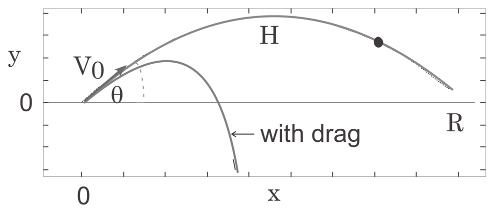
JACOBS UNIVERSITY

2. Realistic Projectile Motion [100 points]



The figure shows trajectories for a projectile shot at inclination Θ and with an initial velocity v_0. If we ignore air resistance, the projectile has only the (height-independent) force of gravity acting on it and the trajectory will be a parabola with range $R = 2v_0^2 \sin(\Theta)\cos(\Theta)/g$ and maximum height $H = \frac{1}{2}v_0^2 \sin^2(\Theta)/g$. Because a parabola is symmetric about its midpoint, it does not describe what appears to be a sharp, nearly vertical, drop-off of baseballs and golf balls near the end of their trajectories.

Investigate several models for the frictional force $\underline{F}^{(f)} = -m|v|^n \frac{\underline{v}}{|v|}$. Here the $\underline{v}/|v|$ factor ensures that the frictional force is always in a direction opposite that of the velocity.

a) Show that for this friction model, the equations of motion are

$$\frac{d^2x}{dt^2} = -kv_x^n \frac{v_x}{|v|}\,, \qquad \frac{d^2y}{dt^2} = -kv_y^n \frac{v_y}{|v|} - g\,, \qquad |v| = \sqrt{v_x^2 + v_y^2}\,.$$

Rewrite these equations into first-order differential equations.

b) Modify your Euler (or Euler-Richardson) code so that it solves the simultaneous ODEs for projectile motion. As parameters and initial conditions try $n = 1$, $v_0 = 22$ m/s, $\Theta = 34$, $g = 9.8$ m/s^2, k=0.8. The actual dimension of the latter depends on the choice of n.

c) The model with n = 1 is applicable only for low velocities. Now modify your program to handle $n = 3/2$ (medium-velocity friction) and $n = 2$ (high- velocity friction). Adjust the value of k for the latter two cases such that the initial force of friction kv_0^n is the same for all three cases.

d) Solve the equations of motion for several initial conditions and powers n and plot the results. How does friction affect the range R and the time aloft T? What conclusion can you draw regarding the observation of balls appearing to fall straight down out of the sky?

Please make sure that the employed time steps are small enough!

General remarks for all Projects

You will have to (i) analyze the problem, (ii) select an algorithm (if not specified), (iii) write a Python program, (iv) run the program, (v) visualize the data numerical data, and (vi) extract an answer to the physics question from the data.

Which checks did you perform to validate the code? State the results you got for these tests.

For each project you will submit a short report describing the physics problem, your way of attacking it, and the results you obtained. Provide the documented Python code in such a form that we can run the code. A Jupyter Notebook including the code and report is fine but not necessary.