Mathematical Foundations for Computer Applications
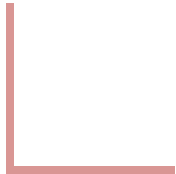
Inference rules-Problems

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Using Rules of Inference to Build Arguments

1. Show that the following argument is valid."Ramu works hard", "If Ramu works hard, then he is a dull boy", and " If Ramu is a dull boy, then he will not get the job".
Therefore "Ramu will not get the job"

Let p=Ramu works hard, q = Ramu is a dull boy, r=Ramu will get the job

$$p$$
$$p \rightarrow q$$
$$q \rightarrow \neg r$$
$$\therefore \neg r$$

Problems: Solution

1) p   -----H1

2) $p \rightarrow q$ -----H2

3) q   ---- Modus ponens 1 & 2

4) $q \rightarrow \neg r$ ---- H3

5) $\neg r$ ---- Modus ponens 3 &4

2.Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

- Let **p ---**"It is sunny this afternoon,"

  q --- "It is colder than yesterday,"

  r--- "We will go swimming,"

  s--- "We will take a canoe trip,"

  t --- "We will be home by sunset."

Problems

Premises become

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$s \rightarrow t$$

The conclusion is t

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Step	Reason
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

3. Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

Let p --- "You send me an e-mail message,"

q --- "I will finish writing the program,"

r ---"I will go to sleep early,"

s --- "I will wake up feeling refreshed

Problems

Premises are $p \to q, \quad \neg p \to r, \quad$ **and** $r \to s.$

Conclusion is $\neg q \to s.$

- **Step**   **Reason**

1. $p \to q$ Premise

2. $\neg q \to \neg p$ Contrapositive of (1)

3. $\neg p \to r$ Premise

4. $\neg q \to r$ Hypothetical syllogism using (2) and (3)

5. $r \to s$ Premise

6. $\neg q \to s$ Hypothetical syllogism using (4) and (5)

4. Establish the validity of the following arguments and specify the rule of inference used.

$H_1 : \neg s \wedge c$

$H_2 : w \rightarrow s$

$H_3 : \neg w \rightarrow t$

$H_4 : t \rightarrow h$

$\therefore \; h$

Problems

Step	Reason
1. ¬s ∧ c	Hypothesis
2. ¬s	Simplification
3. w → s	Hypothesis
4. ¬w	Modus tollens of 2 and 3
5. ¬w → t	Hypothesis
6. t	Modus ponens of 4 and 5
7. t → h	Hypothesis
8. h	Modus ponens of 6 and 7

5. Determine the validity of the following arguments

p→⅂q , q ∨ r , ⅂s →p , ⅂r   ∴ s

1)   q ∨ r----H2

2)   ⅂r -----H4

3)   q ---- Disjunctive syllogism of 1 and 2

4)   p→⅂q ----H1

5)   ⅂p -----Modus Tollens of 3 and 4

6)   ⅂s →p ----H3

7)   s -----Modus Tollens of 5 and 6

Hence the given arguments are valid.

6. Show that the following argument is valid "If it does not rain or if it not foggy, then the sailing race will be held and the lifesaving demonstration will go on", "If the sailing race is held, then the trophy will be awarded" and " The trophy was not awarded" Therefore "It rained"

Let   p=It rains,

            q=it is foggy ,

            r=the sailing race will be held,

            s=the lifesaving demonstration will go on

             t=trophy will be awarded

˥p ∨ ˥q → r ∧ s

r → t

˥t

∴ p

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Problems

1) r → t   (H2)

2) ⅂t   (H3)

3) ⅂r   (Modus tollens)

4) ⅂r v ⅂s   (addition)

5) ⅂(r ∧ s)   {Demorgan's }

6) ⅂p v ⅂q → (r ∧ s)   (H1)

7) ⅂(⅂p v ⅂q)   {Modus tollens 5, 6)

8) (p ∧ q)   {Demorgan's }

9) p   {Simplication}

7.Show that the following argument is valid. If today is Tuesday, then I have a test in maths or CS. If my CS professor is sick, then I will not have a test in CS. Today is Tuesday and my CS professor is sick. Therefore I have a test in maths.

Problems

$$P \rightarrow q \lor r$$

$$s \rightarrow \neg r$$

$$p \land s$$

$$\therefore q$$

Problems

1. p ∧ s (H3)

2. P (Simplification)

3. P → qVr (H1)

4. qvr (MP Rule)

5. s → ¬r (H2)

6. ¬r v ¬s (from step 5)

7. r v q (step 4 commutative)

8. ¬s v q (Resolution for step 6 & 7)

9. ¬(s ∧ ¬q) (D morgan's)

10. ¬(¬q)   (simplification)

11. q

THANK YOU

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