

2nd Problem

Suspension for a wheel on a bumpy road is shown.

Let vehicle mass $\rightarrow m_1$

wheel mass $\rightarrow m_2$

Spring in suspension has a spring const k_1

Tire also has a spring const k_2

Damping constant $\rightarrow b$

wkt Transfer func $\frac{X_1(s)}{X(s)} \Rightarrow$ which is required answer

wkt

$$\text{Inertia} + \text{restoring force} + \text{Damping force} = \text{Exciting Force}$$

I

First

Forces on m_1 :

$$\text{inertia} = m_1 \frac{d^2 y_1}{dt^2}$$

$$\text{Restoring } F = k_1 (y_1 - y_2)$$

$$\text{Damping } F = b \frac{d(y_1 - y_2)}{dt}$$

$$\text{exciting} = 0$$

Forces on m_2 :

$$\text{inertia} = m_2 \frac{d^2 y_2}{dt^2}$$

$$\text{Restoring } F = k_1 (y_1 - y_2)$$

$$\text{Damping } F = b \frac{d(y_2 - y_1)}{dt}$$

$$\text{Exciting force} = k_2 x$$

now with help of eq (I)

$$m_1 \frac{d^2 y}{dt^2} + b \frac{d(y_1 - y_2)}{dt} + k_1 (y_1 - y_2) = 0 \quad (\text{for } m_1)$$

$$m_2 \frac{d^2 y}{dt^2} + b \frac{d(y_2 - y_1)}{dt} + k_1 (y_2 - y_1) + k_2 y_2 = k_2 x \quad (\text{for } m_2)$$

now apply Laplace Transformation ; zero initial conditions

$$m_1 s^2 Y_1(s) + bs(Y_1(s) - Y_2(s)) + k_1(Y_1(s) - Y_2(s)) = 0$$

→ (1)

$$m_2 s^2 Y_2(s) + bs(Y_2(s) - Y_1(s)) + k_1(Y_2(s) - Y_1(s)) + k_2 Y_2(s)$$

$$= k_2 x(s)$$

→ (2)

after rearranging ; $Y_2(s) = \frac{(m_1 s^2 + bs + k_1)}{(bs + k_1)} Y_1(s)$

$$\therefore \frac{Y_1(s)}{X(s)} = \frac{k_2 (bs + k_1)}{[-(bs + k_1)^2 + (m_2 s^2 + bs + k_1 + k_2)(m_1 s^2 + bs + k_1)]}$$

Transfer function