Mathematical Foundations for Computer Applications

Sub Sets

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Subset

- The set **A is a subset of B** if every element of A is also an element of B.

- ie $A \subseteq B$ (A is a subset of the set B).
  - $\forall x (x \in A \rightarrow x \in B)$ is true.

Note -- show that **A is not a subset of B** need only find one element $x \in A$ with $x \notin B$.

A={1,2,3} ,B={ 2,4} , Is $B \subseteq A$?

Subsets-Examples

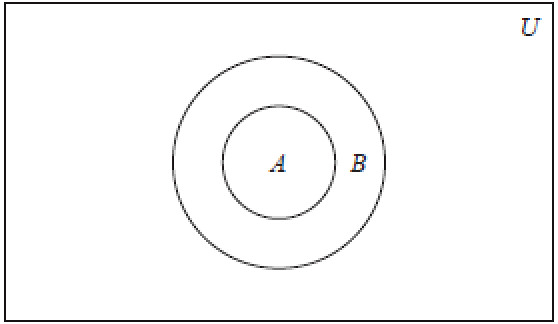
1. A = {red, blue} and B = {red, white, blue}, $A \subseteq B$

2. Let $C = \{a, b, c\}$ and $D = \{b, c, d, e\}$. Then $C \nsubseteq D$

3. $A \subseteq A$ and $\emptyset \subseteq A$.
    - A set is a subset of itself .
    - The empty set is a subset of every set.

Venn Diagram

- **Venn Diagram Showing that *A* Is a Subset of *B*.**



Types of Subsets

Subsets are classified as

- Proper Subset
- Improper Subset

✓ **A proper subset** is one that contains few elements of the original set

✓ **An improper subset**, contains every element of the original set.

Types of Subsets

For example, if set A = {2, 4, 6}, then,

Number of subsets: {2}, {4}, {6}, {2,4}, {4,6}, {2,6}, {2,4,6} ,Φ

Proper Subsets: {}, {2}, {4}, {6}, {2,4}, {4,6}, {2,6}

Improper Subset: {2,4,6}

Proper Subsets

Set A is considered to be a proper subset of Set B if Set B contains at least one element that is not present in Set A.

Example: If set A has elements as {12, 24} and set B has elements as {12, 24, 36}, then set A is the proper subset of B because 36 is not present in the set A.

ie, $A \subset B$

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Power Sets

- Given a set S, **the power set of S** is the set of all subsets of the set S. The power set of S is denoted by P(S).
- If a set has *n* elements, then its power set has $\mathbf{2^n}$ elements

Problems

1.What is the power set of the set {0, 1, 2}?

Solution: The power set P({0, 1, 2}) is the set of all subsets of {0, 1, 2}.

Hence, $2^3 = 8$ elements in the power set

P({0, 1, 2}) = {Ø, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}.

Problems

2. (i)What is the power set of the empty set?

   (ii) What is the power set of the set {Ø}?

Solution: (i) The empty set has exactly one subset,

$P(\emptyset) = \{\emptyset\}$.

(ii) The set {Ø} has exactly two subsets, namely, Ø and the set {Ø} itself.

Therefore,

$P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$.

Cartesian product

- Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$

- $A \times B \neq B \times A$
- The Cartesian product of more than two sets can also be defined

Cartesian product

Eg--Let A represent the set of all students at a university, and let B represent the set of all courses offered at the university. What is the Cartesian product $A \times B$ and how can it be used?

Solution: The Cartesian product $A \times B$ consists of all the ordered pairs of the form (a, b), where a is a student at the university and b is a course offered at the university. One way to use the set $A \times B$ is to represent all possible enrolments of students in courses at the university.

Problems

1. What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

2. What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

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Problems

3. Suppose that $A = \{1, 2\}$ find A^2 and A^3

$A^2 = A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

$A^3 = A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}.$

Cartesian product

- A subset R of $A \times B$ is called a **relation** from the set A to the set B.
- The elements of R are ordered pairs.
- Example, $\{a, b, c\}$ to the set $\{0, 1, 2, 3\}$.
- $R = \{(a, 0), (a, 1), (a, 3), (b, 1), (b, 2), (c, 0), (c, 3)\}$ is a relation from the set

Problems

1. What are the ordered pairs in the less than or equal to relation, which contains *(a, b)* if $a \leq b$, on the set {0, 1, 2, 3} ?

The ordered pair *(a, b)* belongs to *R* if and only if both *a* and *b* belong to {0, 1, 2, 3} and $a \leq b$.

$R = \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3),(2,2), (2, 3), (3, 3)\}$

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THANK YOU

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