# Mathematical Foundations for Computer Applications

## Propositional Equivalences

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## Propositional Equivalences-- Introduction

**Definition 1** A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a *tautology*.

A compound proposition that is always false is called a *contradiction*.

A compound proposition that is neither a tautology nor a contradiction is called a *contingency*.

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## Propositional Equivalences– Introduction

- Examples of a Tautology and a Contradiction.

| $p$ | $\neg p$ | $p \vee \neg p$ | $p \wedge \neg p$ |
|---|---|---|---|
| T | F | T | F |
| F | T | T | F |

## Propositional Equivalences-- Logical Equivalences

**Definition 2** The compound propositions *p* and *q* are called *logically equivalent* if $\boldsymbol{p \leftrightarrow q}$ **is a tautology.**

The notation $p \equiv q$ denotes that *p* and *q* are **logically equivalent.**

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.
- Example: $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent
- Truth tables are the simplest way to prove such facts.

## Problems

1. Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $p \rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

2. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent. **(De Morgan's Law)**

| $p$ | $q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

It follows that $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$ is a tautology and that these compound propositions are logically equivalent

## Problems

3. Show that $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ are logically equivalent. (This is the *distributive law* of disjunction over conjunction)

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## Problems- Prove the Logically Equivalent.

| | |
|---|---|
| $p \vee q \equiv q \vee p$<br>$p \wedge q \equiv q \wedge p$ | Commutative laws |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$<br>$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative laws |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive laws |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan's laws |
| $p \vee (p \wedge q) \equiv p$<br>$p \wedge (p \vee q) \equiv p$ | Absorption laws |
| $p \vee \neg p \equiv \mathbf{T}$<br>$p \wedge \neg p \equiv \mathbf{F}$ | Negation laws |

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# THANK YOU

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