# Unit-2 Counting Principles

# (Q and A)

1. A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

The procedure of assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the office assigned to Sanchez, which can be done in 11 ways. By the product rule, there are $12 \cdot 11 = 132$ ways to assign offices to these two employees.

2. How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 =$ 17,576,000 possible license plates.

3. A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

The student can choose a project by selecting a project from the first list, the second list, or the third list. Because no project is on more than one list, by the sum rule there are 23 + 15 + 19 = 57 ways to choose a project

4. There is one position available for a faculty job at Irvine. The applicant must come from either Berkeley which has 20 candidates or UCLA which has 50 candidates. What is the total number of possible candidates for the position?

A: The applicant must be from Berkeley OR UCLA. So we have 20+50=70 possible applicants.

5. At a resaurant, there are 18 dinners with meat, 10 different dinners with fish, and 5 vegetarian dinners? How many dinners to choose from?

A: Each dinner is meat OR fish OR vegetarian. So we have 18+10+5=33 choices.

6. How many permutations of the letters *ABCDEFGH* contain the string *ABC* ?
*Solution:* Because the letters *ABC* must occur as a block, we can find the answer by finding the number of permutations of six objects, namely, the block *ABC* and the individual letters *D*, *E*,*F*, *G*, and *H*. Because these six objects can occur in any order, there are 6! = 720 permutations of the letters *ABCDEFGH* in which *ABC* occurs as a block.

7. How many ways can 8 boys and 8 girls be seated in a row
   a) If any person can sit next to any other
   b) If boy and girl must occupy alternate seats.

Ans a) =16!
   b) Either a boy can occupy seat 1 , 8! *8!
   OR a girl can occupy seat 1 , 8! *8!
   Therefore 8! *8! +8! *8! =2(8! * 8!)

8. How many ways can 3 exams be scheduled within a 5 days period. so that no two exams are scheduled on same date.
   =5*4*3=60

## Combinations

1. At a consultant mixer with 42 people, everyone shakes everyone else's hand exactly once. How many handshakes occur?

   **There exists one handshake between any two people, so one for each pair. There are C(42, 2) different ways to choose pairs.**

2. Three different friends are splitting 9 different donuts amongst themselves equally so each person gets 3. How many ways are there to do this?

   There are C(9, 3) different ways to choose donuts for the first person,
   Then C(6,3) for the next, and C(3,3) for the last.
   So a total of C(9, 3) * C(6, 3)* C(3,3) different ways.

Another way to get this is to note that this is the same number of Anagrams of AAABBBCCC where the for example in the anagram ABCABCABC,
Donut numbers 1, 4, 7 are given to A.
There are a total of $9! / (3!)^3$ different ways to do this, which is the same answer.

3. How many four digit numbers exist such that their digits are in strictly increasing order?

With any selection of 4 digits from 1 through 9, we can make such a four Digit number and every four digit number is made that way. So there are a total of C(9, 4) different ways.

4. There are 9 points on a circle and lines connect all pairs of points. At how many places inside the circle do these lines intersect?

Each intersection inside is determined by four points on the outside. Each selection of 4 points gives a unique intersection point. Thus, there are a total of C(9, 4) different intersection points.

## Permutations

1. How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other?
Once you place the men, there are 9 spots for the women. We can choose one for each woman to stand in and since the order matters, the _nal number is
P(8, 8) * P(9, 5).

2. How many anagrams of MISSISSIPPI exist?

There are a total of 11 letters with 1 M, 2 P, 4 S, 4 I.

The Final answer is 11! / 2! 4! 4!

3. How many anagrams of BEAD exist so that the vowels appear all next to each other?

We can group them together at first to get (4 - 1)! = 6 different ways to anagram. But then inside the block of vowels, we can arrange two ways

so 2 * 6 = 12

4. How many anagrams of ROYZHAO exist so that the consonants appear next to each other (Y is a vowel)?

4 vowels and 3 consonants. Block the consonants together to get a total of 5 blocks with two repeating (2 Os) so 5!/2! different ways to arrange. Within the block, there are 3! ways to arrange giving a total of

5! / 2! * 3! = 360

5. How many ways are there to choose a delegation out of 10 males and 10 females if the delegation is made up of 2 males and 3 females?

P(10,2) ways to choose the males and P(10,3) ways to choose the females giving a total of P(10,2)* P(10,3) total ways