Mathematical Foundations for Computer Applications

Laws of Set Theory

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Set Identities

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Commutative Law

For all sets A and B, Prove That AUB=BUA

Proof : Let x ∈ AUB

- x∈AUB. Then x∈A or x∈B.
- Which implies x∈B or x∈A.
- Hence x∈BUA.
- Thus AUB⊆BUA ----(1)
- Similarly, we can show that BUA⊆AUB ---(2)

From (1) and (2) Therefore, **AUB=BUA.**

Commutative Law

For all sets A and B, Prove That $A \cap B = B \cap A$

Distributive Law

For all sets A,B and C, Prove That $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof Let $x \in A \cap (B \cup C)$

- Then $x \in A$ and $x \in B \cup C$.
- Thus $x \in A$ and ($x \in B$ or $x \in C$)
- Which implies ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)
- Hence $x \in (A \cap B) \cup (A \cap C)$.
- Thus $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ -------(1)
- Similarly, we can show that $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$---(2)
- From (1) and (2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Distributive Law

For all sets A,B and C, Prove That $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

De Morgan's law

i. The complement of the union of two sets is equal to the intersection of their complements

ii. The complement of the intersection of two sets is equal to the union of their complements. These are called **De Morgan's laws**.

For any two finite sets A and B

(i) $(A \cup B)' = A' \cap B'$ (which is a De Morgan's law of union).

(ii) $(A \cap B)' = A' \cup B'$ (which is a De Morgan's law of intersection).

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De Morgan's law

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$

(We will prove that the two sets $\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$ are equal by showing that each set is a subset of the other)

show that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

Let $x \in \overline{A \cap B}$

=> $x \notin A \cap B$. By the definition of complement

Using the definition of intersection, the proposition

$\neg((x \in A) \wedge (x \in B))$ is true.

By applying De Morgan's law for propositions,

$\neg(x \in A)$ or $\neg(x \in B)$.

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De Morgan's law

$\neg(x \in A)$ or $\neg(x \in B)$.

$x \notin A$ or $x \notin B$.

$x \in \overline{A}$ or $x \in \overline{B}$. definition of the complement

$x \in \overline{A} \cup \overline{B}$

$\overline{\mathbf{A} \cap \mathbf{B}} \subseteq \overline{\mathbf{A}} \cup \overline{\mathbf{B}}$ ------(i)

Next, we will show that $\overline{\mathbf{A}} \cup \overline{\mathbf{B}} \subseteq \overline{\mathbf{A} \cap \mathbf{B}}$

Proof

Let $x \in \overline{A} \cup \overline{B}$

$x \in \overline{A}$ or $x \in \overline{B}$. definition of the complement

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De Morgan's law

$x \notin A$ or $x \notin B$.

$\neg(x \in A) \vee \neg(x \in B)$.

$\neg((x \in A) \wedge (x \in B))$ is true. By De Morgan's law for propositions

$\neg(x \in A \cap B)$ By the definition of intersection

$x \in \overline{A \cap B}$

$\mathbf{\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}}$ **--------(ii)**

From (i) and (ii)

$\mathbf{\overline{A \cap B} = \bar{A} \cup \bar{B}}$

De Morgan's law

- Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$

We can prove this identity with the following steps.

$\overline{\mathbf{A} \cap \mathbf{B}}$ = {x | x $\notin$ A ∩ B} by definition of complement

= {x | $\neg$(x $\in$ (A ∩ B))} by definition of does not belong symbol

= {x | $\neg$(x $\in$ A $\wedge$ x $\in$ B)} by definition of intersection

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De Morgan's law

$= \{x \mid \neg(x \in A) \lor \neg(x \in B)\}$ by the De Morgan law for logical equivalences

$= \{x \mid x \notin A \lor x \notin B\}$ by definition of does not belong symbol

$= \{x \mid x \in \overline{A} \lor x \in \overline{B}\}$ by definition of complement

$= \{x \mid x \in \overline{A} \cup \overline{B}\}$ by definition of union

$= \overline{A} \cup \overline{B}$ by meaning of set builder notation

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Problems

Let A, B, and C be sets. Show that

$$\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}.$$

LHS

$\overline{A \cup (B \cap C)} = \bar{A} \cap \overline{(B \cap C)}$ by the De Morgan law

$= \bar{A} \cap (\bar{B} \cup \bar{C})$ by the De Morgan law

$= (\bar{B} \cup \bar{C}) \cap \bar{A}$ by the commutative law for intersection

$= (\bar{C} \cup \bar{B}) \cap \bar{A}$ by the commutative law for union

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Problems

1. Let U = {1, 2, 3, 4, 5, 6, 7, 8}, A ={3, 4, 5}, B = {4, 5, 6}.
   Show that (AUB)’ = A’∩B’.
2. If U = {a, b, c, d, e, f, g, h}, P = {a, c, d}, Q = {a, b, f, g}. Prove De morgan’s Laws

THANK YOU

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