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Theory & Formuale used –

Continuously compounded zero coupon yield is given by:

$$y(t,T) = -\frac{\log(P(t,T))}{T-t}$$

where, P(t, T) is zero coupon bond price , which is calculated using different short rate models like Vasicek model or Cox-Ingersoll-Ross (CIR) model.

1. Vasicek Model:

- The risk neutral process for r given by the model is:

$$dr = \beta(\mu - r)dt + \sigma dW^Q$$

- Zero-coupon bond prices in Vasicek's model are given by:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

where, $B(t,T) = \frac{1-e^{-\beta(T-t)}}{\beta}$, and

$$A(t,T) = \exp\left(\frac{(B(t,T)-T+t)\left(\beta^2\mu-\frac{\sigma^2}{2}\right)}{\beta^2} - \frac{\sigma^2 B(t,T)^2}{4\beta}\right)$$

2. Cox-Ingersoll-Ross (CIR) Model:

- The risk neutral process for r given by the model is:

$$dr = \beta(\mu - r)dt + \sigma\sqrt{r}dW^Q$$

- Zero-coupon bond prices in Vasicek's model are given by:

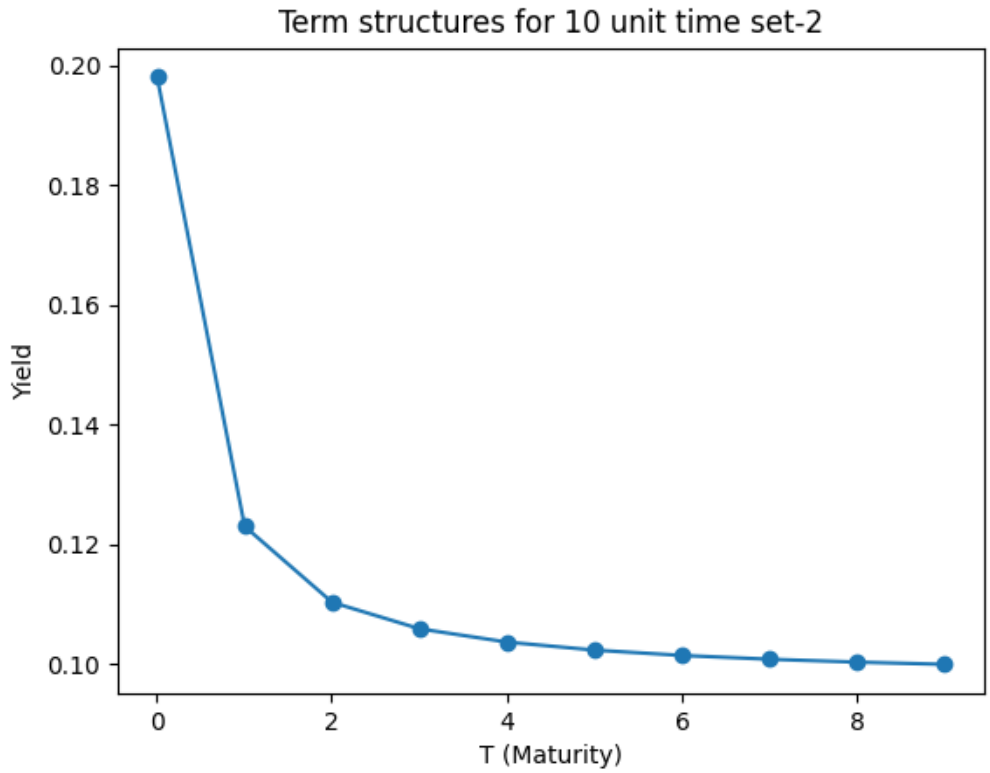
$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

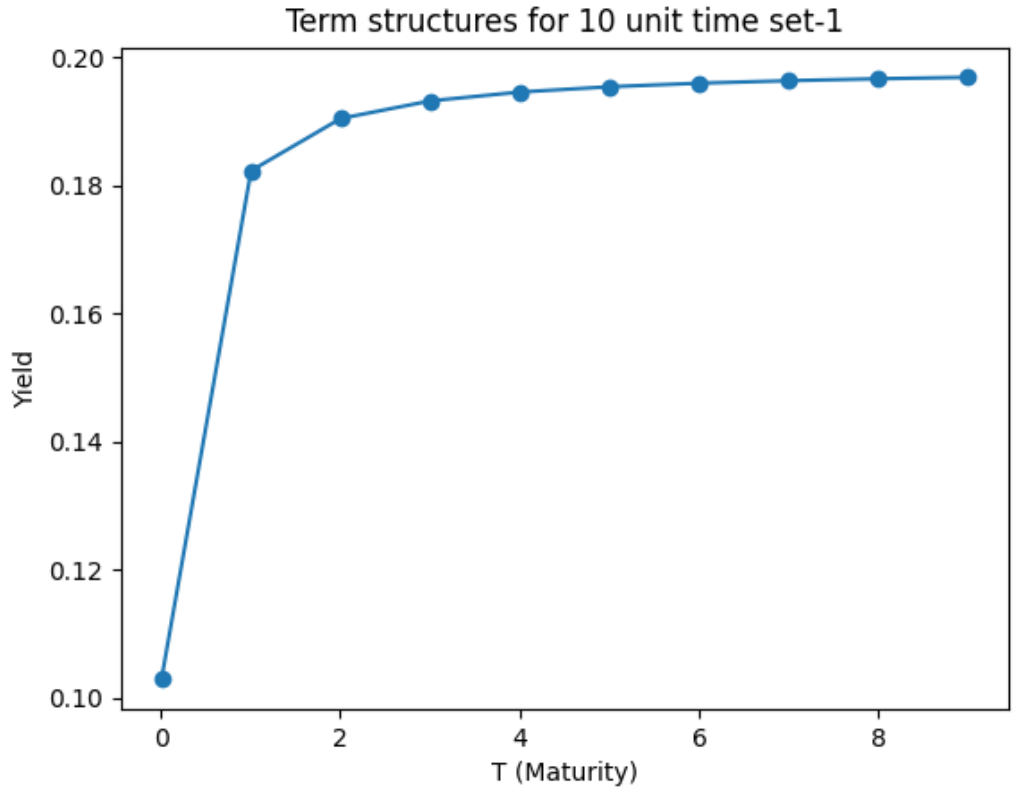
where, $B(t,T) = \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+\beta)(e^{\gamma(T-t)}-1)+2\gamma}$,

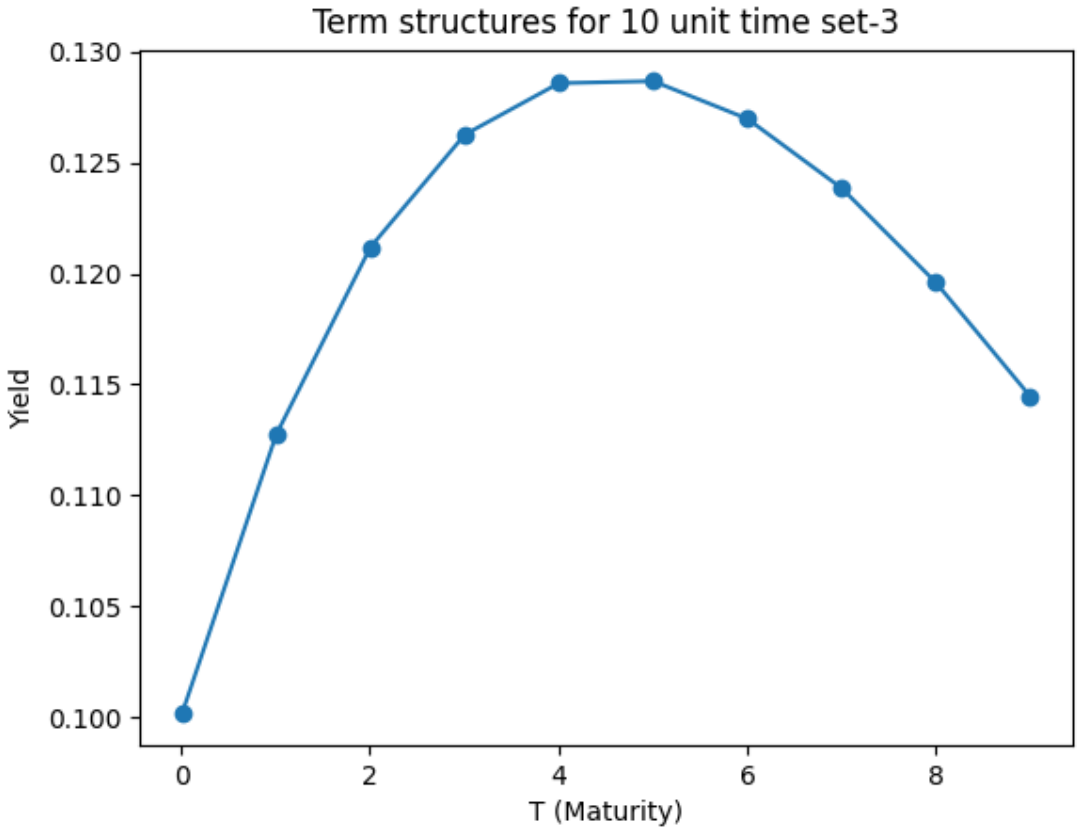
$$A(t,T) = \left[\frac{2\gamma e^{(\beta+\gamma)(T-t)/2}}{(\gamma+\beta)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2\beta\mu/\sigma^2}$$, and

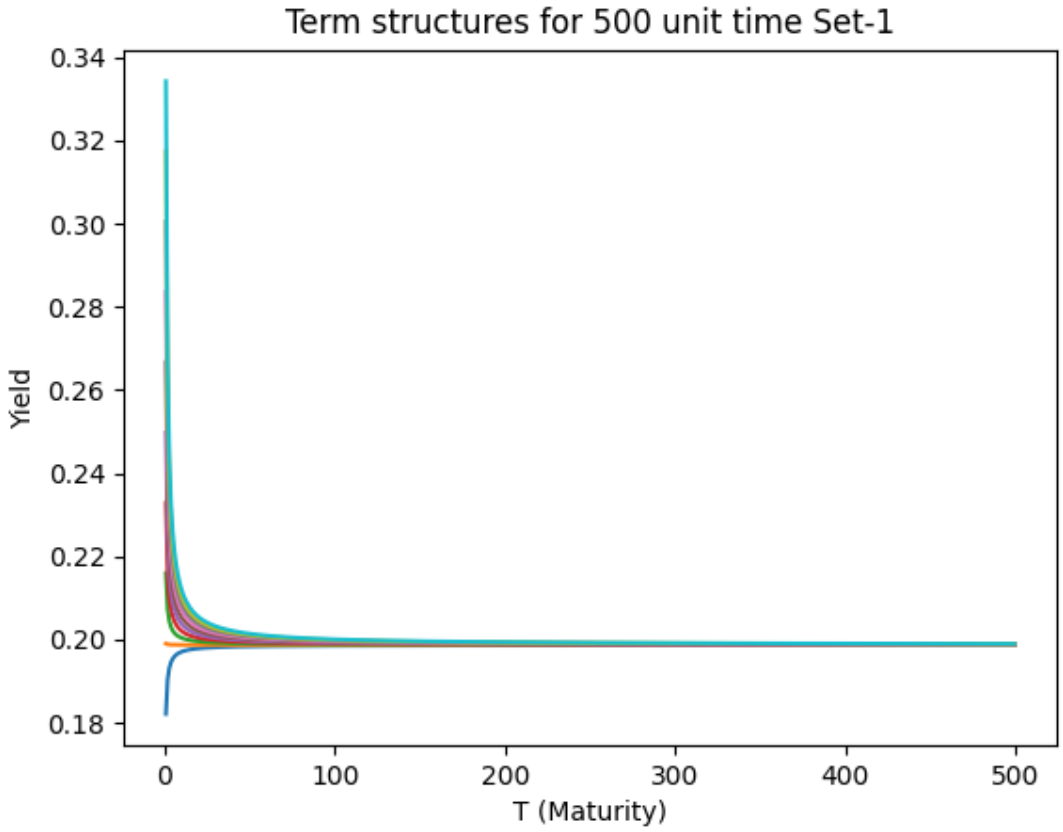
$$\gamma = \sqrt{\beta^2 + 2\sigma^2}$$

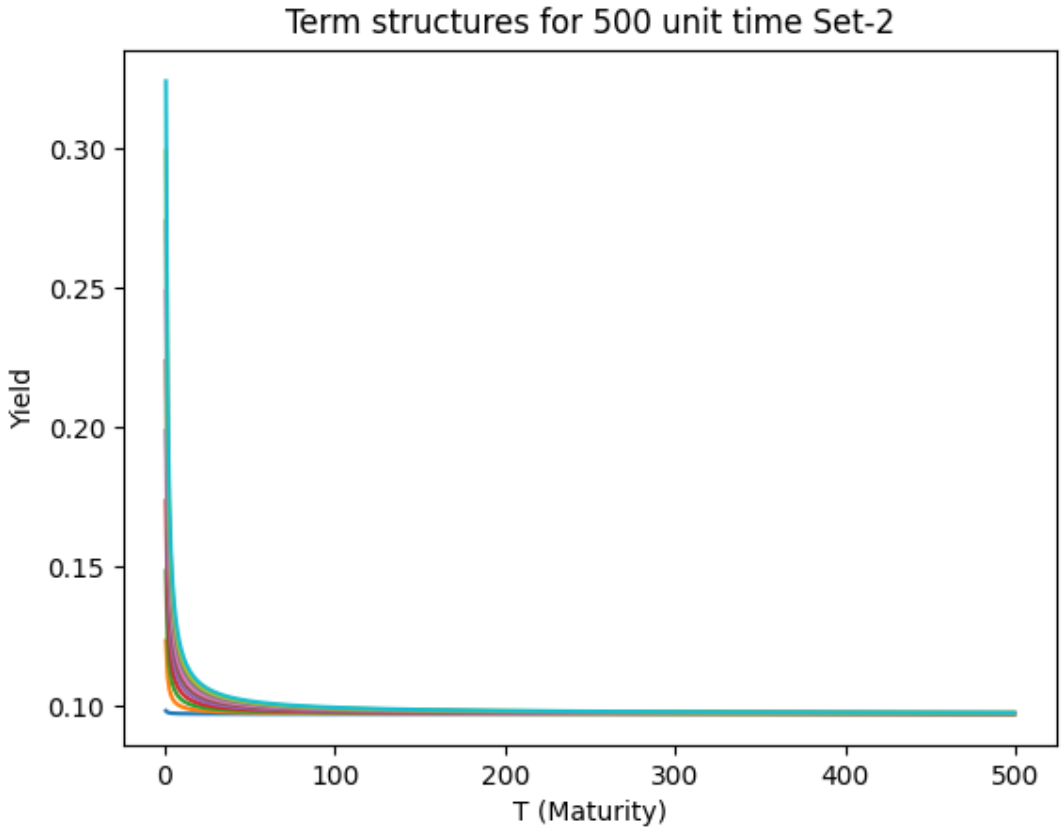
Q1)

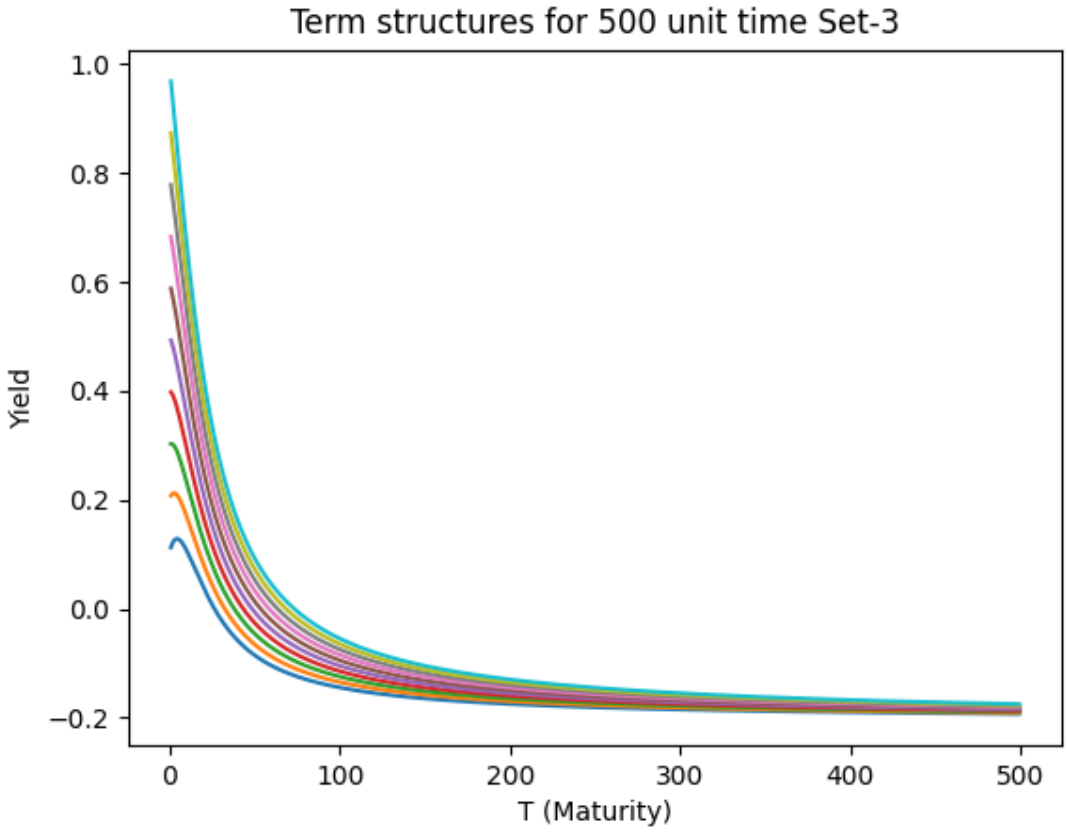

Term structures for 10 unit time set-2
Yield
T (Maturity)
0.20
0.18
0.16
0.14
0.12
0.10
0
2
4
6
8


Term structures for 10 unit time set-1
Yield
T (Maturity)
0.20
0.18
0.16
0.14
0.12
0.10
0
2
4
6
8


Term structures for 10 unit time set-3
0.130
0.125
0.120
0.115
0.110
0.105
0.100
Yield
0
2
4
6
8
T (Maturity)


Term structures for 500 unit time Set-1
0.34
0.32
0.30
0.28
0.26
0.24
0.22
0.20
0.18
Yield
0
100
200
300
400
500
T (Maturity)


Term structures for 500 unit time Set-2
Yield
0.30
0.25
0.20
0.15
0.10
0
100
200
300
400
500
T (Maturity)

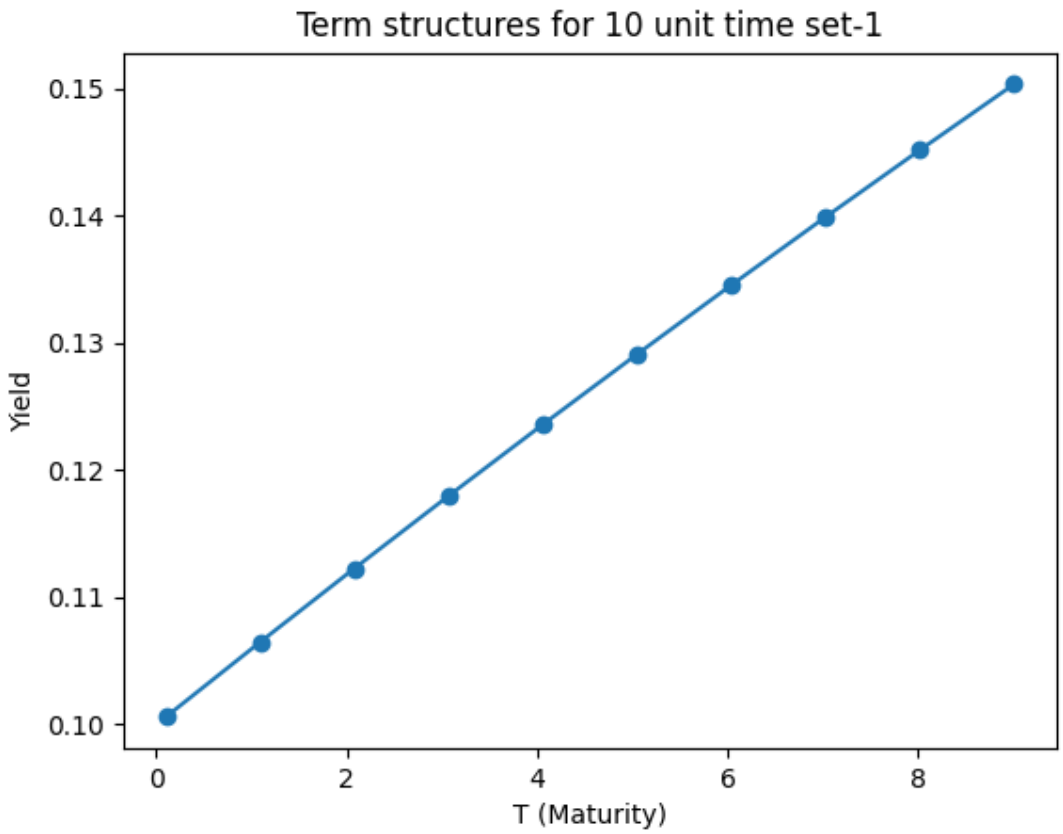

Term structures for 500 unit time Set-3
Yield
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
100
200
300
400
500
T (Maturity)

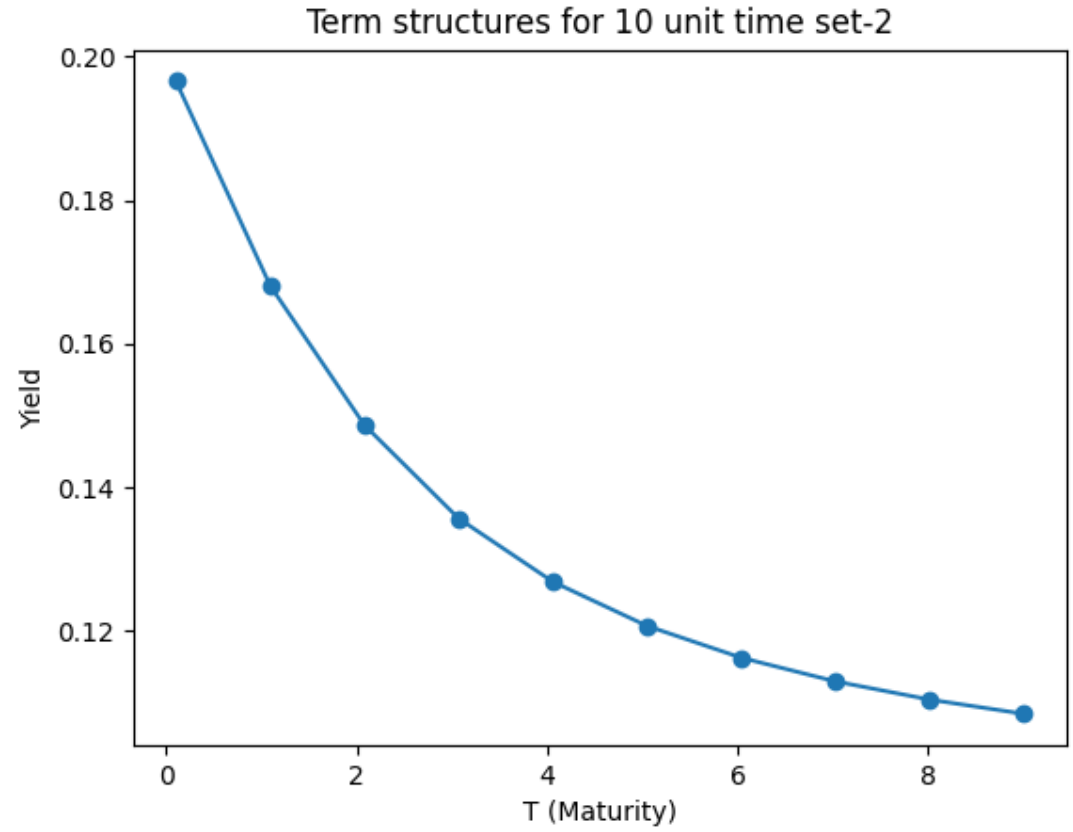
Observations:

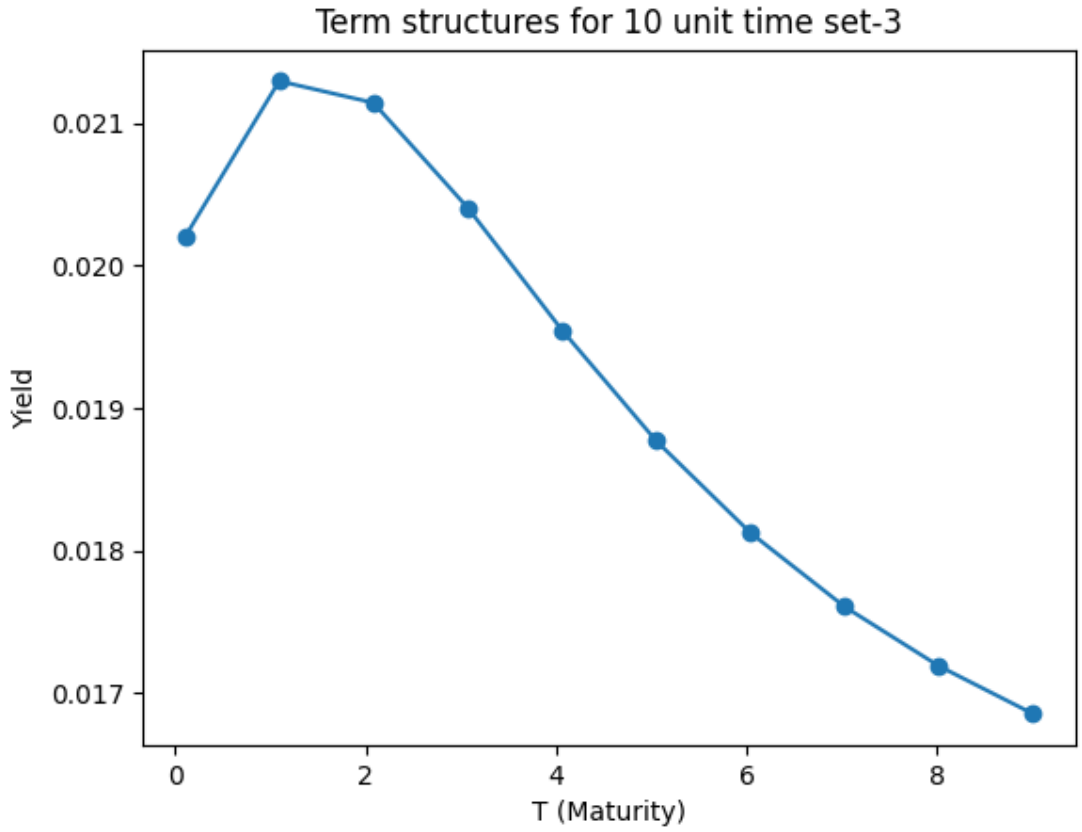
-Term structure for the 1st curve increases and then converges, for the second curve it decreases and then converges, for the third curve it increases then decreases.

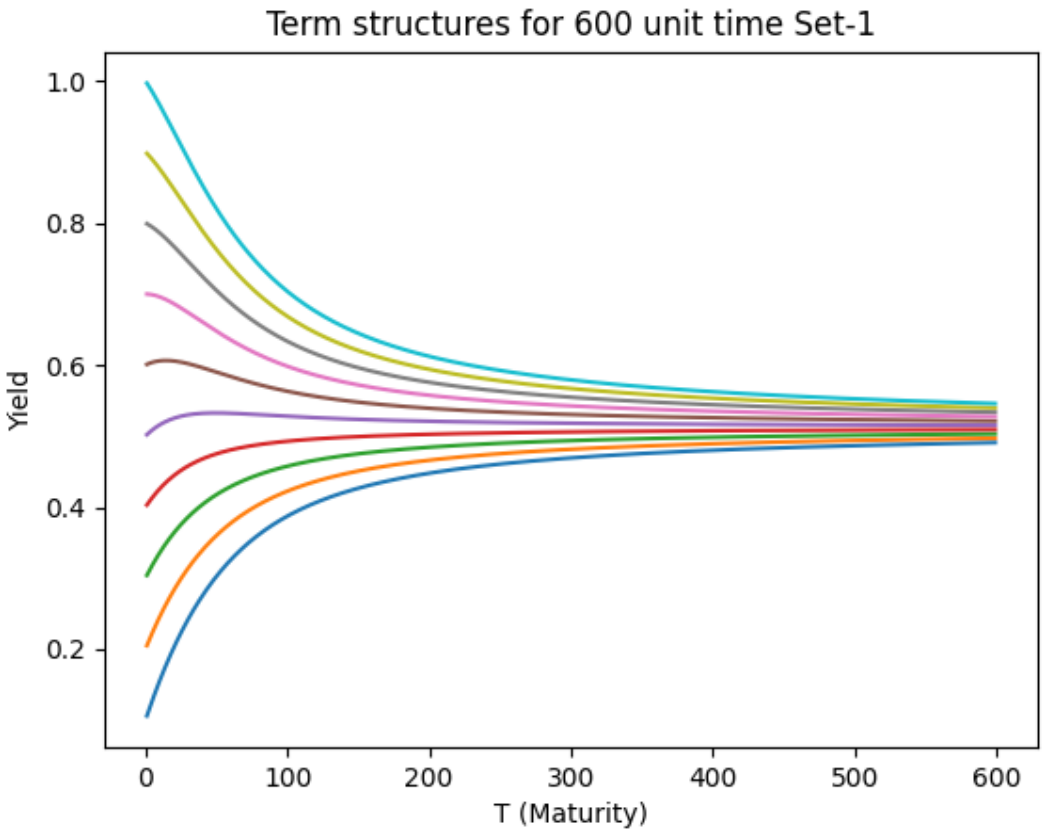
-Yield of bond prices converges to a particular value as T increases, irrespective of r(0).

Q2)




Term structures for 10 unit time set-2
0.20
0.18
0.16
0.14
0.12
Yield
0
2
4
6
8
T (Maturity)


Term structures for 10 unit time set-3
0.021
0.020
0.019
0.018
0.017
Yield
0
2
4
6
8
T (Maturity)



Observations:

-Term structure for the 1st curve increases and then converges, for the 2nd curve it decreases and then converges, for the 3rd curve it increases then decreases.

-Yield of bond prices converges to a particular value as T increases, irrespective of r(0).

Some comparisons between Vasicek and CIR Models:

1. CIR has a volatility drift term that increases as r increases, while Vasicek model assumes constant volatility.
2. Both models are one-factor modelling methods. However, Vasicek model allows for negative interest rate since it does not include a square root component.
3. Both models exhibit Mean Reversion phenomenon.