Mathematical Foundations for Computer Applications

Mathematical Induction

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Mathematical Induction

- Mathematical Induction is a special way of proving things. It has only 2 steps:

Step 1. Show it is true for the **first one**

Step 2. Show that if **any one** is true then the **next one** is true

Then **all** are true

- In the world of numbers we say:

Step 1. Show it is true for first case, usually **n=1**

Step 2. Show that if **n=k** is true then **n=k+1** is also true

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Mathematical Induction--How to Do it ?

Step 1 is usually easy, we just have to prove it is true for **n=1** **(Basic Step)**

Step 2 is best done this way: **(Induction Step)**

Assume it is true for **n=k**

Prove it is true for **n=k+1**

(we can use the **n=k** case as a **fact**.)

Mathematical Induction--Problems

1. Prove that $1 + 3 + 5 + \dots + (2n-1) = n^2$ using Mathematical Induction .(Adding up Odd Numbers)

1.Show it is true for **n=1**

$$1 = 1^2 \text{ is True}$$

2. Assume it is true for **n=k**

$1 + 3 + 5 + \dots + (2k-1) = k^2$ is True

Now, prove it is true for "k+1"

$1 + 3 + 5 + \dots + (2k-1) + (2(k+1)-1) = (k+1)^2$ **?**

Mathematical Induction--Problems

We know that $\mathbf{1 + 3 + 5 + \ldots + (2k-1) = k^2}$ (the assumption above), so we can do a replacement for all but the last term:

$\mathbf{k^2} + (2(k+1)-1) = (k+1)^2$

$k^2 + 2k + 2 - 1 = k^2 + 2k+1$

$k^2 + 2k + 1 = k^2 + 2k + 1$

They are the same! So it is true.

So:

$1 + 3 + 5 + \ldots + (2(k+1)-1) = (k+1)^2$ is True

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Mathematical Induction--Problems

2. Adding up Cube Numbers

Prove that: $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

1. Show it is true for **n=1**

$$1^3 = \frac{1^2(1+1)^2}{4}$$

$1 = 1$ is True

2. Assume it is true for **n=k**

$1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$ is True (An assumption!)

Now, prove it is true for "k+1"

$$1^3 + 2^3 + 3^3 + \ldots + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4} \quad ?$$

We know that $1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$ (the assumption above), so we can do a replacement for all but the last term:

$$\frac{k^2(k+1)^2}{4} + (k + 1)^3 = \frac{(k+1)2(k+2)^2}{4}$$

Mathematical Induction--Problems

Multiply all terms by 4:

$k^2(k + 1)^2 + 4(k + 1)^3 = (k + 1)^2(k + 2)^2$

All terms have a common factor $(k + 1)^2$, so it can be cancelled:

$k^2 + 4(k + 1) = (k + 2)^2$

$k^2 + 4k + 4 = k^2 + 4k + 4$ They are the same! So it is true.

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$ is TRUE

THANK YOU

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