

PERMUTATIONS AND COMBINATIONS

7.1 Overview

The study of permutations and combinations is concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects, without actually listing them. There are some basic counting techniques which will be useful in determining the number of different ways of arranging or selecting objects. The two basic counting principles are given below:

Fundamental principle of counting

7.1.1 Multiplication principle (Fundamental Principle of Counting)

Suppose an event E can occur in m different ways and associated with each way of occurring of E, another event F can occur in n different ways, then the total number of occurrence of the two events in the given order is $m \times n$.

7.1.2 Addition principle

If an event E can occur in m ways and another event F can occur in n ways, and suppose that both can not occur together, then E or F can occur in $m + n$ ways.

7.1.3 Permutations

A permutation is an arrangement of objects in a definite order.

7.1.4 Permutation of n different objects

The number of permutations of n objects taken all at a time, denoted by the symbol ${}^n P_n$, is given by

$${}^n P_n = |n|, \quad \dots (1)$$

where $|n| = n(n - 1)(n - 2) \dots 3.2.1$, read as factorial n , or n factorial.

The number of permutations of n objects taken r at a time, where $0 < r \leq n$, denoted by ${}^n P_r$, is given by

$${}^n P_r = \frac{|n|}{|n - r|}$$

We assume that $|0| = 1$

7.1.5 When repetition of objects is allowed The number of permutations of n things taken all at a time, when repetition of objects is allowed is n^n .

The number of permutations of n objects, taken r at a time, when repetition of objects is allowed, is n^r .

7.1.6 Permutations when the objects are not distinct The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and

the rest if any, are of different kinds is $\frac{n!}{p_1!p_2!...p_k!}$

7.1.7 Combinations On many occasions we are not interested in arranging but only in selecting r objects from given n objects. A combination is a selection of some or all of a number of different objects where the order of selection is immaterial. The number of selections of r objects from the given n objects is denoted by nC_r , and is given by

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Remarks

1. Use permutations if a problem calls for the number of arrangements of objects and different orders are to be counted.
2. Use combinations if a problem calls for the number of ways of selecting objects and the order of selection is not to be counted.

7.1.8 Some important results

Let n and r be positive integers such that $r \leq n$. Then

- (i) ${}^nC_r = {}^nC_{n-r}$
- (ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- (iii) $n {}^{n-1}C_{r-1} = (n-r+1) {}^nC_{r-1}$

7.2 Solved Examples

Short Answer Type

Example 1 In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection?

Solution Here the teacher is to perform two operations:

- (i) Selecting a boy from among the 27 boys and
- (ii) Selecting a girl from among 14 girls.

The first of these can be done in 27 ways and second can be performed in 14 ways. By the fundamental principle of counting, the required number of ways is $27 \times 14 = 378$.

Example 2

- How many numbers are there between 99 and 1000 having 7 in the units place?
- How many numbers are there between 99 and 1000 having atleast one of their digits 7?

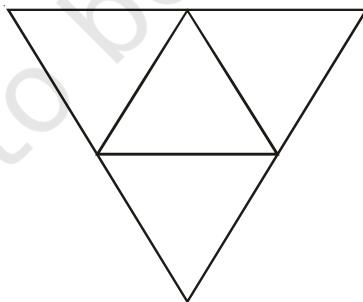
Solution

- First note that all these numbers have three digits. 7 is in the unit's place. The middle digit can be any one of the 10 digits from 0 to 9. The digit in hundred's place can be any one of the 9 digits from 1 to 9. Therefore, by the fundamental principle of counting, there are $10 \times 9 = 90$ numbers between 99 and 1000 having 7 in the unit's place.
- Total number of 3 digit numbers having atleast one of their digits as 7 = (Total numbers of three digit numbers) – (Total number of 3 digit numbers in which 7 does not appear at all).

$$\begin{aligned} &= (9 \times 10 \times 10) - (8 \times 9 \times 9) \\ &= 900 - 648 = 252. \end{aligned}$$

Example 3 In how many ways can this diagram be coloured subject to the following two conditions?

- Each of the smaller triangle is to be painted with one of three colours: red, blue or green.
- No two adjacent regions have the same colour.



Solution These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next paint the remaining 3 triangles, with any one of the remaining two colours.

By the fundamental principle of counting, this can be done in $3 \times 2 \times 2 \times 2 = 24$ ways.

Example 4 In how many ways can 5 children be arranged in a line such that (i) two particular children of them are always together (ii) two particular children of them are never together.

Solution

- (i) We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in $4! = 24$ ways. Again two particular children taken together can be arranged in two ways. Therefore, there are $24 \times 2 = 48$ total ways of arrangement.
- (ii) Among the $5! = 120$ permutations of 5 children, there are 48 in which two children are together. In the remaining $120 - 48 = 72$ permutations, two particular children are never together.

Example 5 If all permutations of the letters of the word AGAIN are arranged in the order as in a dictionary. What is the 49th word?

Solution Starting with letter A, and arranging the other four letters, there are $4! = 24$ words. These are the first 24 words. Then starting with G, and arranging A, A, I and N

in different ways, there are $\frac{4!}{2!1!1!} = 12$ words. Next the 37th word starts with I.

There are again 12 words starting with I. This accounts up to the 48th word. The 49th word is NAAGI.

Example 6 In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.

Solution First we take books of a particular subject as one unit. Thus there are 4 units which can be arranged in $4! = 24$ ways. Now in each of arrangements, mathematics books can be arranged in $3!$ ways, history books in $4!$ ways, chemistry books in $3!$ ways and biology books in $2!$ ways. Thus the total number of ways = $4! \times 3! \times 4! \times 3! \times 2! = 41472$.

Example 7 A student has to answer 10 questions, choosing atleast 4 from each of Parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

Solution The possibilities are:

- 4 from Part A and 6 from Part B
- or 5 from Part A and 5 from Part B
- or 6 from Part A and 4 from Part B.

Therefore, the required number of ways is

$$\begin{aligned} {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4 \\ = 105 + 126 + 35 = 266. \end{aligned}$$

Long Answer Type

Example 8 Suppose m men and n women are to be seated in a row so that no two women sit together. If $m > n$, show that the number of ways in which they can be seated is

$$\frac{m!(m+1)!}{(m-n+1)!}$$

Solution Let the men take their seats first. They can be seated in mP_m ways as shown in the following figure

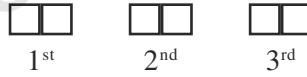


From the above figure, we observe, that there are $(m+1)$ places for n women. It is given that $m > n$ and no two women can sit together. Therefore, n women can take their seats ${}^{(m+1)}P_n$ ways and hence the total number of ways so that no two women sit together is

$$({}^mP_m) \times ({}^{m+1}P_n) = \frac{m!(m+1)!}{(m-n+1)!}$$

Example 9 Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Solution Let us denote married couples by S_1, S_2, S_3 , where each couple is considered to be a single unit as shown in the following figure:



Then the number of ways in which spouses can be seated next to each other is $3! = 6$ ways.

Again each couple can be seated in $2!$ ways. Thus the total number of seating arrangement so that spouses sit next to each other $= 3! \times 2! \times 2! \times 2! = 48$.

Again, if three ladies sit together, then necessarily three men must sit together. Thus, ladies and men can be arranged altogether among themselves in $2!$ ways. Therefore, the total number of ways where ladies sit together is $3! \times 3! \times 2! = 144$.

Example 10 In a small village, there are 87 families, of which 52 families have atmost 2 children. In a rural development programme 20 families are to be chosen for assistance, of which atleast 18 families must have at most 2 children. In how many ways can the choice be made?

Solution It is given that out of 87 families, 52 families have at most 2 children so other 35 families are of other type. According to the question, for rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have atmost 2 children. Thus, the following are the number of possible choices:

$${}^{52}C_{18} \times {}^{35}C_2 \text{ (18 families having atmost 2 children and 2 selected from other type of families)}$$

$${}^{52}C_{19} \times {}^{35}C_1 \text{ (19 families having at most 2 children and 1 selected from other type of families)}$$

$${}^{52}C_{20} \quad (\text{All selected 20 families having atmost 2 children})$$

Hence, the total number of possible choices is

$${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$$

Example 11 A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

Solution Let us make the following cases:

Case (i) Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So the number of possible choices is ${}^6C_1 = 6$.

Case (ii) Boy does not borrow Mathematics Part II, then the number of possible choices is ${}^7C_3 = 35$.

Hence, the total number of possible choices is $35 + 6 = 41$.

Example 12 Find the number of permutations of n different things taken r at a time such that two specific things occur together.

Solution A bundle of 2 specific things can be put in r places in $(r - 1)$ ways (Why?) and 2 things in the bundle can be arranged themselves into $|2$ ways. Now $(n - 2)$ things will be arranged in $(r - 2)$ places in ${}^{n-2}P_{r-2}$ ways.

Thus, using the fundamental principle of counting, the required numer of permutations will be $|2 \cdot (r - 1) \cdot {}^{n-2}P_{r-2}$.

Objective Type Questions

Choose the correct answer out of four options given against each of the following Examples (M.C.Q.).

Example 13 There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?

- (A) 72 (B) 144 (C) 14 (D) 19

Solution (A) is the correct answer. In the following figure:



there are 4 bus routes from A to B and 3 routes from B to C. Therefore, there are $4 \times 3 = 12$ ways to go from A to C. It is round trip so the man will travel back from C to A via B. It is restricted that man can not use same bus routes from C to B and B to A more than once. Thus, there are $2 \times 3 = 6$ routes for return journey. Therefore, the required number of ways = $12 \times 6 = 72$.

Example 14 In how many ways a committee consisting of 3 men and 2 women, can be chosen from 7 men and 5 women?

- (A) 45 (B) 350 (C) 4200 (D) 230

Solution (B) is the correct choice. Out of 7 men, 3 men can be chosen in $7C_3$ ways and out of 5 women, 2 women can be chosen in $5C_2$ ways. Hence, the committee can be chosen in $7C_3 \times 5C_2 = 350$ ways.

Example 15 All the letters of the word ‘EAMCOT’ are arranged in different possible ways. The number of such arrangements in which no two vowels are adjacent to each other is

- (A) 360 (B) 144 (C) 72 (D) 54

Solution (B) is the correct choice. We note that there are 3 consonants and 3 vowels E, A and O. Since no two vowels have to be together, the possible choice for vowels are the places marked as ‘X’. X M X C X T X, these vowels can be arranged in 4P_3 ways. 3 consonants can be arranged in 3P_3 ways. Hence, the required number of ways = $3! \times {}^4P_3 = 144$.

Example 16 Ten different letters of alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have atleast one letter repeated is

- (A) 69760 (B) 30240 (C) 99748 (D) 99784

Solution (A) is correct choice. Number of 5 letters words (with the condition that a letter can be repeated) = 10^5 . Again number of words using 5 different letters is ${}^{10}P_5$. Therefore, required number of letters

$$\begin{aligned}&= \text{Total number of words} - \text{Total number of words in which no letter is repeated} \\&= 10^5 - {}^{10}P_5 = 69760.\end{aligned}$$

Example 17 The number of signals that can be sent by 6 flags of different colours taking one or more at a time is

- (A) 63 (B) 1956 (C) 720 (D) 21

Solution The correct answer is B.

Number of signals using one flag = ${}^6P_1 = 6$

Number of signals using two flags = ${}^6P_2 = 30$

Number of signals using three flags = ${}^6P_3 = 120$

Number of signals using four flags = ${}^6P_4 = 360$

Number of signals using five flags = ${}^6P_5 = 720$

Number of signals using all six flags = ${}^6P_6 = 720$

Therefore, the total number of signals using one or more flags at a time is

$$6 + 30 + 120 + 360 + 720 + 720 = 1956 \text{ (Using addition principle).}$$

Example 18 In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answer correct is

- (A) 11 (B) 12 (C) 27 (D) 63

Solution The correct choice is (D). There are three multiple choice question, each has four possible answers. Therefore, the total number of possible answers will be $4 \times 4 \times 4 = 64$. Out of these possible answer only one will be correct and hence the number of ways in which a student can fail to get correct answer is $64 - 1 = 63$.

Example 19 The straight lines l_1 , l_2 and l_3 are parallel and lie in the same plane. A total numbers of m points are taken on l_1 ; n points on l_2 , k points on l_3 . The maximum number of triangles formed with vertices at these points are

- | | |
|-----------------------------------|---|
| (A) ${}^{(m+n+k)}C_3$ | (B) ${}^{(m+n+k)}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$ |
| (C) ${}^mC_3 + {}^nC_3 + {}^kC_3$ | (D) ${}^mC_3 \times {}^nC_3 \times {}^kC_3$ |

Solution (B) is the correct answer. Here the total number of points are $(m + n + k)$ which must give ${}^{(m+n+k)}C_3$ number of triangles but m points on l_1 taking 3 points at a time gives mC_3 combinations which produce no triangle. Similarly, nC_3 and kC_3

number of triangles can not be formed. Therefore, the required number of triangles is $(m + n + k)C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$.

7.3 EXERCISE

Short Answer Type

- Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.
[Hint: 2 women occupy the chair, from 1 to 4 in 4P_2 ways and 3 men occupy the remaining chairs in 6P_3 ways.]
- If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT ?
[Hint: In each case number of words beginning with A, C, H, I is 5!]
- A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.
- Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.
[Hint: Number of straight lines = ${}^{18}C_2 - {}^5C_2 + 1$.]
- We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can selections be made?
- How many committee of five persons with a chairperson can be selected from 12 persons.
[Hint: Chairman can be selected in 12 ways and remaining in ${}^{11}C_4$.]
- How many automobile license plates can be made if each plate contains two different letters followed by three different digits ?
- A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.
- Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.
- Find the number of different words that can be formed from the letters of the word ‘TRIANGLE’ so that no vowels are together.
- Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

- 12.** There are 10 persons named $P_1, P_2, P_3, \dots, P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.
[Hint: Required number of arrangement = ${}^7C_4 \times 5!$]
- 13.** There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated.
[Hint: Required number = $2^{10} - 1$].
- 14.** A box contains two white, three black and four red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw.
[Hint: Required number of ways = ${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_2 + {}^3C_3$]
- 15.** If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find $'C_2$.
[Hint: Form equation using $\frac{{}^nC_r}{{}^nC_{r+1}}$ and $\frac{{}^nC_r}{{}^nC_{r-1}}$ to find the value of r .]
- 16.** Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.
[Hint: Besides 4 digit integers greater than 7000, five digit integers are always greater than 7000.]
- 17.** If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?
- 18.** In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?
- 19.** In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.
- 20.** A convex polygon has 44 diagonals. Find the number of its sides.
[Hint: Polygon of n sides has $({}^nC_2 - n)$ number of diagonals.]

Long Answer Type Questions

- 21.** 18 mice were placed in two experimental groups and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?
- 22.** A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if (a) they can be of any colour (b) two must be white and two red and (c) they must all be of the same colour.

- 23.** In how many ways can a football team of 11 players be selected from 16 players?
 How many of them will
 (i) include 2 particular players?
 (ii) exclude 2 particular players?
- 24.** A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?
- 25.** A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
 (i) no girls
 (ii) at least one boy and one girl
 (iii) at least three girls.

Objective Type Questions

Choose the correct answer out of the given four options against each of the Exercises from 26 to 40 (M.C.Q.).

- 26.** If ${}^nC_{12} = {}^nC_8$, then n is equal to
 (A) 20 (B) 12 (C) 6 (D) 30
- 27.** The number of possible outcomes when a coin is tossed 6 times is
 (A) 36 (B) 64 (C) 12 (D) 32
- 28.** The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is
 (A) 120 (B) 96 (C) 24 (D) 100
- 29.** The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is
 (A) 432 (B) 108 (C) 36 (D) 18
- 30.** Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
 (A) 60 (B) 120 (C) 7200 (D) 720
- 31.** A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is
 (A) 216 (B) 600 (C) 240 (D) 3125
- [**Hint:** 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]

- 32.** Every body in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is
 (A) 11 (B) 12 (C) 13 (D) 14
- 33.** The number of triangles that are formed by choosing the vertices from a set of 12 points, seven of which lie on the same line is
 (A) 105 (B) 15 (C) 175 (D) 185
- 34.** The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
 (A) 6 (B) 18 (C) 12 (D) 9
- 35.** The number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 4 of them is
 (A) ${}^{16}C_{11}$ (B) ${}^{16}C_5$ (C) ${}^{16}C_9$ (D) ${}^{20}C_9$
- 36.** The number of 5-digit telephone numbers having atleast one of their digits repeated is
 (A) 90,000 (B) 10,000 (C) 30,240 (D) 69,760
- 37.** The number of ways in which we can choose a committee from four men and six women so that the committee includes at least two men and exactly twice as many women as men is
 (A) 94 (B) 126 (C) 128 (D) None
- 38.** The total number of 9 digit numbers which have all different digits is
 (A) $10!$ (B) $9!$ (C) $9 \times 9!$ (D) $10 \times 10!$
- 39.** The number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place is
 (A) 1440 (B) 144 (C) 7! (D) ${}^4C_4 \times {}^3C_3$
- 40.** Given 5 different green dyes, four different blue dyes and three different red dyes, the number of combinations of dyes which can be chosen taking at least one green and one blue dye is
 (A) 3600 (B) 3720 (C) 3800 (D) 3600
[Hint: Possible numbers of choosing or not choosing 5 green dyes, 4 blue dyes and 3 red dyes are 2^5 , 2^4 and 2^3 , respectively.]
- Fill in the Blanks in the Exercises 41 to 50.
- 41.** If ${}^nP_r = 840$, ${}^nC_r = 35$, then $r = \underline{\hspace{2cm}}$.
- 42.** ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 = \underline{\hspace{2cm}}$.
- 43.** The number of permutations of n different objects, taken r at a time, when repetitions are allowed, is $\underline{\hspace{2cm}}$.

44. The number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together is _____.

[Hint: Number of ways of arranging 6 consonants of which two are alike is $\frac{6!}{2!}$
and number of ways of arranging vowels = ${}^7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$.]

45. Three balls are drawn from a bag containing 5 red, 4 white and 3 black balls. The number of ways in which this can be done if at least 2 are red is _____.
46. The number of six-digit numbers, all digits of which are odd is _____.
47. In a football championship, 153 matches were played. Every two teams played one match with each other. The number of teams, participating in the championship is _____.
48. The total number of ways in which six '+' and four '-' signs can be arranged in a line such that no two signs '-' occur together is _____.

49. A committee of 6 is to be chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. In how many different ways can this be done if two particular women refuse to serve on the same committee.

[Hint: At least 3 men and 2 women: The number of ways = ${}^{10}C_3 \times {}^7C_3 + {}^{10}C_4 \times {}^7C_2$.
For 2 particular women to be always there: the number of ways = ${}^{10}C_4 + {}^{10}C_3 \times {}^5C_1$.
The total number of committees when two particular women are never together
= Total – together.]

50. A box contains 2 white balls, 3 black balls and 4 red balls. The number of ways three balls be drawn from the box if at least one black ball is to be included in the draw is _____.

State whether the statements in Exercises from 51 to 59 True or False? Also give justification.

51. There are 12 points in a plane of which 5 points are collinear, then the number of lines obtained by joining these points in pairs is ${}^{12}C_2 - {}^5C_2$.
52. Three letters can be posted in five letterboxes in 3^5 ways.
53. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^mP_m$.
54. In a steamer there are stalls for 12 animals, and there are horses, cows and calves (not less than 12 each) ready to be shipped. They can be loaded in 3^{12} ways.
55. If some or all of n objects are taken at a time, the number of combinations is $2^n - 1$.

- 56.** There will be only 24 selections containing at least one red ball out of a bag containing 4 red and 5 black balls. It is being given that the balls of the same colour are identical.
- 57.** Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. The number of ways in which the seating arrangements can be made is $\frac{11!}{5!6!}(9!)(9!)$.

[**Hint:** After sending 4 on one side and 3 on the other side, we have to select out of 11; 5 on one side and 6 on the other. Now there are 9 on each side of the long table and each can be arranged in 9! ways.]

- 58.** A candidate is required to answer 7 questions out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. He can choose the seven questions in 650 ways.
- 59.** To fill 12 vacancies there are 25 candidates of which 5 are from scheduled castes. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, the number of ways in which the selection can be made is ${}^5C_3 \times {}^{20}C_9$.

In each of the Exercises from 60 to 64 match each item given under the column C₁ to its correct answer given under the column C₂.

- 60.** There are 3 books on Mathematics, 4 on Physics and 5 on English. How many different collections can be made such that each collection consists of :

- | C ₁ | C ₂ |
|---|----------------|
| (a) One book of each subject; | (i) 3968 |
| (b) At least one book of each subject : | (ii) 60 |
| (c) At least one book of English: | (iii) 3255 |

- 61.** Five boys and five girls form a line. Find the number of ways of making the seating arrangement under the following condition:

- | C ₁ | C ₂ |
|--|-------------------------|
| (a) Boys and girls alternate: | (i) $5! \times 6!$ |
| (b) No two girls sit together : | (ii) $10! - 5!6!$ |
| (c) All the girls sit together | (iii) $(5!)^2 + (5!)^2$ |
| (d) All the girls are never together : | (iv) $2!5!5!$ |

- 62.** There are 10 professors and 20 lecturers out of whom a committee of 2 professors and 3 lecturer is to be formed. Find :

C₁

- (a) In how many ways committee : can be formed
- (b) In how many ways a particular : professor is included
- (c) In how many ways a particular : lecturer is included
- (d) In how many ways a particular : lecturer is excluded

C₂

- (i) ${}^{10}C_2 \times {}^{19}C_3$
- (ii) ${}^{10}C_2 \times {}^{19}C_2$
- (iii) ${}^9C_1 \times {}^{20}C_3$
- (iv) ${}^{10}C_2 \times {}^{20}C_3$

- 63.** Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find

C₁

- (a) how many numbers are formed?
- (b) how many numbers are exactly divisible by 2?
- (c) how many numbers are exactly divisible by 25?
- (d) how many of these are exactly divisible by 4?

C₂

- (i) 840
- (ii) 200
- (iii) 360
- (iv) 40

- 64.** How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

C₁

- (a) 4 letters are used at a time
- (b) All letters are used at a time
- (c) All letters are used but the first is a vowel

C₂

- (i) 720
- (ii) 240
- (iii) 360

