Mathematical Foundations for Computer Applications

Negating Quantified Expressions

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Negating Quantified Expressions

Ex-“Every student in your class has taken a course in c++.”

- This statement is a universal quantification, $\forall x P(x)$,
- where $P(x)$ is the statement “x has taken a course in c++” and the domain consists of the students in your class.
- The **negation** of this statement is “It is not the case that every student in your class has taken a course in c++.”

Negating Quantified Expressions

- The **negation** of this statement is "It is not the case that every student in your class has taken a course in c++." OR
- "There is a student in your class who has not taken a course in c++." ----This is the **existential quantification** of the negation of the original propositional function, namely, $\exists x \neg P(x)$.
- This example illustrates the following logical equivalence:

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Negating Quantified Expressions

- $\neg\forall x\, P(x) \equiv \exists x\, \neg P(x)$.
- $\neg\exists x\, Q(x) \equiv \forall x\, \neg Q(x)$.
- The rules for negations for quantifiers are called **De Morgan's laws for quantifiers**. These rules are summarized in Table 2.

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

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Negation of quantification

Positive Universal $\forall x \in D, P(x)$	**Positive Existential** $\exists x \in D, P(x)$
Negative Universal $\forall x \in D, \neg P(x)$	**Negative Existential** $\exists x \in D, \neg P(x)$

$$\neg\left(\forall x \in D, P(x)\right) \equiv \exists x \in D, \neg P(x)$$

$$\neg\left(\exists x \in D, P(x)\right) \equiv \forall x \in D, \neg P(x)$$

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Negation Of Universal Quantifier

- Let's negate the following statements with P(x) being "like homework."

Statement

"Not all students like homework."

Negation

"There is at least one student who does not like homework"

$$\neg\left(\forall x \in D, P(x)\right) \equiv \exists x \in D, \neg P(x)$$

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Negation Of Existential Quantifier

- Let’s negate the following statements with P(x) being “like homework.”

Statement	**Negation**
“It is **not** the case that **some** students like homework.”	“**All** students do **not** like homework.”

$$\neg\left(\exists x \in D, P\left(x\right)\right) \equiv \forall x \in D, \neg P\left(x\right)$$

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Problems

1. What are the negations of the statements "There is an honest politician" and "All Americans eat cheeseburgers"?

- *Solution:* Let $H(x)$ denote "x is honest." Then the statement "There is an honest politician" is represented by $\exists x\, H(x)$, where the domain consists of all politicians.
- The negation of this statement is $\neg\exists x\, H(x)$, which is equivalent to $\forall x\, \neg H(x)$.
- This negation can be expressed as "Every politician is not honest."

Problems

- Let $C(x)$ denote "*x* eats cheeseburgers." Then the statement "All Americans eat cheeseburgers" is represented by $\forall x\, C(x)$, where the domain consists of all Americans.
- The negation of this statement is $\neg\forall x\, C(x)$, which is equivalent to $\exists x\, \neg C(x)$.
- This negation *can be expressed "Some American does not eat cheeseburgers" OR* "There is an American who does not eat cheeseburgers."

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Problems

2. What are the negations of the statements $\forall x(x^2 > x)$ and $\exists x(x^2 = 2)$?

- The negation of $\forall x(x^2 > x) = \neg\forall x(x^2 > x) = \exists x\neg(x^2 > x)$
  $= \exists x(x^2 \leq x)$.
- The negation of $\exists x(x^2 = 2) = \neg\exists x(x^2 = 2) = \forall x\neg(x^2 = 2)$.
  $= \forall x(x^2 \neq 2)$.

The truth values of these statements depend on the domain.

Problems

3. Show that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \land \neg Q(x))$ are logically equivalent.

- We know that $\neg(P(x) \rightarrow Q(x))$ and $P(x) \land \neg Q(x)$ are logically equivalent for every x.
- It follows that $\neg\forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \land \neg Q(x))$ are logically equivalent.

Translating from English into Logical Expressions

1. Express the statement “Every student in this class has studied calculus” using predicates and quantifiers.

Solution: “For every student *x* in this class, *x* has studied calculus.”

- $C(x)$, which is the statement “*x* has studied calculus.” Consequently, if the domain for *x* consists of the students in the class,
- **Statement** $\forall x\, C(x)$.

Translating from Logical Expressions into English statement

2. Translate these statements into English, where $C(x)$ is “x is a comedian” and $F(x)$ is “x is funny” and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) $\forall x(C(x) \land F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \land F(x))$

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Translating from Logical Expressions into English statement

a) $\forall x(C(x) \rightarrow F(x))$, b) $\forall x(C(x) \land F(x))$,c) $\exists x(C(x) \rightarrow F(x))$,d) $\exists x(C(x) \land F(x))$

a) All comedians are funny.

b) Everyone is a funny comedian.(All people are funny comedians)

c) There's someone who, if they are a comedian, then they are funny.

(There exist a comedian that is funny)

d) There is at least one comedian who is actually funny.

(A person is a funny comedian)

Translating from English into Logical Expressions

2. Express the statements “Some student in this class has visited Mexico” and “Every student in this class has visited either Canada or Mexico” using predicates and quantifiers.

(i) “There is a student *x* in this class having the property that *x* has visited Mexico.”

- $M(x)$ is the statement “*x* has visited Mexico.” If the domain for *x* consists of the students in this class, we can translate this first statement as $\exists x M(x)$.

Translating from English into Logical Expressions

- "There is a person *x* having the properties that *x* is a student in this class and *x* has visited Mexico."
- In this case, the domain for the variable *x* consists of all people. $S(x)$ to represent "*x* is a student in this class."
- Solution becomes $\exists x(S(x) \land M(x))$

Translating from English into Logical Expressions

(ii) Second statement can be expressed as

$$\forall x(C(x) \vee M(x)).$$

"For every person *x*, if *x* is a student in this class, then *x* has visited Mexico or *x* has visited Canada."

- The statement can be expressed as

$$\forall x(S(x) \rightarrow (C(x) \vee M(x))).$$

THANK YOU

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