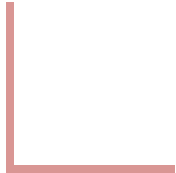


Mathematical Foundations for Computer Applications

Mathematical Induction

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Mathematical Foundations for Computer Applications

Mathematical Induction

- Mathematical Induction is a special way of proving things. It has only 2 steps:

Step 1. Show it is true for the **first one**

Step 2. Show that if **any one** is true then the **next one** is true

Then **all** are true

- In the world of numbers we say:

Step 1. Show it is true for first case, usually **$n=1$**

Step 2. Show that if **$n=k$** is true then **$n=k+1$** is also true

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Mathematical Induction--How to Do it ?

Step 1 is usually easy, we just have to prove it is true for **$n=1$** (**Basic Step**)

Step 2 is best done this way: (**Induction Step**)

Assume it is true for **$n=k$**

Prove it is true for **$n=k+1$**

(we can use the **$n=k$** case as a **fact**.)

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Mathematical Induction--Problems

1. Prove that $1 + 3 + 5 + \dots + (2n-1) = n^2$ using
Mathematical Induction .(Adding up Odd Numbers)

1. Show it is true for **$n=1$**

$$1 = 1^2 \text{ is True}$$

2. Assume it is true for **$n=k$**

$$1 + 3 + 5 + \dots + (2k-1) = k^2 \text{ is True}$$

Now, prove it is true for " $k+1$ "

$$1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = (k+1)^2 \quad ?$$

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Mathematical Induction--Problems

We know that $1 + 3 + 5 + \dots + (2k-1) = k^2$ (the assumption above), so we can do a replacement for all but the last term:

$$k^2 + (2(k+1)-1) = (k+1)^2$$

$$k^2 + 2k + 2 - 1 = k^2 + 2k + 1$$

$$k^2 + 2k + 1 = k^2 + 2k + 1$$

They are the same! So it is true.

So:

$$1 + 3 + 5 + \dots + (2(k+1)-1) = (k+1)^2 \text{ is True}$$

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Mathematical Induction--Problems

2. Adding up Cube Numbers

Prove that: $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

1. Show it is true for **n=1**

$$1^3 = \frac{1^2(1+1)^2}{4}$$

$1 = 1$ is True

2. Assume it is true for **n=k**

$1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$ is True (An assumption!)

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Mathematical Induction--Problems

Now, prove it is true for "k+1"

$$1^3 + 2^3 + 3^3 + \dots + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4} \quad ?$$

We know that $1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}$ (the assumption above), so we can do a replacement for all but the last term:

$$\frac{k^2(k+1)^2}{4} + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4}$$

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Mathematical Induction--Problems

Multiply all terms by 4:

$$k^2(k+1)^2 + 4(k+1)^3 = (k+1)^2(k+2)^2$$

All terms have a common factor $(k+1)^2$, so it can be cancelled:

$$k^2 + 4(k+1) = (k+2)^2$$

$k^2 + 4k + 4 = k^2 + 4k + 4$ They are the same! So it is true.

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \text{ is TRUE}$$



THANK YOU

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