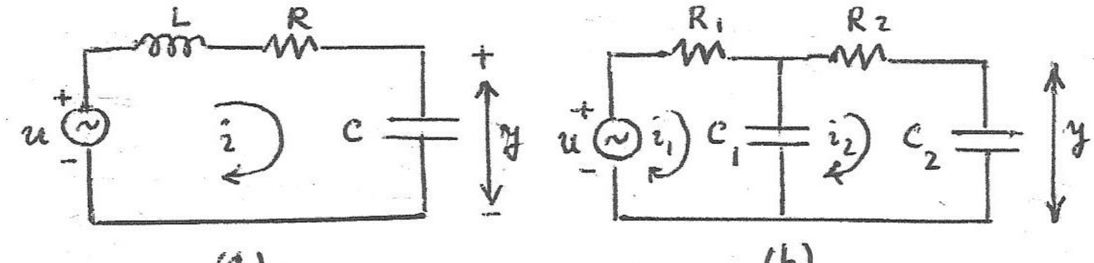
B. Shafai, ME 5250

Homework # 4

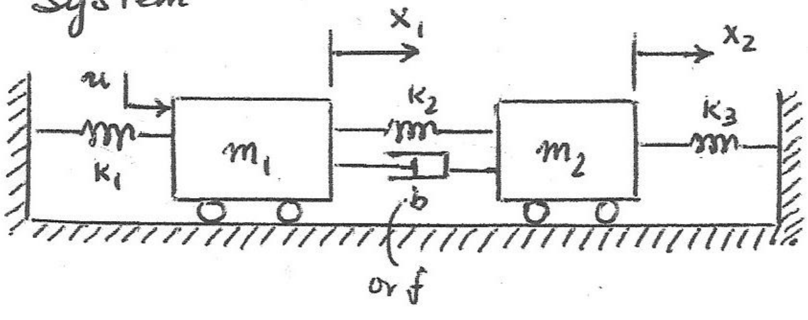
1. Consider the simple electrical circuits below



(a) (b)

Write the differential equations, state equations, and transfer functions for both circuits. Suppose the capacitor C_1 is replaced by an inductor. Obtain its transfer function and write the state equation.

2. Write the differential equation, transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the following mechanical system



3. Consider a system described by

$$\dot{x} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$$

(a) Obtain the transfer function of the system.

(b) Find the response of the system with initial conditions $x_1(0) = x_2(0) = 1$ and unit step input $u(t) = 1$. Use MATLAB to plot the response.

(c) Check the stability of the system. Suppose the term -5 is changing to $-5 + \delta$. Obtain the largest possible δ before the system becomes unstable.

4. Consider the second order system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $\omega_n = 5$ rad/sec, $\zeta = 0.4$, and $K = 1$. Obtain the unit step response of the system analytically and numerically by using MATLAB.

5. Consider the dynamical equation of a two links manipulator derived in my lecture notes and in your text-book sections 6.7 and 6.8

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta)$$

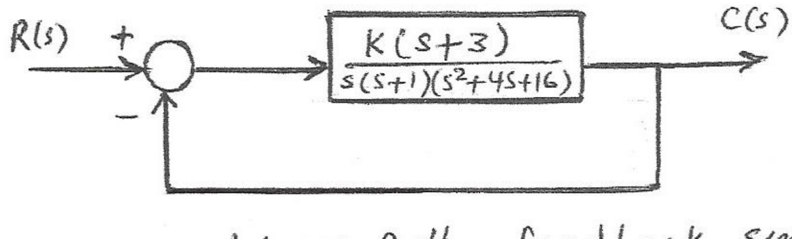
(a) Derive this equation once more yourself to understand the underlying steps.

(b) Write the state equation of the system by defining $\theta_1 = x_1$, $\dot{\theta}_1 = x_2$, $\theta_2 = x_3$, and $\dot{\theta}_2 = x_4$.

(c) Use reasonable parameter values for l_1, l_2, m_1, m_2 and find the response of the system with an arbitrary input τ of your choice. Use MATLAB to obtain your response.

6. Solve Problem 6.5 of your text-book and use reasonable values for the parameters to obtain the response of the system to an arbitrary input of your choice. Use MATLAB to obtain your response.

7. Consider the system shown below



(a) Plot the root locus of the feedback system for a range of [-6 6] in both real and imaginary axes. Use MATLAB function rlocus (num, den) to plot it. Note that once you specify the closed-loop transfer function $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$, it is sufficient to specify $1 + G(s)H(s) = 1 + k\frac{num}{den}$ and perform the task.

(b) Obtain the limiting value for k before the closed-loop system becomes unstable.

8. Consider the unstable system $G(s) = \frac{1}{(s-1)^2}$. Use a phase_lead controller to stabilize the system. Solve the problem both in transfer function and state_space formulations. Obtain the step response of the system before and after the implementation of the controller using MATLAB.