

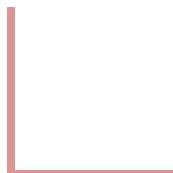
# **Mathematical Foundations for Computer Applications**

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## **Mathematical Induction**

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# Mathematical Foundations for Computer Applications

## Mathematical Induction

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- Mathematical Induction is a special way of proving things. It has only 2 steps:

Step 1. Show it is true for the **first one**

Step 2. Show that if **any one** is true then the **next one** is true

Then **all** are true

- In the world of numbers we say:

Step 1. Show it is true for first case, usually  $n=1$

Step 2. Show that if  $n=k$  is true then  $n=k+1$  is also true

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## Mathematical Induction--How to Do it ?

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**Step 1** is usually easy, we just have to prove it is true for **n=1 (Basic Step)**

**Step 2** is best done this way: **(Induction Step)**

**Assume** it is true for **n=k**

**Prove** it is true for **n=k+1**

(we can use the **n=k** case as a **fact.**)

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## Mathematical Induction--Problems

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1. Prove that  $1 + 3 + 5 + \dots + (2n-1) = n^2$  using Mathematical Induction .(Adding up Odd Numbers)

**1.**Show it is true for  $n=1$

$$1 = 1^2 \text{ is True}$$

**2.** Assume it is true for  $n=k$

$$1 + 3 + 5 + \dots + (2k-1) = k^2 \text{ is True}$$

Now, prove it is true for " $k+1$ "

$$1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = (k+1)^2 \ ?$$

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## Mathematical Induction--Problems

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We know that  $1 + 3 + 5 + \dots + (2k-1) = k^2$  (the assumption above), so we can do a replacement for all but the last term:

$$k^2 + (2(k+1)-1) = (k+1)^2$$

$$k^2 + 2k + 2 - 1 = k^2 + 2k + 1$$

$$k^2 + 2k + 1 = k^2 + 2k + 1$$

**They are the same! So it is true.**

So:

$$1 + 3 + 5 + \dots + (2(k+1)-1) = (k+1)^2 \text{ is True}$$

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## Mathematical Induction--Problems

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### 2. Adding up Cube Numbers

Prove that:  $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

#### 1. Show it is true for $n=1$

$$1^3 = \frac{1^2(1+1)^2}{4}$$

1 = 1 is True

#### 2. Assume it is true for $n=k$

$$1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4} \text{ is True (An assumption!)}$$

# Mathematical Foundations for Computer Applications

## Mathematical Induction--Problems

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Now, prove it is true for "k+1"

$$1^3 + 2^3 + 3^3 + \dots + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4} \quad ?$$

We know that  $1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$  (the assumption above), so we can do a replacement for all but the last term:

$$\frac{k^2(k+1)^2}{4} + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4}$$

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## Mathematical Induction--Problems

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Multiply all terms by 4:

$$k^2(k+1)^2 + 4(k+1)^3 = (k+1)^2(k+2)^2$$

All terms have a common factor  $(k+1)^2$ , so it can be cancelled:

$$k^2 + 4(k+1) = (k+2)^2$$

$k^2 + 4k + 4 = k^2 + 4k + 4$  They are the same! So it is true.

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$
 is TRUE



**THANK YOU**

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