Emergence of social hierarchies in a society of two competitive classes

Keywords: Agent-based models, Social hierarchies, Inequalities, Stochastic processes, Complex Systems

Extended Abstract

In recent years, agent-based modelling has been implemented to a broad number of scientific fields, where it preserves to be still an exciting approach. In fact, computational social science recently has been of great interest and growth thanks to the availability of large datasets and computational platforms to social scientists [1]. In addition, statistical physics approach is giving us a unique position for understanding a large variety of socioeconomic systems and, in particular, cities. The need for better interpretable models for cities is critical: understanding the micro-motives behind human behaviour is a necessary step to explain their macroscopic social behaviour and to be useful in the decision-making process. Several populations and social cohesion are vital for sustainable urban evolution, but cities are facing increasing segregation and inequality. One of the topics which is worth tackling is the observation, study and understanding of the formation of hierarchies and social inequalities which could appear inside cities [2, 3]. The hierarchy of social organization is an omnipresent property of animal and human aggregations, related to many features of the system such as collective decisions, intrinsic properties of the individuals, spatial characteristics, and so on. Nevertheless, in 1951 Landau pointed out that intrinsic properties of individuals such as weight and aggressiveness were not enough to give rise to the hierarchies observed, and that the interactions between individuals are essential in the formation of inequalities [4]. Consequently, based on empirical observations where dominance relationships looks to be established by the result of competition between agents, and in particular, with the existence of a positive feedback mechanism [5], Bonabeau et al. proposed a pioneer computationally-based model which shows that self-organized hierarchies can emerge spontaneously from an egalitarian society through a simple algorithm of struggles between individuals diffused on a square grid [6].

We here proceed in a different manner than in the original Bonabeau model. We assume that the society has two different classes which compete each other. Our aim is to construct a minimalist interacting agent model that accounts for the development of social diversity in cities. Agents are only allowed to exchange with agents that belong to the opposite class. We designate by $f_i(t)$, $i = 1,\ldots,N_f$, the fitness of the members of the first class, and by $F_j(t)$, $j = 1,\ldots,N_F$, the fitness of the members of the second class. The variation of an agent fitness can just change by competition. To proceed with a simple interaction, the fitness of each involved agent is standardized under the minimum value of its class, that is to say, it could be understood as the prestige/reputation which an agent has in its own community. In this standpoint, the probability that an individual i of one class wins a random competition against an individual j of the other class at time t is [6]:

$$P_{ij}(t) = \frac{1}{1+\exp\left(\eta\left(\hat{F}_j(t) - \hat{f}_i(t)\right)\right)} \tag{1}$$

where $\hat{f}_i$ is the standardized agent fitness of one class and $\hat{F}_j$ for the other one. In addition, $\eta > 0$ is a free parameter which controls the strength of the interaction. After the interaction, they exchange a certain amount of their fitness proportional to parameter $0 \leq x < 1$. Then, the total fitness $\mathscr{F}_{total} = f_{total}(t) + F_{total}(t)$ in the society always remains constant.

To solve the model numerically, Monte Carlo simulations with N_f and N_F individuals performing a stochastic random walk on a 2D square lattice is implemented. To reproduce the moment at which a moves occurs and also to determine what class of agent is shifting, a residence time algorithm has been applied. We prove that the results are invariant in the system size, and consequently on the density, and they just depend on the number of agents of each class. The main result is that for a broad range of values of η, the fitnesses of the agents of each class show a clear decays in time except for one or very few agents which capture almost all the fitness of the society. Wherefore, the results show a behavioural change in several observables of the system as a function of η. The studied observables at the stationary regime have been: the maximum agent fitness of each class normalized by the sum over the fitness values for each class (see Fig. 1), the maximum agent fitness normalized by the total fitness in the society, and finally the total fitness of each class under the total fitness in the society (see Fig. 2). As a matter of fact, for a critical value of η_c, the results do not depend on the number of agents in the population. Finally, a universal scaling sigmoidal functions for the first two observables are a good candidates for the data collapsing on the same curves independent of the agents' quantity.

We find surprising results in this model that can be interpreted properly as the non-existence of phase transition in this version of Bonabeau model, but a changing in fixed point structure. The existence of many scenarios in cities where there are two interacting groups is very large, such as racial, cultural, income groups segregation, and so on. [3, 2, 7]. Many further variations of the model, such as other winners' probabilities, different kind of interactions between agents and even additional more classes could also be implemented and studied in detail.

References

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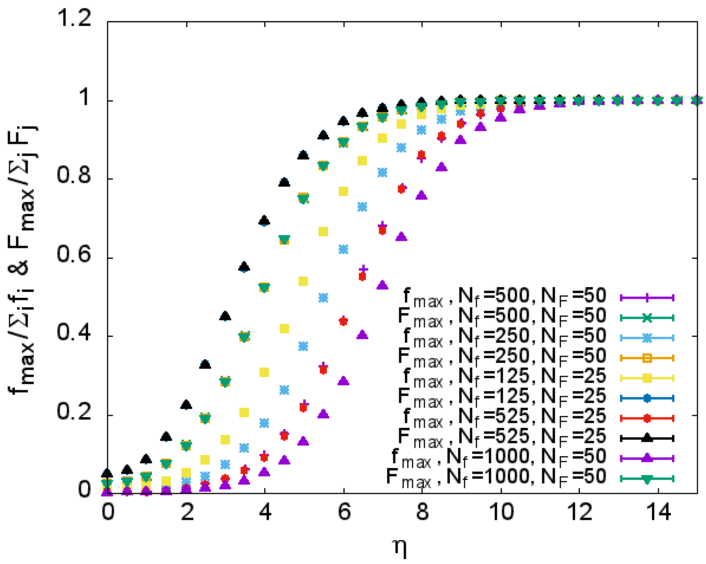

Figure 1: **Emergence of one leader in each class as η increases.** Maximum agent fitness of each class normalized by the sum of all agent fitness of the same class as a function of the parameter η. Simulations are done for several values of N_f and N_F with the exchanged proportion x fixed at 0.01. Results are computed at the stationary regime and averaging under time evolution together with 50 different runs.

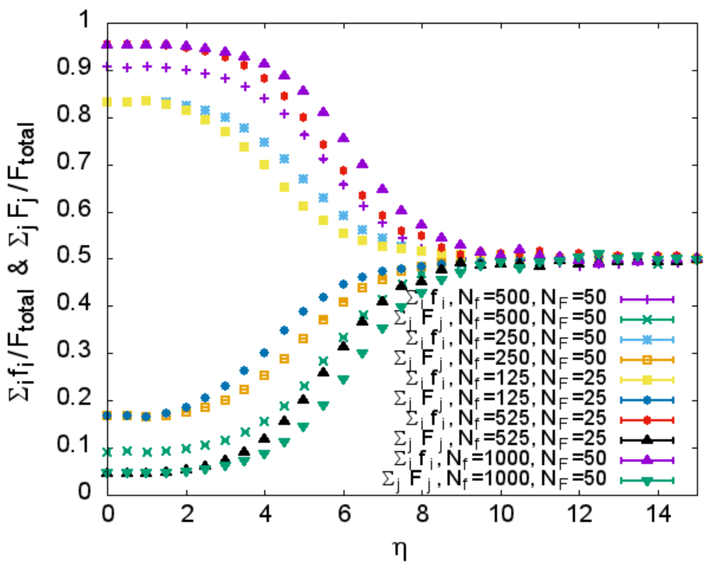

Figure 2: **Distribution of the total fitness between both classes.** Total fitness of each class under the total fitness in the society as a function of the parameter η. Simulations are done for several values of N_f and N_F with the exchanged proportion x fixed at 0.01. Results are computed at the stationary regime and averaging under time evolution together with 50 different runs.