

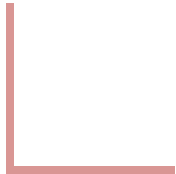
# Mathematical Foundations for Computer Applications

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## Negating Quantified Expressions

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# Mathematical Foundations for Computer Applications

## Negating Quantified Expressions

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Ex-“Every student in your class has taken a course in c++.”

- This statement is a universal quantification,  $\forall xP(x)$ ,
- where  $P(x)$  is the statement “x has taken a course in c++” and the domain consists of the students in your class.
- The **negation** of this statement is “It is not the case that every student in your class has taken a course in c++.”

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## Negating Quantified Expressions

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- The **negation** of this statement is “It is not the case that every student in your class has taken a course in c++.” OR
- “There is a student in your class who has not taken a course in c++.” ----This is the **existential quantification** of the negation of the original propositional function, namely,  $\exists x \neg P(x)$ .
- This example illustrates the following logical equivalence:

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## Negating Quantified Expressions

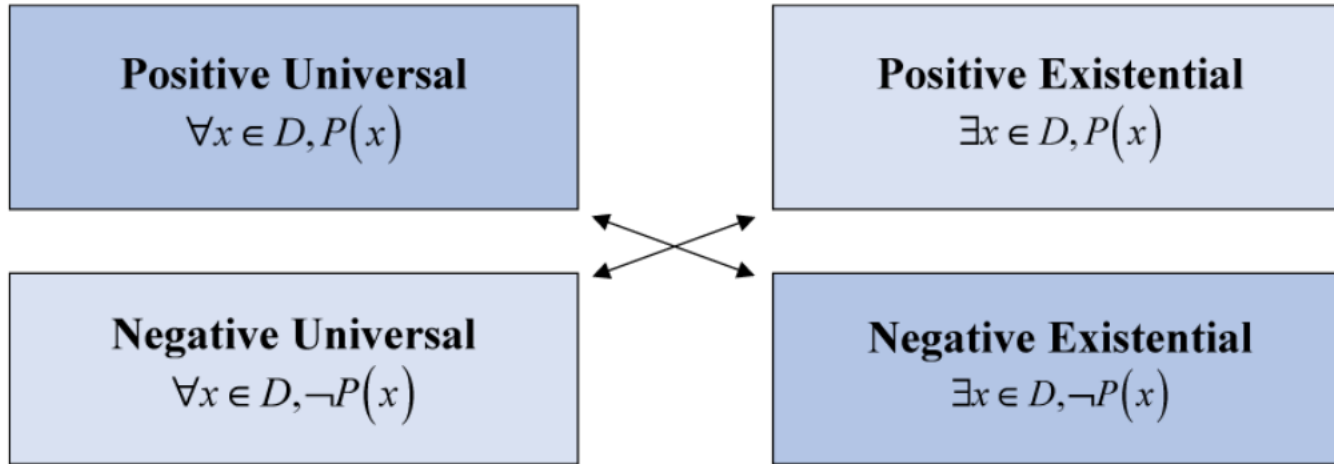
- $\neg \forall x P(x) \equiv \exists x \neg P(x)$ .
- $\neg \exists x Q(x) \equiv \forall x \neg Q(x)$ .
- The rules for negations for quantifiers are called **De Morgan's laws for quantifiers**. These rules are summarized in Table 2.

**TABLE 2** De Morgan's Laws for Quantifiers.

<i>Negation</i>	<i>Equivalent Statement</i>	<i>When Is Negation True?</i>	<i>When False?</i>
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every $x$ , $P(x)$ is false.	There is an $x$ for which $P(x)$ is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an $x$ for which $P(x)$ is false.	$P(x)$ is true for every $x$ .

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## Negation of quantification



$$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$$

$$\neg(\exists x \in D, P(x)) \equiv \forall x \in D, \neg P(x)$$

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## Negation Of Universal Quantifier

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- Let's negate the following statements with  $P(x)$  being "like homework."

### Statement

"Not all students like homework."

### Negation

"There is at least one student who does  
not like homework"

$$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$$

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## Negation Of Existential Quantifier

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- Let's negate the following statements with  $P(x)$  being "like homework."

### Statement

"It is **not** the case that **some** students like homework."

### Negation

"**All** students do **not** like homework."

$$\neg(\exists x \in D, P(x)) \equiv \forall x \in D, \neg P(x)$$

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## Problems

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1. What are the negations of the statements “There is an honest politician” and “All Americans eat cheeseburgers”?
- *Solution*: Let  $H(x)$  denote “ $x$  is honest.” Then the statement “There is an honest politician” is represented by  $\exists x H(x)$ , where the domain consists of all politicians.
  - The negation of this statement is  $\neg \exists x H(x)$ , which is equivalent to  $\forall x \neg H(x)$ .
  - This negation can be expressed as “Every politician is not honest.”



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## Problems

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- Let  $C(x)$  denote “ $x$  eats cheeseburgers.” Then the statement “**All Americans eat cheeseburgers**” is represented by  $\forall x C(x)$ , where the domain consists of all Americans.
- The negation of this statement is  $\neg \forall x C(x)$ , which is equivalent to  $\exists x \neg C(x)$ .
- This **negation** can be expressed “*Some American does not eat cheeseburgers*” OR “*There is an American who does not eat cheeseburgers.*”

# Mathematical Foundations for Computer Applications

## Problems

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**2.** What are the negations of the statements  $\forall x(x^2 > x)$  and  $\exists x(x^2 = 2)$ ?

- The negation of  $\forall x(x^2 > x) = \neg \forall x(x^2 > x) = \exists x \neg(x^2 > x)$   
 $= \exists x(x^2 \leq x).$
- The negation of  $\exists x(x^2 = 2) = \neg \exists x(x^2 = 2) = \forall x \neg(x^2 = 2).$   
 $= \forall x(x^2 \neq 2).$

The truth values of these statements depend on the domain.

# Mathematical Foundations for Computer Applications

## Problems

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3. Show that  $\neg \forall x(P(x) \rightarrow Q(x))$  and  $\exists x(P(x) \wedge \neg Q(x))$  are logically equivalent.

- We know that  $\neg(P(x) \rightarrow Q(x))$  and  $P(x) \wedge \neg Q(x)$  are logically equivalent for every  $x$ .
- It follows that  $\neg \forall x(P(x) \rightarrow Q(x))$  and  $\exists x(P(x) \wedge \neg Q(x))$  are logically equivalent.

# Mathematical Foundations for Computer Applications

## Translating from English into Logical Expressions

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**1.** Express the statement “Every student in this class has studied calculus” using predicates and quantifiers.

*Solution:* “For every student  $x$  in this class,  $x$  has studied calculus.”

- $C(x)$ , which is the statement “ $x$  has studied calculus.”  
Consequently, if the domain for  $x$  consists of the students in the class,
- **Statement**  $\forall x C(x)$ .

# Mathematical Foundations for Computer Applications

## Translating from Logical Expressions into English statement

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2. Translate these statements into English, where  $C(x)$  is “ $x$  is a comedian” and  $F(x)$  is “ $x$  is funny” and the domain consists of all people.

a)  $\forall x(C(x) \rightarrow F(x))$

b)  $\forall x(C(x) \wedge F(x))$

c)  $\exists x(C(x) \rightarrow F(x))$

d)  $\exists x(C(x) \wedge F(x))$

# Mathematical Foundations for Computer Applications

## Translating from Logical Expressions into English statement

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a)  $\forall x(C(x) \rightarrow F(x))$ , b)  $\forall x(C(x) \wedge F(x))$ , c)  $\exists x(C(x) \rightarrow F(x))$ , d)  $\exists x(C(x) \wedge F(x))$

a) All comedians are funny.

b) Everyone is a funny comedian. (All people are funny comedians)

c) There's someone who, if they are a comedian, then they are funny.

(There exist a comedian that is funny)

d) There is at least one comedian who is actually funny.

(A person is a funny comedian)

# Mathematical Foundations for Computer Applications

## Translating from English into Logical Expressions

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2. Express the statements “Some student in this class has visited Mexico” and “Every student in this class has visited either Canada or Mexico” using predicates and quantifiers.

(i) “There is a student  $x$  in this class having the property that  $x$  has visited Mexico.”

- $M(x)$  is the statement “ $x$  has visited Mexico.” If the domain for  $x$  consists of the students in this class, we can translate this first statement as  $\exists x M(x)$ .

# Mathematical Foundations for Computer Applications

## Translating from English into Logical Expressions

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- “There is a person  $x$  having the properties that  $x$  is a student in this class and  $x$  has visited Mexico.”
- In this case, the domain for the variable  $x$  consists of all people.  $S(x)$  to represent “ $x$  is a student in this class.”
- Solution becomes  $\exists x(S(x) \wedge M(x))$



# Mathematical Foundations for Computer Applications

## Translating from English into Logical Expressions

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(ii) Second statement can be expressed as

$$\forall x(C(x) \vee M(x)).$$

“For every person  $x$ , if  $x$  is a student in this class, then  $x$  has visited Mexico or  $x$  has visited Canada.”

- The statement can be expressed as

$$\forall x(S(x) \rightarrow (C(x) \vee M(x))).$$



# THANK YOU

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