1)

Apply second law of motion M_1

$$M_1 \frac{d^2y_1}{dt^2} = F - k_1 y_1 - b\frac{dy_1}{dt} - k_{12}(y_1 - y_2)$$

rearrange terms of eqn

$$M_1 \frac{d^2y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + k_{12}(y_1 - y_2) = F \quad \text{—— (1)}$$

apply Newton's II law of M_2

$$M_2 \frac{d^2y_2}{dt^2} + k_{12}(y_2 - y_1) = 0$$

$$\Rightarrow M_2 \frac{d^2y_2}{dt^2} + k_{12} y_2 = k_{12} y_1 \quad \text{—— (2)}$$

Laplace on both eqn

$$(1) \rightarrow \left[M_1 s^2 + bs + (k_1 + k_{12})\right] Y_1(s) = F(s) + k_{12} Y_2(s) \quad \text{—— (3)}$$

$$(2) \rightarrow M_2 s^2 Y_2(s) + k_{12} Y_2(s) = k_{12} Y_1(s)$$

$$\rightarrow Y_2(s) = \frac{k_{12}}{M_2 s^2 + k_{12}} Y_1(s) \quad \text{—— (4)}$$

now given

$f(t) = a \sin \omega_0 t$

Laplace $\longrightarrow F(s) = \dfrac{a\omega_0}{s^2 + \omega_0^2}$

For the mass M_1 to not vibrate

$y_1(t) = 0$

$Y_1(s) = 0$

$\therefore$ From (4)

$\underset{0}{\underline{Y_1(s)}} = \left(\dfrac{M_2 s^2 + k_{12}}{k_{12}}\right) Y_2(s)$

$\therefore \quad M_2 s^2 = -k_{12} \quad$ (or) $\boxed{k_{12} = -M_2 s^2}$

but $s = j\omega_0$

$k_{12} = -(-\omega_0^2) M_2$

$\boxed{k_{12} = \omega_0^2 M_2.}$ ✓

PTO

Sub $M_2 \omega^2$ for K_{12} in (1)

$$M_1 \frac{d^2 y_1}{dt^2} + b \frac{dy_1}{dt} + k_1 y_1 + M_2 \omega_0^2 (y_1 - y_2) = F$$

$$M_1 \frac{d^2 y_1}{dt^2} + b \frac{dy_1}{dt} + k_1 y_1 + M_2 \omega_0^2 y_1$$
$$= F + M_2 \omega_0^2 y_2$$

but $F(t) = a \sin(\omega_0 t)$

&

$M_2 \omega_0^2 = k_{12}$

$$\boxed{M_2 \frac{d^2 y_2}{dt^2} + M_2 \omega_0^2 y_2 = M_2 \omega_0^2 y_1}$$

$$M_1 \frac{d^2 y_1}{dt^2} + b \frac{dy_1}{dt} + k_1 y_1 + M_2 \omega_0^2 y_1$$
$$= a \sin \omega_0 t + M_2 \omega_0^2 y_2$$