Computational Physics II
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Discrete Fourier Transform (DFT)

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1 Question Formulation

The Discrete Fourier Transformation (DFT) of a Gaussian function is to be calculated.

The DFT of a function $f(n)$ results in a set of discrete values of function $g(k)$ given by:

$$g(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} f(n) e^{-i2\pi n \frac{k}{N}} \tag{1}$$

The Inverse Discrete Fourier Transformation (IDFT) of $g(k)$ gives the original function $f(n)$

$$f(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} g(k) e^{-i2\pi n \frac{k}{N}} \tag{2}$$

2 Implementation and Results

2.1 DFT of Gaussian

A simple gaussian function,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

with

$$\mu = 0$$
$$\sigma = 0.5$$

was used for testing.

The results of the DFT of the above function are shown in figure (1).
In figure (1), the real and imaginary parts of the solutions of DFT are also shown. It can be seen that the magnitude of the result of DFT of Gaussian is also a Gaussian. By construction, our original function had only real values, however the transformed Gaussian has imaginary components as well.

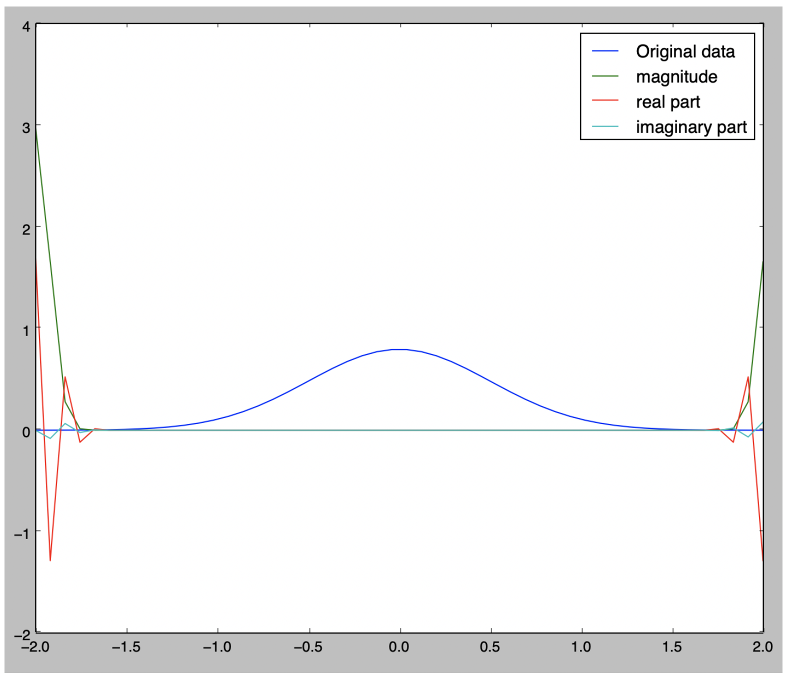

Figure 1: Discrete Fourier Transformation of a Gaussian function

2.2 DFT and IDFT

The DFT and IDFT were successively applied to a function

$$f(x) = \sin^2(x) \cdot e^{-(x-\pi/2)^2} \tag{4}$$

The results are shown in figure (2).

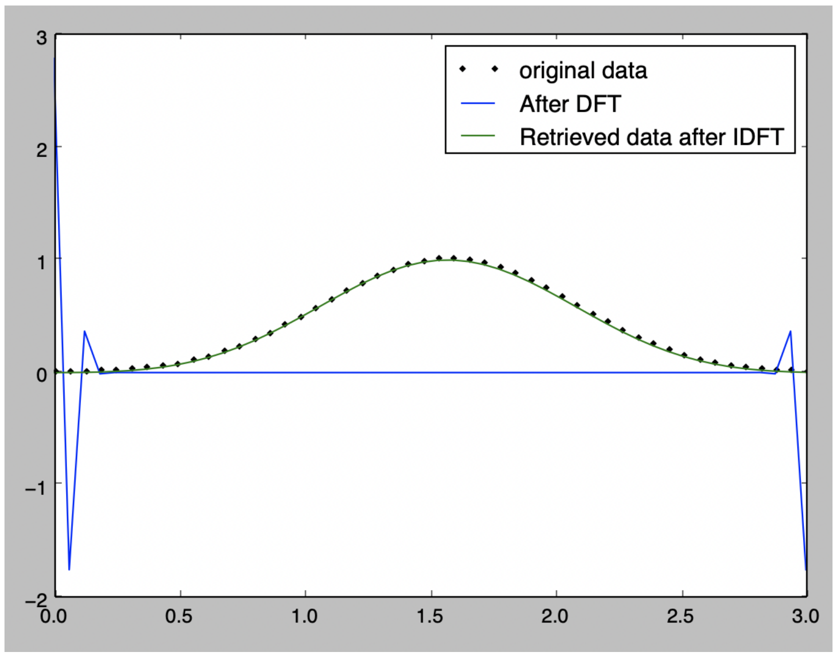

Figure 2: Back-transform with DFT and IDFT

In figure (2), the original data has been plotted as discrete points instead of a continuous line to avoid overlap with the retrieved data. As seen here, applying a Discrete Fourier Transform and Inverse Discrete Fourier Transform in succession results in retrieval of orginal data/signal.