Mathematical Foundations for Computer Applications

Problems- Logically Equivalent

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Problems- Prove the Logically Equivalent.

$$p \to q \equiv \neg p \vee q$$

$$p \to q \equiv \neg q \to \neg p$$

$$p \vee q \equiv \neg p \to q$$

$$p \wedge q \equiv \neg(p \to \neg q)$$

$$\neg(p \to q) \equiv p \wedge \neg q$$

$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$

$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$

$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$

$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Logical Equivalences Involving Conditional / Biconditional Statements.

Problems-without TT

1. Show that $\neg(p \to q)$ and $p \land \neg q$ are logically equivalent.

$\neg(p \to q) \equiv \neg(\neg p \lor q)$ use $p \to q \equiv \neg p \lor q$

$\equiv \neg(\neg p) \land \neg q$ by the De Morgan's law

$\equiv p \land \neg q$ by the double negation law

Problems

2. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	De Morgan's law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	De Morgan's law
	$\equiv \neg p \wedge (p \vee \neg q)$	double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	$\neg p \wedge p \equiv \mathbf{F}$
	$\equiv \neg p \wedge \neg q$	identity law for **F**

Problems

3. Show that $(p \land q) \to (p \lor q)$ is a tautology.

$(p \land q) \to (p \lor q) \equiv \neg(p \land q) \lor (p \lor q)\,)$ *use* $p \to q \equiv \neg p \lor q$

$\equiv (\neg p \lor \neg q) \lor (p \lor q)$ De Morgan's law

$\equiv (\neg p \lor \neg q) \lor p) \lor ((\neg p \lor \neg q) \lor q)$ *Distributive* Law

$\equiv ((\neg p \lor p) \lor \neg q) \lor (\neg p \lor (\neg q \lor q))$

(Associative & commutative)

$\equiv \mathbf{T} \lor \mathbf{T}$

$\equiv \mathbf{T}$

THANK YOU

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