Mathematical Foundations for Computer Applications

Inclusion-Exclusion Principle

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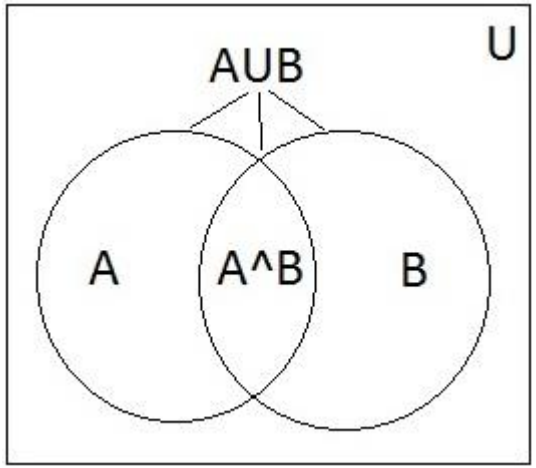
Mathematical Foundations for Computer Applications

Inclusion-Exclusion Principle with Two Sets

--Addition Principle

Let A and B be two finite sets then cardinality of $A \cup B$ is

$$|A \cup B| = |A| + |B| - |A \cap B|$$



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Problems

1. consider a group of people where 10 own dogs, 12 own cats and 5 own both. How many people own either or both.

$$|D \cup C| = |D| + |C| - |D \cap C|$$

$$|D \cup C| = 10 + 12 - 5 = 17$$

The result is 17 people that own either dogs or cats or both.

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Problems

2. A group of 100 students. 20 are taking discrete math (D), 30 are taking Java programming (J) and 6 are taking both classes. How many are taking *neither* of the class.

$$|D \cup J| = |D| + |J| - |D \cap J| = 20 + 30 - 6 = 44.$$

100-44 =56 students are taking neither of the class.

Note-suppose we want to count the number of students taking discrete math but not Java. (20-6=14)

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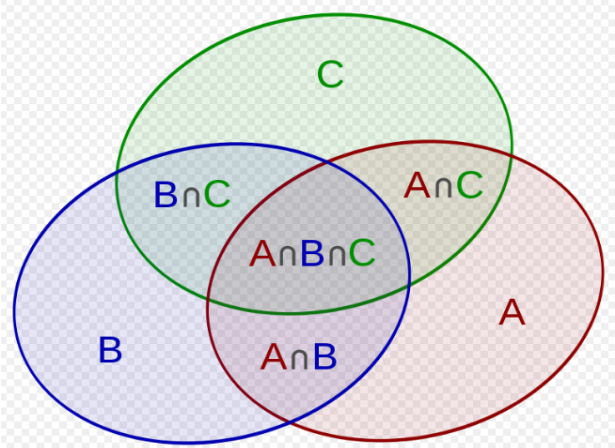
Extended Addition Principle

If there are **three** sets, the principle of inclusion and exclusion states

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$



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Problems

1.There are exactly three types of students in a school: the geeks, the wannabees and the athletes. Each student is classified into at least one of these categories. And the total number of students in the school is 1000. Suppose that the following is given:
The total number of students who are geeks is 310.
The total number of students who are wannabees is 650.
The total number of students who are athletes is 440.
The total number of students who are both geeks and wannabees is 170.
The total number of students who are both geeks and athletes is 150.
The total number of students who are both wannabees and athletes is 180.
What is the total number of students who fit into all 3 categories?

Solution

Let A,B,C denote the set for geeks, wannabees, and athletes, respectively.

Then by the principle of inclusion and exclusion, we have

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

$1000 = 310 + 650 + 440 - 170 - 150 - 180 + |A \cap B \cap C|$

$|A \cap B \cap C| = 1000\text{-}900 = 100$

So the total number of students who **fit into all 3 categories is 100.**

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Problems

2. A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch basketball and hockey games. 50 do not watch any three kinds of games. Find:

i) How many in the survey watch all 3 kinds of games?

ii) How many watch exactly one of the sports?

iii) Draw Venn Diagram showing results of the survey.

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Solution

Let F denote people watch football.
H denotes people watch hockey.
B denotes people watch basketball.

|F|=285 ,|H|=195,|B|=115
|F∩H| =70,|F∩B| =45,|H∩B| =50
$\overline{(|F\ U\ H\ U\ B|)}$ =50

(|F U H U B|)=U- $\overline{(|F\ U\ H\ U\ B|)}$ = 500-50=450

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Solution

(i) $|F \cap H \cap B| = |F \cup H \cup B| - |F| - |H| - |B| + |F \cap H| + |H \cap B| + |F \cap B|$

$= 450 - 285 - 195 - 115 + 70 + 50 + 45 = 20$

Hence, 20 viewers watch all 3 kinds of games.

(ii)

$|F - H - B| = |F| - |F \cap H| - |F \cap B| + |F \cap H \cap B| = 190$

Similarly , $|H - B - F| = 95$, $|B - F - H| = 40$

No of viewers who watch exactly one of the sports = 190+95+40 = 325

Exercise -Number of people who watch exactly 2 games=50+30+25=105

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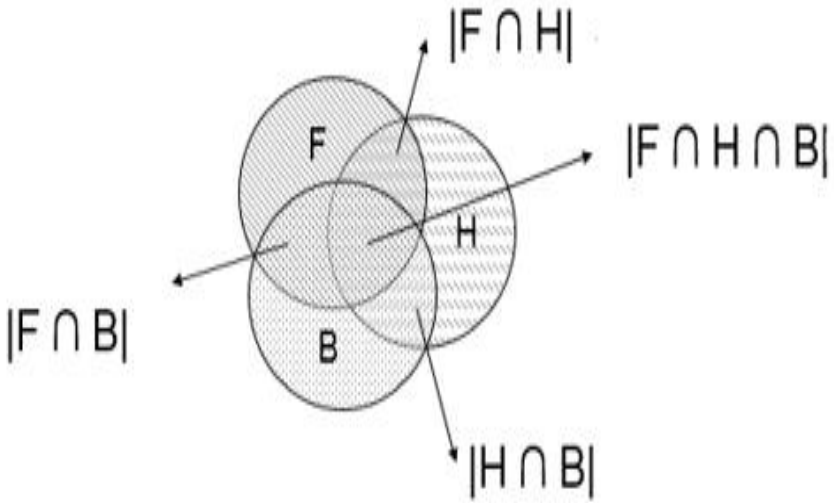
Solution



THANK YOU

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