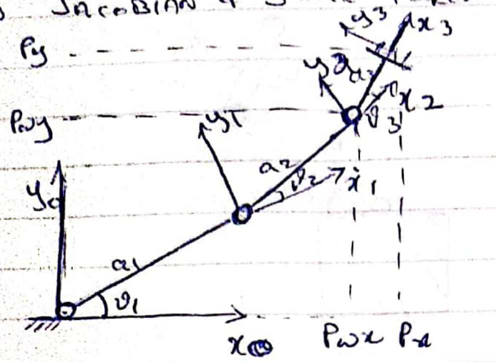
HOMEWORK ON JACOBIAN & STATIC FORCES

1.



=> All joints are revolute

Linear: $R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_n^0 - d_{i-1}^0)$

Rotational: $R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$R_0^0 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}, \quad R_1^0 = \begin{bmatrix}c\vartheta_1 & -s\vartheta_1 & 0\\ s\vartheta_1 & c\vartheta_1 & 0\\ 0 & 0 & 1\end{bmatrix}, \quad R_2^1 = \begin{bmatrix}c\vartheta_2 & -s\vartheta_2 & 0\\ s\vartheta_2 & c\vartheta_2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R_2^0 = \begin{bmatrix}c\vartheta_1 c\vartheta_2 - s\vartheta_1 s\vartheta_2 & -c\vartheta_1 s\vartheta_2 - s\vartheta_1 c\vartheta_2 & 0\\ s\vartheta_1 c\vartheta_2 + c\vartheta_1 s\vartheta_2 & -s\vartheta_1 s\vartheta_2 + c\vartheta_1 c\vartheta_2 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}C_{12} & -S_{12} & 0\\ S_{12} & C_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R_3^2 = \begin{bmatrix}c\vartheta_3 & -s\vartheta_3 & 0\\ s\vartheta_3 & c\vartheta_3 & 0\\ 0 & 0 & 1\end{bmatrix}, \quad R_3^0 = \begin{bmatrix}c\vartheta_3 C_{12} - s\vartheta_3 S_{12} & -c\vartheta_3 S_{12} - s\vartheta_3 C_{12} & 0\\ s\vartheta_3 C_{12} + c\vartheta_3 S_{12} & -s\vartheta_3 S_{12} + c\vartheta_3 C_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$R_3^0 = \begin{bmatrix}C_{123} & -S_{123} & 0\\ S_{123} & C_{123} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$T_0^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^0 = \begin{bmatrix} c\vartheta_1 & -s\vartheta_1 & 0 & a_1 c\vartheta_1 \\ s\vartheta_1 & c\vartheta_1 & 0 & a_1 s\vartheta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} c\vartheta_2 & -s\vartheta_2 & 0 & a_2 c\vartheta_2 \\ s\vartheta_2 & c\vartheta_2 & 0 & a_2 s\vartheta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_2 c\vartheta_2 \; a_1(c_1 c_2 - s_1 \ldots \\ s_{12} & c_{12} & 0 & a_1(c_1 s_2 + s_1 c_2 \ldots + a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_3(c_3 c_{12} - s_{12} s_3) + a_2 c_2 + a_1(c_1 c_2 - s_1 s \ldots \\ s_{123} & c_{123} & 0 & a_3(s_{12} c_3 + c_{12} s_3) + a_1(c_1 s_2 + s_1 c_2) + a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & a_3 c_{123} + a_2 c_2 + a_1 c_{12} \\ s_{123} & c_{123} & 0 & a_3 s_{123} + a_1 s_3 + a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{123} & -s_{123} & 0 & R_1 \\ s_{123} & c_{123} & 0 & R_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -R_2 & a_1 s_1 - R_2 & a_1 s_{12} + a_2 s_2 - R_2 \\ R_1 & R_1 - a_1 c_1 & R_1 - (a_1 c_{12} + a_2 c_2) \\ 0 & 0 & \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

2. $T_0^0 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$ $T_1^0 = \left[\begin{array}{ccc|c} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & a_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

$$R_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T_2^1 = \left[\begin{array}{ccc|c} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$ $T_3^2 = \left[\begin{array}{ccc|c} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$

$$T_2^0 = \left[\begin{array}{ccc|c} C_1C_2 & -C_1S_2 & S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & -C_1 & a_2S_1C_2 \\ S_2 & C_2 & 0 & a_1+a_2S_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T_3^0 = \left[\begin{array}{ccc|c} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$R_{11} = C_1C_2C_3 - C_1S_2S_3$ $R_{21} = S_1C_2C_3 - S_1S_2S_3$ $R_{31} = S_2C_3 + C_2S_3$

$R_{12} = -C_1C_2S_3 - C_1S_2C_3$ $R_{22} = -S_1C_2S_3 - S_1S_2C_3$ $R_{32} = -S_2S_3 + C_2C_3$

$R_{13} = S_1$ $R_{23} = -C_1$ $R_{33} = 0$

$P_x = a_3(C_1C_2C_3 - C_1S_2S_3) = X_3$

$P_y = a_3(S_1C_2C_3 - S_1S_2S_3) = Y_3$

$P_z = a_3(S_2C_3 + C_2S_3) + a_2S_2 + a_1 = Z_3$

$$d_3^0 = \begin{bmatrix} a_3(c_1c_2c_3 - c_1s_2s_3) \\ a_3(s_1c_2c_3 - s_1s_2s_3) \\ a_3(s_2c_3 + c_2s_3) + a_2s_2 + a_1 \end{bmatrix} \qquad d_2^0 = \begin{bmatrix} a_2c_1c_2 \\ a_2s_1c_2 \\ a_1 + a_2s_2 \end{bmatrix}$$

$$d_1^0 = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix} \qquad d_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -y_3 & c_1(a_1 - z_3) & c_1(a_1 + a_2s_2 - z_3) \\ x_3 & s_1(z_3 - a_1) & s_1(z_3 - a_2s_2 - a_1) \\ 0 & s_1y_3 + c_1x_3 & s_1(y_3 - a_2s_1s_2) + c_1(x_3 - a_2c_1c_2) \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

3. From Exercise 3.

$$T_3^0 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & L_1c_1 + L_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & L_1s_1 + L_2s_1c_2 \\ s_{23} & c_{23} & 0 & L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \cancel{T_4^0 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & L_1c_1 + L_2c_1c_2 + L_3c_1c_{23} \\ s_1c_{23} & -s_1s_{23} & -c_1 & L_1s_1 + L_2s_1c_2 \\ s_{23} & c_{23} & 0 & L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & x_4 \\ s_1c_{23} & -s_1s_{23} & -c_1 & y_4 \\ s_{23} & c_{23} & 0 & z_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x_4 = L_1c_1 + L_2c_1c_2 + L_3c_1c_{23}$

$y_4 = L_1s_1 + L_2s_1c_2 + L_3s_1c_{23}$

$z_4 = L_3s_{23} + L_2s_2$

$$T_0^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & l_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & l_1S_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} \cancel{+Y_H} & \cancel{Y_H} & \cancel{-S_1Z_H} & & -S_1(Z_H - L_2S_2) \\ \cancel{X_H} & \cancel{X_H} & \cancel{S_1Z_H} & & S_1(Z_H - L_2S_2) \\ 0 & 0 & \cancel{S_1(X_H - l_1C_1) + C_1(Y_H - l_1S_1)} & & Z_J \\ 0 & 0 & \cancel{S_1} & \cancel{0} & S_1 \\ 0 & 0 & \cancel{-C_1} & \cancel{0} & -C_1 \\ 1 & 1 & 0 & \cancel{0} & 0 \end{bmatrix}$$

$$Z_J = S_1(Y_H - L_1S_1 - L_2S_1C_2) + C_1(X_H - L_1C_1 - L_2C_1C_2)$$

$$J = \begin{bmatrix} Y_H & Y_H & 0 & 0 \\ X_H & X_H & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

4. $\tau = {}^3J^T \, {}^3F$

$\Rightarrow {}^3F = {}^3J^{-T} \tau$

$${}^3J = \begin{bmatrix} L_1 S_2 & 0 \\ L_1 C_2 + L_2 & L_2 \end{bmatrix}$$

$${}^3J^T = \begin{bmatrix} L_1 S_2 & L_1 C_2 + L_2 \\ 0 & L_2 \end{bmatrix} \Rightarrow ({}^3J^T)^{-1} = \frac{1}{L_1 L_2 S_2} \begin{bmatrix} L_2 & -(L_1 C_2 + L_2) \\ 0 & L_1 S_2 \end{bmatrix}$$

$\Rightarrow$ Transformation map: $\Rightarrow \frac{1}{L_1 L_2 S_2} \begin{bmatrix} L_2 & -L_1 C_2 - L_2 \\ 0 & L_1 S_2 \end{bmatrix}$

5. $${}^A_BT = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Velocity : ${}^AV =$

$${}^AV = [0 \quad 2 \quad -3 \quad 1.414 \quad 1.414 \quad 0]^T \qquad {}^BV = ?$$

$${}^BV = \begin{bmatrix} R_A^B & -R_A^B \, {}^AP_x \\ 0 & R_A^B \end{bmatrix} {}^AV$$

$${}^B_AR \, P_x^A = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} x_B & y_B & z_B \end{array} \\ \begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix} \end{array} \begin{array}{c} x_A \\ y_A \\ z_A \end{array} = \begin{bmatrix} 2.5 & -4.3 & -5 \\ 4.3 & 2.5 & -8.6 \\ 0 & 10 & 0 \end{bmatrix}$$

$$
{}^{B}V = \begin{bmatrix} 0.866 & 0.5 & 0 & -2.5 & 4.3 & 5 \\ -0.5 & 0.866 & 0 & -4.3 & -2.5 & 8.6 \\ 0 & 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.414 \\ 1.414 \\ 0 \end{bmatrix}
$$

$$
{}^{B}V = \begin{bmatrix} 3.52 & -7.8 & \overset{-17.1}{\cancel{\;}} & 1.91 & 0.5 & 0 \end{bmatrix}^T
$$

6. $$
{}^{0}J(\theta) = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix} \qquad {}^{0}F = 10\,\hat{x}_0
$$

$$
\tau = {}^{0}J^{T}(\theta)\,{}^{0}F
$$

$$
= \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -\cancel{l_2 S_{12}} + l_2 C_{12} \\ \cancel{l_1 C_1} - l_2 S_{12} & l_2 C_{12} \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix}
$$

$$
\tau = \begin{bmatrix} 10\,[-l_1 S_1 - l_2 S_{12}] \\ -10\, l_2 S_{12} \end{bmatrix} \Rightarrow \begin{aligned} \tau_1 &= -10\, l_1 S_1 - 10\, l_2 S_{12} \\ \tau_2 &= -10\, l_2 S_{12} \end{aligned}
$$

7. $$
{}^{0}_{3}T = \begin{bmatrix} C_1 C_{23} & -C_1 S_{23} & S_1 & l_1 C_1 + l_2 C_1 C_2 \\ S_1 C_{23} & -S_1 S_{23} & -C_1 & l_1 S_1 + l_2 S_1 C_2 \\ S_{23} & C_{23} & 0 & l_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}J(\theta) = ?
$$

since velocity vector is linear velocity only the displacement column is considered. Take it's differentiation.

$$
{}^{0}J(\theta) = \begin{array}{c} \begin{array}{ccc} \quad\partial\theta_1 \quad & \quad\quad\partial\theta_2 \quad\quad & \partial\theta_3 \end{array} \\ \begin{bmatrix} -l_1 S_1 - l_2 S_1 C_2 & -l_2 C_1 S_2 & 0 \\ l_1 C_1 + l_2 C_1 C_2 & -l_2 S_1 S_2 & 0 \\ 0 & l_2 C_2 & 0 \end{bmatrix} \end{array}
$$

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8. ${}^0P_{org} = [a_1c_1 - d_2s_1 \quad a_1s_1 + d_2c_1 \quad 0]^T$

$$\begin{array}{cc} \partial\theta & \partial d \end{array}$$

$${}^0J(\theta) = \begin{bmatrix} -a_1s_1 - d_2c_1 & -s_1 \\ a_1c_1 - d_2s_1 & c_1 \end{bmatrix}$$

$$\cancel{\det({}^0J(\theta)) = \frac{1}{c_1(-a_1s_1 - d_2c_1)}}$$

$$\det({}^0J(\theta)) = c_1(-a_1s_1 - d_2c_1) + s_1(a_1c_1 - d_2s_1)$$
$$= -\cancel{a_1s_1c_1} - d_2c_1^2 + \cancel{a_1c_1s_1} - d_2s_1^2$$
$$= -d_2\underbrace{(c_1^2 + s_1^2)}_{1}$$
$$= -d_2$$

∴ Singular is when $\boxed{d_2 = 0}$