

Unit 3: Fundamentals of Logic and Proofs(QB)

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

- a) Boston is the capital of Massachusetts.
- b) Miami is the capital of Florida.
- c) $2 + 3 = 5$.
- d) $5 + 7 = 10$.
- e) $x + 2 = 11$.
- f) Answer this question.

2. Which of these are propositions? What are the truth values of those that are propositions?

- a) Do not pass go.
- b) What time is it?
- c) There are no black flies in Maine.
- d) $4 + x = 5$.
- e) The moon is made of green cheese.
- f) $2n \geq 100$.

3. What is the negation of each of these propositions?

- a) Mei has an MP3 player.
- b) There is no pollution in New Jersey.
- c) $2 + 1 = 3$.
- d) The summer in Maine is hot and sunny.

4. What is the negation of each of these propositions?

- a) Jennifer and Teja are friends.
- b) There are 13 items in a baker's dozen.
- c) Abby sent more than 100 text messages every day.
- d) 121 is a perfect square.

5. What is the negation of each of these propositions?

- a) Steve has more than 100 GB free disk space on his laptop.
- b) Zach blocks e-mails and texts from Jennifer.
- c) $7 \cdot 11 \cdot 13 = 999$.
- d) Diane rode her bicycle 100 miles on Sunday.

6. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

- a) Smart phone B has the most RAM of these three smartphones.
- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

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- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

7. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

- a) Quixote Media had the largest annual revenue.
- b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
- c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
- d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
- e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

8. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

- | | | |
|---------------------------|-------------------------------|--------------------------------|
| a) $\neg p$ | b) $p \vee q$ | c) $p \rightarrow q$ |
| d) $p \wedge q$ | e) $p \leftrightarrow q$ | f) $\neg p \rightarrow \neg q$ |
| g) $\neg p \wedge \neg q$ | h) $\neg p \vee (p \wedge q)$ | |

9. Let p and q be the propositions “Swimming at the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.

- | | | |
|-------------------------------|------------------------------------|--------------------------------|
| a) $\neg q$ | b) $p \wedge q$ | c) $\neg p \vee q$ |
| d) $p \rightarrow \neg q$ | e) $\neg q \rightarrow p$ | f) $\neg p \rightarrow \neg q$ |
| g) $p \leftrightarrow \neg q$ | h) $\neg p \wedge (p \vee \neg q)$ | |

10. Let p and q be the propositions “The election is decided” and “The votes have been counted,” respectively. Express each of these compound propositions as an English sentence.

- | | |
|--------------------------------|------------------------------------|
| a) $\neg p$ | b) $p \vee q$ |
| c) $\neg p \wedge q$ | d) $q \rightarrow p$ |
| e) $\neg q \rightarrow \neg p$ | f) $\neg p \rightarrow \neg q$ |
| g) $p \leftrightarrow q$ | h) $\neg q \vee (\neg p \wedge q)$ |

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11. Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

- a) It is below freezing and snowing.
- b) It is below freezing but not snowing.
- c) It is not below freezing and it is not snowing.
- d) It is either snowing or below freezing (or both).
- e) If it is below freezing, it is also snowing.
- f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
- g) That it is below freezing is necessary and sufficient for it to be snowing.

12. Let p , q , and r be the propositions

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

Express each of these propositions as an English sentence.

- a) $p \rightarrow q$
- b) $\neg q \leftrightarrow r$
- c) $q \rightarrow \neg r$
- d) $p \vee q \vee r$
- e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
- f) $(p \wedge q) \vee (\neg q \wedge r)$

13. Let p and q be the propositions

p : You drive over 65 miles per hour.

q : You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

- a) You do not drive over 65 miles per hour.
- b) You drive over 65 miles per hour, but you do not get a speeding ticket.
- c) You will get a speeding ticket if you drive over 65 miles per hour.
- d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
- e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
- f) You get a speeding ticket, but you do not drive over 65 miles per hour.
- g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

14. Let p , q , and r be the propositions

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p , q , and r and logical connectives (including negations).

- a) You get an A in this class, but you do not do every exercise in this book.
- b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- c) To get an A in this class, it is necessary for you to get an A on the final.

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- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

15. Let p , q , and r be the propositions

p : Grizzly bears have been seen in the area.

q : Hiking is safe on the trail.

r : Berries are ripe along the trail.

Write these propositions using p , q , and r and logical connectives (including negations).

- a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
- b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
- c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
- d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
- e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
- f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

16. Determine whether these biconditionals are true or false.

- a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
- b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
- c) $1 + 1 = 3$ if and only if monkeys can fly.
- d) $0 > 1$ if and only if $2 > 1$.

17. Determine whether each of these conditional statements is true or false.

- a) If $1 + 1 = 2$, then $2 + 2 = 5$.
- b) If $1 + 1 = 3$, then $2 + 2 = 4$.
- c) If $1 + 1 = 3$, then $2 + 2 = 5$.
- d) If monkeys can fly, then $1 + 1 = 3$.

18. Determine whether each of these conditional statements is true or false.

- a) If $1 + 1 = 3$, then unicorns exist.
- b) If $1 + 1 = 3$, then dogs can fly.
- c) If $1 + 1 = 2$, then dogs can fly.
- d) If $2 + 2 = 4$, then $1 + 2 = 3$.

19. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

- a) Coffee or tea comes with dinner.
- b) A password must have at least three digits or be at least eight characters long.

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- c) The prerequisite for the course is a course in number theory or a course in cryptography.
d) You can pay using U.S. dollars or euros.

20. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

- a) Experience with C++ or Java is required.
b) Lunch includes soup or salad.
c) To enter the country you need a passport or a voter registration card.
d) Publish or perish.

21. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

- a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.
c) Dinner for two includes two items from column A or three items from column B.
d) School is closed if more than 2 feet of snow falls or if the wind chill is below -100.

22. Write each of these statements in the form "if p , then q " in English.

- a) It is necessary to wash the boss's car to get promoted.
b) Winds from the south imply a spring thaw.
c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
d) Willy gets caught whenever he cheats.
e) You can access the website only if you pay a subscription fee.
f) Getting elected follows from knowing the right people.
g) Carol gets seasick whenever she is on a boat.

23. Write each of these statements in the form "if p , then q " in English.

- a) It snows whenever the wind blows from the northeast.
b) The apple trees will bloom if it stays warm for a week.
c) That the Pistons win the championship implies that they beat the Lakers.
d) It is necessary to walk 8 miles to get to the top of Long's Peak.
e) To get tenure as a professor, it is sufficient to be world famous.
f) If you drive more than 400 miles, you will need to buy gasoline.
g) Your guarantee is good only if you bought your CD player less than 90 days ago.
h) Jan will go swimming unless the water is too cold.

24. Write each of these statements in the form "if p , then q " in English.

- a) I will remember to send you the address only if you send me an e-mail message.
b) To be a citizen of this country, it is sufficient that you were born in the United States.
c) If you keep your textbook, it will be a useful reference in your future courses.
d) The Red Wings will win the Stanley Cup if their goalie plays well.
e) That you get the job implies that you had the best credentials.
f) The beach erodes whenever there is a storm.

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- g)** It is necessary to have a valid password to log on to the server.
h) You will reach the summit unless you begin your climb too late.

25. Write each of these propositions in the form “ p if and only if q ” in English.

- a)** If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.
b) For you to win the contest it is necessary and sufficient that you have the only winning ticket.
c) You get promoted only if you have connections, and you have connections only if you get promoted.
d) If you watch television your mind will decay, and conversely.
e) The trains run late on exactly those days when I take it.

26. Write each of these propositions in the form “ p if and only if q ” in English.

- a)** For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.
b) If you read the newspaper every day, you will be informed, and conversely.
c) It rains if it is a weekend day, and it is a weekend day if it rains.
d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

27. State the converse, contrapositive, and inverse of each of these conditional statements.

- a)** If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

28. State the converse, contrapositive, and inverse of each of these conditional statements.

- a)** If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

29. How many rows appear in a truth table for each of these compound propositions?

- a)** $p \rightarrow \neg p$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$
c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

30. How many rows appear in a truth table for each of these compound propositions?

- a)** $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$
b) $(p \vee \neg t) \wedge (p \vee \neg s)$
c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$
d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

31. Construct a truth table for each of these compound propositions.

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- a) $p \wedge \neg p$
- b) $p \vee \neg p$
- c) $(p \vee \neg q) \rightarrow q$
- d) $(p \vee q) \rightarrow (p \wedge q)$
- e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

32. Construct a truth table for each of these compound propositions.

- a) $p \rightarrow \neg p$
- b) $p \leftrightarrow \neg p$
- c) $p \oplus (p \vee q)$
- d) $(p \wedge q) \rightarrow (p \vee q)$
- e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
- f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

33. Construct a truth table for each of these compound propositions.

- a) $(p \vee q) \rightarrow (p \oplus q)$
- b) $(p \oplus q) \rightarrow (p \wedge q)$
- c) $(p \vee q) \oplus (p \wedge q)$
- d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
- e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
- f) $(p \oplus q) \rightarrow (p \oplus \neg q)$

34. Construct a truth table for each of these compound propositions.

- a) $p \oplus p$
- b) $p \oplus \neg p$
- c) $p \oplus \neg q$
- d) $\neg p \oplus \neg q$
- e) $(p \oplus q) \vee (p \oplus \neg q)$
- f) $(p \oplus q) \wedge (p \oplus \neg q)$

35. Construct a truth table for each of these compound propositions.

- a) $p \rightarrow \neg q$
- b) $\neg p \leftrightarrow q$
- c) $(p \rightarrow q) \vee (\neg p \rightarrow q)$
- d) $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
- e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
- f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

36. Construct a truth table for each of these compound propositions.

- a) $(p \vee q) \vee r$
- b) $(p \vee q) \wedge r$
- c) $(p \wedge q) \vee r$
- d) $(p \wedge q) \wedge r$
- e) $(p \vee q) \wedge \neg r$
- f) $(p \wedge q) \vee \neg r$

37. Construct a truth table for each of these compound propositions.

- a) $p \rightarrow (\neg q \vee r)$
- b) $\neg p \rightarrow (q \rightarrow r)$
- c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$
- d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

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e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

38. Construct a truth table for $((p \rightarrow q) \rightarrow r) \rightarrow s$.

39. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

40. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001

b) 1111 0000, 1010 1010

c) 00 0111 0001, 10 0100 1000

d) 11 1111 1111, 00 0000 0000

41. Evaluate each of these expressions.

a) 1 1000 \wedge (0 1011 \vee 1 1011)

b) (0 1111 \wedge 1 0101) \vee 0 1000

c) (0 1010 \oplus 1 1011) \oplus 0 1000

d) (1 1011 \vee 0 1010) \wedge (1 0001 \vee 1 1011)

1. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

b) Yoshiko knows Java and calculus.

c) James is young and strong.

d) Rita will move to Oregon or Washington.

2. Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

c) $[p \wedge (p \rightarrow q)] \rightarrow q$

d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

3. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

4. Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

5. Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

6. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

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7. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.
8. Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.
9. Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
10. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.
11. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.
12. Show that $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are not logically equivalent.

Predicate Calculus

1. Let $P(x)$ denote the statement “ $x \leq 4$.” What are these truth values?
 - a) $P(0)$
 - b) $P(4)$
 - c) $P(6)$
2. Let $P(x)$ be the statement “the word x contains the letter a .” What are these truth values?
 - a) $P(\text{orange})$
 - b) $P(\text{lemon})$
 - c) $P(\text{true})$
 - d) $P(\text{false})$
3. Let $Q(x, y)$ denote the statement “ x is the capital of y .” What are these truth values?
 - a) $Q(\text{Denver, Colorado})$
 - b) $Q(\text{Detroit, Michigan})$
 - c) $Q(\text{Massachusetts, Boston})$
 - d) $Q(\text{NewYork, NewYork})$
4. State the value of x after the statement **if** $P(x)$ **then** $x := 1$ is executed, where $P(x)$ is the statement “ $x > 1$,” if the value of x when this statement is reached is
 - a) $x = 0$.
 - b) $x = 1$.
 - c) $x = 2$.
5. Let $P(x)$ be the statement “ x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.
 - a) $\exists x P(x)$
 - b) $\forall x P(x)$
 - c) $\exists x \neg P(x)$
 - d) $\forall x \neg P(x)$

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6. Let $N(x)$ be the statement “ x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.

- a) $\exists xN(x)$
- b) $\forall xN(x)$
- c) $\neg \exists xN(x)$
- d) $\exists x \neg N(x)$
- e) $\neg \forall xN(x)$
- f) $\forall x \neg N(x)$

7. Translate these statements into English, where $C(x)$ is “ x is a comedian” and $F(x)$ is “ x is funny” and the domain consists of all people.

- a) $\forall x(C(x) \rightarrow F(x))$
- b) $\forall x(C(x) \wedge F(x))$
- c) $\exists x(C(x) \rightarrow F(x))$
- d) $\exists x(C(x) \wedge F(x))$

8. Translate these statements into English, where $R(x)$ is “ x is a rabbit” and $H(x)$ is “ x hops” and the domain consists of all animals.

- a) $\forall x(R(x) \rightarrow H(x))$
- b) $\forall x(R(x) \wedge H(x))$
- c) $\exists x(R(x) \rightarrow H(x))$
- d) $\exists x(R(x) \wedge H(x))$

9. Let $P(x)$ be the statement “ x can speak Russian” and let $Q(x)$ be the statement “ x knows the computer language C++.” Express each of these sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

- a) There is a student at your school who can speak Russian and who knows C++.
- b) There is a student at your school who can speak Russian but who doesn't know C++.
- c) Every student at your school either can speak Russian or knows C++.
- d) No student at your school can speak Russian or knows C++.

10. Let $C(x)$ be the statement “ x has a cat,” let $D(x)$ be the statement “ x has a dog,” and let $F(x)$ be the statement “ x has a ferret.” Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives.

Let the domain consist of all students in your class.

- a) A student in your class has a cat, a dog, and a ferret.
- b) All students in your class have a cat, a dog, or a ferret.
- c) Some student in your class has a cat and a ferret, but not a dog.
- d) No student in your class has a cat, a dog, and a ferret.
- e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

11. Let $P(x)$ be the statement “ $x = x_2$.” If the domain consists of the integers, what are these truth values?

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- a) $P(0)$
d) $P(-1)$

- b) $P(1)$ c) $P(2)$
e) $\exists x P(x)$ f) $\forall x P(x)$

12. Let $Q(x)$ be the statement " $x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

- a) $Q(0)$ b) $Q(-1)$ c) $Q(1)$
d) $\exists x Q(x)$ e) $\forall x Q(x)$ f) $\exists x \neg Q(x)$
g) $\forall x \neg Q(x)$

13. Determine the truth value of each of these statements if the domain consists of all integers.

- a) $\forall n (n + 1 > n)$ b) $\exists n (2n = 3n)$
c) $\exists n (n = -n)$ d) $\forall n (3n \leq 4n)$

14. Determine the truth value of each of these statements if the domain consists of all real numbers.

- a) $\exists x (x^3 = -1)$ b) $\exists x (x^4 < x^2)$
c) $\forall x ((-x)^2 = x^2)$ d) $\forall x (2x > x)$

15. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

- a) $\forall n (n^2 \geq 0)$ b) $\exists n (n^2 = 2)$
c) $\forall n (n^2 \geq n)$ d) $\exists n (n^2 < 0)$

16. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

- a) $\exists x (x^2 = 2)$ b) $\exists x (x^2 = -1)$
c) $\forall x (x^2 + 2 \geq 1)$ d) $\forall x (x^2 = x)$

17. Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

- a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$ e) $\neg \exists x P(x)$ f) $\neg \forall x P(x)$

18. Suppose that the domain of the propositional function $P(x)$ consists of the integers -2, -1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.

- a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \neg P(x)$

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d) $\forall x \neg P(x)$

e) $\neg \exists x P(x)$

f) $\neg \forall x P(x)$