# Computational Physics II
# Spring 2021

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**Spectral Analysis**

## Contents



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# 1 Question Formulation

The frequency spectrum of the Duffing oscillator with the differential equation

$$\frac{\partial^2 x}{\partial t^2} + k\frac{\partial x}{\partial t} + x^3 = B\ \cos(t) \tag{1}$$

The second order differential equation can be solved by noting that:

$$v = \frac{\partial x}{\partial t} \tag{2}$$

$$a = \frac{\partial v}{\partial t} = \frac{\partial^2 x}{\partial t^2} \tag{3}$$

$$\tag{4}$$

So, equation (1) can be written as:

$$a(t) + kv(t) + x^3 = B\ \cos(t) \tag{5}$$

First, the trajectory $x(t)$ was calculated by performing numerical integration using Euler's iteration scheme with the initial condition:

$$x(t=0) = 1$$
$$v(t=0) = -1$$

The spectra was then obtained by Fourier transform of the obtained $x(t)$ trajectories. The DFT of a function $f(n)$ results in a set of discrete values of function $g(k)$ given by:

$$g(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} f(n)e^{-i2\pi n\frac{k}{N}} \tag{6}$$

# 2 Implementation and Results

Using a timestep of $dt = 0.05$ and $N = 200$ points, the analysis was performed for $k = 0, 1$ and $B = 1, 5, 12$. The following results were obtained.

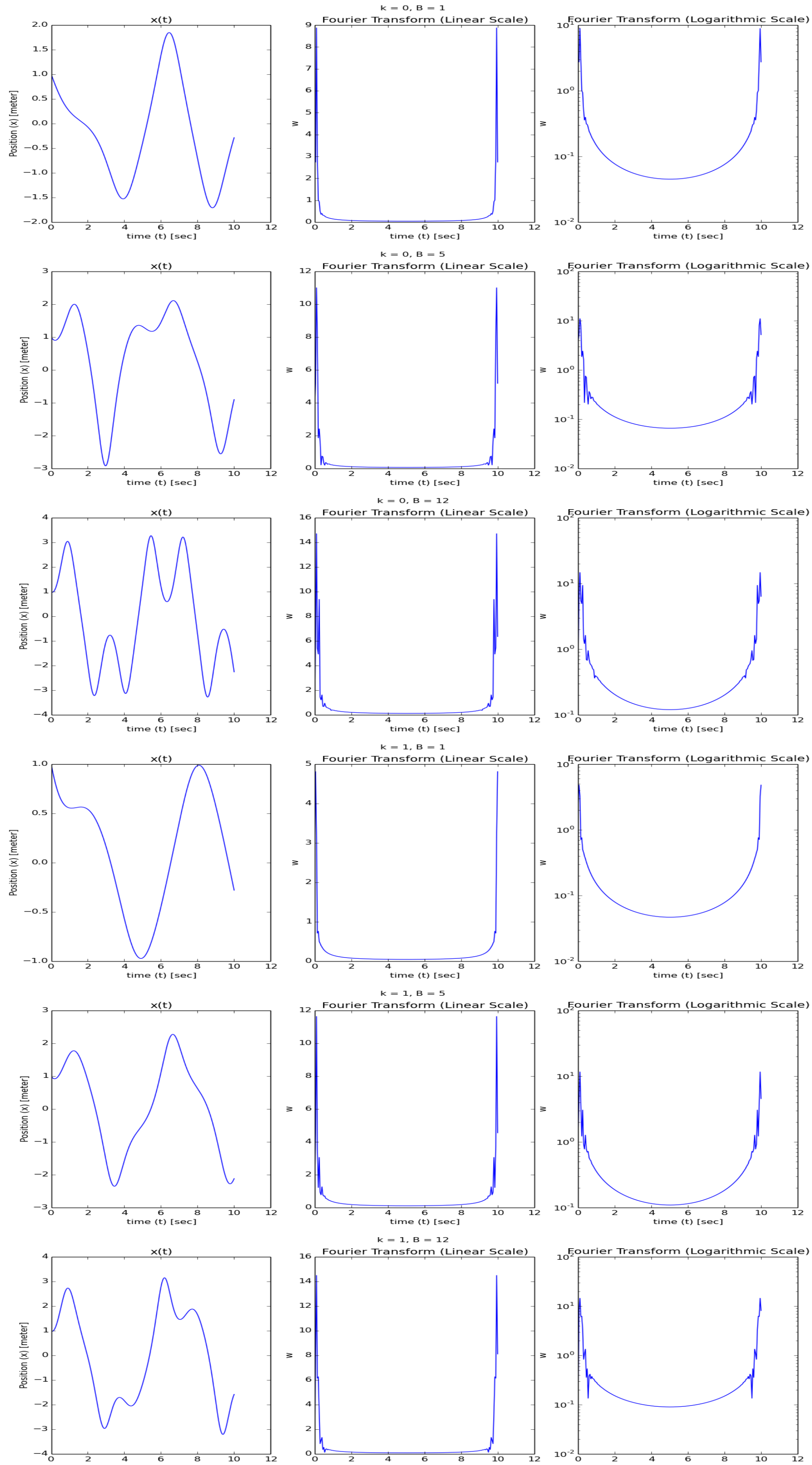


Figure 1: Trajectory of Duffing oscillator and its Fourier Transform, N = 200, dt = 0.05

## 2.1 Variation with $dt$ and $N$

### 2.1.1 N = 200, dt = 0.1

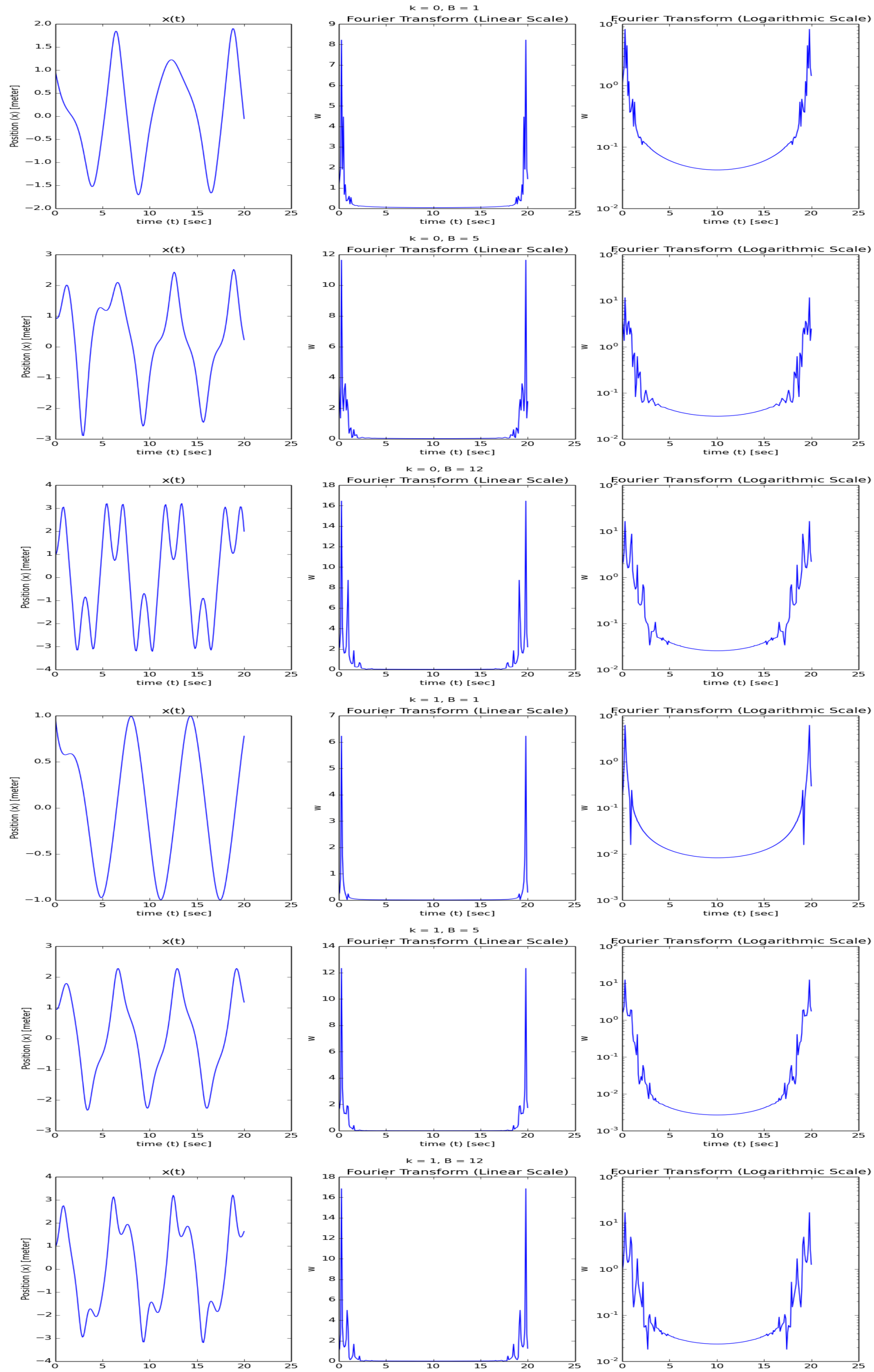

Figure 2: Trajectory of Duffing oscillator and its Fourier Transform, N = 200, dt = 0.1

### 2.1.2 N = 200, dt = 0.25

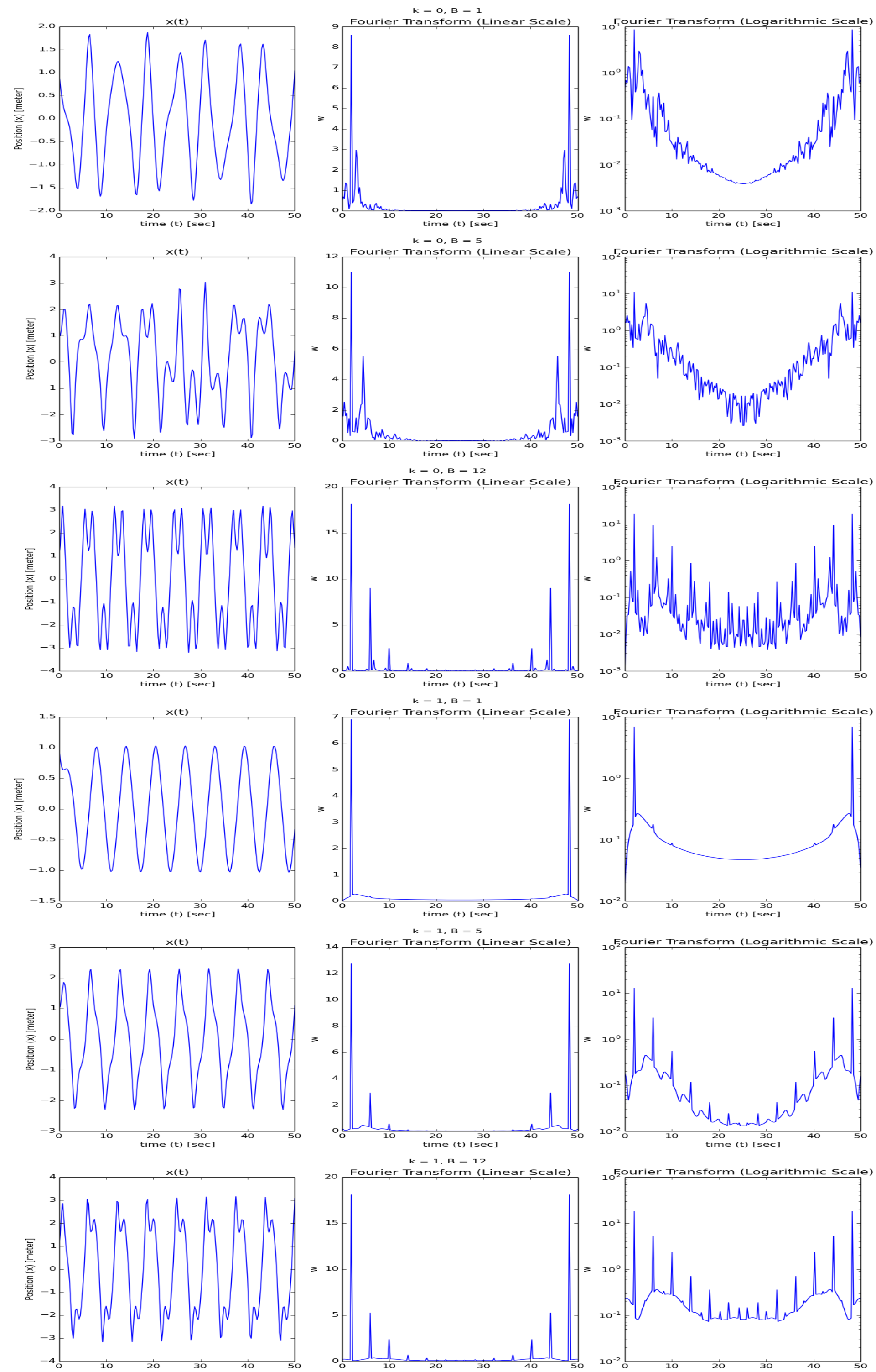


Figure 3: Trajectory of Duffing oscillator and its Fourier Transform, N = 200, dt = 0.25

### 2.1.3 N = 1000, dt = 0.05

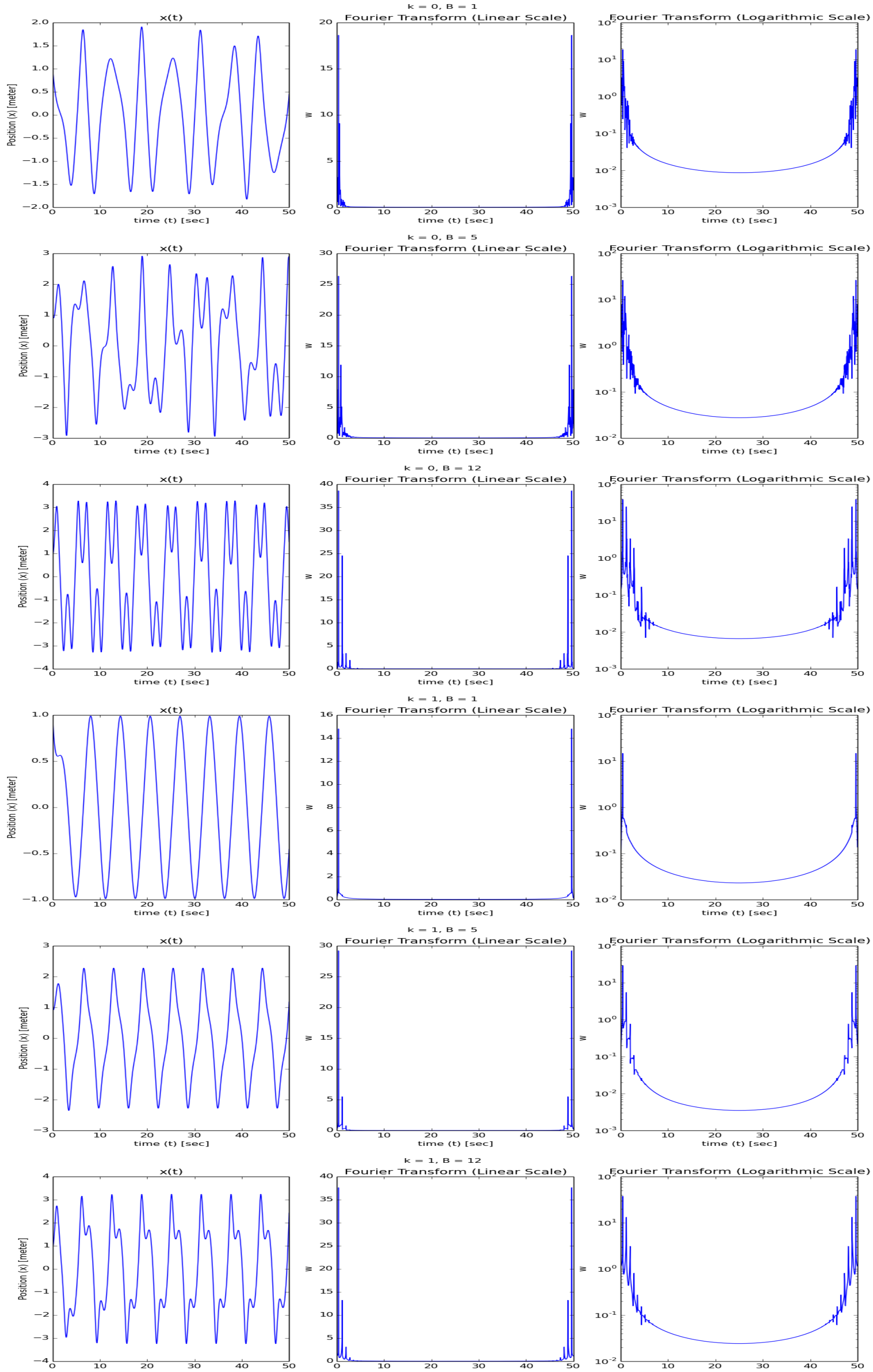


Figure 4: Trajectory of Duffing oscillator and its Fourier Transform, N = 1000, dt = 0.05

### 2.1.4 N = 750, dt = 0.05

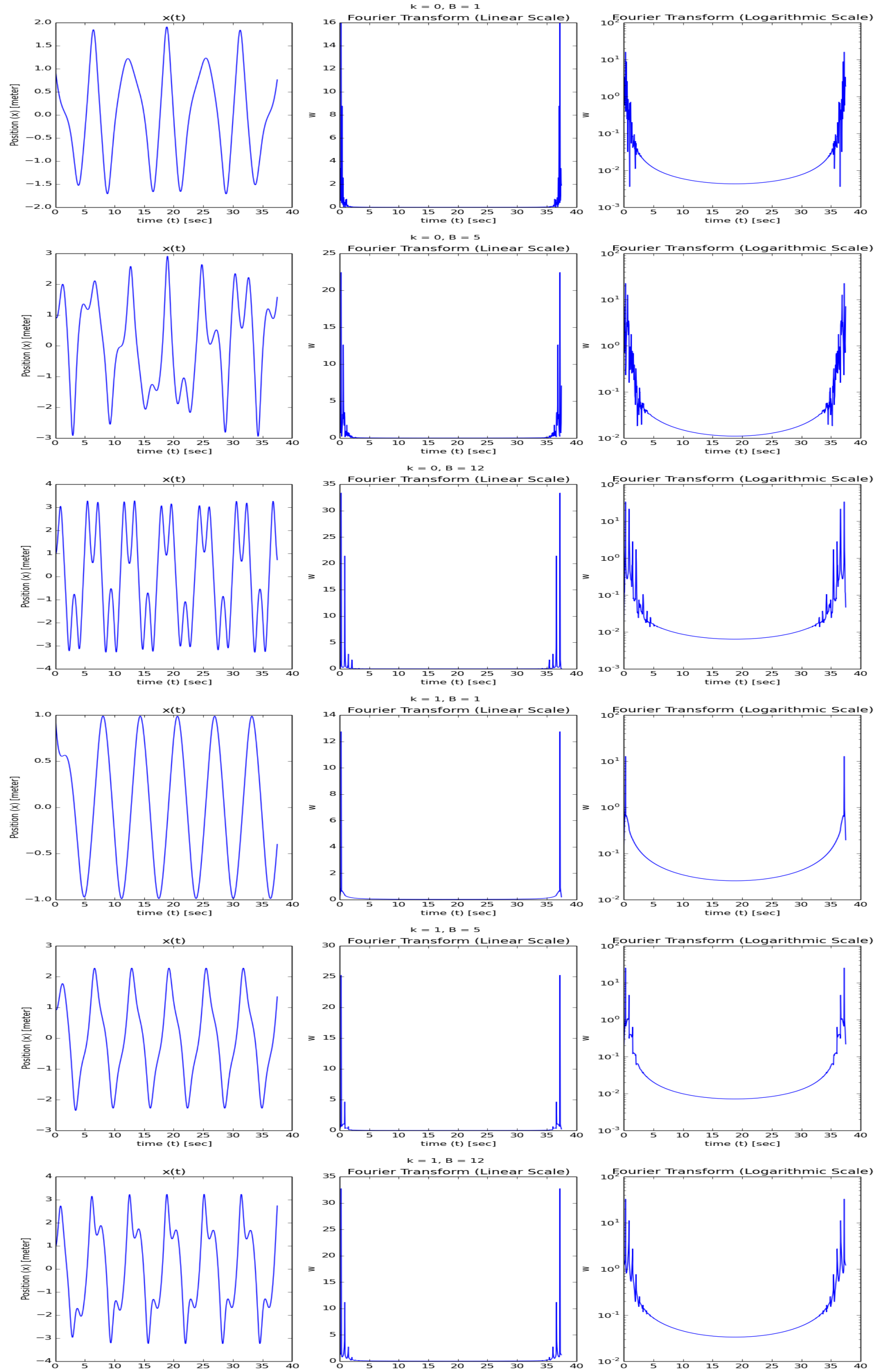


Figure 5: Trajectory of Duffing oscillator and its Fourier Transform, N = 750, dt = 0.05

### 2.2 Conclusion

As seen from graphs above, for some combination of $k$ and $B$ values, the Duffing oscillator shows chaotic behaviour. It can also be seen that (see figure 3 for instance) for same value of B, the spectra obtained with k = 1 has more ordered and distinct frequency peaks than one obtained with k = 0. For a constant k, increasing the value of B results in more number of peaks implying diverse frequency spectra.

By comparing the plots obtained in section 2.1, it can also be noticed that upon increasing the number of data points, $N$ or the time step $dt$, the number of oscillation in the trajectory increases. Consequently, more (secondary) peaks are seen in the frequency spectra.