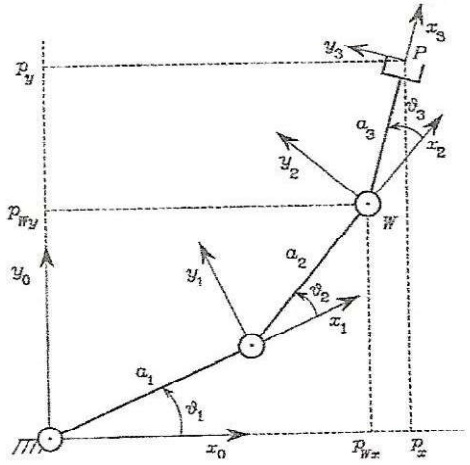
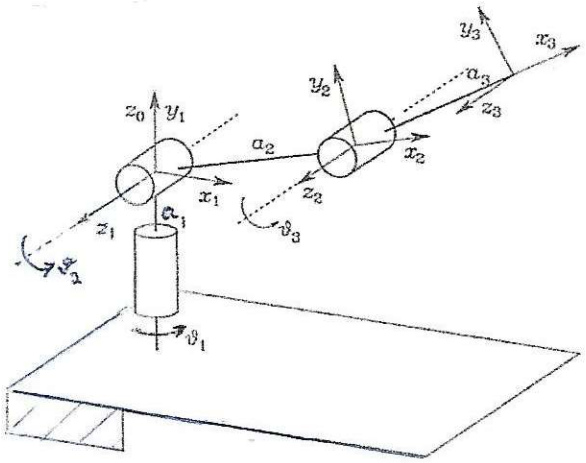
B. Shafai, Robotics ME5250

Practice Homework on Jacobian and Static Forces

1. Consider the three-link Planar Arm as shown below and obtain its Jacobian



2. Consider the following configuration of a manipulator. Derive all rotation matrices and displacement vectors, as well as the associated homogeneous transformation matrices. Then, obtain the Jacobian matrix of this manipulator.



3. Find the Jacobian of the manipulator with three degrees of freedom from exercize 3 of chapter 3. Write it in terms of a frame {4} located at the tip of the hand and having the same orientation as frame {3}.

4. For the two-link manipulator of example 5.3 of your textbook, give the transformation that would map joint torques into a 2x1 force vector, ${}^3F$ at hand.

5. Given

$$ {}^A_B T = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

If the velocity vector at the origin of {A} is ${}^A V$ given by ${}^A V = [0 \quad 2 \quad -3 \quad 1.414 \quad 1.414 \quad 0]^T$. Find the 6x1 velocity vector with reference point the origin of {B}.

6. A certain two-link manipulator has the following Jacobian:

$$ {}^0 J(\theta) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} $$

Ignoring gravity, what are the joint torques required in order that the manipulator will apply a static force vector ${}^0 F = 10 \hat{X}_0$.

7. The homogeneous transformation of a 3R robot is given by

$$ {}^0_3 T = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & l_1 c_1 + l_2 c_1 c_2 \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & l_1 s_1 + l_2 s_1 c_2 \\ s_{23} & c_{23} & 0 & l_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Find ${}^0 J(\theta)$, which, when multiplied by the joint velocity vector, gives the linear velocity of the origin of {3} relative to {0}.

8. The position of the origin of Link 2 for an RP manipulator is given by ${}^0 P_{2ORG} = [a_1 c_1 - d_2 s_1 \quad a_1 s_1 + d_2 c_1 \quad 0]^T$. Obtain the 2x2 Jacobian that relates the two joint rates to the linear velocity of the origin of frame {2}. Give a value of θ for singularity.