Unit 3: Fundamentals of Logic and Proofs(QA)

1. Use informal methods. If the argument is valid, explain why. If it is invalid, give a counterexample.

(a) Today is Monday

tomorrow must be Tuesday. Valid.

By definition, the day after Monday is Tuesday and the day after today is tomorrow.

2. Apply the truth table test to each of the following argument forms.

(a) ~(P ∧ Q), R ↔ ~P, P → ~R, Q

P

P	Q	R	$((((\sim P \wedge Q) \wedge (R \leftrightarrow \sim P)) \wedge (P \to \sim R)) \wedge Q) \to P$							
T	T	T	F	T	F	F	F	F	F	T
T	T	F	F	T	F	T	F	T	F	T
T	F	T	T	F	F	F	F	F	F	T
T	F	F	T	F	T	T	T	T	F	T
F	T	T	T	F	T	T	T	T	T	F
F	T	F	T	F	F	F	F	T	F	T
F	F	T	T	F	T	T	T	T	F	T
F	F	F	T	F	F	F	F	T	F	T

Since the conditional is contingent, the test fails.

3. Find the converse, inverse, and contrapositive of "It raining is a sufficient condition for my not going to town."

Solution:

converse: If I do not go to town, then it is raining.

inverse: If it is not raining, then I will go to town.

contrapositive: If I go to town, then it is not raining.

4. How many rows are there in a truth table with n propositional variables?

Solution : 2^n

5. Convert an English sentence to a statement in propositional logic

“If I go to Harry’s or to the country, I will not go shopping.”

p: I go to Harry’s

q: I go to the country.

r: I will go shopping.

If p or q then not r.

$$(p \lor q) \to \neg r$$

6. Express in propositional logic: “The automated reply cannot be sent when the file system is full”

Solution: Let p denote “The automated reply can be sent” and q denote “The file system is full.”

$q \to \neg p$

7. What is tautology, contradiction and contingency? give an example.

A tautology is a proposition which is always true.

Example: $p \lor \neg p$

A contradiction is a proposition which is always false.

Example: $p \land \neg p$

A contingency is a proposition which is neither a tautology nor a contradiction

Example: p

7. Show that $\neg(p \lor (\neg p \land q)$ is logically equivalent to $(\neg p \land \neg q)$

$$\begin{array}{llll} \neg(p \vee (\neg p \wedge q)) & \equiv & \neg p \wedge \neg(\neg p \wedge q) & \text{by the second De Morgan law} \\ & \equiv & \neg p \wedge [\neg(\neg p) \vee \neg q] & \text{by the first De Morgan law} \\ & \equiv & \neg p \wedge (p \vee \neg q) & \text{by the double negation law} \\ & \equiv & (\neg p \wedge p) \vee (\neg p \wedge \neg q) & \text{by the second distributive law} \\ & \equiv & F \vee (\neg p \wedge \neg q) & \text{because } \neg p \wedge p \equiv F \\ & \equiv & (\neg p \wedge \neg q) \vee F & \text{by the commutative law} \\ & & & \text{for disjunction} \\ & \equiv & (\neg p \wedge \neg q) & \text{by the identity law for } \mathbf{F} \end{array}$$

8. Negate each of the following propositions:

1.∀x p(x)∧ ∃ y q(y)

Sol: ~.∀x p(x)∧ ∃ y q(y))
≅~∀ x p(x)∨~∃yq (y) (∴~(p∧q)=~p∨~q)
≅ ∃ x ~p(x)∨∀y~q(y)

2. (∃x∈U) (x+6=25)

Sol: ~(∃ x∈U) (x+6=25)
≅∀ x∈U~ (x+6)=25
≅(∀ x∈U) (x+6)≠25

3. ~(∃ x p(x)∨∀ y q(y)

Sol: ~(∃ x p(x)∨∀ y q(y))
≅~∃ x p(x)∧~∀ y q(y) (∴~(p∨q)= ~p∧~q)
≅ ∀ x ~ p(x)∧∃y~q(y))