HW-3

1. Compute the kinematics equation of the planar arm in example 3.3.

Link Parameters.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0 α_0	0 a_0	0	θ_1
2	0 α_1	L_1 a_1	0	θ_2
3	0 α_2	L_2 a_2	0	θ_3.

Modified form:

$$T_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = T_1^0 \, T_2^1 \, T_3^2$$

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \cancel{T_2^0} = \left[\ \cancel{c\theta_1 c\theta_2 - s\theta_1 s\theta_2} \right.$$

$$\cancel{T_2^0 = \left[\ c(\theta_1+\theta_2) - s(\theta_1+\theta_2) \right.}$$

$$T_2^0 = \begin{array}{c} \\ \end{array}\begin{array}{ccccc} C_{12} & & -S_{12} & & \end{array}$$

$$T_2^0 = \begin{bmatrix} C\theta_1 C\theta_2 - S\theta_1 S\theta_2 & -C\theta_1 S\theta_2 - S\theta_1 C\theta_2 & 0 & L_1 C\theta_1 \\ \underbrace{S\theta_1 C\theta_2 + C\theta_1 S\theta_2}_{S_{12}} & \underbrace{-S\theta_1 S\theta_2 + C\theta_1 C\theta_2}_{C_{12}} & 0 & L_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

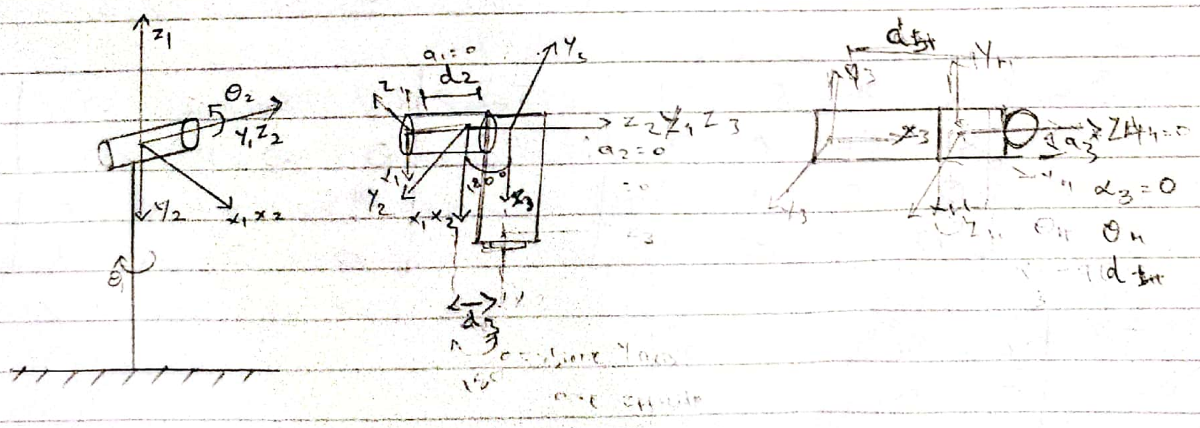
$$T_3^0 = \begin{bmatrix} C_{12} C\theta_3 - S_{12} S\theta_3 & -C_{12} S\theta_3 - S_{12} C\theta_3 & 0 & L_2 C_{12} + L_1 C\theta_1 \\ S_{12} C\theta_3 + C_{12} S\theta_3 & -S_{12} S\theta_3 + C_{12} C\theta_3 & 0 & S_{12} L_2 + L_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1 C_1 + L_2 C_{12} \\ S_{123} & C_{123} & 0 & L_1 S_1 + L_2 S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T$$

$C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$ $\quad C_{12} = \cos(\theta_1 + \theta_2)$ $\quad C_1 = \cos\theta$

$S_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$ $\quad S_{12} = \sin(\theta_1 + \theta_2)$ $\quad S_1 = \sin\theta$.

2.



Link parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90°	0	d_2	θ_2
3	90	0	d_3	180°
4	0°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Modified Form

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -S\theta_2 & -C\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ +S_3 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} C_4 & -S_4 & 0 & a_3 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5^4 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^5 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_6 & -C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & -d_2S_1 \\ S_1C_2 & -S_1S_2 & C_1 & d_2C_1 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^0 = \begin{bmatrix} -C_1C_2 & C_1S_2 & S_1 & d_2S_1 \\ S_2 & C_2 & 0 & -d_3 \\ -S_1C_2 & S_1S_2 & -C_1 & -d_2C_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} -C_1C_2 & S_1 & C_1S_2 & -d_2S_1+d_3C_1S_2 \\ -S_1C_2 & -C_1 & S_1S_2 & d_2C_1+d_3S_1S_2 \\ S_2 & 0 & C_2 & d_3C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^3 = \begin{bmatrix} C_4C_5C_6-S_4S_6 & -C_4C_5S_6-S_4C_6 & -C_4S_5 & a_3 \\ S_4C_5C_6+C_4S_6 & -S_4C_5S_6+C_4C_6 & -S_4S_5 & 0 \\ S_5C_6 & -S_5S_6 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = T_6^0 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = -dS_1 + C_1S_2(d_3+d_4) - a_3C_1C$

$P_y = d_2C_1 + S_1S_2(d_3+d_4) - a_3S_1C_2$

$P_z = (d_3+d_4)C_2 + a_3S_2$

$R_{11} = -C_1C_2C_4C_5C_6 + C_1C_2S_4S_6 + S_1S_4C_5C_6 + S_1C_4S_6 + C_1S_2S_5S_6$

$R_{12} = C_1C_2C_4C_5S_6 + C_1C_2S_4C_6 - S_1S_4C_5S_6 + S_1C_4C_6 - S_1S_2S_5S_6$

$R_{13} = C_1C_2C_4S_5 - S_1S_4S_5 + C_1C_2C_5$

$R_{21} = S_1C_2S_4S_6 - S_1C_2C_4C_5C_6 - C_1S_4C_5C_6 - C_1C_4S_6 + S_1S_2S_5C_6$

$R_{22} = S_1C_2C_4C_5S_6 + S_1C_2S_4C_6 + C_1S_4C_5S_6 - C_1C_4C_6 - S_1S_2S_5S_6$

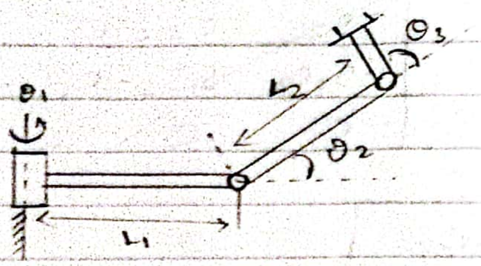
$R_{23} = S_1C_2C_4S_5 + C_1S_4S_5 + S_1S_2C_5$

$R_{31} = S_2C_4C_5C_6 - S_2S_4S_6 + C_2S_5C_6$

$R_{32} = -S_2C_4C_5S_6 - S_2S_4C_6 - C_2S_5S_6$

$R_{33} = -S_2C_4C_5 + C_2C_5$

3.



Link parameter

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	l_1	0	θ_2
3	0	l_2	0	θ_3

Modified form:

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

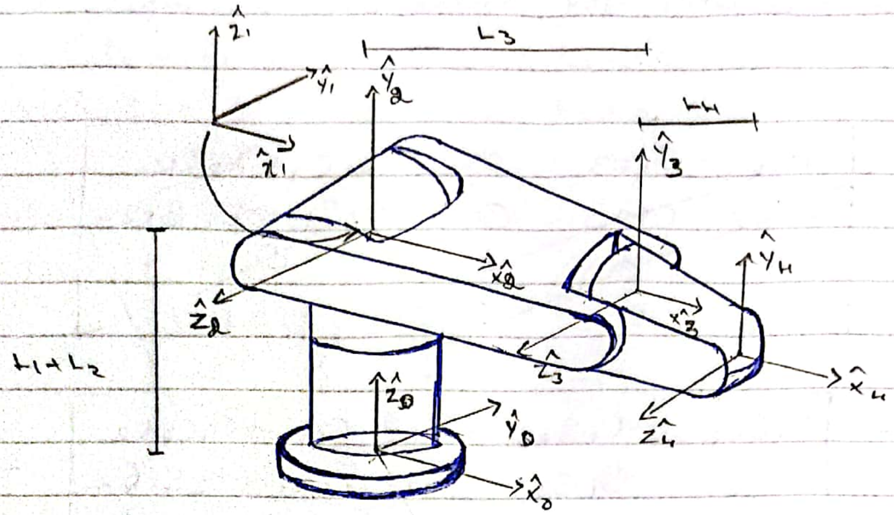
$$T_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & l_1 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} C_3 & -S_3 & 0 & l_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^0 = \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & C_1l_1 \\ S_1C_2 & -S_1S_2 & -C_1 & S_1l_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} C_1C_2C_3 - S_3 C S_2 C_1 & + C_1 S_3 C_2 - C_1 S_2 S_3 & S_1 & C_1C_2l_2 + C_1l_1 \\ S_1C_2C_3 - S_1S_2S_3 & -S_1C_2S_3 - S_1S_2C_3 & -C_1 & S_1C_2l_2 + S_1l_1 \\ S_2C_3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & S_2l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{B}_{W}T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1l_1 + C_1C_2l_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & l_1S_1 + l_2S_1C_2 \\ S_{23} & C_{23} & 0 & l_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.



{1}, {2} have same origin.

Link Parameters.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L_1+L_2	θ_1
2	90°	0	~~L_1+L_2~~ 0	θ_2
3	0	L_3	0	θ_3
4	0	L_4	0	0

Modified form

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ 0 & 0 & -1 & \cancel{L_1+L_2}\ 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & \cancel{L_1+L_2}\ 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. derive inverse kinematics for three link manipulator of exam 3.3

$$T = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1C_1 + L_2C_{12} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1l_1 + C_1C_2l_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & l_1S_1 + l_2S_1S_2 \\ S_{23} & C_{23} & 0 & l_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -①$$

let

$$T = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad -② \quad \Rightarrow \left[\begin{array}{c|c} R(\phi) & t \\ \hline 0 & 1 \end{array}\right]$$

from ① & ②

$P_x = C_1l_1 + C_1C_2l_2$

$P_y = l_1S_1 + l_2S_1S_2$

$P_z = l_2S_2$

for θ_1

$R_{13} = \sin\theta_1$

$R_{23} = -\cos\theta_1$

$$\frac{\sin\theta_1}{\cos\theta_1} = \frac{-R_{13}}{R_{23}} \Rightarrow \tan\theta_1 = \frac{-R_{13}}{R_{23}}$$

if $r_{13}, r_{23} = 0$,
$\Rightarrow$ no solution.

$$\theta_1 = \tan^{-1}\left(\frac{-R_{13}}{R_{23}}\right)$$

$$= \operatorname{atan2}(-R_{13}, R_{23})$$

for θ_2:

$$P_x = C_1(l_1 + l_2 C_2)$$
$$P_y = S_1(l_1 + l_2 C_2)$$

if $S_1 = 0$

$$C_2 = \frac{1}{l_2}\left(\frac{P_x}{C_1} - l_1\right)$$

if $C_1 = 0$

$$C_2 = \frac{1}{l_2}\left[\frac{P_y}{S_1} - l_1\right]$$

$$P_z = S_2 L_2$$

$$S_2 = \frac{P_z}{l_2}$$

$$\frac{S_2}{C_2} = \tan\theta_2 \Rightarrow \theta_2 = \tan^{-1}\left[\frac{P_z/l_2}{C_2}\right]$$

$$\boxed{\theta_2 = \operatorname{atan2}\left(\frac{P_z}{l_2}, C_2\right)}$$

To find θ_3:

$$R_{31} = S_{23}$$
$$R_{32} = C_{23}$$

$$\theta_{23} = \tan^{-1}\left[\frac{R_{31}}{R_{32}}\right] = \operatorname{atan2}(R_{31}, R_{32})$$

$$\theta_{23} = \theta_2 + \theta_3$$

$$\theta_3 = \theta_{23} - \theta_2$$

$$\theta_3 = \tan^{-1}\left[\frac{R_{31}}{R_{32}}\right] - \tan^{-1}\left[\frac{P_z/l_2}{C_2}\right]$$

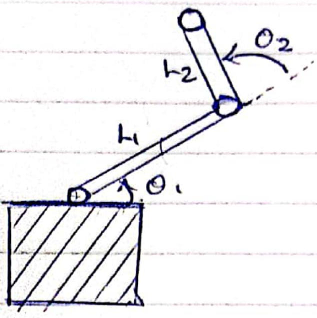
$$\boxed{\theta_3 = \operatorname{atan}(R_{31}, R_{32}) - \theta_2}$$

if $R_{31}, R_{32} = 0$
θ_3 is unsolvable

6. $l_1 = 2l_2$

$0 < \theta_1 < 180$

$-90 < \theta_2 < 180$



work space