Unit 1: Set Theory and Matrix Theory

1. **Write the following sets in Set-Builder Form :**

(i) A = {1, 3 ,5, 7, 9}

(ii) B = {16, 25, 36, 49, 64}

(iii) C = {5,10,15,20,25}

(iv) D = {violet, indigo, blue, green, yellow, orange, red}

(v) E = {January, March, May, July, August, October, December}

2. **Find A U B and A ⋂ B and A – B.**

If A = {a, b, c, d} and B = {c, d}.

3. If A = {1, 3, 5}, B = {3, 5, 6} and C = {1, 3, 7}

(i) Verify that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) Verify A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

4. Let A = {a, b, d, e}, B = {b, c, e, f} and C = {d, e, f, g}

(i) Verify A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

(ii) Verify A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

5. Let A = {3, 5, 7}, B = {2, 3, 4, 6} and C = {2, 3, 4, 5, 6, 7, 8}

(i) Verify (A ∩ B)' = A' ∪ B'

(ii) Verify (A ∪ B)' = A' ∩ B'

6. Given three sets P, Q and R such that:

P = {x : x is a natural number between 10 and 16},

Q = {y : y is a even number between 8 and 20} and

R = {7, 9, 11, 14, 18, 20}

(i) Find the difference of two sets P and Q

(ii) Find Q - R

(iii) Find R - P

(iv) Find Q – P

7. Write the cardinal number of each of the following sets:

(i) X = {letters in the word MALAYALAM}

(ii) Y = {5, 6, 6, 7, 11, 6, 13, 11, 8}

(iii) Z = {natural numbers between 20 and 50, which are divisible by 7}

8. If the number of elements in a set is 2, find the number of subsets and proper subsets.

9. If P and Q are two sets such that P ∪ Q has 40 elements, P has 22 elements and Q has 28 elements, how many elements does P ∩ Q have?

10. In a class of 40 students, 15 like to play cricket and football and 20 like to play cricket. How many like to play football only but not cricket?

11. There is a group of 80 persons who can drive scooter or car or both. Out of these, 35 can drive scooter and 60 can drive car. Find how many can drive both scooter and car? How many can drive scooter only? How many can drive car only?

12. It was found that out of 45 girls, 10 joined singing but not dancing and 24 joined singing. How many joined dancing but not singing? How many joined both?

13. Let U = {John, Mary, Dave, Lucy,Peter,Larry} , A= {John, Mary, Dave}, and B={John,Larry,Lucy}. Find A∩B, A∪B , A−B , B−A , $\bar{A}$ and $\bar{B}$.

14. At a construction site, Jorge is in charge of hiring skilled workers for the project. Out of 80 candidates that he interviewed, he found that
45 were painters,
50 were electricians,
50 were plumbers,
15 had skills in all three areas, and
all of them had skills in at least one of these areas.
If he hired everyone who was skilled in exactly 2 areas, how many candidates were hired?

15. There are 350 farmers in a large region. 260 farm beetroot, 100 farm yams, 70 farm radish, 40 farm beetroot and radish, 40 farm yams and radish, and 30 farm beetroot and yams. Let B, Y, and R denote the set of farms that farm beetroot, yams and radish respectively. Determine the number of farmers that farm beetroot, yams, and radish.

16.F ind the power set of the following sets.

i. {a,b}
ii. {4,7}

iii. {x,y,z,w}
iv. {{a}}

17. In a class of 30 students, some studied physics and some chemistry. If 20 studied physics, 18 studied chemistry and 4 studied neither, calculate the number of students that studied both subjects.

Mathematical Induction

1. Show that $1^2+2^2+.....+n^2 = \frac{n(n+1)(2n+1)}{6}$ by mathematical Induction
2. Show that $1+2+2^2+.....+ 2^n = (2^{n+1} -1)$ by mathematical Induction
3. Show that $n < 2^n$ by mathematical Induction
4. Show that $n^2 \geq 2n+1$, for all $n \geq 3$, by mathematical Induction