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Course: CIS 502
Assignment: HW7

Question 1

First of all, in order to ensure that we have enough sequence of x and sequence of y to choose from, let $xs = x^n$ and $ys = y^n$, such that both xs and ys will be no shorter than s.

Let $m(i, j)$ denote the feasibility of getting the first $i + j$ symbols after interleaving first i symbols of xs and first j symbols of ys. We know $m(0, 0) = 1$, and we will have the formula,

$$m(i,j) = \begin{cases} 1 & \text{if } (m(i-1,j) = 1 \text{ and } xs[i] == s[i+j]) \text{ or } (m(i,j-1) = 1 \text{ and } ys[j] == s[i+j]) \\ 0 & \text{otherwise} \end{cases}.$$

We will have to search all the combination of i ,j and compute the corresponding $m(i, j)$, such that for $k \in [1..n]$,$i + j = k$. At last, if there is at least one $m(i', j') = 1$, where $i' + j' = n$, we can say s is an interleaving of x and y.

Because the length of s is n, the maximum value of i or j should be n. Therefore, we need to compute a $n \times n$ matrix of m. The runtime is $O(n^2)$.

Question 2

Observation 1: we have to order some gas on day 1. Because the tank is empty at the end of day 0.

Observation 2: we do not need to consider the oil cost. Because for each possible scheme, the total cost of oil is always $\sum_{i=1}^{n} g_i$.

Observation 3: the amount of oil we order should be $\sum_{i=a}^{b} g_i$, where $a \leq b$.

Proof. Suppose not. If we order A gallon oil on day a, we know $A > g_a$ and $A = \sum_{i=a}^{b} g_i + \Delta$, where $a \leq b$. So we have to order again on day $a + b + 1$. But we can actually save $c(a + b)\Delta$ by only ordering $\sum_{i=a}^{b} g_i$ on day a. It contradicts our goal that we want to save as much as we can. □

Question 3

a) For a bipartite graph $G = (V, E)$, each person $p_i \in V$, each night $d_i \in V$, and there is a edge $(p_i, d_j) \in E$ if $d_j \notin S_i$.

If G has a perfect matching, each person p_i will be paired with exact one night d_j, and no person is left alone and no night is left alone. Therefore, for each matching (p_i, d_j), the person p_i is able to cook on night d_j, and this provides a feasible schedule.

b)

Question 4