Unit 1: Set Theory and Matrix Theory

1. Write the following sets in Set-Builder Form :

(i) A = {1, 3 ,5, 7, 9}

(ii) B = {16, 25, 36, 49, 64}

(iii) C = {5,10,15,20,25}

(iv) D = {violet, indigo, blue, green, yellow, orange, red}

(v) E = {January, March, May, July, August, October, December}

Answers: (i) A = {x | x is an odd number less than 10}.

(ii) B = {x | x is a perfect square natural number between 15 and 65}

(iii) C = {x | x is a multiple of 5 less than 30}.

(iv) D = {x | x is a colour in rainbow}.

(v) E = {x | x is a month having 31 days}.

2. Write the following sets in Roster Form :

(i) The first four odd natural numbers each divisible by 3.

(ii) The first four odd natural numbers each divisible by 5.

(iii) The counting numbers between 15 and 35, each of which is divisible by 6.

(iv) The names of the last three days of a week.

(v) The names of the last four months of a year.

Ans: (i) {3, 9, 15, 21}

(ii) {5, 15, 25, 35}

(iii) {18, 24, 3}

(iv) {Friday, Saturday, Sunday}

(v) {September, October, November, December}

3. How do you find a proper subset?

A proper subset of a set A is a subset of A that is not equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B. For example, if A={1,3,5} then B={1,5} is a proper subset of A.

4. **Find A U B and A ⋂ B and A – B.**

If A = {a, b, c, d} and B = {c, d}.

Solution:

A = {a, b, c, d} and B = {c, d}

A U B = {a, b, c, d}

A ⋂ B = {c, d} and

A – B = {a, b}

5. **Let ξ = {1, 2, 3, 4, 5, 6, 7} and A = {1, 2, 3, 4, 5} B = {2, 5, 7} show that**

(a) (A ∪ B)' = A' ∩ B'

(b) (A ∩ B)' = A' ∪ B'

(c) (A ∩ B) = B ∩ A

(d) (A ∪ B) = B ∪ A

Ans: (a) L.H.S. = R. H. S = {6}
(b) L.H.S. = R. H. S = {1, 3, 4, 6, 7}
(c) {2, 5}
(d) {1, 2, 3, 4, 5, 7}

6.Show that For all sets A and B , A∪B=B∪A and A∩B=B∩A (Commutative Law)

Proof

Let $x \in A \cup B$. Then $x \in A$ or $x \in B$. Which implies $x \in B$ or $x \in A$. Hence $x \in B \cup A$. Thus $A \cup B \subseteq B \cup A$. Similarly, we can show that $B \cup A \subseteq A \cup B$. Therefore, $A \cup B = B \cup A$.

Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. Which implies $x \in B$ and $x \in A$. Hence $x \in B \cap A$. Thus $A \cap B \subseteq B \cap A$. Similarly, we can show that $B \cap A \subseteq A \cap B$. Therefore, $A \cap B = B \cap A$.

6. For all sets A,B and C , $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Distributive Law

Proof

Let $x \in A \cap (B \cup C)$.

Then $x \in A$ and $x \in B \cup C$.

Thus $x \in A$ and $x \in B$ or $x \in C$.

Which implies $x \in A$ and $x \in B$ or $x \in A$ and $x \in C$.

Hence $x \in (A \cap B) \cup (A \cap C)$. Thus $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$. Similarly, we can show that $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$. Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

7**. Determine the truth values of these expressions.**

i. $\emptyset \in \emptyset$
ii. $1 \subseteq \{1\}$
iii. $\emptyset \in \{\emptyset\}$

Answer

i. By definition, an empty set contains no element. Consequently, the statement $\emptyset \in \emptyset$ is false.
ii. A subset relation only exists between two sets. To the left of the symbol $\subseteq$, we have only a number, which is not a set. Hence, the statement is false. In fact, this expression is syntactically incorrect.
iii. The set $\{\emptyset\}$ contains one element, which happens to be an empty set. Compare this to an empty box inside another box. The outer box is described by the pair of set brackets $\{\ldots\}$, and the (empty) box inside is $\emptyset$. It follows that $\emptyset \in \{\emptyset\}$ is a true statement.

1. If A **=** {1, 3, 5}, B = {3, 5, 6} and C = {1, 3, 7}

(i) Verify that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) Verify A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Solution:

(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

L.H.S. = A ∪ (B ∩ C)

B ∩ C = {3}

A ∪ (B ∩ C) = {1,3,5} ……………….. (1)

R.H.S. = (A ∪ B) ∩ (A ∪ C)

A∪B = {1,3,5,6}

A∪C = {1,3,5,7}

(A ∪ B) ∩ (A ∪ C) = {1,3,5} ……………….. (2)

From (1) and (2), we conclude that;

A ∪ (B ∩ C) = A ∪ B ∩ (A ∪ C) [*verified*]

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

L.H.S. = $A \cap (B \cup C)$

$B \cup C = \{1,3,5,6\}$

$A \cap (B \cup C) = \{1,3,5\}$ ……………….. (1)

R.H.S. = $(A \cap B) \cup (A \cap C)$

$A \cap B = \{3,5\}$

$A \cap C = \{1,3\}$

$(A \cap B) \cup (A \cap C) = \{1,3,5\}$ ……………….. (2)

From (1) and (2), we conclude that;

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [*verified*]

2. Let $A = \{a, b, d, e\}$, $B = \{b, c, e, f\}$ and $C = \{d, e, f, g\}$

(i) Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. = $A \cap (B \cup C)$

$B \cup C = \{b, c, d, e, f, g\}$

$A \cap (B \cup C) = \{b, d, e\}$ ……………….. (1)

R.H.S. = $(A \cap B) \cup (A \cap C)$

$A \cap B = \{b, e\}$

$A \cap C = \{d, e\}$

$(A \cap B) \cup (A \cap C) = \{b, d, e\}$ ………………… (2)

From (1) and (2), we conclude that;

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [*verified*]

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S. = $A \cup (B \cap C)$

$B \cap C = \{e, f\}$

$A \cup (B \cap C) = \{a, b, d, e, f\}$ ………………… (1)

R.H.S. = $(A \cup B) \cap (A \cup C)$

$A \cup B = \{a, b, c, d, e, f\}$

$A \cup C = \{a, b, d, e, f, g\}$

$(A \cup B) \cap (A \cup C) = \{a, b, d, e, f\}$ ………………… (2)

From (1) and (2), we conclude that;

$A \cup (B \cap C) = A \cup B \cap (A \cup C)$ [*verified*]

3. Let $A = \{3, 5, 7\}$, $B = \{2, 3, 4, 6\}$ and $C = \{2, 3, 4, 5, 6, 7, 8\}$

(i) Verify $(A \cap B)' = A' \cup B'$

(ii) Verify $(A \cup B)' = A' \cap B'$

Solution:

(i) $(A \cap B)' = A' \cup B'$

L.H.S. = $(A \cap B)'$

$A \cap B = \{3\}$

$(A \cap B)' = \{2, 4, 5, 6, 7, 8\}$ ………………… (1)

R.H.S. = $A' \cup B'$

$B' = \{5, 7, 8\}$

$A' = \{2, 4, 6, 8\}$

$A' \cup B' = \{2, 4, 5, 6, 7, 8\}$ ………………… (2)

From (1) and (2), we conclude that;

$(A \cap B)' = (A' \cup B')$

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. = $(A \cup B)'$

$A \cup B = \{2, 3, 4, 5, 6, 7\}$

$(A \cup B)' = \{8\}$ ………………… (1)

R.H.S. = $A' \cap B'$

$A' = \{2, 4, 6, 8\}$

$B' = \{5, 7, 8\}$

$A' \cap B' = \{8\}$ ………………… (2)

From (1) and (2), we conclude that;

$(A \cup B)' = A' \cap B'$

4. Given three sets P, Q and R such that:

P = {x : x is a natural number between 10 and 16},

Q = {y : y is a even number between 8 and 20} and

R = {7, 9, 11, 14, 18, 20}

(i) Find the difference of two sets P and Q

(ii) Find Q - R

(iii) Find R - P

(iv) Find Q – P

Solution:

According to the given statements:

P = {11, 12, 13, 14, 15}

Q = {10, 12, 14, 16, 18}

R = {7, 9, 11, 14, 18, 20}

(i) P – Q = {Those elements of set P which are not in set Q}

= {11, 13, 15}

(ii) Q – R = {Those elements of set Q not belonging to set R}

= {10, 12, 16}

(iii) R – P = {Those elements of set R which are not in set P}

= {7, 9, 18, 20}

(iv) Q – P = {Those elements of set Q not belonging to set P}

= {10, 16, 18}

5. Write the cardinal number of each of the following sets:

(i) X = {letters in the word MALAYALAM}

(ii) Y = {5, 6, 6, 7, 11, 6, 13, 11, 8}

(iii) Z = {natural numbers between 20 and 50, which are divisible by 7}

Solution:

(i) Given, X = {letters in the word MALAYALAM}

Then, X = {M, A, L, Y}

Therefore, cardinal number of set X = 4, i.e., n(X) = 4

(ii) Given, Y = {5, 6, 6, 7, 11, 6, 13, 11, 8}

Then, Y = {5, 6, 7, 11, 13, 8}

Therefore, cardinal number of set Y = 6, i.e., n(Y) = 6

(iii) Given, Z = {natural numbers between 20 and 50, which are divisible by 7}

Then, Z = {21, 28, 35, 42, 49}

Therefore, cardinal number of set Z = 5, i.e., n(Z) = 5

6. Find the cardinal number of a set from each of the following:

(i) P = {x | x ∈ N and x22 < 30}

(ii) Q = {x | x is a factor of 20}

Solution:

(i) Given, P = {x | x ∈ N and x22 < 30}

Then, P = {1, 2, 3, 4, 5}

Therefore, cardinal number of set P = 5, i.e., n(P) = 5

(ii) Given, Q = {x | x is a factor of 20}

Then, Q = {1, 2, 4, 5, 10, 20}

Therefore, cardinal number of set Q = 6, i.e., n(Q) = 6

7. If the number of elements in a set is 2, find the number of subsets and proper subsets.

Solution:

Number of elements in a set = 2

Then, number of subsets = 2^2 = 4
Also, the number of proper subsets = 2^2 - 1

$= 4 – 1 = 3$

8. If P and Q are two sets such that P ∪ Q has 40 elements, P has 22 elements and Q has 28 elements, how many elements does P ∩ Q have?

Solution:

Given $n(P \cup Q) = 40$, $n(P) = 18$, $n(Q) = 22$

We know that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

So, $40 = 22 + 28 - n(P \cap Q)$

$40 = 50 - n(P \cap Q)$

Therefore, $n(P \cap Q) = 50 – 40$

$= 10$

9. In a class of 40 students, 15 like to play cricket and football and 20 like to play cricket. How many like to play football only but not cricket?

Solution:

Let C = Students who like cricket

F = Students who like football

C ∩ F = Students who like cricket and football both

C - F = Students who like cricket only

F - C = Students who like football only.

$n(C) = 20$ $n(C \cap F) = 15$ $n (C \cup F) = 40$ $n (F) = ?$

$n(C \cup F) = n(C) + n(F) - n(C \cap F)$

$40 = 20 + n(F) - 15$

$40 = 5 + n(F)$

$40 – 5 = n(F)$

Therefore, $n(F)= 35$

Therefore, $n(F - C) = n(F) - n (C \cap F)$

$= 35 – 15$

$= 20$

Therefore, Number of students who like football only but not cricket = 20

10. There is a group of 80 persons who can drive scooter or car or both. Out of these, 35 can drive scooter and 60 can drive car. Find how many can drive both scooter and car? How many can drive scooter only? How many can drive car only?

Solution:

Let S = {Persons who drive scooter}

C = {Persons who drive car}

Given, $n(S \cup C) = 80$ $n(S) = 35$ $n(C) = 60$

Therefore, $n(S \cup C) = n(S) + n(C) - n(S \cap C)$

$80 = 35 + 60 - n(S \cap C)$

$80 = 95 - n(S \cap C)$

Therefore, $n(S \cap C) = 95 – 80 = 15$

Therefore, 15 persons drive both scooter and car.

Therefore, the number of persons who drive a scooter only = $n(S) - n(S \cap C)$

$= 35 – 15$

$= 20$

Also, the number of persons who drive car only = $n(C) - n(S \cap C)$

$= 60 - 15$

$= 45$

11. It was found that out of 45 girls, 10 joined singing but not dancing and 24 joined singing. How many joined dancing but not singing? How many joined both?

Solution:

Let S = {Girls who joined singing}

D = {Girls who joined dancing}

Number of girls who joined dancing but not singing = Total number of girls - Number of girls who joined singing

$45 – 24$

$= 21$

Now, $n(S - D) = 10$ $\quad n(S) =24$

Therefore, $n(S - D) = n(S) - n(S \cap D)$

$\Rightarrow n(S \cap D) = n(S) - n(S - D)$

$= 24 - 10$

$= 14$

Therefore, number of girls who joined both singing and dancing is 14.

Problem 1. Find the Boolean product of A and B, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Solution.

$$A \odot B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge 1) \vee (0 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \vee 0 \vee 0 \vee 1 & 0 \vee 0 \vee 0 \vee 0 \\ 0 \vee 0 \vee 0 \vee 1 & 0 \vee 1 \vee 0 \vee 0 \\ 1 \vee 0 \vee 1 \vee 1 & 0 \vee 1 \vee 1 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Problem 2. Let A be a 3×3 zero-one matrix. Let I be a 3×3 identity matrix. Show that $A \odot I = I \odot A = A$.

Solution. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Then

$$A \odot I = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (a \wedge 1) \vee (b \wedge 0) \vee (c \wedge 0) & (a \wedge 0) \vee (b \wedge 1) \vee (c \wedge 0) & (a \wedge 0) \vee (b \wedge 0) \vee (c \wedge 1) \\ (d \wedge 1) \vee (e \wedge 0) \vee (f \wedge 0) & (d \wedge 0) \vee (e \wedge 1) \vee (f \wedge 0) & (d \wedge 0) \vee (e \wedge 0) \vee (f \wedge 1) \\ (g \wedge 1) \vee (h \wedge 0) \vee (i \wedge 0) & (g \wedge 0) \vee (h \wedge 1) \vee (i \wedge 0) & (g \wedge 0) \vee (h \wedge 0) \vee (i \wedge 1) \end{bmatrix}$$

$$= \begin{bmatrix} a \vee 0 \vee 0 & 0 \vee b \vee 0 & 0 \vee 0 \vee c \\ d \vee 0 \vee 0 & 0 \vee e \vee 0 & 0 \vee 0 \vee f \\ g \vee 0 \vee 0 & 0 \vee h \vee 0 & 0 \vee 0 \vee i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = A;$$

$$I \odot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= \begin{bmatrix} (1 \wedge a) \vee (0 \wedge d) \vee (0 \wedge g) & (1 \wedge b) \vee (0 \wedge e) \vee (0 \wedge h) & (1 \wedge c) \vee (0 \wedge f) \vee (0 \wedge i) \\ (0 \wedge a) \vee (1 \wedge d) \vee (0 \wedge g) & (0 \wedge b) \vee (1 \wedge e) \vee (0 \wedge h) & (0 \wedge c) \vee (1 \wedge f) \vee (0 \wedge i) \\ (0 \wedge a) \vee (0 \wedge d) \vee (1 \wedge g) & (0 \wedge b) \vee (0 \wedge e) \vee (1 \wedge h) & (0 \wedge c) \vee (0 \wedge f) \vee (1 \wedge i) \end{bmatrix}$$

$$\begin{bmatrix} a \vee 0 \vee 0 & b \vee 0 \vee 0 & c \vee 0 \vee 0 \\ 0 \vee d \vee 0 & 0 \vee e \vee 0 & 0 \vee f \vee 0 \\ 0 \vee 0 \vee g & 0 \vee 0 \vee h & 0 \vee 0 \vee i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = A.$$