Spring 2013

$\boxed{\#3}$

$$2 - \tan\theta \cdot \cot\theta = 2 - \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}$$

$$= 2 - 1$$

$$= 1$$

ALTERNATIVELY

$$2 - \tan\theta \cdot \cot\theta = 2 - \cancel{\tan\theta} \cdot \frac{1}{\cancel{\tan\theta}}$$

$$= 2 - 1$$

$$= 1$$

Spring 2013

$\boxed{\#4}$

$$\tan\theta = \frac{\sqrt{2}}{2} \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}}{2}$$

$$\rightarrow \quad \sin\theta = \frac{\sqrt{2}}{2}\cos\theta$$

Now

$$\sin^2\theta + \cos^2\theta = 1$$

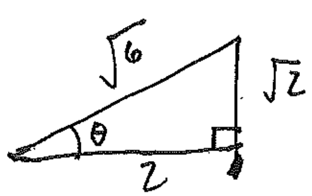
$$\left(\frac{\sqrt{2}}{2}\cos\theta\right)^2 + \cos^2\theta = 1$$

$$\frac{2}{4}\cos^2\theta + \cos^2\theta = 1 \quad \longrightarrow \quad \cos^2\theta = \frac{2}{3}$$

$$\rightarrow \quad \cos\theta = \pm\sqrt{\frac{2}{3}} \quad \text{since} \quad \pi < \theta < \frac{3\pi}{2}$$

$$\cos\theta < 0 \quad \text{so} \quad \boxed{\begin{aligned} \cos\theta &= -\sqrt{\frac{2}{3}} \\ \sin\theta &= -\frac{\sqrt{2}}{2}\cdot\sqrt{\frac{2}{3}} \\ &= -\frac{1}{\sqrt{3}} \end{aligned}}$$

ALTERNATIVELY



$$\cos\theta = -\frac{2}{\sqrt{6}} = -\sqrt{\frac{2}{3}}$$

$$\sin\theta = \frac{-\sqrt{2}}{\sqrt{6}} = -\frac{1}{\sqrt{3}}$$

Spring 2013

$\boxed{\#5}$ $\ln c = -1$

So $e^{\ln c} = e^{-1} \rightarrow \boxed{c = e^{-1}}$

Spring 2012 Exam 2

$\boxed{\#3}$ $P(t) = P_0 e^{rt}$

Growth Rate 20% $\rightarrow$ $r = .2$

Current Population 50 $\rightarrow$ $P_0 = 50$

a) $P(t) = 50e^{.2t}$

b) $P(5) = 50e^{.2 \cdot 5} = 50e^1$

c) $50e^{.2t} = 500$

$\rightarrow$ $e^{.2t} = 10$

$\rightarrow$ $\ln e^{.2t} = \ln 10$

$\rightarrow$ $.2t = \ln 10$

$\rightarrow$ $t = \dfrac{\ln 10}{.2} = \dfrac{\ln 10}{\frac{2}{10}} = \dfrac{10 \ln 10}{2} = 5 \ln 10$

Spring 2012 Exam 2

$\boxed{\#4}$ Line: $y = -x$

Circle: $x^2 + y^2 = 1$

Substitute

$\longrightarrow$ $x^2 + (-x)^2 = 1$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \frac{\pm 1}{\sqrt{2}}$

So points are $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Spring 2012 Exam 2

$\boxed{\#5a}$

$$\ln x + \ln(x+1) = \ln 20$$

$$\ln x\cdot(x+1) = \ln 20$$

$$\rightarrow \cancel{e^{\ln}}\, x(x+1) = \cancel{e^{\ln}}\, 20$$

$$\rightarrow x(x+1) = 20$$

$$\rightarrow x^2 + x - 20 = 0$$

$$(x-4)(x+5) = 0$$

$$x = 4 \quad , \quad x = -5$$

But $\ln -5$ is not defined!

$$\boxed{x = 4}$$

Fall 2011 Exam 2 Malitsky

$\boxed{\#3}$ a) $\cosh(x) = \dfrac{e^{x}+e^{-x}}{2}$

$$\cosh(0) = \frac{e^{0}+e^{-0}}{2} = \frac{1+1}{2} = 1$$

b) $\cosh(5) - \cosh(-5)$

$$= \frac{e^{5}+e^{-5}}{2} - \frac{e^{-5}+e^{5}}{2}$$

$$= \frac{e^{5}+e^{-5}-(e^{-5}+e^{5})}{2} = \frac{e^{5}+e^{-5}-e^{-5}-e^{5}}{2}$$

$$= \boxed{0}$$

Fall 2011 Exam 2 Malitsky

$\boxed{5a}$ $e^{2t} + e^{t} = 6$

Let $u = e^{t}$ so $u^2 = e^{2t}$

$u^2 + u = 6 \rightarrow u^2 + u - 6 = 0$

$\rightarrow (u-2)(u+3) = 0$

$\rightarrow e^{t} = 2 \quad , \quad \cancel{e^{t} = -3}$

$\rightarrow \boxed{t = \ln 2}$

$\boxed{5b}$

Continuous Growth rate $\rightsquigarrow$ $P = P_0 e^{rt}$

After 1 hour

$$2\cancel{P_0} = \cancel{P_0} e^{r} \rightarrow t = 1$$

$$2 = e^{r}$$

$$\rightarrow \ln 2 = \ln e^{r}$$

$$\ln 2 = r \cdot 1 \rightarrow r = \ln 2$$

So $P = P_0 e^{t \ln 2}$

Now solve

$$3\cancel{P_0} = \cancel{P_0} e^{t \ln 2}$$

$$3 = e^{t \ln 2}$$

$$\ln 3 = \ln e^{t \ln 2}$$

$$\ln 3 = t \ln 2$$

$$\rightarrow t = \frac{\ln 3}{\ln 2}$$

Review Problems <u>pg 311</u>

$\boxed{4.}$ $f(x) = \log_2(5x+1)$

So $5x+1 > 0 \longrightarrow x > \frac{-1}{5}$ or $x \in \left(\frac{-1}{5}, +\infty\right)$

$\boxed{9}$ $\log_3(4^x+1) = 2$

3 3

$\rightarrow 4^x + 1 = 3^2$

$\rightarrow 4^x = 9-1 = 8$

$\rightarrow \log_4 4^x = \log_4 8$

$\rightarrow x = \log_4 8 = \log_4(4\cdot 2)$

$= \log_4 4 + \log_4 2$

$= 1 + \frac{1}{2} = \frac{3}{2}$

Alternatively

$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

$\boxed{11}$ $\log_5 \sqrt{125} = \log_5 125^{1/2}$

$= \frac{1}{2}\log_5 125$

$= \frac{3}{2}$

$\boxed{19}$ $\log x - \log y = \log \dfrac{x}{y}$

$\dfrac{\log x}{\log y} \rightarrow$ CAN'T BE SIMPLIFIED

$\boxed{20}$ $f(x) = 4 + 5\log_3(7x+2)$

$y = 4 + 5\log_3(7x+2)$

Switch xy: $x = 4 + 5\log_3(7y+2)$

$x - 4 = 5\log_3(7y+2)$

$\rightarrow \dfrac{x-4}{5} = \log_3(7y+2) \rightarrow 3^{\frac{x-4}{5}} = 7y + 2$

$\rightarrow \boxed{\dfrac{3^{\frac{x-4}{5}} - 2}{7} = y}$

$\boxed{25}$

$$\log_7\left(\frac{49w^2}{z^3}\right) = \log_7(49w^2) - \log_7 z^3$$

$$= \log_7 49 + \log_7 w^2 - \log_7 z^3$$

$$= 2 + 2\log_7 w - 3\log_7 z$$

$$= 2 + 2 \cdot 3.1 - 3 \cdot 2.2$$

$$= 2 + 6.2 - 6.6$$

$$= 8.2 - 6.6 = 1.6$$

$\boxed{42}$

$$e^{2w-7} = 6 \quad \to \quad \ln e^{2w-7} = \ln 6$$

$$\to 2w - 7 = \ln 6$$

$$\to 2w = \ln 6 + 7$$

$$w = \frac{\ln 6 + 7}{2}$$

$\boxed{50}$ Remember $e^x \approx 1 + x$ for tiny $|x|$

$$\frac{e^{1000.002}}{e^{1000}} = e^{1000.002 - 1000}$$

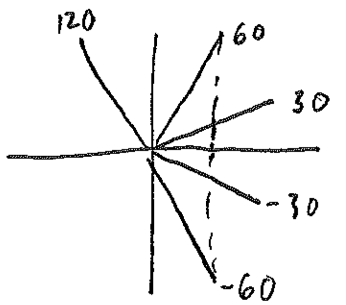
$$= e^{.002} = 1 + .002$$

$$= 1.002$$

Review Problems pg 390

$\boxed{5}$ $27 \cdot \dfrac{\pi \text{ rad}}{180 \text{ deg}} = \dfrac{\cancel{27}^{\cancel{9}\ 3}\pi}{\cancel{180}\ \cancel{60}\ 20} = \dfrac{3\pi}{20}$

$\boxed{9}$



$\boxed{60°, -60°, 120°}$

$\boxed{17}$ $\tan x = -4 \rightsquigarrow \dfrac{\sin x}{\cos x} = -4$

$\rightsquigarrow \sin x = -4\cos x$

$\sin^2 x + \cos^2 x = 1$

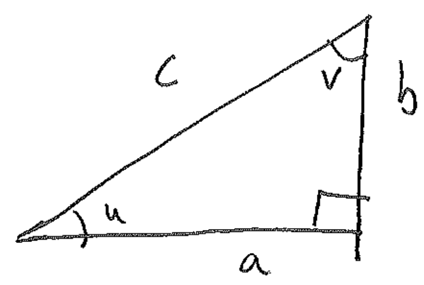
$\to$ ~~[illegible]~~ $(-4\cos x)^2 + \cos^2 x = 1$

$\to 17\cos^2 x = 1 \to \cos^2 x = \dfrac{1}{17}$

$\to \cos x = -\sqrt{\dfrac{1}{17}} \leftsquigarrow \dfrac{\pi}{2} < x < \pi$

$\sin x = 4\sqrt{\dfrac{1}{17}}$

$\boxed{19}$



Suppose $a = 4$, $b = 9$

a) $c = \sqrt{4^2 + 9^2} = \sqrt{52} = 2\sqrt{13}$

b) $\cos u = \dfrac{4}{2\sqrt{13}} = \dfrac{2}{\sqrt{13}}$

c) $\sin u = \dfrac{9}{2\sqrt{13}}$

d) $\tan u = \dfrac{9}{4}$

e) $\cos v = \dfrac{9}{2\sqrt{13}}$

f) $\sin v = \dfrac{4}{2\sqrt{13}} = \dfrac{2}{\sqrt{13}}$

g) $\tan v = \dfrac{4}{9}$

$\boxed{23}$

$\cos\theta = \cos(-\theta) = \dfrac{3}{8}$

~~cos u~~

$\boxed{31}$

$\tan\theta = -\tan(360 - \theta)$

$\tan 20 = -\tan(360 - 20)$

$= -\tan(340)$

So $\tan 20 + \tan 340 = 0$

$\boxed{32}$ $\log(\cos\theta)$ when ~~cosθ~~ $\theta \in (0, \frac{\pi}{2})$

$\cos\theta \in (0, 1)$

$\log x < 0$ for $x \in (0, 1)$