University of Toronto

Department of Electrical and Computer Engineering

ECE411S – Real-time Computer Control

LAB 3

IMPLEMENTATION ISSUES IN DIGITAL CONTROL

1 Purpose

The purpose of this experiment is to investigate issues that arise in the implementation of digital controllers. The effects of quantization and finite word-length on signal representation as well as system parameters are studied. Different forms for the realization of digital controllers and their numerical accuracy are investigated. The possibility of quantization-induced limit cycles is demonstrated. Finally, a sampled-data control system is implemented with 16-bit accuracy and its performance evaluated.

2 Introduction

In our study of digital control design, we have implicitly assumed that computations can be done with infinite precision. In an actual implementation, the digital control law is realized through a microcontroller or a digital signal processing chip. These would have finite precision, usually 8-bit or 16-bit. The analog-to-digital converter (ADC) as well as the digital-to-analog converter (DAC) also have finite precision, usually 8-bit, 10-bit, or 12-bit. The digital control law calculation is often done in fixed-point arithmetic, although we do not go into the details of the implementation.

The finite word-length issue has a significant impact on how to implement a digital controller. To evaluate the performance of a sampled-data control system, we need to quantize all controller coefficients as well as signals coming out from the ADC and DAC. In this experiment, we examine typical situations to illustrate the finite word-length effects. You will also implement the sampled-data control system described in Section 4.6 of the course notes.

3 Preparation

Familiaze yourself with the operation of the quantizer in simulink, which can be found in the Discontinuities library. In a directory which is in the path for Matlab, add the m-file quant.m, consisting of the lines:

```
function [xq]=quant(x,q)
xq=q*round(x/q);
```

Here q is the quantization value. You will use quant.m to quantize the coefficients of the control law. Next, complete the following preparation exercises and hand in your answers, **on**

a separate sheet, to your T.A. at the beginning of the lab. Every student **must hand in his/her own work**. There is space provided below to record your preparation work for the experiment.

3.1 Control Design Calculations

1. In preparation for the design calculations for the sampled-data control problem described in Section 4.6, enter the A, B, and D matrices as described on p.123 into Matlab. Take the sampling interval $T = 0.1$ sec. Compute $[A_d, B_d] = c2d(A, B, T)$ and partition the matrices of the resulting discrete time system into the form

$$x' = A_d x + B_d u \quad e = Dx$$

with

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A_d = \begin{bmatrix} A_1 & A_3 \\ 0 & A_2 \end{bmatrix}, \quad B_d = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} D_1 & D_2 \end{bmatrix}$$

The controller can be designed as in the design method on p. 125 in the course notes.

2. Choose F_1 to place the poles of $A_1 + B_1 F_1$ at $0, 0$. Repeat the calculation to place the poles of $A_1 + B_1 F_1$ at $0.8 \pm j0.2$.

3. Determine the matrices needed to solve the linear algebraic equation (4.8) in the course notes for the 2 choices of F_1. Determine the corresponding solutions for X and F_2. Note that the command to vectorize a matrix A in Matlab is $A(:)$, and the Kronecker product of A and B is $kron(A, B)$.

4. Determine the gain for the full order observer to place the poles of $A_d + LD$ at $0, 0, 0, 0, 0$. Repeat to place the poles at $0.1, 0.1 \pm j0.3, 0.2 \pm j0.2$. The controller is given as (see Step 3 on page 125)

$$\hat{x}' = (A_d + B_d F + LD)\hat{x} - Le, \qquad u = F\hat{x}$$

where $F = \begin{bmatrix} F_1 & F_2 \end{bmatrix}$. For future reference, call the output feedback controller with all poles placed at 0, Controller 1. The second one is Controller 2. Determine their respective transfer functions. Save your data also to a usb stick for use in the lab.

5. Sketch the simulink diagram that you would use to simulate the sampled-data control system, incorporating explicitly the quantizer after the zero-order holds. Put also the sketch in the box below for reference during the experiment.

4 Experiment

4.1 Effects of Quantization

1. We illustrate the effects of quantization with an example. Suppose the plant is given by the transfer function $P(z) = \frac{1}{z-1}$, and the controller is given by $C(z) = \frac{(z-0.522)(z+0.45)}{(z-0.96)(z-0.94)(z-0.92)}$, with the gain $K = 10^{-5}$ in the single loop feedback configuration:

   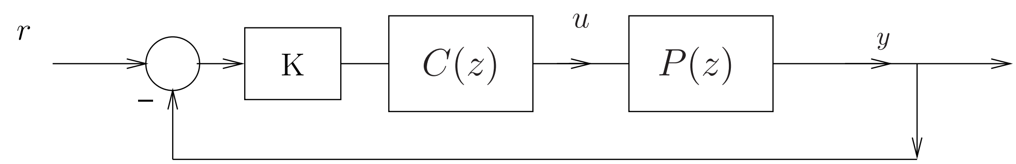

   Figure 1: Single loop feedback system

   Determine the poles of the closed loop system and record them in the space below. Are they stable?

   Construct the simulink diagram to simulate the step response of this discrete-time control system. Print and label the Simulink model, and print and label the output of the closed loop system for inclusion in your report. Note that the response of the system is quite slow and you need to run the simulation for at least 2000 steps.

2. Let the quantization level $q = 2^{-16}$. This corresponds to using 16-bit processor for computation. Quantize the gain K, and the numerator and denominator of $C(z)$ if you use a transfer function representation, or the zeros and poles of $C(z)$ if you use a pole-zero representation for $C(z)$. Re-run the simulation with these quantized values for the gain and the controller. Print and label the output of this system for inclusion in your report. Comment on any differences between the outputs and the reasons for the difference in the space below.

4.2 Improving Controller Realization

Realizing a controller using a single transfer function is often referred to as direct form realization. We can improve the numerical properties of the controller by using alternative realization forms.

1. In the case of real poles in the controller, first decompose the controller transfer function into a cascade of first order transfer functions, e.g. $C(z) = C_1(z)C_2(z)C_3(z)$, where $C_1(z) = \frac{1}{z-0.96}$, $C_2(z) = \frac{z-0.522}{z-0.94}$, and $C_3(z) = \frac{z+0.45}{z-0.92}$. Now quantize the coefficients of $C_1(z)$, $C_2(z)$, $C_3(z)$, and re-run the simulation. Print the output response and discuss any differences in the responses in the space below.

2. Although you do not need to work this out, you can also realize $C(z) = C_1(z) + C_3(z) +$

$C_3(z)$, with each $C_i(z)$ a first order transfer function. This will give rise to a parallel implementation of $C(z)$.

4.3 Existence of Limit Cycles

Since the quantizer is a nonlinear operation, the presence of the quantizer in the loop, for example to quantize input and output signals, can trigger nonlinear phenomena. One effect that can be observed is that of a limit cycle. Informally, a limit cycle is a sustained oscillation which does not die out. For control specifications such as step tracking, the presence of limit cycles is undesirable.

Consider the following feedback system

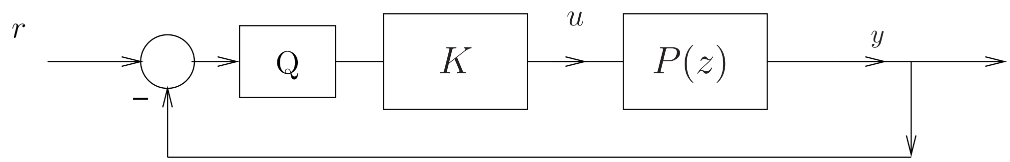

Figure 2: Feedback system with quantizer

where $P(z) = \frac{0.25}{(z-1)(z-0.5)}$, K is the gain, and Q represents the quantizer. For this part, take the quantization interval for Q to be 0.2. Without the quantizer, the closed loop system is stable for $K < 2$, and hence step tracking is achieved. Starting with $1 \leq K < 2$, determine experimentally a value of the gain K such that a limit cycle is observed at the output. Record the value of K in the space below and print the output to demonstrate the existence of the limit cycle.

4.4 Controller Implementation for Step Tracking and Disturbance Rejection

In this section, you complete the controller implementation for the cart/spring system described in the course notes for step tracking and sinusoidal disturbance rejection.

1. Assume that 16-bit ADC and DAC are used and that the controller coefficients are quantized at the quantization level of 2^{-16}. Implement the sampled-data control system for the continuous time cart/spring system based on Controller 1 design, incorporating explicitly quantization wherever appropriate.

2. Repeat for the design based on Controller 2.

3. In the space below, discuss concisely the performance of the 2 control designs. Your discussion should include the accuracy of step tracking and disturbance rejection, the transient

response, the effects of choice of closed loop poles, and any other aspects arising from your experimental work.

Finally, demonstrate to the TA all your experimental results using your Simulink diagrams and the Simulink scope.

- *Name:*
- *Student No.:*
- *Name:*
- *Student No.:*
- *Lab Section, Date, Time:*