Computational Physics II
Spring 2021

Lalit Chaudhary

March 10, 2021

Two Dimensional Numerical Integration

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1 Question Formulation and Analytical solution

The integral of the function,

$$f(x, y) = 2x^2 + 3xy + y^2$$

is to be calculated over the circular domain, D defined by

$$x^2 + y^2 \leq 0.5$$

using numerical integration.

To check the correctness of the implementation, the analytical result can be first evaluated.

$$I = \iint_D f(x, y) dx$$

In polar co-ordinates:

$$I = \int_0^{r=\sqrt{0.5}} \int_0^{2\pi} (r^2 + r^2 \cos\,^2\theta + 3r^2 \,\sin\,\theta\,\cos\,\theta) r dr d\theta$$

The solution to the above integral is:

$$I = \frac{3\pi}{4} \cdot r^4 = 0.58904862$$

2 Implementation and Results

2.1 Midpoint Approximation

The method was implemeted for 4 different grid spacing. The code was validated by plotting the domain at h = 0.1 to avoid clustered datapoints.

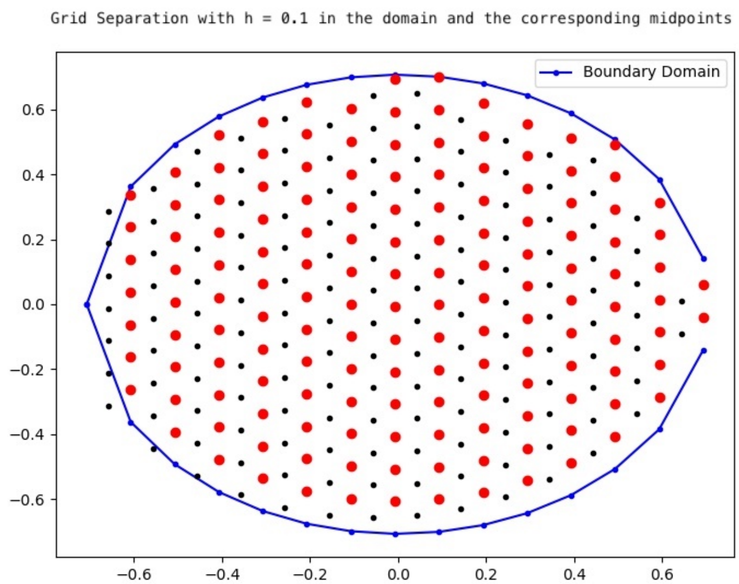

Figure 1: Domain of the integral

In figure (1), the red dots define the lattice points of the grids at approximately uniform spacing of h = 0.1. The black dots are the midpoints of the lattice cells, at which the value of the functions were calculated.

The obtained results are summarised in the screenshot of the output terminal. It is seen that the error decreases with the decrease in spacing. A plot of error vs N, the total number of data points the function is evaluated at, shows an inverse relation.

```
 Using Midpoint Approximation
 ...................


With h =  0.1

Evaluated Integral =  0.5389453854975323
Error =  0.05010323705055397

With h =  0.05

Evaluated Integral =  0.5684406090778241
Error =  0.0206080134702622

With h =  0.025

Evaluated Integral =  0.5772243200807167
Error =  0.011824302467369652

With h =  0.0125

Evaluated Integral =  0.5834494170356452
Error =  0.0055992055124410856

 Using Monte Carlo Method
 ...................
```

Figure 2: Results of Midpoint approximation at different grid spacing

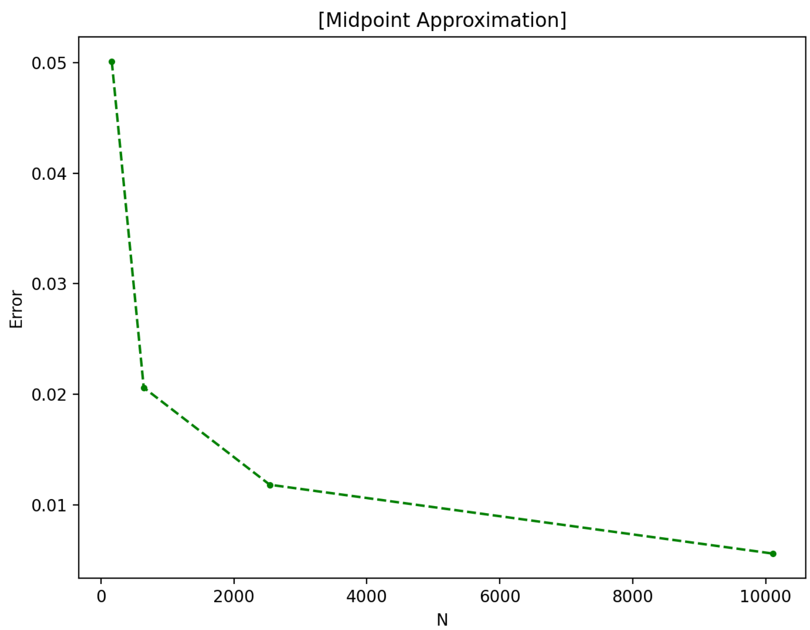

Figure 3: Variation of Error with number of data points/cells

2.2 Monte Carlo Method

For each set of grid spacing(consequently number of points), the measurement was repeated 200 times. The following results were obtained.

```
 Using Monte Carlo Method
 ...................


With N =  162

Evaluated Integral =  0.5863491909975992
Error =  0.0026994315504871302

With N =  643

Evaluated Integral =  0.5893834019012352
Error =  0.00033477935314885077

With N =  2539

Evaluated Integral =  0.588773713606275
Error =  0.00027490894181136927

With N =  10107

Evaluated Integral =  0.5885208701152114
Error =  0.0005277524328749506
```

Figure 4: Results of Monte Carlo Method at different grid spacing

The variation of error with number of points was plotted.

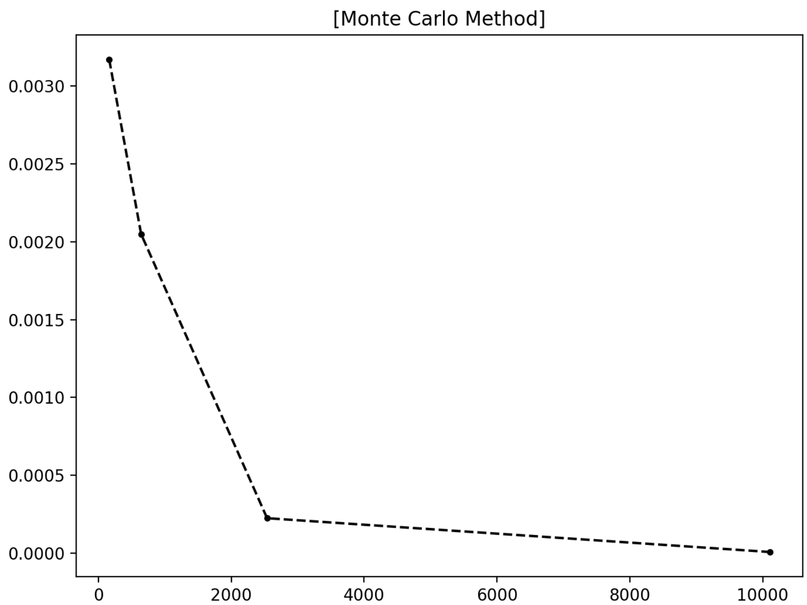

Figure 5: Variation of Error with N [averaged over 200 measurement]

The results of the error comparison between the two methods can be seen from the plot as shown in figure (8). It can be seen that the error decreases

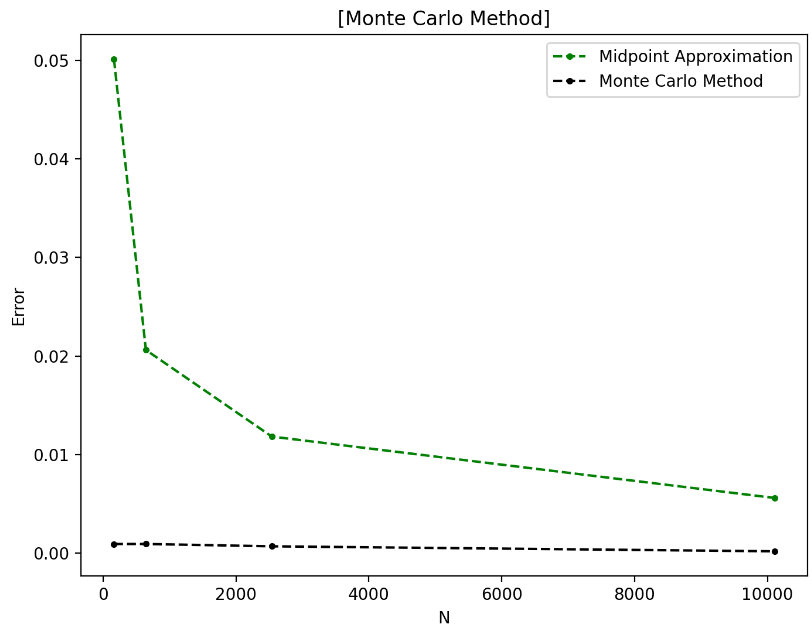

Figure 6: Comparison of two methods

when the measurements are done at more data points. For very large values of N, the two methods produce approximately the same result (extrapolation of figure (6)). However, at smaller values of N, the Monte Carlo method with several repetitions produce more accurate results. A plot of results of Monte carlo simulation with k = 1000 repitition per measurement is shown in figure (7).

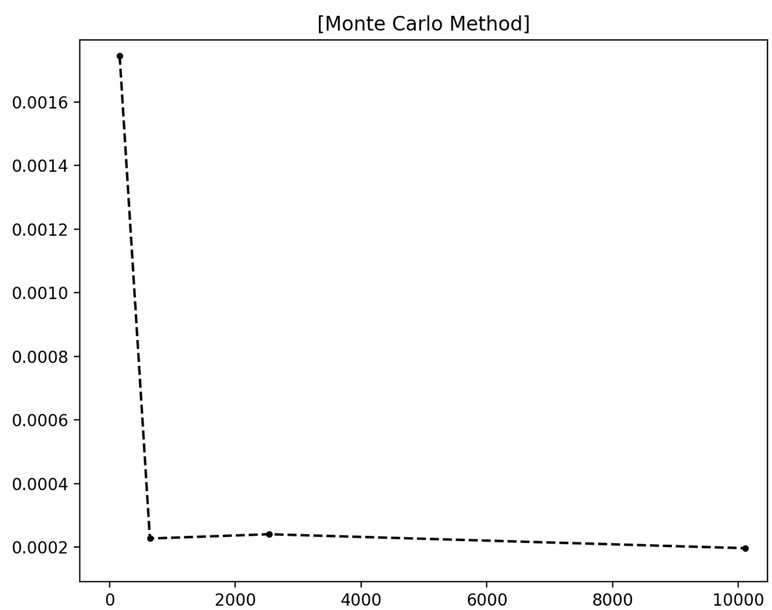

Figure 7: Variation of Error with N [averaged over 1000 measurement]