9^{th} International Conference on Computational Social Science IC^2S^2
July 17-20, 2023, Copenhagen, Denmark

Multiscale semiwalk-based measure of structural balance in signed networks

Keywords: Structural Balance, Signed Networks, Semiwalk-based SBT, Polarization, Social Sciences Theory

Extended Abstract

Many networked system have both positive and negative ties. Examples include relations of friendships and animosity between people or correlations between financial time series. Such signed networks are often analyzed from the perspective of Structural Balance Theory (SBT) [1], which posits that configurations of relations may be balanced or unbalanced depending on the pattern of positive and negative weights. Namely, each cycle is considered either *balanced* if the product over signs of all edges is positive, or *unbalanced* otherwise. This distinction is crucial because of the fundamental result of SBT: Networks with no unbalanced cycles can be decomposed into two groups with negative edges going only between them [1] (strong structure theorem). A relaxed version (weak structure theorem) posits that if a network has no cycles with exactly one negative edge then it can be decomposed into k groups [2].

The central technical problem in SBT is how to measure the overall degree of balance in a network, that is, how close it is to being perfectly clusterable. A natural solution is to measure this in terms of the fraction of balanced cycles.

The problem is that it is not feasible to enumerate all cycles, especially in large graphs. Another problem is how to treat cycles of different lengths as obviously longer cycles are much more numerous but intuitively should be less important [3]. A possible solution, proposed by Estrada & Benzi [4], is to approximate the cycles with closed walks and assign them with weights decreasing with length. This problem is analytically tractable, as such a sum can be approximated by the trace of the exponential of an adjacency matrix, which is always defined and easy to compute based on eigenvalues of the adjacency matrix.

However, SBT for directed networks is formulated in terms of semicycles, which should be approximated not with ordinary walks but semiwalks. Moreover, it was noted already by [4] that walk-based approaches yield results that often suggest much lower levels of structural balance than typically expected, particularly for large networks. Here we extend the approach of [4] to semiwalks and introduce a tunable inverse temperature parameter β controlling the decay rate of the cycle weights (it was briefly considered by [4], but they did not study its properties in detail). We use it to show that the default walk-balance approach ($\beta = 1$) often puts too much weight on very long cycles, explaining why large networks were often found to be surprisingly unbalanced. We present methods for finding the optimal value of β and discuss physical interpretations of the walk-balance formalism, which in turn allows us to find a principled way to handle arbitrary non-uniform edge weights. We also generalize the walk-balance formalism to measure weak structural balance [2] and define pairwise measures allowing for SBT-motivated clustering of signed networks.

We illustrate our methods on a classical dataset representing sympathies and antipathies within a group of monks [5], and show how rising tensions eventually lead to a decomposition of the monastery. We also reconstruct networks of co-sponsorships in the US Congress and measure the increasing polarization within the US legislature.

References

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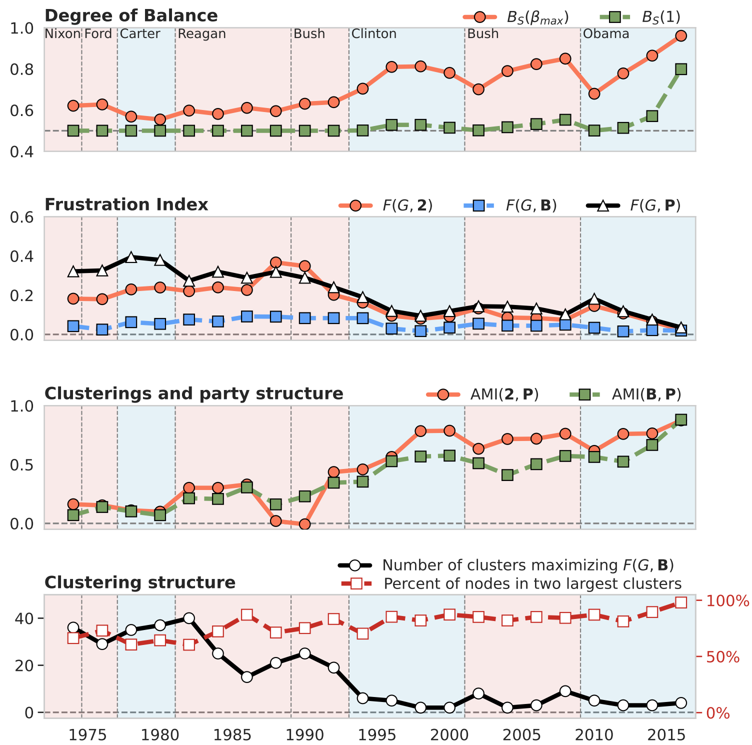

Figure 1: Polarization in the U.S. Senate between 93th and 114th Congress (1973-2016). Starting from the top, **(1st panel)** shows strong DoB time series using our approach and the approach from [4]. **(2nd)** presents values of frustration index for optimal partitions into 2 clusters, $F(G,\mathbf{2})$, optimal partition into 2 or more cluster $F(G,\mathbf{B})$, as well as partitions based on partisan affiliations, $F(G,\mathbf{P})$. **(3rd)** quantifies similarity between party-based partitions and optimal bipartitions, $AMI(\mathbf{2},\mathbf{P})$, as well as optimal clusterings, $AMI(\mathbf{B},\mathbf{P})$, using Adjusted Mutual Information (AMI). **(4th)** shows the number of clusters in the solution minimizing $F(G,\mathbf{B})$ (left y-axis), as well as the fraction of nodes within the two largest clusters (right y-axis).