

Bayesian Shrinkage in High-Dimensional VAR Models: A Comparative Study

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Abstract

High-dimensional vector autoregressive (VAR) models offer a versatile framework for multivariate time series analysis, yet face critical challenges from over-parameterization and uncertain lag order. In this paper, we systematically compare three Bayesian shrinkage priors (horseshoe, lasso, and normal) and two frequentist regularization approaches (ridge and nonparametric shrinkage) under three carefully crafted simulation scenarios. These scenarios encompass (i) overfitting in a low-dimensional setting, (ii) sparse high-dimensional processes, and (iii) a combined scenario where both large dimension and overfitting complicate inference.

We evaluate each method in quality of parameter estimation (root mean squared error, coverage, and interval length) and out-of-sample forecasting (one-step-ahead forecast RMSE). Our findings show that local-global Bayesian methods, particularly the horseshoe, dominate in maintaining accurate coverage and minimizing parameter error, even when the model is heavily over-parameterized. Frequentist ridge often yields competitive point forecasts but underestimates uncertainty, leading to sub-nominal coverage. A real-data application using macroeconomic variables from Canada illustrates how these methods perform in practice, reinforcing the advantages of local-global priors in stabilizing inference when dimension or lag order is inflated.

1 Introduction

Vector autoregressive (VAR) models remain a cornerstone of multivariate time series analysis (Sims, 1980; Chan, 2020; Koop and Korobilis, 2013). A d -dimensional VAR(p) posits that an observed time series $\{\mathbf{y}_t\}_{t=1}^T$ satisfies a regression model where \mathbf{y}_t given p previous observations $\mathbf{y}_{t-p}, \dots, \mathbf{y}_{t-1}$ is modeled as

$$\mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_2 \mathbf{y}_{t-2} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t. \quad (1)$$

When each \mathbf{A}_i is a $d \times d$ coefficient matrix, and we assume that

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \Sigma_{\varepsilon}),$$

is normally distributed white noise, that is, $\boldsymbol{\varepsilon}_t$ has mean $\mathbf{0}$ and covariance Σ_{ε} and the $\boldsymbol{\varepsilon}_t$'s are uncorrelated over time. VAR models see extensive use in macroeconomic data analysis, where the assumption of correlated vector time series is natural; see, for instance, Stock and Watson (2002); Crump et al. (2021); Carriero et al. (2022); Zhou and Chan (2023) for discussions on the ongoing empirical suitability and methodological refinements of VAR structures in economics.

Standard VAR theory relies on the stationarity of the time series. If the characteristic polynomial of $(I_d - \mathbf{A}_1 z - \cdots - \mathbf{A}_p z^p)$ does not have all its roots outside the unit circle, the process may be nonstationary or cointegrated. In such cases, one typically either differences or applies error-correction forms to restore stationarity, or else fits a specialized variant (such as a vector error-correction model).

A single VAR(p) of dimension d has $d^2 p$ parameters, so with increasing d or p , the parameter space quickly becomes high dimensional (Bańbura et al., 2010; Koop, 2013; Korobilis and Pettenuzzo, 2019). Consequently, regularization (often referred to as “shrinkage” in the VAR literature) is crucial not only to prevent overfitting but also to control the high variability in estimated parameters, which can undermine model stability and predictive

accuracy (Stock and Watson, 2002; Huber and Feldkircher, 2017; Chan, 2021). In this context, *shrinkage* describes the penalization of numerous parameters toward zero or toward a simpler structure, so as to enforce parsimony. For VAR models, shrinkage is particularly needed when the number of time series variables (d) or the chosen lag order (p) is large relative to the available sample size, a common scenario in macroeconomic forecasting (Ba  nbara et al., 2010; Bai et al., 2022).

A variety of shrinkage techniques have been developed in both Bayesian and frequentist frameworks. Bayesian local-global priors, such as the horseshoe (Carvalho et al., 2010) and Bayesian lasso (Park and Casella, 2008), aggressively shrink small or irrelevant coefficients toward zero while allowing large signals to remain relatively unshrunk. The spike-and-slab prior (George and McCulloch, 1997) offers an alternative Bayesian framework, combining sparsity with flexibility in signal detection. Hierarchical shrinkage priors, like the horseshoe and its variants (Makalic and Schmidt, 2016; Pr  ser, 2021), adaptively adjust shrinkage intensity and have shown strong performance in time series applications (Pr  ser, 2021). Other recent Bayesian shrinkage methods in time series have been explored (Huber and Koop, 2023; Kowal et al., 2019). Frequentist approaches such as ridge regression (Doan et al., 1984) penalize large coefficients to prevent overfitting, while nonparametric shrinkage approaches can directly estimate an optimal shrinkage factor—often via James–Stein type estimators—and adjust the sample covariance-based estimates accordingly (Giannone et al., 2015; Del Negro and Giannoni, 2015).

Recent literature has refined global-local priors and explored flexible hierarchical frameworks tailored to VARs. Huber and Feldkircher (2019) propose an adaptive shrinkage scheme with a Normal-Gamma prior that learns the degree of shrinkage across coefficients, showing improved forecasts in large VAR systems. Bitto and Fr  hwirth-Schnatter (2019) apply a Horseshoe-type prior in time-varying parameter models, highlighting the ability of heavy-tailed priors to shrink aggressively while preserving large signals. In the realm of block-specific shrinkage, Aprigliano (2020) demonstrates that treating different coefficient groups with distinct shrinkage intensities can further boost forecast accuracy. Recent

computational strategies, such as variational approximations, also facilitate faster Bayesian inference for high-dimensional VARs, as discussed in Gefang et al. (2023).

Lasso-based methods (Tibshirani, 1996) and elastic net (Zou and Hastie, 2005) also appear in the high-dimensional VAR toolkit, providing sparse solutions when many coefficients are truly zero or near zero. Recent innovations, such as dynamic shrinkage models (Griffin and Brown, 2017) and time-varying parameter (TVP) VARs (Nakajima and West, 2011), add further flexibility for capturing structural breaks or changing relationships over time.

Beyond ridge and nonparametric shrinkage, recent research has proposed more flexible or structured penalties to handle the large parameter spaces typical of VAR models. Group lasso and hierarchical lag penalties can exploit the grouped structure of lagged predictors by shrinking or eliminating entire blocks of coefficients (Nicholson et al., 2017; Basu et al., 2019; Nicholson, 2020). Non-convex penalties such as the smoothly clipped absolute deviation (SCAD) and the minimax concave penalty (MCP) balance aggressive shrinkage of small coefficients with reduced bias for large signals (Song and Bickel, 2019; Chen and Chen, 2021). Adaptive lasso variants further improve selection consistency by penalizing large coefficients less, based on preliminary estimates (Kock and Callot, 2015). Lastly, time series-specific modifications—like rolling cross-validation or penalization that accounts for serial dependence (Nicholson et al., 2017; Medeiros et al., 2021)—help tailor these methods to dynamic settings, often improving stability and forecast performance in high-dimensional VARs.

In this paper, we compare five shrinkage approaches for high-dimensional VAR estimation: three Bayesian priors (horseshoe, lasso, and normal) and two frequentist estimators (ridge and nonparametric shrinkage). We evaluate each method’s handling of over-parameterization in both low- and high-dimensional settings, using root mean squared error, interval coverage, interval length, and one-step-ahead RMSE to assess parameter recovery and forecast performance.

We find that local-global priors—particularly the horseshoe—strike a strong balance

between parsimony and flexibility, delivering accurate estimation and consistent coverage even in heavily overfitted scenarios. While ridge regression frequently provides competitive point forecasts, it underestimates uncertainty when the parameter space grows large. Meanwhile, nonparametric shrinkage, though computationally efficient, suffers from undercoverage in complex models.

Application to Canadian macroeconomic time series illustrates how each method behaves with various lag choices and a relatively small sample. These empirical findings corroborate the simulation evidence: local-global priors, especially the horseshoe, remain robust even when the chosen lag order exceeds what is strictly necessary.

The remainder of the paper is structured as follows. Section 2 defines the VAR(p) framework and outlines both Bayesian and frequentist shrinkage estimators. Section 3 describes our simulation designs and metrics, Section 4 presents the simulation results, and Section 5 provides the real-data application to Canadian macroeconomic variables. Finally, we offer concluding remarks.

2 Bayesian and Frequentist Approaches to VAR(p)

We study a d -dimensional VAR(p) of the form

$$\mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad (2)$$

where each \mathbf{A}_i is a $d \times d$ coefficient matrix, and $\boldsymbol{\varepsilon}_t$ is a white-noise process following

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \Sigma_{\varepsilon}), \quad \text{Cov}(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_s) = \mathbf{0} \text{ for } t \neq s.$$

We treat each \mathbf{y}_t as a $d \times 1$ column vector.

Vectorizing the Coefficients. Let

$$\mathbf{B} = [\mathbf{A}_1 \ \mathbf{A}_2 \ \cdots \ \mathbf{A}_p] \in \mathbb{R}^{d \times (dp)},$$

i.e., the horizontal concatenation of the p coefficient matrices with elements β_j , $j = 1, \dots, d \times dp$. Then define the *lagged-regressor* vector

$$\mathbf{X}_t = \begin{pmatrix} \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \\ \vdots \\ \mathbf{y}_{t-p} \end{pmatrix} \in \mathbb{R}^{dp \times 1},$$

so that

$$\mathbf{B} \mathbf{X}_t \in \mathbb{R}^{d \times 1}.$$

The VAR(p) model in (1) can thus be written as

$$\mathbf{y}_t = \mathbf{B} \mathbf{X}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \Sigma_\varepsilon).$$

2.1 Bayesian Shrinkage Priors

In a fully Bayesian treatment, we specify priors for both the coefficient matrix and the error covariance. We gather the coefficients into a matrix $\mathbf{B} \in \mathbb{R}^{d \times (dp)}$, so that

$$\mathbf{y}_t \sim \mathcal{N}(\mathbf{X}_t \mathbf{B}', \Sigma_\varepsilon),$$

where \mathbf{X}_t is the row vector of the lagged responses at time t . To ensure Σ_ε is positive-definite, we use a Cholesky-factor parameterization:

$$\Sigma_\varepsilon = \mathbf{L} \mathbf{L}^\top, \quad \mathbf{L} = \text{diag}(\sigma) \mathbf{L}_\Omega,$$

where \mathbf{L}_Ω is the Cholesky factor of a correlation matrix with an LKJ prior (Lewandowski et al., 2009), and each component of σ follows a half-Cauchy prior. This flexible structure permits correlation among the d error components.

We then place shrinkage priors on each coefficient in \mathbf{B} . Below, we detail three such

priors—normal (ridge), horseshoe, and Bayesian lasso—all of which can be combined with the same LKJ-based prior for Σ_ε :

Normal Prior (Bayesian Ridge). A normal (Gaussian) prior imposes a global ℓ_2 penalty. For each coefficient β_j we set priors,

$$\beta_j \sim \mathcal{N}(0, 1), \quad j = 1, \dots, d^2 p,$$

so that most coefficients are moderately shrunk towards zero. This parallels the frequentist ridge penalty, and one can include an additional scale factor if stronger or weaker global shrinkage is desired depending on the data set.

Horseshoe Prior. The horseshoe prior (Carvalho et al., 2010; Makalic and Schmidt, 2016) introduces more adaptive shrinkage via a local-global hierarchy. Each β_j is modeled as $\beta_j = B_{\text{raw},j} \lambda_j \tau$, where $B_{\text{raw},j} \sim \mathcal{N}(0, 1)$, $\lambda_j \sim C^+(0, 1)$ (local scale), and $\tau \sim C^+(0, 1)$ (global scale). Small coefficients are heavily shrunk by small local scales, while the heavy-tailed Cauchy priors allow some large signals to remain.

Bayesian Lasso Prior. Finally, the Bayesian lasso (Park and Casella, 2008) imposes a Laplace (double-exponential) prior,

$$\beta_j | \eta \sim \text{Laplace}(0, \eta),$$

which corresponds to an ℓ_1 penalty in a frequentist setting. As with the horseshoe, this encourages coefficients to be near zero, possibly leading to sparsity in β .

In all three cases, the covariance matrix Σ_ε is handled by the same LKJ-based prior, thus capturing potential correlations in the innovation process. The posterior distribution factors as

$$\pi(\mathbf{B}, \Sigma_\varepsilon | \mathbf{y}_t) \propto \ell(\mathbf{y}_t | \mathbf{B}, \Sigma_\varepsilon) \pi(\mathbf{B}) \pi(\Sigma_\varepsilon),$$

where ℓ is the Gaussian likelihood induced by the VAR model, and $\pi(\mathbf{B})$, $\pi(\Sigma_\varepsilon)$ encode the chosen shrinkage and covariance priors, respectively. By jointly estimating \mathbf{B} and Σ_ε , this framework avoids the assumption of uncorrelated errors and allows us to examine how different shrinkage priors influence coefficient estimation in a fully multivariate setting.

2.2 Frequentist Methods

Ridge Regression. Classical ridge regression for a VAR(p) solves

$$\min_{\boldsymbol{\beta} \in \mathbb{R}^{d^2 p}} \sum_{t=p+1}^T \left\| \mathbf{y}_t - \sum_{i=1}^p \mathbf{A}_i \mathbf{y}_{t-i} \right\|^2 + \lambda \|\boldsymbol{\beta}\|_2^2, \quad (3)$$

where $\boldsymbol{\beta}$ is just the vectorized collection of $\{\mathbf{A}_i\}$. We set the regularization parameter $\lambda = .1$ in our analysis.

Nonparametric Shrinkage (NS). We use a James–Stein-like shrinkage approach for VAR coefficients (Giannone et al., 2015; Del Negro and Giannoni, 2015), implemented in R via `VARshrink` with `method="ns"`. Instead of explicitly solving (3), the NS method estimates the necessary sample covariances of $\mathbf{y}_{t-i}, \mathbf{y}_t$ and then applies a closed-form shrinkage rule to these covariance estimates, thereby producing a shrunk solution for \mathbf{B} . In this paper, we rely on the default choice for the shrinkage parameter, which `VARshrink` selects via a moment-based (empirical Bayes) formula akin to Stein’s unbiased risk estimate.

3 Simulation Studies

3.1 Data-Generating Processes

We design three VAR data scenarios to examine both low- and high-dimensional settings and the effects of overfitting the lag order. Each dataset is simulated with a 50-observation burn-in (discarded), followed by $T = 200$ observations. Of these 200, the first 180 are the training set used for estimation, and the remaining 20 are held out for forecasting.

Each scenario is replicated $N = 50$ times with independent parameter draws of β_j for each simulation.

Scenario 1 (low dimension, overfit lag). We generate a *3-dimensional* VAR(1) process here but intentionally *fit* a VAR(4). To form the 3×3 transition matrix A , each of its 9 entries is set to zero with probability 0.7 (so on average across simulations, 3 are nonzero) and the nonzeros are drawn uniformly from $(-0.4, 0.4)$. We then rescale A so its largest eigenvalue in modulus is strictly below 1 by dividing by 1.1 times the maximum absolute eigenvalue, and set the noise covariance to $\Sigma = 0.05 I_3$. As a result, there are 4×3^2 parameters to estimate in \mathbf{B} , even though only 3 of them are non-zero on average.

Scenario 2 (high dimension, correct lag). We generate a *20-dimensional* VAR(1) process and also fit a VAR(1). The 20×20 matrix A is constructed where each of its 400 entries is zero with probability 0.7 (so roughly 120 are nonzero), with nonzeros drawn randomly from $(-0.4, 0.4)$. Afterward, we ensure stationarity by dividing by 1.1 times the maximum absolute eigenvalue. We set $\Sigma = 0.1 I_{20}$.

Scenario 3 (high dimension, overfit lag). We use the same data-generating scheme as in Scenario 2 (thus a sparse VAR(1) in 20 dimensions), but this time *fit* a VAR(4). As a result, there are $4 \times 20^2 = 1600$ parameters to estimate, even though only 120 of them are truly nonzero on average across simulations.

3.2 Estimation Methods

Design Matrix Setup. To estimate a VAR(p) in a linear regression framework, we arrange lagged responses into a design matrix $\mathbf{X} \in \mathbb{R}^{(T_{\text{train}}-p) \times (dp)}$. For $t = p+1, \dots, T_{\text{train}}$, the t -th row of \mathbf{X} (denoted \mathbf{X}_t^\top) is formed by horizontally concatenating the transposes of the p lagged column vectors

$$\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{y}_{t-p} \quad (\text{each } d \times 1).$$

Hence, each row of \mathbf{X} is a $1 \times (dp)$ vector. Likewise, the t -th row of the response matrix $\mathbf{Y} \in \mathbb{R}^{(T_{\text{train}}-p) \times d}$ is simply the transpose $\mathbf{y}_t^\top \in \mathbb{R}^{1 \times d}$. Once \mathbf{X} and \mathbf{Y} are formed, any penalized or Bayesian regression method can be applied directly, and the estimated coefficient matrix is then reshaped to match $\mathbf{B} \in \mathbb{R}^{d \times (dp)}$ as defined in Section 2.

Frequentist fits and block-bootstrapped standard errors. We estimate the VAR coefficients in Ridge (`glmnet` with $\alpha = 0$) and NS (`VARshrink` with `method="ns"`) by penalized least squares, and then obtain empirical standard errors via a *block bootstrap* to better respect local time dependence. Specifically we

1. **Partition into blocks:** We group the training data $\{X, Y\}$ into non-overlapping consecutive blocks of length 4. We choose a block size of 4 because our VAR models use up to 4 lags, so each block captures the short-range autocorrelation structure of interest.
2. **Sample blocks with replacement:** To form a bootstrap dataset of the same size as the original, we randomly select blocks *with replacement* until we have at least T_{train} observations in total.
3. **Refit the model:** We refit the Ridge or NS model on this resampled dataset and record the estimated coefficients.
4. **Repeat and aggregate:** Steps (2)–(3) are repeated for 30 bootstrap replications (`n_boot = 30`). The empirical standard error for each coefficient is then taken to be the sample standard deviation of its estimates across these replications.

This procedure retains within-block autocorrelations (up to 4 lags) while randomly mixing which blocks are selected, preserving important time-series structure better than naive row-wise (i.i.d.) resampling. As a result, the resulting intervals yield more realistic coverage for dependent data.

Bayesian fits. We fit the three Bayesian models by calling `Stan` with 4 parallel Markov chains, each run for 2000 total iterations (the first 500 of which are warm-up). We fix `seed=123` for reproducibility and use `{adapt_delta = 0.9, max_treedepth = 12}`. In Stan’s Hamiltonian Monte Carlo (HMC) framework, `adapt_delta` is the target acceptance probability, and increasing it to 0.9 aims for smaller step sizes and more conservative sampling. The `max_treedepth` parameter caps the depth of the binary tree in each iteration’s leapfrog integrator, preventing extremely long trajectories.

For each chain, we obtain posterior draws of the coefficient vector β . We summarize each coefficient by its posterior mean and 95% central credible interval (2.5% and 97.5% quantiles).

3.3 Performance Metrics

We evaluate each method along two dimensions: *parameter estimation* and *forecast* performance.

Parameter estimation. Let β_{true} denote the true parameters in \mathbf{B} . Each frequentist method (Ridge or Nonparametric Shrinkage) estimates β by minimizing a penalized least squares criterion, whereas each Bayesian method (Normal/Ridge, Lasso, Horseshoe) uses the posterior mean from MCMC samples as $\hat{\beta}$. We then compute the root mean squared error (RMSE),

$$\text{RMSE} = \sqrt{\frac{1}{d^2 p} \sum_{j=1}^{d^2 p} (\hat{\beta}_j - \beta_{j,\text{true}})^2},$$

to measure how closely $\hat{\beta}$ matches β_{true} .

Next, we construct 95% intervals for each coefficient by applying a block bootstrap to estimate standard errors and forming approximate normal intervals of the form $\hat{\beta}_j \pm z_{0.975} \text{SE}_j$ for the frequentist approaches, whereas for the Bayesian methods we use the 2.5% and 97.5% posterior quantiles from the MCMC samples. We record the *empirical coverage* (the percentage of intervals that contain the true value) and the *average interval*

length to assess how well each approach quantifies uncertainty.

Forecasting performance. To assess predictive accuracy, we reserve the final 20 observations as a test set. Each method then produces sequential one-step-ahead forecasts by estimating \mathbf{y}_{t+1} at time t based on all data up to \mathbf{y}_t , avoiding the accumulation of multi-step errors. We compute the average root mean squared forecast error (RMSE)

$$\text{Forecast RMSE} = \sqrt{\frac{1}{20d} \sum_{t=T_{\text{train}}+1}^{T_{\text{train}}+20} \|\mathbf{y}_t - \hat{\mathbf{y}}_t\|_2^2},$$

where $\hat{\mathbf{y}}_t$ is the forecast at time t . After 50 replications per scenario, we summarize the average forecasting RMSE and coverage to compare each method's predictive capabilities.

3.4 Simulation Results

The results from the three simulation studies are shown in tables 1–3 and figures 1–4. The percentage of replications in which each method has the lowest forecast RMSE or parameter RMSE is shown in table 4.

3.4.1 Scenario 1 (Low-Dimension, Overfit Lag)

Forecasting The top block of Table 1 has each method's mean forecast RMSE. Horseshoe has the smallest value (0.211), followed by *ns* and Ridge (0.213), Lasso (0.214), and Normal (0.215). Table 4 has the proportion of replications in which each method has the best forecast: Horseshoe leads with 60%, *ns* has 20%, Normal 10%, Ridge 6%, and Lasso 4%.

Parameter Estimation Horseshoe has the lowest overall parameter RMSE (0.0434) and exceeds average nominal coverage (97.2%), with intervals about 8% shorter than those of the next-best method. Lasso (0.0803) and Normal (0.0838) have higher RMSEs but maintain coverage near 94–95%. Both *ns* (0.0693) and Ridge (0.0730) occupy a middle tier; *ns* has coverage of 85.7% but yields narrower intervals (mean length 0.204). Horseshoe has the

best parameter RMSE in all replications (100%), giving strong shrinkage without sacrificing coverage.

3.4.2 Scenario 2 (High-Dimension, Correct Lag)

Forecasting All methods have similar forecasting accuracy (middle block of Table 1). Horseshoe has the smallest mean forecast RMSE (0.325), followed by Lasso (0.326) and Normal, ns , and Ridge (0.327). Horseshoe is the top forecaster in 48% of replications, Lasso and ns each in 20%, Ridge in 10%, and Normal in 2% (Table 4).

Parameter Estimation

Horseshoe again has the lowest parameter RMSE (0.0536). Lasso, Normal, ns , and Ridge cluster between 0.0568 and 0.0598. Coverage remains high (94–95%) for Horseshoe, Lasso, Normal, and ns , but dips to 84% for Ridge. Table 2 has results for zero coefficients, where Horseshoe has an RMSE of 0.0357 and 99.0% coverage. The ns method handles zero parameters well but sometimes underperforms on nonzeros. Horseshoe has a nonzero RMSE of 0.0596, higher than Lasso and Normal (0.0571–0.0576), while maintaining overall coverage of 92.9%. Horseshoe has the best parameter RMSE in 90% of replications, followed by Ridge in 10% (Table 4).

3.4.3 Scenario 3 (High-Dimension, Overfit Lag)

Forecasting In the bottom block of Table 1, Horseshoe has the smallest mean forecast RMSE (0.342), followed by ns and Ridge (0.365–0.366), Lasso (0.404), and Normal (0.418). Horseshoe is the top forecaster in 100% of replications (Table 4), indicating a strong ability to handle overfitting.

Parameter Estimation Horseshoe has the lowest parameter RMSE (0.0394), with better-than-nominal average coverage (97.5%) and intervals about 6% shorter than those of the next-best method. Lasso (0.104) and Normal (0.117) have higher RMSEs but maintain

nominal coverage (95–96%) through wider intervals (0.432–0.464). Both *ns* and Ridge have moderate RMSEs (0.0619–0.0635) but show lower coverage (88.2% and 84.5%) and narrower intervals (0.18–0.20). Horseshoe remains the top performer in parameter RMSE for 100% of the replications.

Overall, Horseshoe consistently has excellent forecast accuracy and parameter recovery, including the lowest RMSE, high coverage, and moderate interval lengths. Ridge occasionally has strong forecasts but frequently undercovers in higher dimensions. Lasso and Normal have intermediate performance for both forecasting and parameter estimation, ensuring reasonable coverage by using somewhat larger intervals. The *ns* approach is computationally efficient and sometimes has precise point estimates, but coverage can be volatile due to overly narrow intervals. These findings reinforce the advantages of local-global shrinkage (Horseshoe) in moderate- and high-dimensional VAR contexts, especially when the lag order is inflated.

4 Data Analysis

We illustrate our methods on the `Canada` dataset from the R package `vars` (Pfaff, 2008), which provides quarterly macroeconomic observations on four Canadian variables spanning $T = 84$ quarters (1980Q1–2000Q4): employment (e , in log-index form), productivity ($prod$, in log-index form measuring labor productivity), real wages (rw , in log-index form), and the unemployment rate (U , in percent). Economic considerations suggest these variables are jointly dependent, making a Vector Auto Regression (VAR)-based approach suitable.

Differencing and Stationarity. To reduce nonstationarity, we difference each series once

$$\Delta \mathbf{y}_t = \mathbf{y}_t - \mathbf{y}_{t-1} \quad (t = 2, \dots, T).$$

We then estimate the VAR on these differenced observations. To obtain forecasts on the original scale, we *invert* the differencing by recursively summing the predicted differences

$$\hat{\mathbf{y}}_{T+1} = \mathbf{y}_T + \widehat{\Delta\mathbf{y}}_{T+1}, \quad \hat{\mathbf{y}}_{T+2} = \hat{\mathbf{y}}_{T+1} + \widehat{\Delta\mathbf{y}}_{T+2}, \quad \text{etc.}$$

This ensures the final forecasts reflect the original scale of the data, while the differencing step helps achieve stationarity and limit spurious trends when fitting the VAR.

Lag Orders from $p = 1$ to $p = 12$. We fit $\text{VAR}(p)$ models for $p \in \{1, 2, \dots, 12\}$ to assess how each shrinkage method adapts to varying lag choices. Specifically, we train each VAR on the first 80 quarters of the differenced data, then produce rolling 1-step-ahead forecasts for quarters 81–84 (the holdout set). After each forecast, we *roll forward* by incorporating the newly observed actual data. We measure forecast accuracy in the original scale by computing both root mean squared error (RMSE), $\text{RMSE} = \sqrt{\frac{1}{4} \sum_{t=81}^{84} (y_t - \hat{y}_t)^2}$ and mean absolute percentage error (MAPE), $\text{MAPE} = \frac{100}{4} \sum_{t=81}^{84} \left| \frac{y_t - \hat{y}_t}{y_t} \right|$ over the 4 test observations.

4.1 Results

Figure 5 (left) shows the original Canadian macroeconomic variables, while the right panel displays their first-differenced versions used for estimation.

4.1.1 Aggregate Performance ($p = 1, \dots, 12$)

Each method's forecast accuracy (RMSE, MAPE) for varying lag orders (p) is shown in Figures 6 and 7. Table 5 reports the mean and standard deviation of these metrics across all 12 fitted $\text{VAR}(p)$ models and all four series.

Overall, *Horseshoe* achieves the most consistent and accurate forecasts, attaining the lowest mean RMSE (0.51). *Ridge* (RMSE = 0.56) and *NS* (RMSE = 0.60) provide somewhat intermediate performance, while *Lasso* (RMSE = 0.60) and *Normal* (RMSE = 0.63) tend to yield slightly larger prediction errors. In terms of variability, Horseshoe's stan-

dard deviation of RMSE (0.22) is comparable to that of Lasso and Ridge, whereas Normal exhibits the largest SD (0.26).

For MAPE, Horseshoe again stands out with an average of 0.71%, followed by NS (0.97%), Ridge (1.06%), Lasso (1.24%), and Normal (1.37%). This pattern suggests that Horseshoe effectively suppresses many small coefficients without overshrinking the larger signals, yielding robust relative-error forecasts even as p grows. By contrast, Normal’s wider prior and Lasso’s strong ℓ_1 shrinkage can lead to higher MAPE in certain lag settings (see Figure 7), and NS remains susceptible to moderate forecast deterioration for larger p .

Shrinkage patterns for a few representative lag orders ($p = 3, 6, 9, 12$) appear in Figure 8. Horseshoe and Lasso consistently display heavy shrinkage toward zero for small coefficients, Normal has moderate Gaussian-like shrinkage, NS exhibits a broader coefficient spread, and Ridge pulls estimates closer to zero but never to an exact zero. Across the different p values, Horseshoe’s local-global prior structure appears to adapt more flexibly, resulting in better overall forecasting metrics.

4.1.2 Case Study: $p = 11$

To illustrate performance at a higher lag, we examine the VAR(11) specification, which delivers the lowest average forecasting errors among the tested orders. Table 6 shows each method’s RMSE and MAPE on the final four quarters of the holdout set. Notably, *Horseshoe* again achieves the best performance on both measures (RMSE = 0.51, MAPE = 0.60%). The next closest method is *Ridge* (RMSE = 0.61, MAPE = 1.66%) and *NS* (RMSE = 0.65, MAPE = 1.26%), while Lasso and Normal both exhibit slightly higher errors (RMSE = 0.70–0.78, MAPE = 1.66–1.81%). This example highlights Horseshoe’s ability to preserve large coefficients and aggressively shrink small ones, maintaining strong predictive accuracy even at high lag orders.

The 1-step-ahead forecasts of the observed data across the four holdout quarters are shown in figure 9. Horseshoe, ns, and Ridge all track the actual values fairly closely. Lasso and Normal lag behind somewhat. Overall, these VAR(11) results echo our broader sim-

ulation findings: Horseshoe’s adaptive local-global prior can maintain strong performance at high lag orders, and Ridge remains reasonably robust as well, whereas *ns*, Lasso, and Normal can become less accurate or more variable depending on the specific error metric.

5 Discussion

Our simulation results lead to several important takeaways about shrinkage estimation in VAR models under varying dimensionality and lag orders. First, the Horseshoe prior stands out for consistently achieving the lowest parameter RMSE and near-nominal coverage, particularly in the most challenging high-dimensional or overfit scenarios. This local-global prior structure successfully suppresses small coefficients while preserving truly large effects, thereby producing stable estimates and competitive forecasts across the board. By contrast, Lasso and Normal priors often deliver mid-range forecast accuracy and parameter estimation, but they maintain coverage near or above the 95% target, albeit with wider intervals in some cases.

Ridge regression remains effective for forecasting in low- to moderate-dimensional scenarios (where the ratio of parameters to observations is not excessively large), frequently ranking second or third in terms of forecast RMSE. However, it underestimates parameter uncertainty in high-dimensional settings e.g., when the dimension-to-sample-size ratio is particularly large, leading to undercoverage. Similarly, Nonparametric Shrinkage (*ns*) provides very short intervals and can achieve strong point forecasts, but it exhibits markedly low coverage in the same high-dimensional regimes, suggesting that its narrower intervals are overly optimistic about uncertainty in heavily over-parameterized models.

In the Canadian macro-economic data application, similar patterns emerge: Horseshoe and Ridge each exhibit strong one-step-ahead forecast accuracy, while Lasso, Normal, and *ns* occasionally lag behind, particularly when the model order is large. Overall, these findings reinforce the benefits of using local-global shrinkage to adapt to large model spaces, especially for practitioners seeking reliable inference and coverage. Frequentist options like

Ridge can still perform competitively in lower-dimensional or less overfit settings but risk severe undercoverage when the parameter space grows.

Taken together, these results underscore that when parameter interpretation and interval validity are paramount, Horseshoe or other local-global Bayesian priors are well-suited to handle high-dimensional or inflated-lag VAR models. If short-term predictive performance alone is the principal goal, Ridge can remain attractive, provided one is willing to accept somewhat lower coverage in complex settings. The Lasso and Normal priors offer middle-ground alternatives, balancing coverage and moderate forecasting performance without fully matching Horseshoe's combination of shrinkage strength and coverage reliability.

Code Availability

All **R** scripts and **Stan** model files used in this study are publicly available at <https://github.com/harrisonekatz/BayesVAR-SimStudy>. In particular, the main simulation script `var_three_sim_script.R` (which orchestrates data generation, frequentist and Bayesian estimation, and result collation) may be found in the repository's `R/` directory. The repository also includes each of the **Stan** model files (`var_normal.stan`, `var_lasso.stan`, `var_horseshoe.stan`), along with examples illustrating their usage. All results and figures in this manuscript can be reproduced by running the scripts found in that repository.

Figures and tables

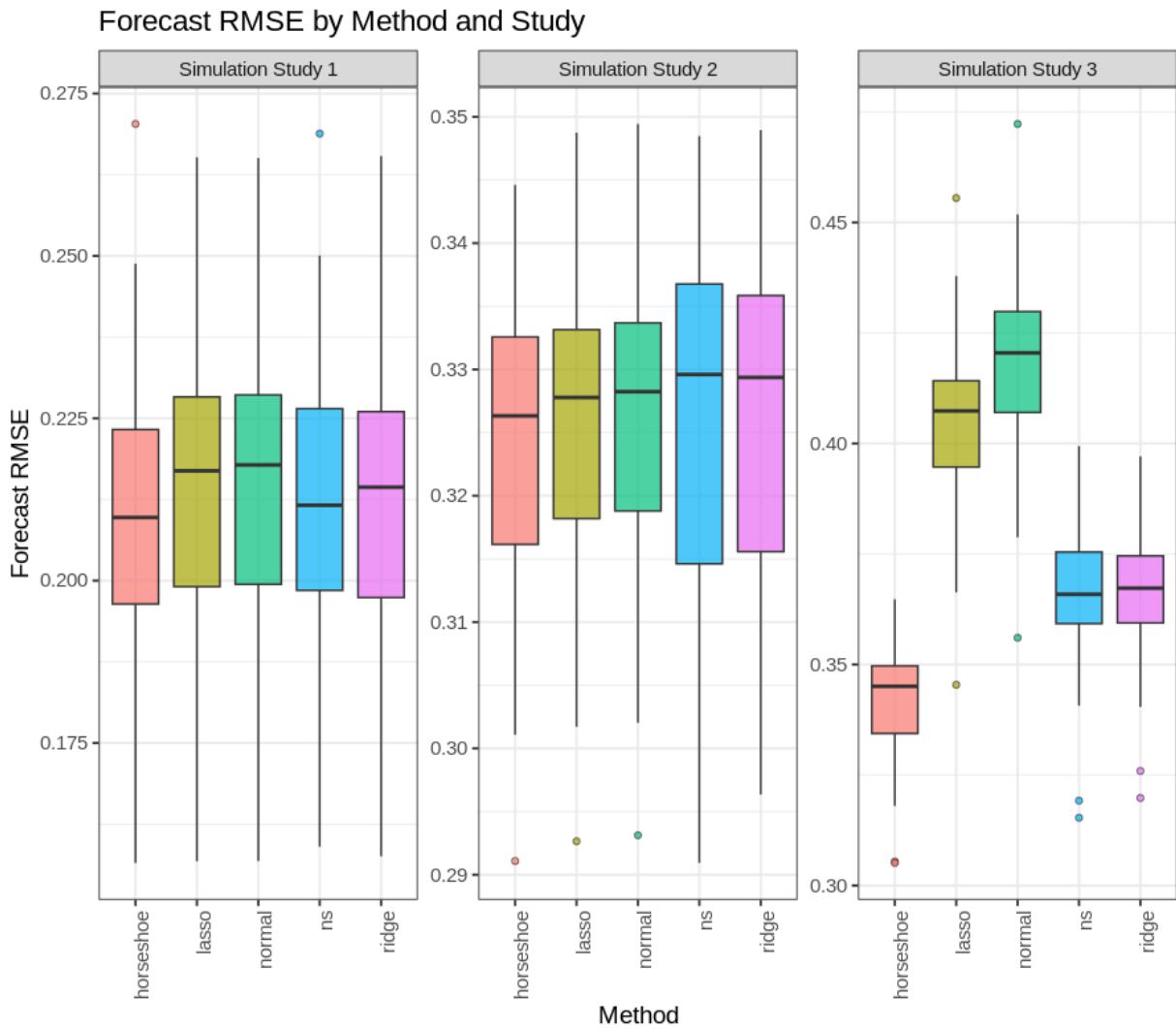


Figure 1: **Forecast RMSE by Method and Study (All Coefficients).** Boxplots reflect the distribution of one-step-ahead RMSE across the 50 replications. Horseshoe achieves or ties for the lowest forecast error, especially in the high-dimension overfit scenario (Study 3).

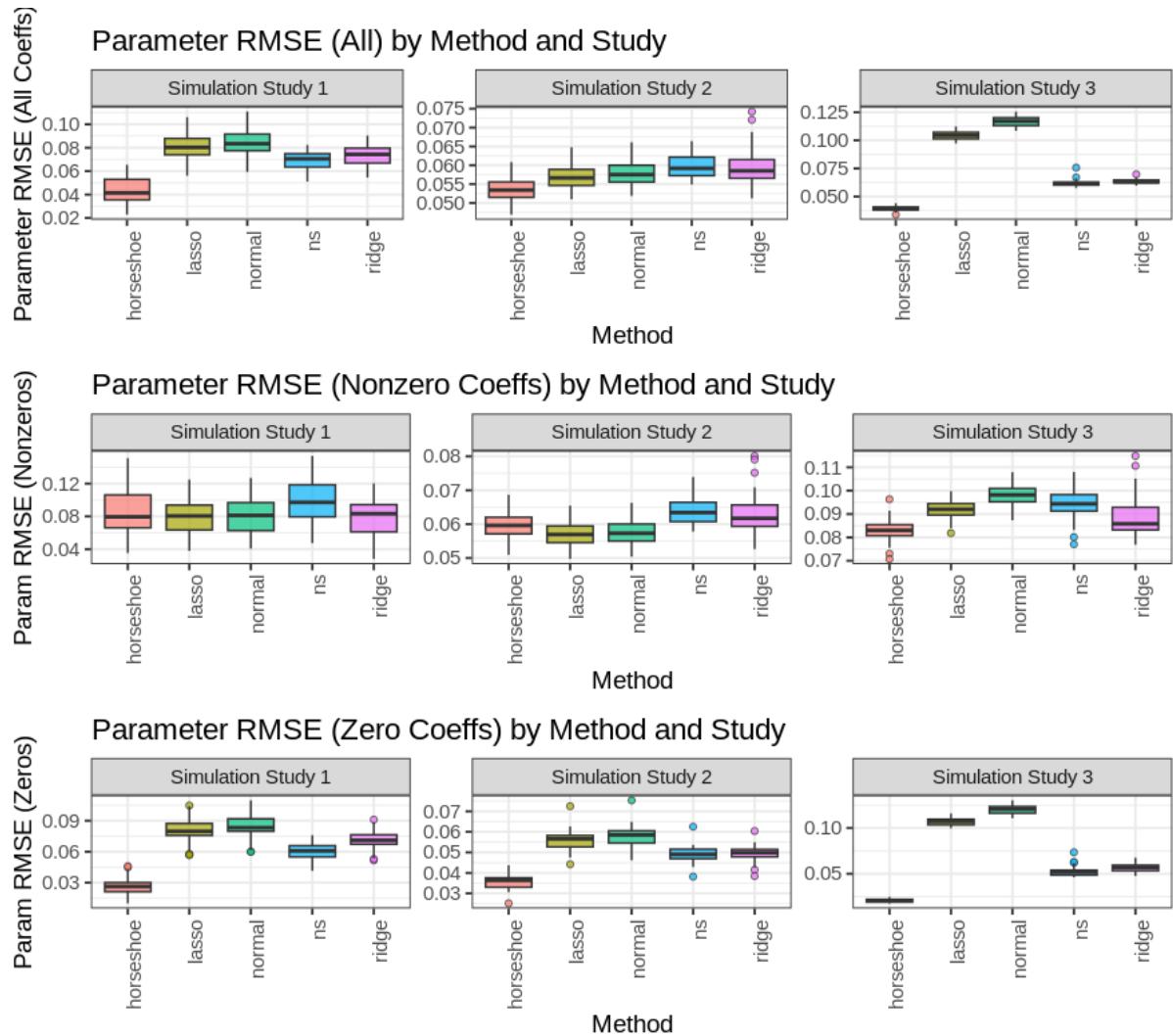


Figure 2: **Parameter RMSE by Method and Study.** Horseshoe is consistently lowest in overall parameter RMSE, while NonparamShrink (*ns*) occasionally performs well but can exhibit greater variance or undercoverage.

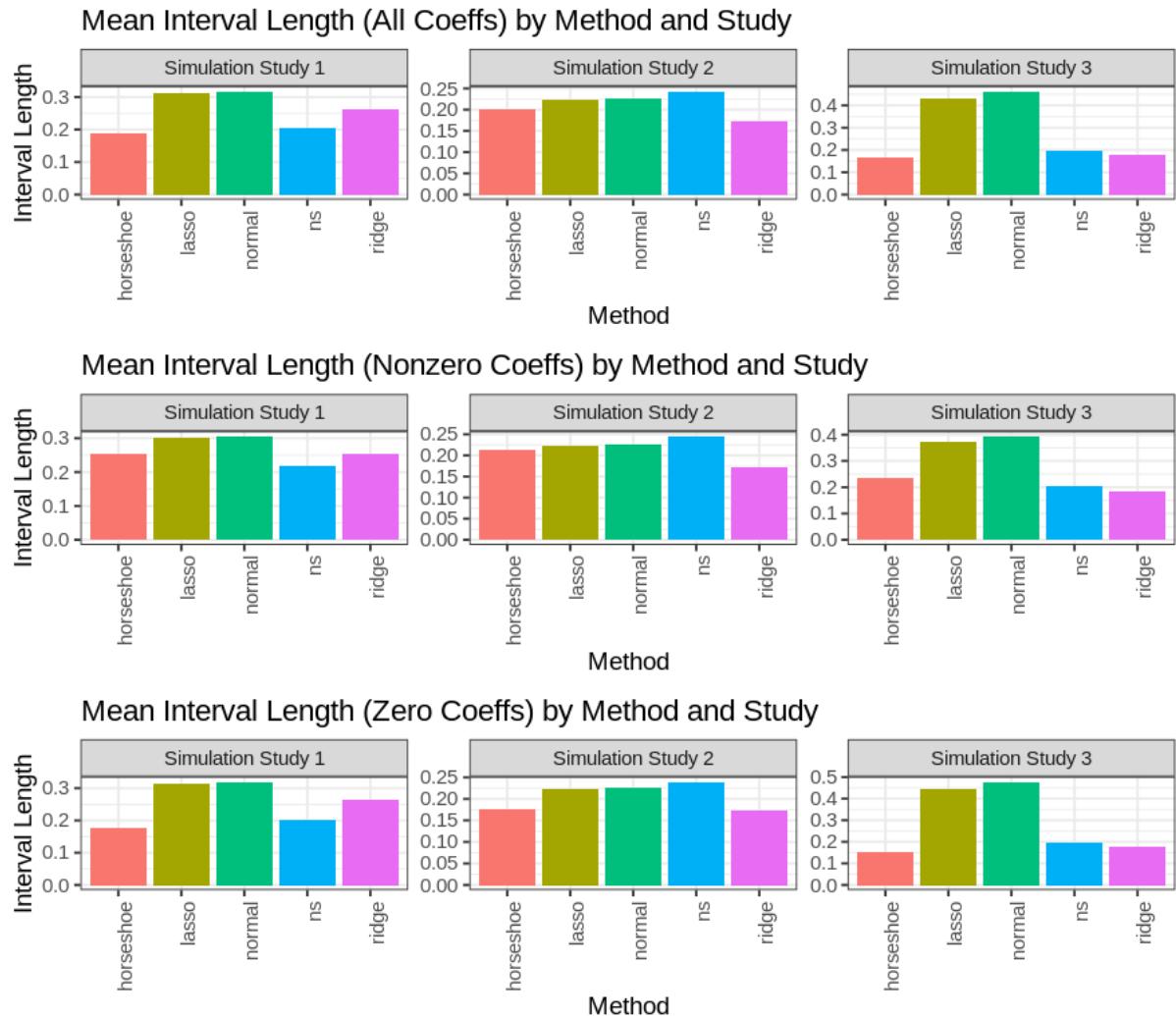


Figure 3: **Mean Interval Length by Method and Study.** Shorter intervals may indicate overconfidence if coverage is below the nominal 95%; for instance, *ns* has narrower intervals but lower coverage in some scenarios.

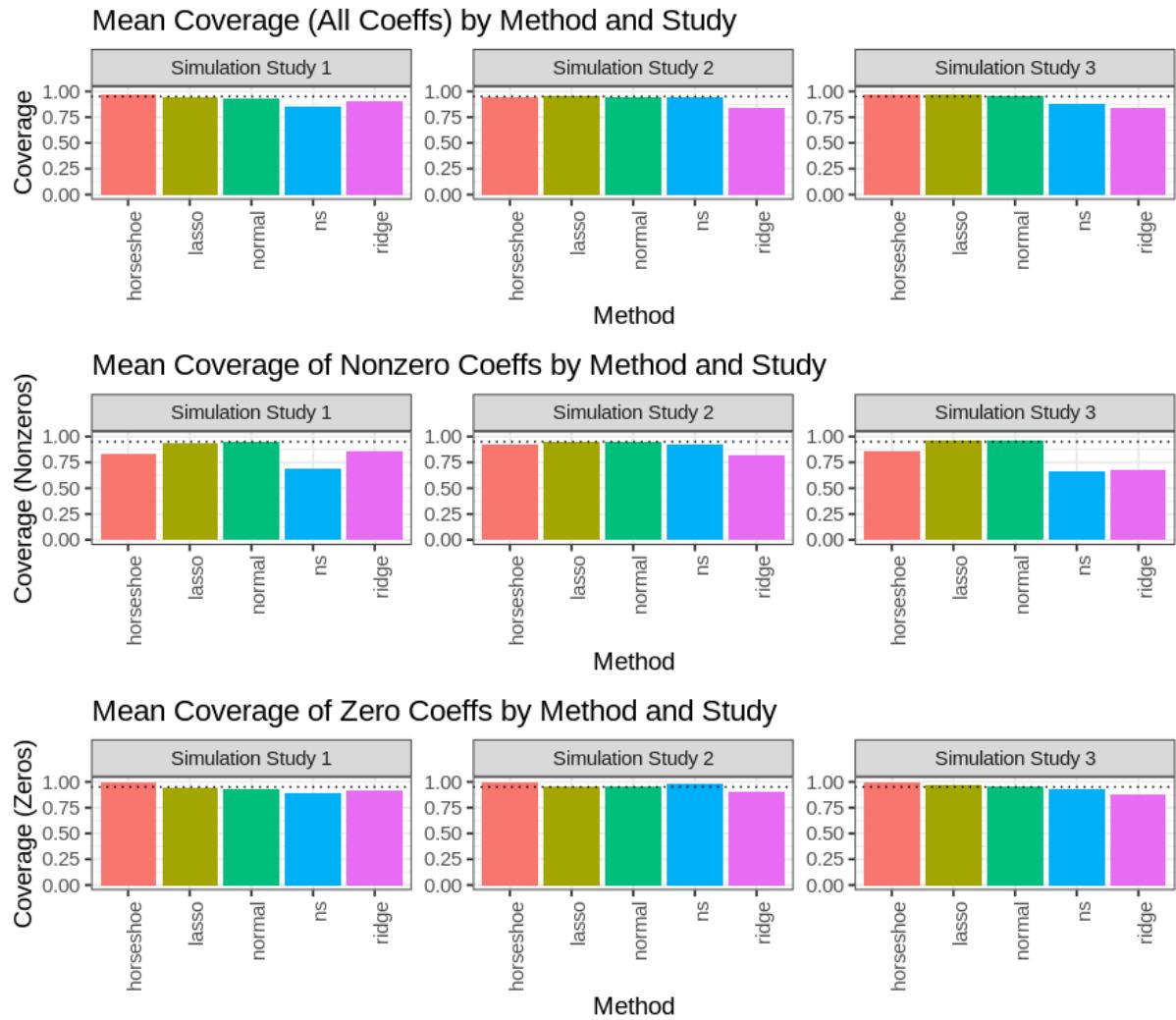


Figure 4: **Coverage by Method and Study.** A dotted line at 0.95 indicates the nominal coverage target. Horseshoe, Lasso, and Normal usually achieve near 95%, while *ns* and Ridge can dip below this level for high-dimensional or overfit scenarios.

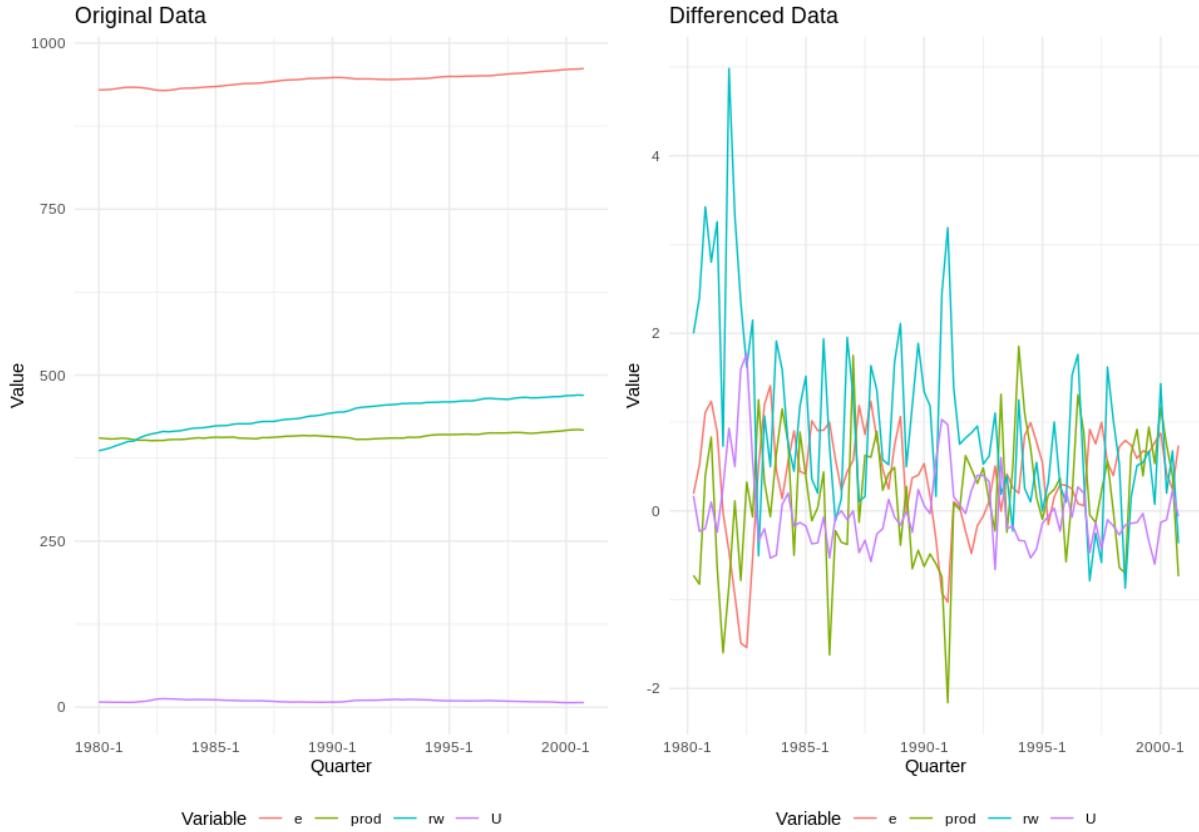


Figure 5: The four Canadian macroeconomic variables in their original form (left) and once-differenced (right). Differencing helps remove trends and stabilize the series prior to VAR model estimation.

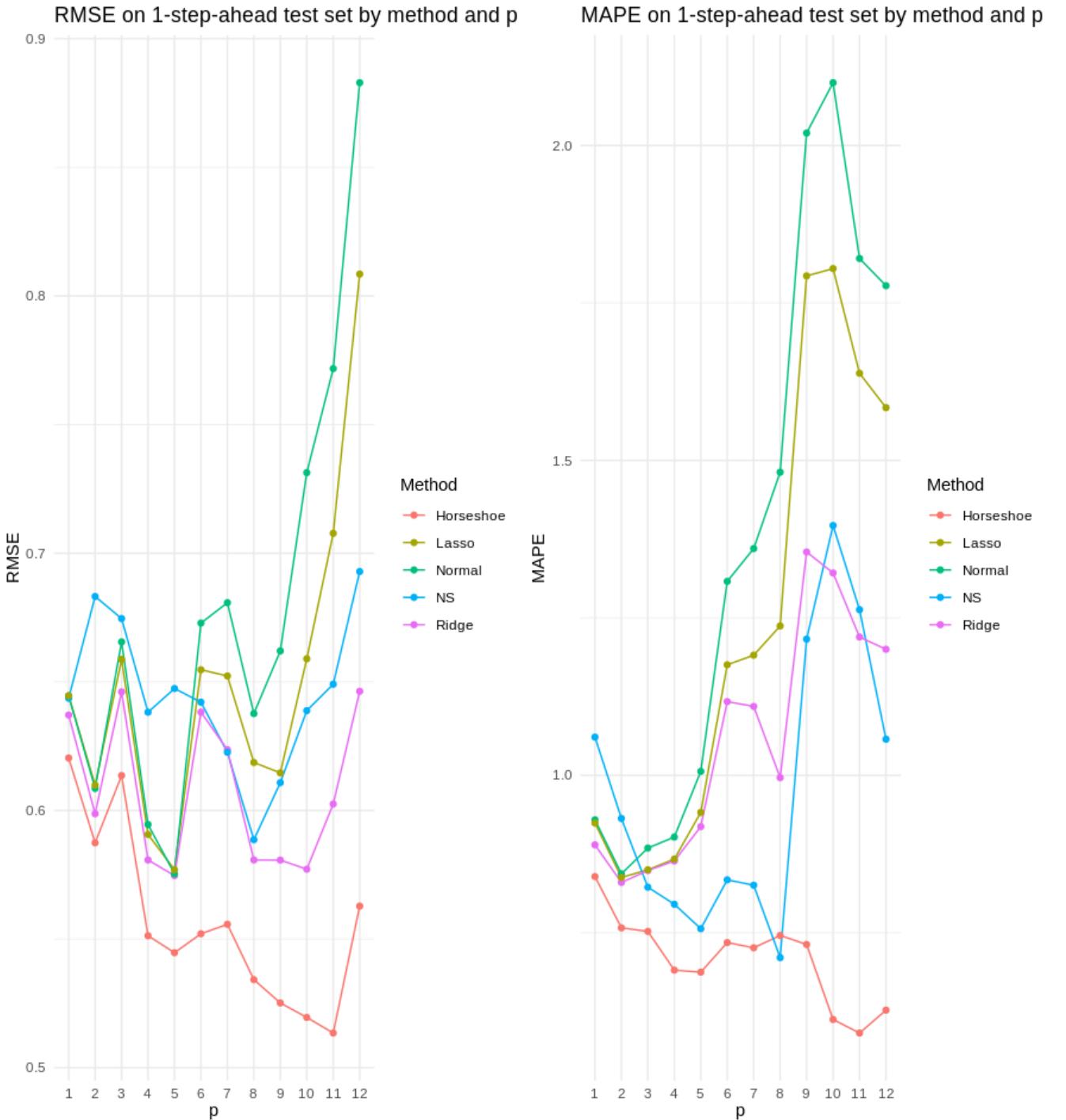


Figure 6: **Forecast performance on Canadian data with $\text{VAR}(p)$, $p = 1, \dots, 12$.** *Left:* Out-of-sample RMSE. *Right:* Out-of-sample MAPE (%). Methods evaluated: Horseshoe, Lasso, ns, Normal, and Ridge. Horseshoe and Ridge consistently achieve lower RMSEs, while Lasso and Normal show moderate performance. ns experiences higher errors, especially at larger p .

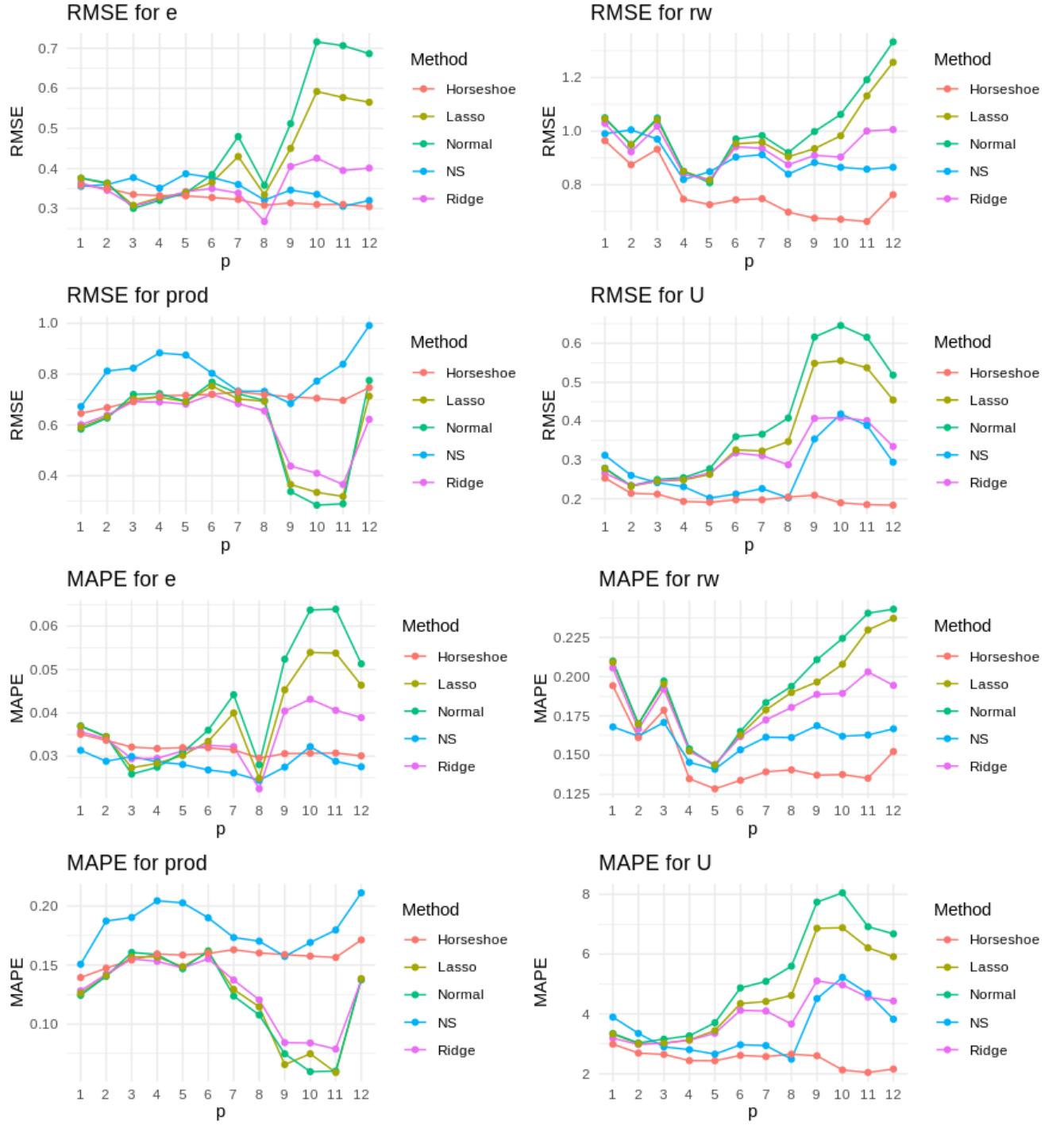


Figure 7: Out-of-sample forecasting accuracy for each Canadian macroeconomic variable—employment (e), real wages (rw), productivity ($prod$), and unemployment (U)—across increasing VAR orders ($p = 1, \dots, 12$). The top row shows the Root Mean Squared Error (RMSE), and the bottom row shows the Mean Absolute Percentage Error (MAPE). Each colored line corresponds to one of five shrinkage methods (Horseshoe, Lasso, Normal, ns, and Ridge). Overall, Horseshoe achieves the lowest MAPE and exhibits relatively stable performance as p increases. Lasso and Ridge perform well but show greater variability at higher orders.

Coefficient Distributions by Method

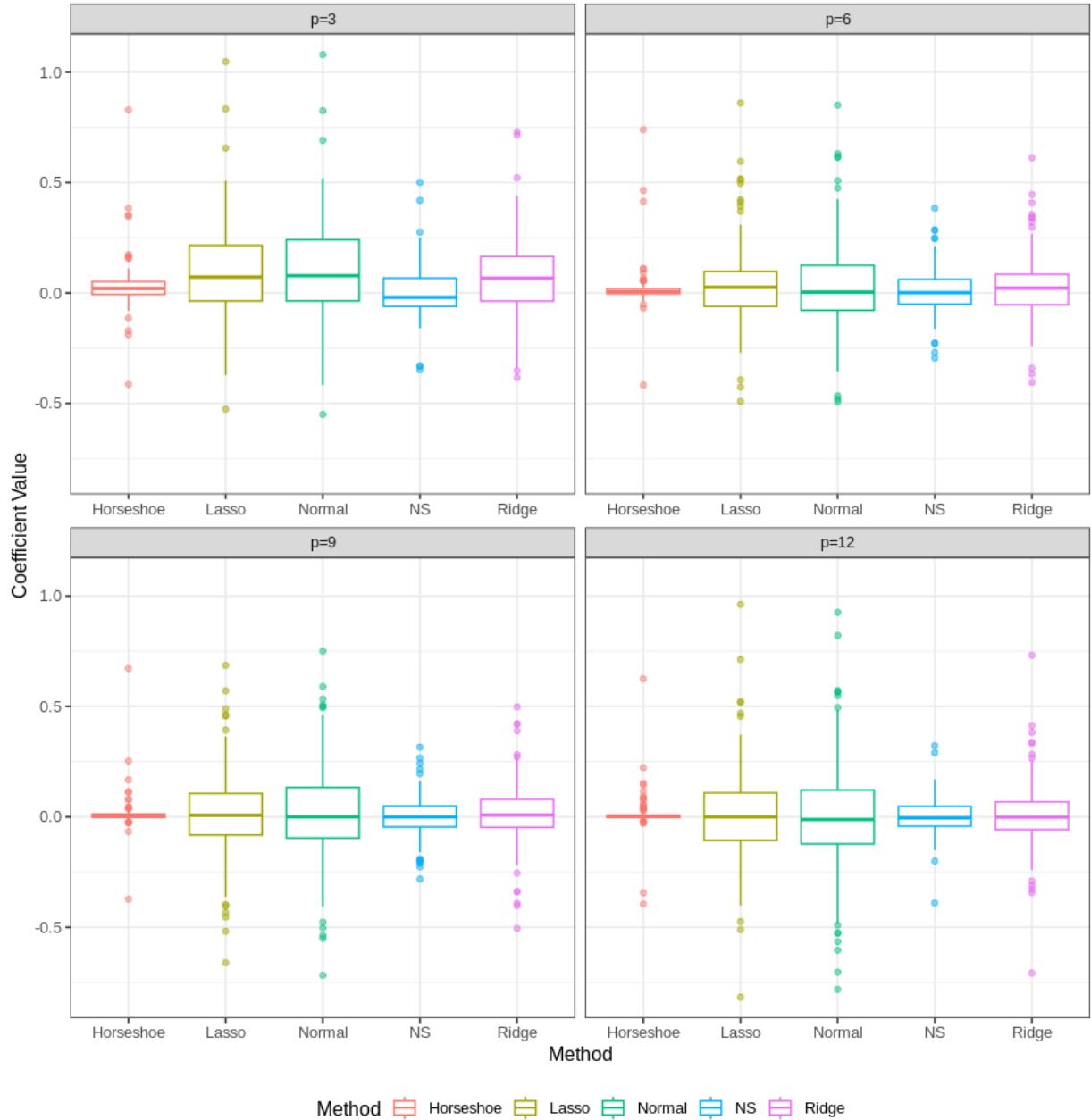


Figure 8: Distribution of estimated VAR coefficients by method. Each point represents one of the $(4 \times 4 \times p)$ parameters, highlighting the degree of shrinkage for each prior.

Forecasts and Actuals on Test Set

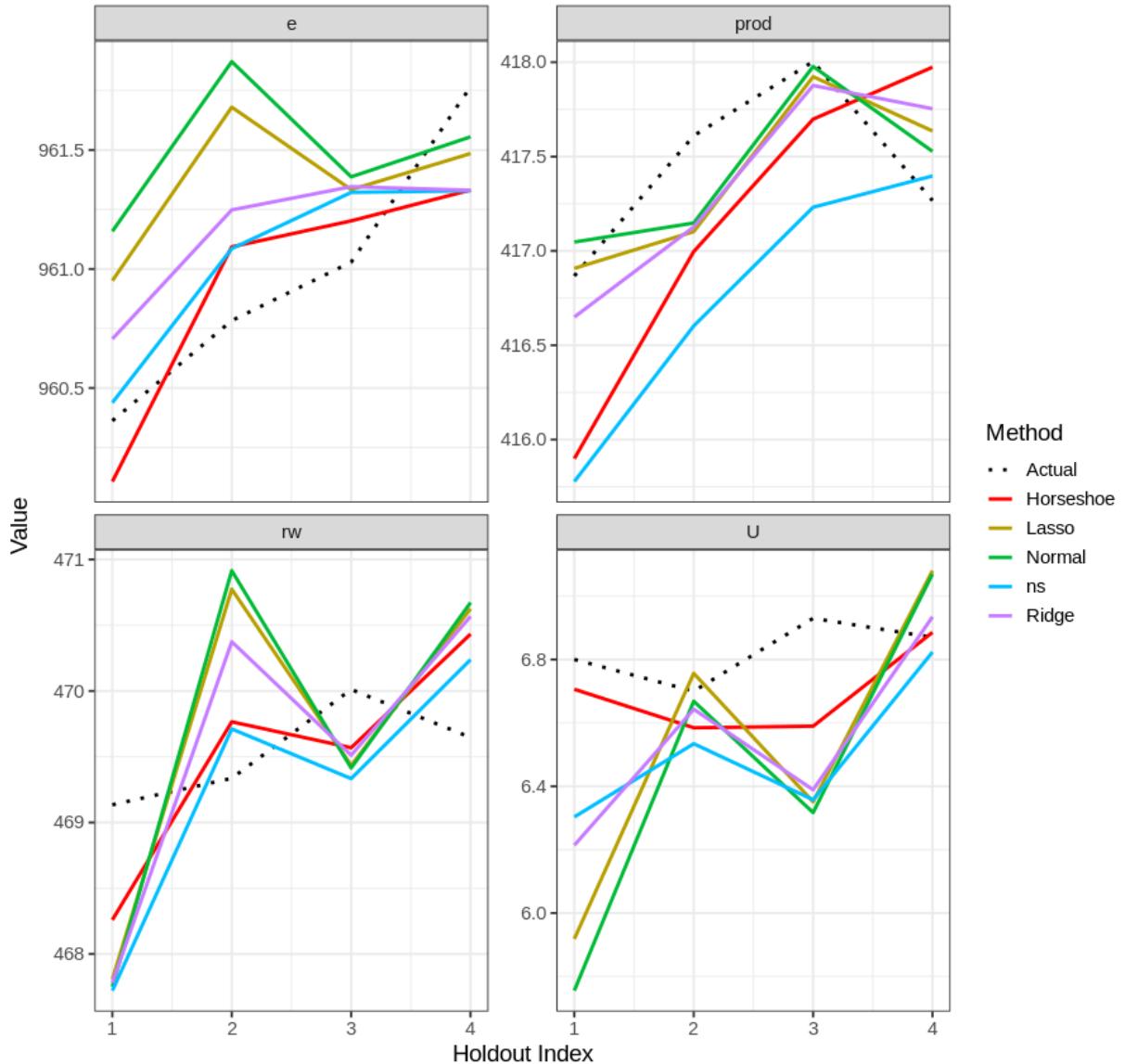


Figure 9: Out-of-sample 1 step ahead forecasts from VAR(11) and actuals for each Canadian macroeconomic variable—employment (*e*), real wages (*rw*), productivity (*prod*), and unemployment (*U*).

Table 1: **Overall Performance (All Coefficients).** Mean and standard deviation (SD) of forecast RMSE (FRMSE), mean and SD of parameter RMSE (PRMSE), mean coverage (Cov), and mean interval length (Int. Length) across the three studies.

Scenario	Method	FRMSE	FRMSE	PRMSE	PRMSE	Cov	Int. Length
		Mean	SD	Mean	SD		
<i>Study 1</i>							
Study 1	Horseshoe	0.211	0.0208	0.0434	0.0105	0.972	0.189
	Lasso	0.214	0.0217	0.0803	0.0111	0.943	0.311
	Normal	0.215	0.0218	0.0838	0.0115	0.936	0.318
	ns	0.213	0.0212	0.0693	0.00781	0.857	0.204
	Ridge	0.213	0.0215	0.0730	0.00897	0.904	0.261
<i>Study 2</i>							
Study 2	Horseshoe	0.325	0.0122	0.0536	0.00330	0.947	0.202
	Lasso	0.326	0.0124	0.0568	0.00319	0.951	0.223
	Normal	0.327	0.0125	0.0577	0.00326	0.949	0.225
	ns	0.327	0.0132	0.0598	0.00319	0.944	0.243
	Ridge	0.327	0.0129	0.0591	0.00487	0.840	0.173
<i>Study 3</i>							
Study 3	Horseshoe	0.342	0.0132	0.0394	0.00229	0.975	0.167
	Lasso	0.404	0.0194	0.104	0.00368	0.963	0.432
	Normal	0.418	0.0208	0.117	0.00446	0.955	0.464
	ns	0.365	0.0164	0.0619	0.00291	0.882	0.197
	Ridge	0.366	0.0153	0.0635	0.00206	0.845	0.181

Table 2: **Performance on Zero Coefficients Only.** Mean and SD of parameter RMSE (PRMSE) for zero coefficients, mean coverage (Cov), and mean interval length (Len).

Scenario	Method	PRMSE	PRMSE	Cov	Len
		Mean	SD		
<i>Study 1</i>					
Study 1	Horseshoe	0.0266	0.0082	0.999	0.176
	Lasso	0.0798	0.0119	0.945	0.313
	Normal	0.0839	0.0124	0.935	0.320
	ns	0.0606	0.0086	0.890	0.201
	Ridge	0.0711	0.0096	0.912	0.262
<i>Study 2</i>					
Study 2	Horseshoe	0.0357	0.0034	0.990	0.177
	Lasso	0.0558	0.0047	0.956	0.221
	Normal	0.0578	0.0049	0.950	0.225
	ns	0.0489	0.0039	0.978	0.240
	Ridge	0.0495	0.0037	0.904	0.173
<i>Study 3</i>					
Study 3	Horseshoe	0.020	0.002	1.000	0.152
	Lasso	0.107	0.004	0.965	0.445
	Normal	0.120	0.005	0.955	0.478
	ns	0.052	0.005	0.928	0.196
	Ridge	0.056	0.004	0.882	0.180

Table 3: **Performance on Nonzero Coefficients Only.** Mean and SD of parameter RMSE (PRMSE) for nonzero coefficients, mean coverage (Cov), and mean interval length (Len).

Scenario	Method	PRMSE	PRMSE	Cov	Len
		Mean	SD		
<i>Study 1</i>					
Study 1	Horseshoe	0.085	0.027	0.84	0.26
	Lasso	0.079	0.022	0.94	0.30
	Normal	0.080	0.023	0.94	0.31
	ns	0.098	0.028	0.70	0.22
	Ridge	0.078	0.023	0.86	0.25
<i>Study 2</i>					
Study 2	Horseshoe	0.0596	0.00391	0.929	0.213
	Lasso	0.0571	0.00327	0.949	0.224
	Normal	0.0576	0.00329	0.949	0.225
	ns	0.0639	0.00396	0.929	0.245
	Ridge	0.0626	0.00587	0.814	0.173
<i>Study 3</i>					
Study 3	Horseshoe	0.083	0.0049	0.86	0.24
	Lasso	0.092	0.0040	0.96	0.37
	Normal	0.098	0.0044	0.96	0.39
	ns	0.094	0.0063	0.67	0.20
	Ridge	0.088	0.0082	0.67	0.18

Table 4: **Times Each Method Is “Best” in Forecast or Parameter RMSE.** For each scenario and replication (50 total), we identify which method attains the lowest forecast RMSE or lowest parameter RMSE. Columns show the percentage of replications in which each method is best.

Best in Forecast RMSE		
Scenario	Method	% of Replications
Study 1	Horseshoe	60%
	ns	20%
	Normal	10%
	Ridge	6%
	Lasso	4%
Study 2	Horseshoe	48%
	Lasso	20%
	ns	20%
	Ridge	10%
	Normal	2%
Study 3	Horseshoe	100%

Best in Parameter RMSE (All Coefficients)		
Scenario	Method	% of Replications
Study 1	Horseshoe	100%
Study 2	Horseshoe	90%
	Ridge	10%
Study 3	Horseshoe	100%

Method	Mean RMSE	SD RMSE	Mean MAPE (%)	SD MAPE
Horseshoe	0.51	0.22	0.71	0.94
Lasso	0.60	0.25	1.24	1.87
NS	0.60	0.25	0.97	1.39
Normal	0.63	0.26	1.37	2.12
Ridge	0.56	0.24	1.06	1.51

Table 5: **Forecast error summaries for $\text{VAR}(p)$, $p = 1, \dots, 12$.** Shown are the mean and standard deviation of RMSE and MAPE (%) across the 12 lag choices. Horseshoe achieves the smallest mean RMSE and MAPE, while Normal exhibits the largest mean RMSE and MAPE. Ridge, NS, and Lasso provide intermediate performance.

Method	RMSE	MAPE (%)
Horseshoe	0.51	0.60
Lasso	0.70	1.66
Normal	0.78	1.81
ns	0.65	1.26
Ridge	0.61	1.66

Table 6: Forecasting accuracy on the Canada data for the VAR(11) model, evaluated on the final four observations. Lower RMSE and MAPE values indicate better performance.

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