

Mathematical Foundations for Computer Applications

Cramer's Rule

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Cramer's rule

- Cramer's rule is one of the important methods applied to **solve a system of equations.**
- In matrices, Cramer's rule is used to find the **solution of a system of linear equations in n variables.**
- In this method, the values of the variables in the system are to be calculated **using the determinants of matrices.**
- Cramer's rule is also known as the determinant method.

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Cramer's rule Formula

- Consider a system of linear equations with n variables $x_1, x_2, x_3, \dots, x_n$ written in the matrix form $AX = B$.
 - A = Coefficient matrix (must be a square matrix)
 - X = Column matrix with variables
 - B = Column matrix with the constants (which are on the right side of the equations)
- Now, we have to find the determinants as:
- $D = |A|, Dx_1, Dx_2, Dx_3, \dots, Dx_n$
 - Here, Dx_i for $i = 1, 2, 3, \dots, n$ is the same determinant as D such that the column is replaced with B .
- Thus,
- $x_1 = Dx_1/D; x_2 = Dx_2/D; x_3 = Dx_3/D; \dots; x_n = Dx_n/D$ {where D is not equal to 0}

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Cramer's Rule 2x2

- Cramer's rule for the 2x2 matrix is applied to solve the system of equations in two variables.
- Let us consider two linear equations in two variables.
- $a_1x + b_1y = c_1$
- $a_2x + b_2y = c_2$
- Let us write these two equations in the form of $AX = B$.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Here,

$$\text{Coefficient matrix } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$\text{Variable matrix } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{Constant matrix } B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$D = |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

And

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1$$

Therefore,

$$x = D_x/D$$

$$y = D_y/D$$

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Example:

1. Solve the following system of equations using Cramer's rule:

$$2x - y = 5$$

$$x + y = 4$$

Given,

$$2x - y = 5$$

$$x + y = 4$$

Let us write these equations in the form $AX = B$.

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now,

$$D = |A|$$

$$= \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 2(1) - (-1)1$$

$$= 2 + 1$$

$$= 3 \neq 0$$

So, the given system of equations has a unique solution.

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Example :

$$D_x = \begin{vmatrix} 5 & -1 \\ 4 & 1 \end{vmatrix}$$
$$= 5(1) - (-1)(4)$$

$$= 5 + 4$$

$$= 9$$

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}$$
$$= 2(4) - 5(1)$$

$$= 8 - 5$$

$$= 3$$

Therefore,

$$x = D_x/D = 9/3 = 3$$

$$y = D_y/D = 3/3 = 1$$

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Cramer's Rule 3x3

To find the Cramer's rule formula for a 3x3 matrix, we need to consider the system of 3 equations with three variables.

Consider:

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Let us write these equations in the form $AX = B$.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Now,

$$D = |A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Therefore, $x = D_x/D$, $y = D_y/D$, $z = D_z/D$; $D \neq 0$

And

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Problems

Solve the following system of equations using Cramer's rule:

$$x + y + z = 6$$

$$y + 3z = 11$$

$$x + z = 2y$$

Let us write these equations in the form $AX = B$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

Now,

$$D = |A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 1(1+6) - 1(0-3) + 1(0-1) = 7 + 3 - 1 = 9$$

$D \neq 0$ so the given system of equations has a unique solution.

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Problems

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 11 & 1 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 6(1+6) - 1(11-0) + 1(-22-0) = 42 - 11 - 22 = 9$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 0 & 11 & 3 \\ 1 & 0 & 1 \end{vmatrix} = 1(11-0) - 6(0-3) + 1(0-11) = 11 + 18 - 11 = 18$$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & 11 \\ 1 & -2 & 0 \end{vmatrix} = 1(0+22) - 1(0-11) + 6(0-1) = 22 + 11 - 6 = 27$$

Thus,

$$x = D_x/D = 9/9 = 1$$

$$y = D_y/D = 18/9 = 2$$

$$z = D_z/D = 27/9 = 3$$

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NOTE

- If $D \neq 0$, we say that the system $AX = B$ has a unique solution.
- If $D=0$, system has “no solution” or “infinite number of solutions”.

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Problems

Solve using Cramer's Rule:

- 1) $3x - 4y = 10$ and $5x + 2y = 8$.
- 2) $12x - 10y = 46$ and $3x + 20y = -11$
- 3) $3x - 4y + 8z = 34$, $4x + y - 2z = 1$, $-6x - 13y + 20z = 61$
- 4) $3x - 8y + 10z = 8$, $-x + 10y + 9z = -15$, $2x - 6y + z = 11$

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Assignment Problems

1. Solve the following system of equations by Cramer's rule:
 $2x - 3y + 5z = 11$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

2. Solve the following system of linear equations using Cramer's rule:
 $5x + 7y = -2$

$$4x + 6y = -3$$

3. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion, 2 kg wheat, and 3 kg rice is Rs 70. Find the cost of each item per kg by Cramer's rule.



THANK YOU

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