Computational Physics
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Fall term 2020
Project 5, due November 22, 2020 at 11:55 pm
to be uploaded to https://moodle.jacobs-university.de

5. Wave equation [10 points]

In the lecture we looked at a discretization of the wave equation with a space discretization h and time discretization δ_t

$$\frac{u_i^{j+1} - u_i^{j-1} - 2u_i^j}{c^2 \delta_t^2} = \frac{u_{i+1}^j + u_{i-1}^j - 2u_i^j}{h^2} \, .$$

A rearrangement leads to a "leapfrog" algorithm that predicts $u(x,t)$ at a future time in terms of the known values at present and past times and three nearby positions

$$u_i^{j+1} = 2u_i^j - u_i^{j-1} + \frac{c^2}{c'^2}(u_{i+1}^j + u_{i-1}^j - 2u_i^j)$$

with the lattice velocity $c' = h/\delta_t$ (ratio of numerical parameters).
Write a program that implements the leapfrog algorithm and plots up the motion of the string (or optionally (!) produces an animation of the motion). Please use the sample parameters given in the lecture as one example and take half of the grid spacing as another example including the given initial conditions. Do you see differences when reducing the time step?
Please provide plots of the string at different times.
When a string is plucked near its end, a pulse reflects off the ends and bounces back and forth. Change the initial conditions of the program to one corresponding to a string pucked near one of the end. Again, please, provide plot(s).

General remarks for all Projects

You will have to (i) analyze the problem, (ii) select an algorithm (if not specified), (iii) write a Python program, (iv) run the program, (v) visualize the data numerical data, and (vi) extract an answer to the physics question from the data.
Which checks did you perform to validate the code? State the results you got for these tests.
For each project you will submit a short report describing the physics problem, your way of attacking it, and the results you obtained. Provide the documented Python code in such a form that we can run the code. A Jupyter Notebook including the code and report is fine but not necessary.