Northeastern University
Department of Electrical and Computer Engineering

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Final Exam

1. Consider a robotic cart that tries to balance a pendulum at the vertical position by appropriate control u as shown in Figure 1. Let H and V be, respectively, the horizontal and vertical forces exerted by the cart to the pendulum.

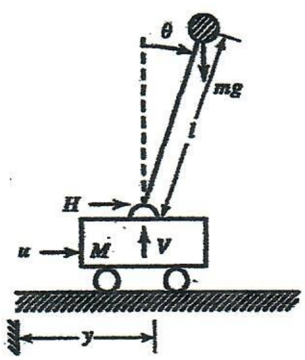

Figure 1. Inverted Pendulum on a Cart

(a) Derive the dynamical equations of this system and show that they are nonlinear.

(b) Under the assumption that θ is small, show that the dynamical equations of the system can be represented by two coupled linear differential equations as

$$(M+m)\,\ddot{y}+m\,l\,\ddot{\theta}=u$$

$$m\,l^2\ddot{\theta}+m\,l\,\ddot{y}=m\,g\,l\,\theta$$

(c) Find the transfer functions of the pendulum $\theta(s)/U(s)$ and $Y(s)/U(s)$. Obtain the poles of the system with respect to each transfer function and show that the system is open loop unstable.

(d) Write the state equations of the system using the coupled differential equations from part (b) by defining the state variables $\theta=x_1$, $\dot{\theta}=x_2$, $y=x_3$, $\dot{y}=x_4$.

(e) Use a state feedback control law $u=v+k\,x$ to stabilize the system with desired eigenvalues -1, -2, -3, -4 where $M=2$ kg, $m=0.1$ kg, $l=0.5$ m. Use command "place" from MATLAB to perform the task. Note that $u=v-k\,x$ is used in MATLAB to obtain k.

2. Consider the two-link planar manipulator shown in Figure 2.

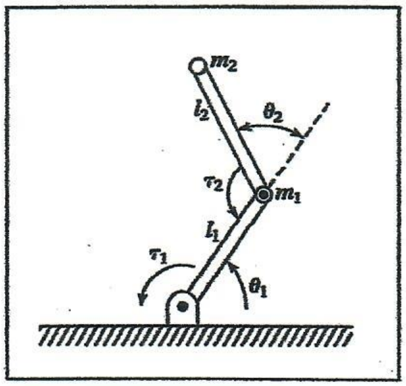

Figure 2. Two-link Planar Manipulator

(a) Using the Jacobian of the two-link manipulator, calculate the tool point (end-effector) linear velocity if joint 1 is rotating at 1 rad/s and joint 2 is rotating at 3 rad/s for two cases of ($\theta_1 = 60^0$, $\theta_2 = 30^0$) and ($\theta_1 = 167^0$, $\theta_2 = -156^0$), where $l_1 = 2$m and $l_2 = 3$m.

(b) Calculate the resulting joint torque τ, if two different forces F = (30, 20) and F = (30, -20) are applied to the tool point corresponding to the two cases in part (a). Compare the results of two cases and make your conclusion.

(c) Consider the dynamical equation of two-link manipulator derived in your text-book and my lecture notes, which can be summarized as follows

$$\tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta),$$

$$M(\Theta) = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2 c_2 + l_1^2(m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix}.$$

$$V(\Theta, \dot{\Theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix}. \qquad G(\Theta) = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix}.$$

Apply the method of computed torque (control-law partitioning approach) to stabilize the system such that the arm is critically damped over its entire work space. Use your own numerical values to perform simulation with the aid of MATLAB/SIMULINK. If you are not familiar with Simulink, obtain the matrices K_p and K_d, and draw the block diagram implementation.

3. Consider a one link manipulator described by the nonlinear differential equation

$$\tau = ml^2\ddot{\theta} + f\dot{\theta} + mgl\cos\theta$$

(a) Use the method of computed torque (control-law partitioning approach) to construct the control law to keep the system critically damped with natural frequency of $\omega_n = 10$. Let $m = 1$, $l = 1$, $f = 7$, and $g = 10$ to solve the problem.

(b) Suppose the goal is to track the desired trajectory $\theta_d = A\sin(2\pi t/T)$, where $A = 0.1$ rad (6 degrees) and T=2s. Draw the block diagram implementation. Perform simulation if possible.

(c) The above nonlinear system can be linearized around the equilibrium point. Use the open literature to obtain the linearized model of the above system in state equation representation and investigate its stability status.