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Homework on Preliminary Mathematical Background

<u>Homework 1</u>

1. Are the following vectors linearly independent with respect to their associated field?

(a) $\begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 13 \\ 10 \end{bmatrix}$, $\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1+i \\ 2+3i \end{bmatrix}$, $\begin{bmatrix} 10+2i \\ 4-i \end{bmatrix}$, $\begin{bmatrix} -i \\ 3 \end{bmatrix}$.

2. Which of the following subset of vector space (R^3, R) are actually subspaces?

(a) The plane of vectors with first component $x_1 = 0$

(b) The plane of vectors with first component $x_1 = 1$

(c) The plane that goes through the end point of orthogonal basis vectors

(d) All combinations of two given vectors $u = [1\ 1\ 0]^T$ and $v = [2\ 0\ 1]^T$.

(e) The vectors with components x_1, x_2, x_3 that satisfy $3x_1 - x_2 + x_3 = 0$

3. Consider the two dimensional vector space (R^2, R) with vectors $x = [1\ \ 3]^T$, $e_1 = [1\ \ 0]^T$, $e_2 = [0\ \ 1]^T$, $\bar{e}_1 = [1\ \ 4]^T$, $\bar{e}_2 = [3\ \ 2]^T$. Find the representation of x with respect to $[e_1\ e_2]$ and $[\bar{e}_1\ \bar{e}_2]$. Perform this task analytically and graphically.

4. Find the representation of a linear operator defined by Reflection of a point in two dimensional vector space with respect to y-axis. Suppose the basis of the vector space is changed from the orthogonal basis $[1\ \ 0]^T$, $[0\ \ 1]^T$ to $[2\ \ 1]^T$, $[1\ \ 1]^T$. What is the representation in this case?

5. Find the representation of the linear operator

$$L:(R^3,R)\to(R^2,R)\ :\ L[x]=\begin{bmatrix}x_1-x_2\\ x_2-x_3\end{bmatrix}$$

with respect to the basis $v_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $v_2=\begin{bmatrix}1\\0\\1\end{bmatrix}$, $v_3=\begin{bmatrix}0\\1\\-1\end{bmatrix}$ of (R^3,R)

and the basis $w_1=\begin{bmatrix}1\\1\end{bmatrix}$, $w_2=\begin{bmatrix}1\\-1\end{bmatrix}$ of (R^2,R).

6. Consider the algebraic equation $Ax=0$, where

$$A=\begin{bmatrix}0&1&1&2&-1\\1&2&3&4&-1\\2&0&2&0&2\end{bmatrix}$$

specify the basis for the null space of A and find the general solution of $Ax=0$ in terms of the basis vectors. Does the equation $Ax=b$ have always solution for any b? If, No, why not? If, Yes, give one possible b.

7. Find the general solution of the following algebraic equations

(a) $$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 1 & 0 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix}$$

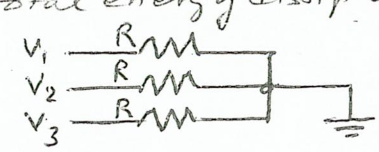
(b) $$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$$

(c) $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

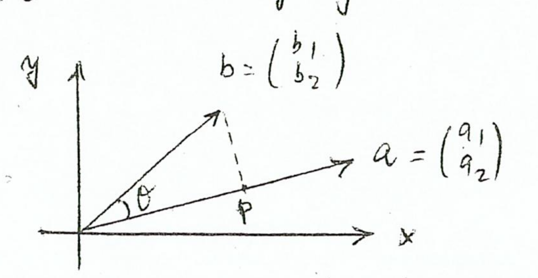
8. (a) A small microcomputer has five terminals connected to it. The fraction of the time devoted to each terminal is x_i, so $x_1 + x_2 + x_3 + x_4 + x_5 = 1$. The programmer at terminal 2 types four times as fast as the programmer at terminal 1. They are both typing in the same code and must finish at the same time. Both terminals 3 and 4 are sending mail files to 5. Find the best possible solution for allocating CPU time.

(b) Find the equation of a parabola $C + Dx + Ex^2$ that best fits the points 1, 2, 4, 5, 8 at $-2, -1, 0, 1, 2$.

(c) A specified amount of constant current, i, must be delivered to the ground point as shown in figure below. Specify v_1, v_2, v_3 so that the total energy dissipated in the resistors is minimized.



9. Find the angle between two vectors a and b, as well as the projection vector p of b on a.



10. (a) Find a basis for the subspace of the vector space (R^4, R) in which $x_1 = x_2 = x_3$.

(b) If the vectors $x_1, x_2, \ldots, x_m$ span a subspace S, then can we conclude that $\dim S = m$.

11. Find a third column so that the matrix

$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & * \\ 1/\sqrt{3} & 0 & * \\ 1/\sqrt{3} & -1/\sqrt{2} & * \end{bmatrix}$$

is orthogonal.

12. If u is a unit vector, prove that $Q = I - 2uu^T$ is an orthogonal matrix. Show this with an example by taking $u = [\, 1/\sqrt{3} \;\; 1/\sqrt{3} \;\; 1/\sqrt{3} \,]^T$.