Mathematical Foundations for Computer Applications

Matrix Theory

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Matrix

Matrix- A matrix is an ordered rectangular array of numbers.
Order-A matrix having ‘m’ rows and ‘n’ columns is called as order of a matrix of m $\times$ n.

Types of Matrices

- **Row Matrix**
- **Column Matrix**
- **Square Matrix**
- **Diagonal matrix**
- **Identity matrix**

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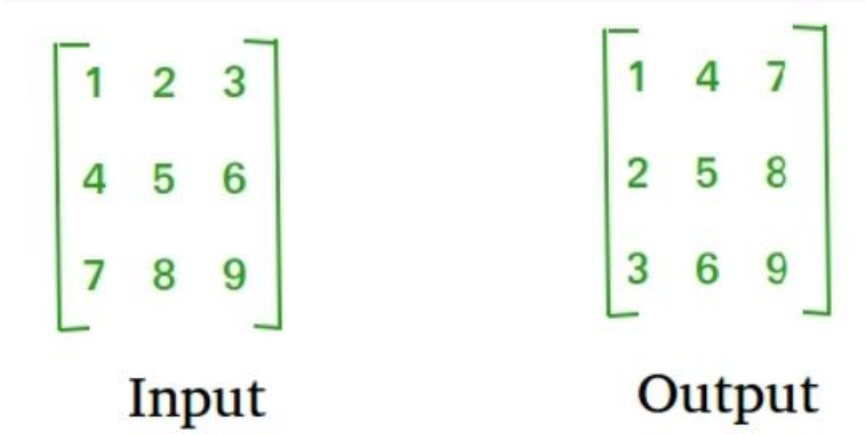
Operations on Matrices

Transpose of a matrix-- is obtained by changing rows to columns and columns to rows.



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Operations on Matrices

Addition of Matrices

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

Subtraction of Matrices

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} - \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 - a_2 & b_1 - b_2 \\ c_1 - c_2 & d_1 - d_2 \end{bmatrix}$$

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Operations on Matrices

Multiplication of Matrices

Example: $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & 4 \\ 5 & 1 & -1 \end{bmatrix}$ $B = \begin{pmatrix} 1 & 6 \\ 3 & -3 \\ 5 & 2 \end{pmatrix}$

$3x3$ $3x2$

$$A*B = \begin{bmatrix} 1*1+3*3+2*5 & 1*6+3*-3+2*2 \\ 0*1+(-2*3)+4*5 & 0*6+(-2*-3)+4*2 \\ 5*1+1*3+(-1*5) & 5*6+1*-3+(-1*2) \end{bmatrix}$$

$$A * B = \begin{bmatrix} 20 & 1 \\ 14 & 14 \\ 3 & 25 \end{bmatrix}$$

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Operations on Matrices

Multiplication of Matrices

If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & -2 \end{bmatrix}$

find AB and BA if possible.

Type equation here.

AB= $\begin{pmatrix} 16 & -9 \\ 3 & -10 \\ 3 & 0 \end{pmatrix}$

BA is not possible since the number of columns of B ≠ the number of rows of A.

Determinants

To every square matrix A = $[a_{ij}]$ of order n, we can associate a number (real or complex) called determinant of the square matrix A, where a_{ij} = $(i, j)^{th}$ element of A.

- Only square matrices have determinants

Determinant of a matrix of order one

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.

Determinants

Determinant of a matrix of order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2×2,

then the determinant of A is defined as:

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Determinants

Example 1 Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$.

Solution We have $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2(2) - 4(-1) = 4 + 4 = 8.$

Example 2 Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

Solution We have

$$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x(x) - (x+1)(x-1) = x^2 - (x^2 - 1) = x^2 - x^2 + 1 = 1$$

Determinants

Determinant of a matrix of order 3 × 3

There are various methods exist for calculating a matrix's determinant. The most common approach is by breaking a given 3 × 3 matrix into smaller 2 × 2 determinants. This simplifies the process of finding the determinant and is widely used in linear algebra.

Let's take a 3 × 3 square matrix which is written as,

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Determinants

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|A| = a(ei-fh) - b(di-fg) + c(dh-eg)$$

Determinants- Problems

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$$

Determinant=|A|=2(0+1)-1(8-2)+3(-4-0)=2 – 6 + (-12) = -16

$$\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

Determinant = 1(0-0)-2(0-0)+4(-1-12)= – 52

Determinants- Problems

Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

THANK YOU

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