Enmity Paradox

Keywords: social networks, friendship paradox, enmity paradox, generalized paradox, signed graphs

Extended Abstract

The “friendship paradox” of social networks states that, on average, “your friends have more friends than you do” [1]. Friendship paradoxes can be explained as sampling biases because high-degree individuals are counted more than low-degree individuals. Recently, the friendship paradox has been generalized to other non-topological characteristics in the network, such as happiness and wealth [2], and has been studied in relation to other topological properties of networks, such as betweenness, closeness, eigenvector, and Katz centrality [3], as well as extensions to directed networks [3], and also be considered a special case of human social sensing, for example, individuals with more social connections are more likely to be able to predict early trends than the average member of the population for many societal phenomena, from the spread of disease to the spread of information. It is also possible to exploit the friendship paradox to develop effective strategies to intervene in human networks even when the structure of the network is unclear.

This phenomenon has only been investigated from the vantage of positive networks, however. And little attention has been paid to questions regarding a counterfactual world in which negative ties exist or a mixed world in which both positive and negative ties exist. As a result, it is unclear whether we can still observe these paradoxes in a world involving antagonistic ties. For instance, does a negative world manifest the same paradoxes? If so, what mechanisms would cause such paradoxes to occur in a negative world? Here, we mathematically clarify and empirically investigate the existence, origins, and manifestations of these paradoxes. Here, we theoretically and empirically explore a new paradox we refer to as the “enmity paradox.” We use empirical data from 24,687 people living in 176 villages in rural Honduras.

Here, we find for undirected networks constructed by symmetrizing the edges (*us*), we observe almost similar strengths in the enmity and friendship paradoxes in negative and positive worlds, respectively (Figure 1, A and D). The results for the mixed worlds are also presented in Fig. 1. In *us* networks, our friends are more likely to have enemies than we are (Fig. 1B) and our enemies are more likely to have friends than we are (Fig. 1C).

To investigate the relationship between the nodal contribution in the local enmity and friendship paradoxes and its topological features, we plot the node-level heatmap of paradoxes with respect to the location of the nodes in a network. The location of the nodes is embedded using a simple fact in networks that the nodes in the center of a network have a smaller distance from other nodes while the nodes in the periphery have a larger distance from other nodes. Therefore, if we embed every node using its average and standard deviation of distances from other nodes in a network normalized by its diameter, the embedded points in the lower left of the figure are corresponding to the central locations while the upper right points are in the periphery. The results of this analysis are provided in Fig. 2, where we aggregate all heatmaps from all 176 Honduras villages for both enmity and friendship paradoxes.

In order to understand these paradoxes in greater depth, we also examine them in higher orders as well as in directed networks besides the generalized enmity paradox for nontopological attributes in real data.

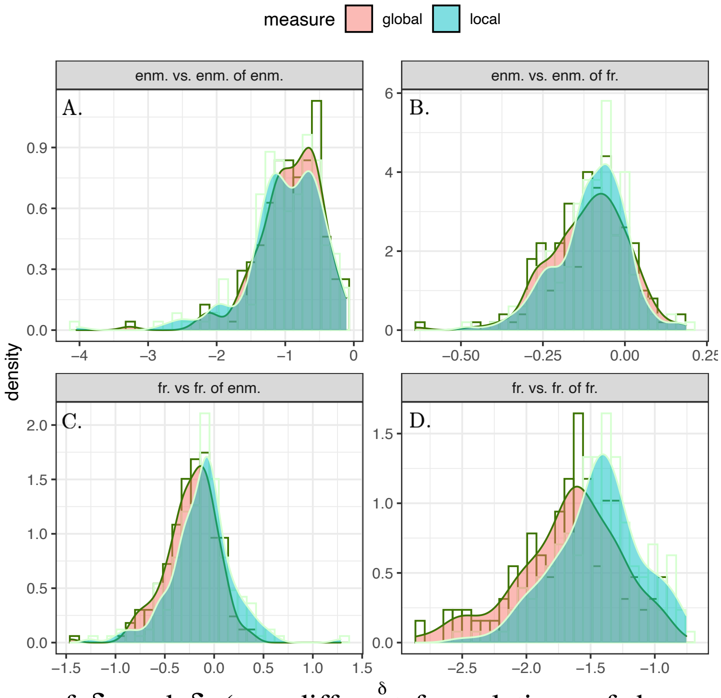

Figure 1: Histogram of δ_g and δ_l (two different formulations of the enmity and friendship paradoxes [1]) for undirected networks constructed via symmetrizing the edges (us) among 176 village networks. The histogram of friendship and enmity paradoxes for undirected networks utilizing symmetrized edges is provided in panels A and D. Other panels represent the results for the mixed worlds. For example, the histogram in panel B/C shows the global and local paradox distributions for the difference between the number of our enemies/friends and the number of enemies/friends of our friends/enemies.

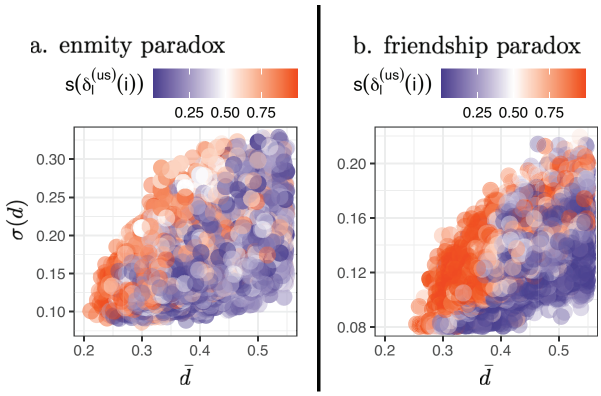

Figure 2: The relationship between the nodal contribution of paradoxes in the local enmity and friendship paradoxes and the location of the nodes in a network. Here, we utilize a sigmoid function $s(x) = {}^{1}/_{1+e^{-x}}$ in order to increase the small positive and negative differences and reduce the larger differences. The peripheral nodes (upper right) are in larger paradoxical strength in comparison with the central units (lower left).

References

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