Mathematical Foundations for Computer Applications

Pigeonhole Principle

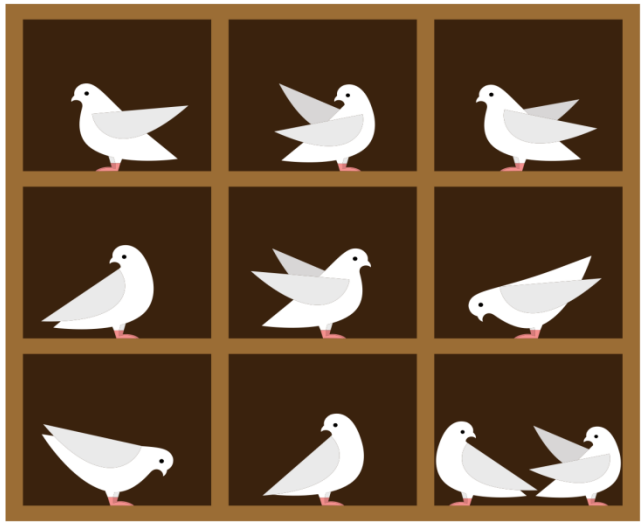
Dr. Premalatha H M

Department of Computer Applications

Mathematical Foundations for Computer Applications

The Pigeonhole Principle-Introduction

- If a flock of 10 pigeons roosts in a set of 9 pigeonholes, one of the pigeonholes must have more than 1 pigeon.



There are more Pigeons than Pigeonholes.

The Pigeonhole Principle

- If *k* is a positive integer and *k* + 1 or more objects are placed into *k* boxes, then there is at least one box containing two or more of the objects.
- **Proof:** We use a proof by **contraposition.** Suppose none of the k boxes has more than one object. Then the total number of objects would be at most k. This contradicts the statement that we have k + 1 objects.

The Generalized Pigeonhole Principle

- **If N objects are placed into k boxes, then there is at least one box containing at least ⌈N/k⌉ objects.**

Proof: We use a proof by contraposition.

Suppose that none of the boxes contains more than ⌈N/k⌉ − 1 objects.

Then the total number of objects is at most

$$k\left(\left\lceil\frac{N}{k}\right\rceil-1\right)<k\left(\left(\frac{N}{k}+1\right)-1\right)=N,$$

The Generalized Pigeonhole Principle

- where the inequality $\lceil N/k \rceil < \lceil N/k \rceil + 1$ has been used.
- This is a contradiction because there are a total of n objects.

Example: Among 100 people there are at least $\lceil 100/12 \rceil = 9$ who were born in the same month.

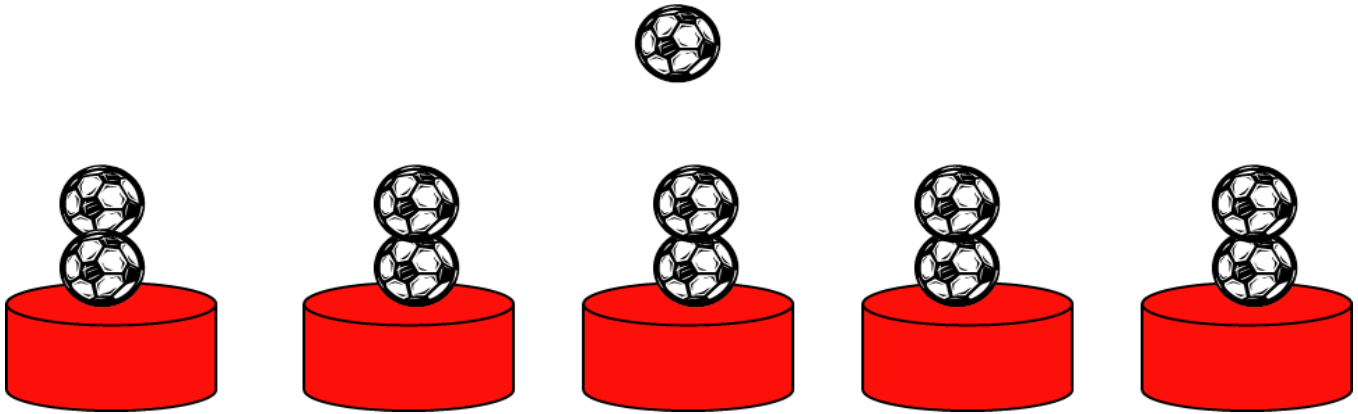
Mathematical Foundations for Computer Applications

Counting: Pigeonhole Principles

Example:

k=5, n=11

$\lceil 11/5 \rceil = 3$



Examples

1. In a group of 366 people there are at least 2 people who have their birthday on the same day.

- $\lceil 366/365 \rceil = 2$.

2. We have 5 possible grades: A,B,C,D,E. How many students do we need to be sure at least 6 get the same grade?

Examples

Try to fill the boxes evenly: If we have 25 students, then we can have 5 A's, 5 B's etc.

So if we have 26 students, we need to add a student to some grade, and this grade will then have 6 students.

- Check: $\lceil 26 / 5 \rceil = 6$, but $\lceil 25 / 5 \rceil = 5$

Examples

3. How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Solution: There are 101 possible scores on the final.

The pigeonhole principle shows that among any 102 students there must be at least 2 students with the same score.

Examples

4. Show that if 7 colors are used to paint 50 bicycles at least 8 bicycle paint in same color.

- N=50, k=7
- =n/k = 50/7 =8

THANK YOU

Dr. Premalatha H M

Department of Computer Applications

Premalatha.hm@pes.edu

+91 80 26721983 Extn 224