Computational Physics II
Prof. Ulrich Kleinekathöfer
Spring term 2021
Project 5, due April 28, 2021 at 11:55 pm
to be uploaded to https://moodle.jacobs-university.de

5. Spectral analysis [100 points]

Study the frequency spectrum of the Duffing oscillator with the differential equation

$$\frac{\partial^2 x}{\partial t^2} + k\frac{\partial x}{\partial t} + x^3 = B\cos t \ .$$

The behaviour of the oscillator strongly depends on the choice of the parameters. The following values shall be used: $k = 0, 1$ and $B = 1, 5, 12$.
a) Calculate the trajectories $x(t)$ for equidistant time steps $t = n\tau$ with $n = 0, 1, 2, \ldots, N-1$ by performing a numerical integration using the Euler or the Euler-Richardson method. Plot the results.
b) Determine the spectra, i.e., the Fourier transforms of the time trajectories. How does the choice of the time step τ (it needs to be small enough for numerical convergence in any case!) and the number of data points N influence the spectrum? Compare the results for increasing τ and N by factors of 2 and 5 (independently for the two parameters). Plot the spectra on a linear and a logarithmic scale.

General remarks for all Projects

You will have to (i) analyze the problem, (ii) select an algorithm (if not specified), (iii) write a Python program, (iv) run the program, (v) visualize the data numerical data, and (vi) extract an answer to the physics question from the data.
Which checks did you perform to validate the code? State the results you got for these tests.
For each project you will submit a short report describing the physics problem, your way of attacking it, and the results you obtained. Provide the documented Python code in such a form that we can run the code. A Jupyter Notebook including the code and report is fine but not necessary.