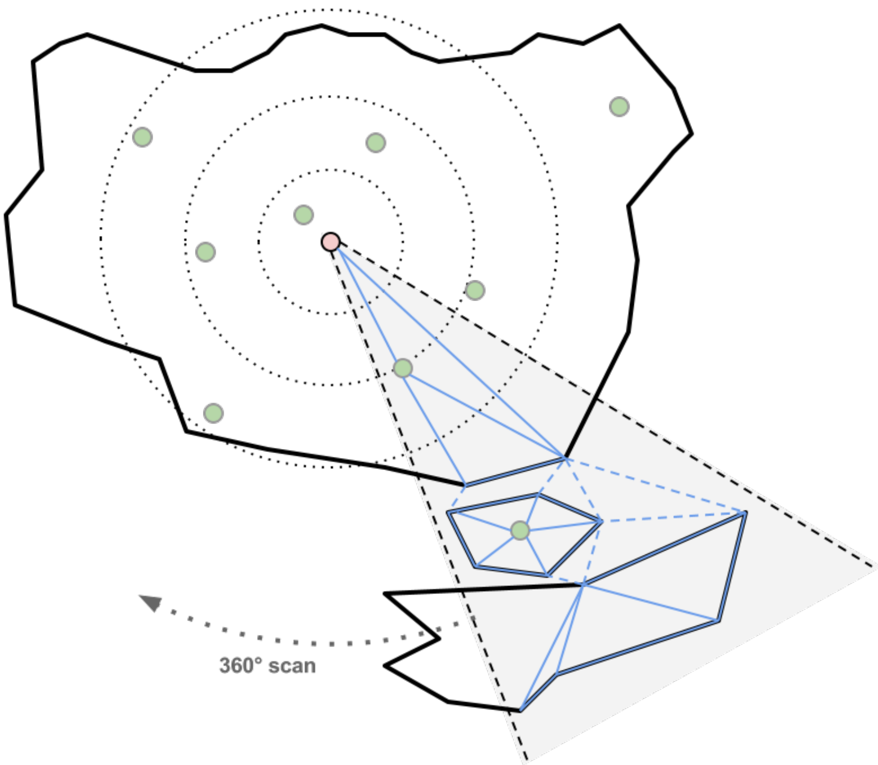


Figure 5: Ray scanning for triangulation

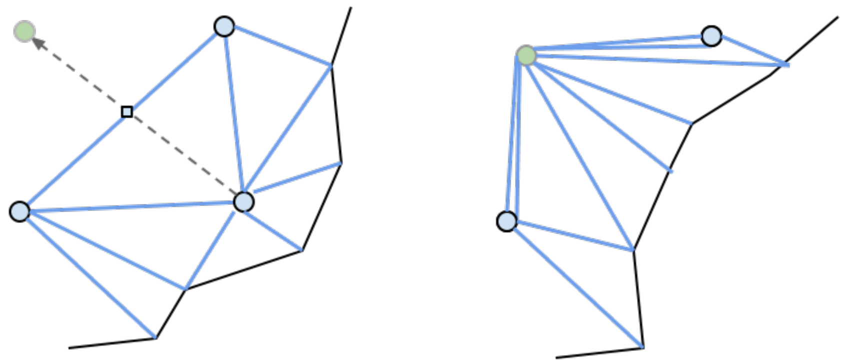


Figure 6: Spring tension when traversing a triangle edge

movement vectors.

Triangulation algorithms are usually implemented in $O(n \log n)$ time, but there are many optimizations to reduce this bound. Chazelle (1991) proved a linear time is achievable for this operation, and the ideas have been already implemented in some of the libraries available.

Steps 5 and 6 involve simulating a spring network, whose complexity depends a lot on the input data (variables K, ε, average number of triangle traversals). In general, this would achieve a running time complexity of $O(n(n+m)^2)$ since for each city (n) we need to do an adaptation of a breadth-first search ($n+m$ nodes) and checking segment intersections (for traversals) would take another $n+m$ operations.

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