EXERCISES

9.1 [20] For a second-order differential equation with complex roots

$$s_1 = \lambda + \mu i,$$
$$s_2 = \lambda - \mu i,$$

show that the general solution

$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t},$$

can be written

$$x(t) = c_1 e^{\lambda t}\cos(\mu t) + c_2 e^{\lambda t}\sin(\mu t).$$

✓ **9.2** [13] Compute the motion of the system in Fig. 9.2 if the parameter values are $m = 2$, $b = 6$, and $k = 4$, and the block (initially at rest) is released from the position $x = 1$.

✓ **9.3** [13] Compute the motion of the system in Fig. 9.2 if the parameter values are $m = 1$, $b = 2$, and $k = 1$, and the block (initially at rest) is released from the position $x = 4$.

o ✓ **9.4** [13] Compute the motion of the system in Fig. 9.2 if the parameter values are $m = 1$, $b = 4$, and $k = 5$, and the block (initially at rest) is released from the position $x = 2$.

✓ **9.5** [15] Compute the motion of the system in Fig. 9.2 if the parameter values are $m = 1$, $b = 7$, and $k = 10$, and the block is released from the position $x = 1$ with an initial velocity of $x = 2$.

o ✓ **9.6** [15] Use the (1, 1) element of (6.60) to compute the variation (as a percentage of the maximum) of the inertia "seen" by joint 1 of this robot as it changes configuration. Use the numerical values

$$l_1 = l_2 = 0.5 \text{ m},$$
$$m_1 = 4.0 \text{ Kg},$$
$$m_2 = 2.0 \text{ Kg}.$$

Consider that the robot is direct drive, and that the rotor inertia is negligible.

9.7 [17] Repeat Exercise 9.6 for the case of a geared robot (use $\eta = 20$) and a rotor inertia of $I_m = 0.01$ Kg m^2.

o ✓ **9.8** [18] Consider the system of Fig. 9.6 with the parameter values $m = 1$, $b = 4$, and $k = 5$. The system is also known to possess an unmodeled resonance at $\omega_{res} = 6.0$ radians/second. Determine the gains k_v and k_p that will critically damp the system with as high a stiffness as is reasonable.

o ✓ **9.20** [16] Determine α, β, k_p, and k_p required for critical damping and $\omega_n = \frac{1}{2}\omega_{res}$ if the system shown in Fig. 9.6 has mass $m = 7$, damping coefficient $b = 1$, and stiffness $k = 9$.

✓ **9.21** [13] Compute the motion of the system in Fig. 9.2 if the parameter values are $m = 3$, $b = 5$, and $k = 2$, and the block (initially at rest) is released from the position $x = 3$.

o ✓ **9.22** [18] Consider the system of Fig. 9.6 with the parameter values $m = 2$, $b = 3$, and $k = 8$. The system is also known to possess an unmodeled resonance at $\omega_{res} = 6.0$ radians/second. Determine the gains k_v and k_p that will critically damp the system with as high a stiffness as is reasonable.

9.23 [25] A shaft of stiffness 600 Nt-m/radian drives the input of a rigid gear pair with $\eta = 12$. The shaft has an inertia of 0.15 kg-m^2. The output of the gears drives a rigid link of inertia 2 Kg-m^2. What is the ω_{res} caused by flexibility of the shaft?

9.24 [25] A steel shaft of length 25 cm and diameter 0.5 cm drives the input gear of a reduction of $\eta = 10$. The rigid output gear drives a steel shaft of length 35 cm and diameter 1 cm. What is the range of resonant frequencies observed if the load inertia varies between 0.1 and 0.5 kg $\cdot$ m^2?

EXERCISES

10.1 [15] Give the nonlinear control equations for an α, β-partitioned controller for the system

$$\tau = (2\sqrt{\theta} + 1)\ddot{\theta} + 3\dot{\theta}^2 - \sin(\theta).$$

Choose gains so that this system is always critically damped with $k_{CL} = 10$.

10.2 [15] Give the nonlinear control equations for an α, β-partitioned controller for the system

$$\tau = 5\theta\dot{\theta} + 2\ddot{\theta} - 13\dot{\theta}^3 + 5.$$

Choose gains so that this system is always critically damped with $k_{CL} = 10$.

10.3 [19] Draw a block diagram showing a joint-space controller for the two-link arm from Section 6.7, such that the arm is critically damped over its entire workspace. Show the equations inside the blocks of a block diagram.

10.4 [20] Draw a block diagram showing a Cartesian-space controller for the two-link arm from Section 6.7, such that the arm is critically damped over its entire workspace (see Example 6.6). Show the equations inside the blocks of a block diagram.

10.6 [17] For the control system designed for the one-link manipulator in Example 10.3, give an expression for the steady-state position error as a function of error in the mass parameter. Let $\psi_m = m - \hat{m}$. The result should be a function of l, g, θ, ψ_m, $\hat{m}$, and k_p. For what position of the manipulator is this at a maximum?

10.7 [26] For the two-degree-of-freedom mechanical system of Fig. 10.17, design a controller that can cause x_1 and x_2 to follow trajectories and suppress disturbances in a critically damped fashion.

10.10 [15] Design a control system for the system

$$f = 5x\dot{x} + 2\ddot{x} - 12.$$

Choose gains so that this system is always critically damped with a closed-loop stiffness of 20.

10.11 [20] Consider a position-regulation system that (without loss of generality) attempts to maintain $\Theta_d = 0$. Prove that the control law

$$\tau = -K_p\Theta - M(\Theta)K_v\dot{\Theta} + G(\Theta)$$

yields an asymptotically stable nonlinear system. You may take K_v to be of the form $K_v = k_v I_n$ where k_v is a scalar, and I_n is the $n \times n$ identity matrix. *Hint*: This is similar to example 10.6.

10.15 [28] Consider a position-regulation system that (without loss of generality) attempts to maintain $\Theta_d = 0$. Prove that the control law

$$\tau = -K_p\Theta - K_v\dot{\Theta}$$

yields a stable nonlinear system. Show that stability is not asymptotic and give an expression for the steady-state error. *Hint*: This is similar to Example 10.6.

10.16 [30] Prove the global stability of the Jacobian-transpose Cartesian controller introduced in Section 10.8. Use an appropriate form of velocity feedback to stabilize the system. *Hint*: See [18].