Measuring Performance

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Ways of Measuring

Ways of Measuring

Timing with stopwatch

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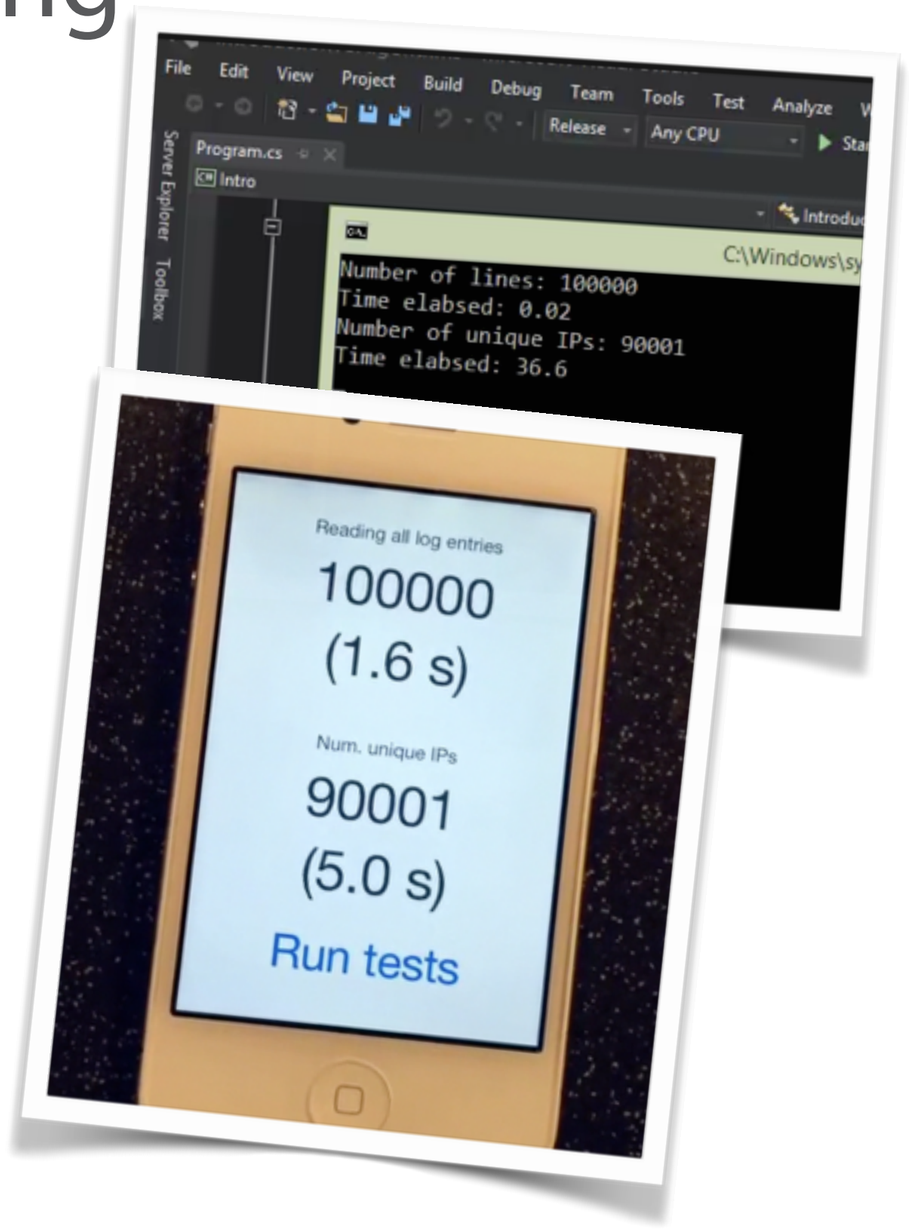
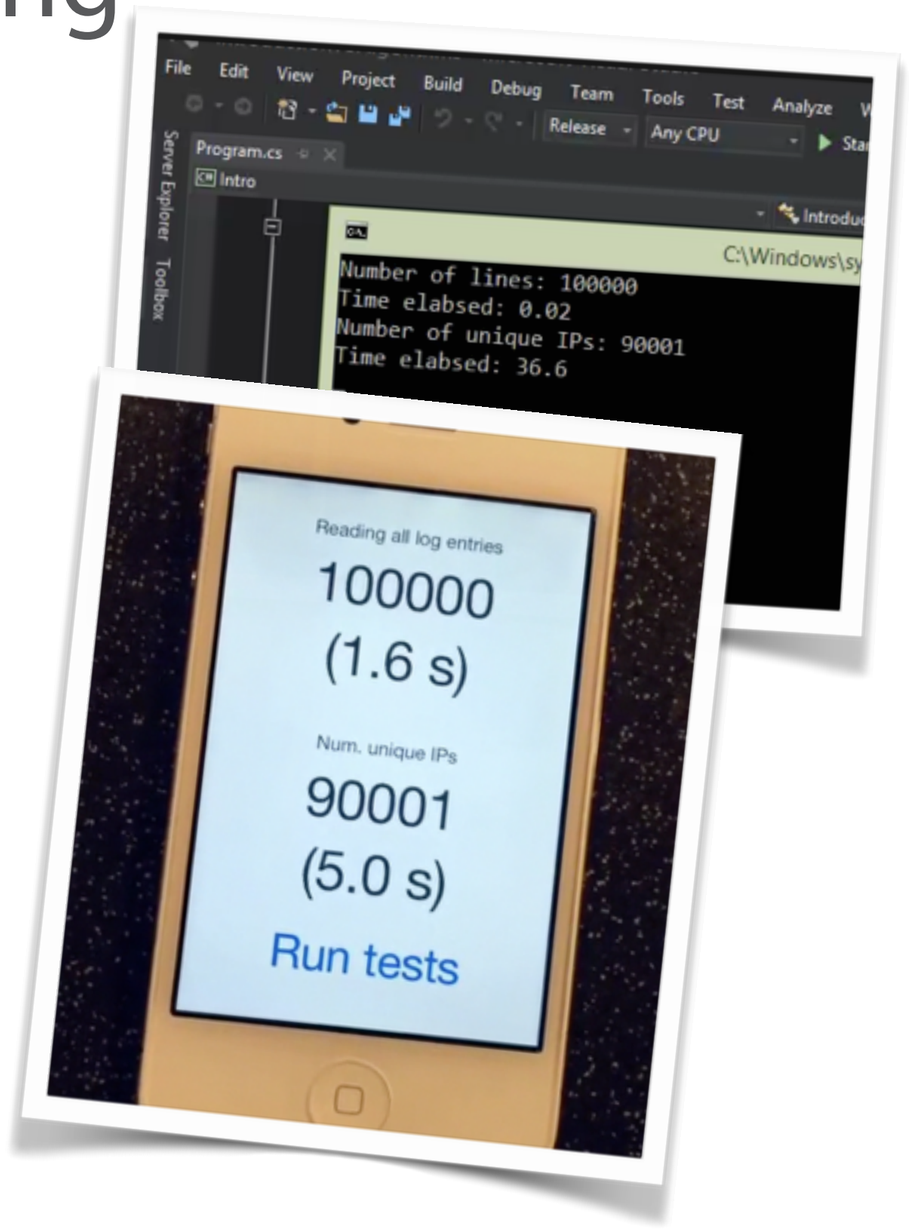
Timing with stopwatch



Ways of Measuring

Timing with stopwatch

Depends on…
hardware,
programming language,
environment,
etc.



Ways of Measuring

Timing with stopwatch

Counting instructions

Ways of Measuring

```
func readAllLogs() -> Int
{
    var reader = LogReader();
    var linesSeen = 0;
    for logLine in reader.GetLogLines() {
        logLine.getIP();
        linesSeen++;
    }
    return linesSeen;
}
```

Counting instructions

Ways of Measuring

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Counting instructions

100,000 lines:
2 + 100,000 · 4 + 1 instructions

Ways of Measuring

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    }
    return linesSeen;
}
```

Counting instructions

100,000 lines:
2 + 100,000 · 4 + 1 instructions

N lines:
2 + *N* · 4 + 1 instructions

Ways of Measuring

```
func readAllLogs() -> Int
{
    var reader = LogReader();
    var linesSeen = 0;
    for logLine in reader.GetLogLines() {
        logLine.getIP();
        linesSeen++;
    }
    return linesSeen;
}
```

Counting instructions

How does execution time grow, when *N* grows?

100,000 lines:
$2 + 100{,}000 \cdot 4 + 1$ instructions

N lines:
$2 + N \cdot 4 + 1$ instructions

Ways of Measuring

Timing with stopwatch

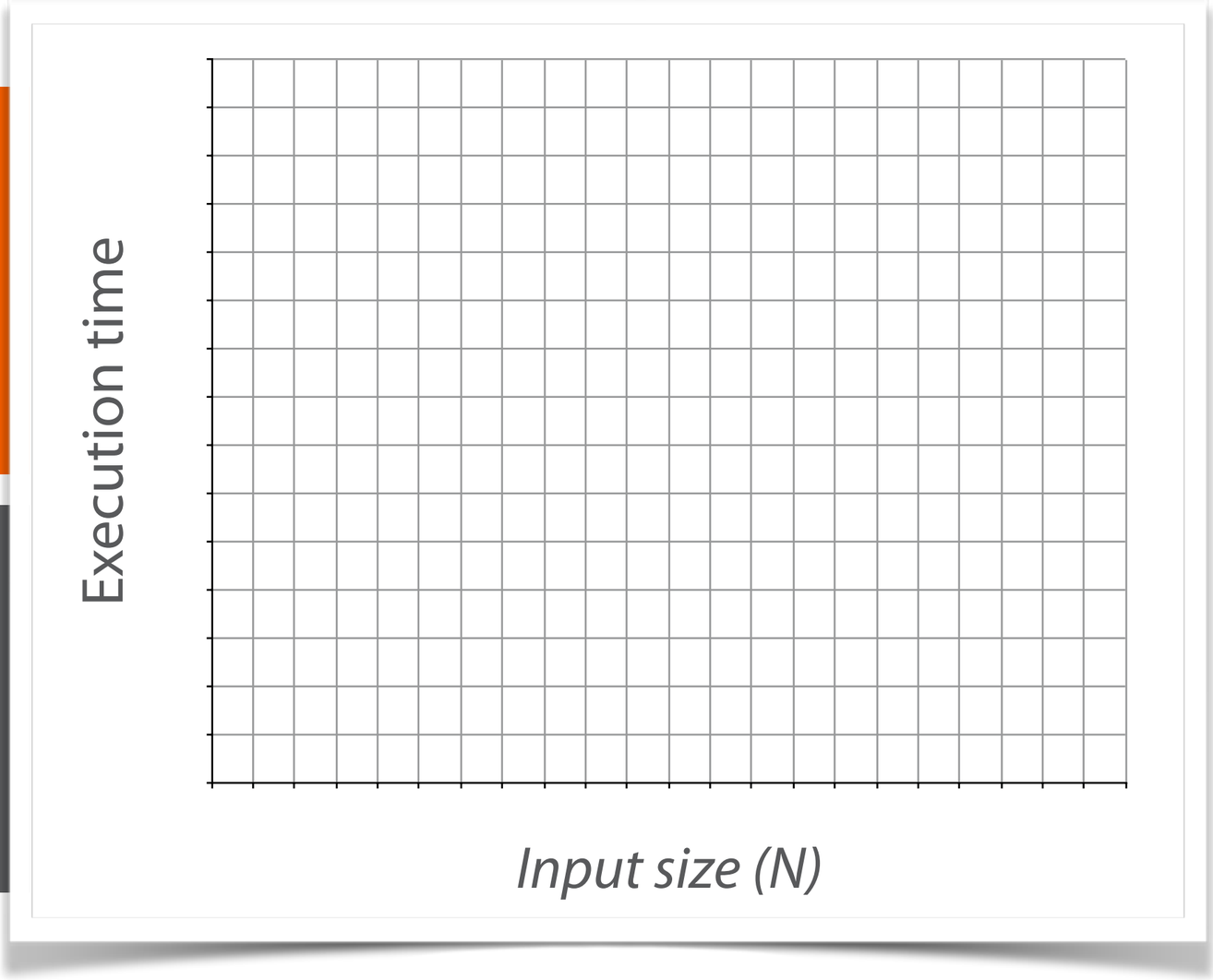
Counting instructions

Looking at the curve

Ways of Measuring

Timing with stopwatch

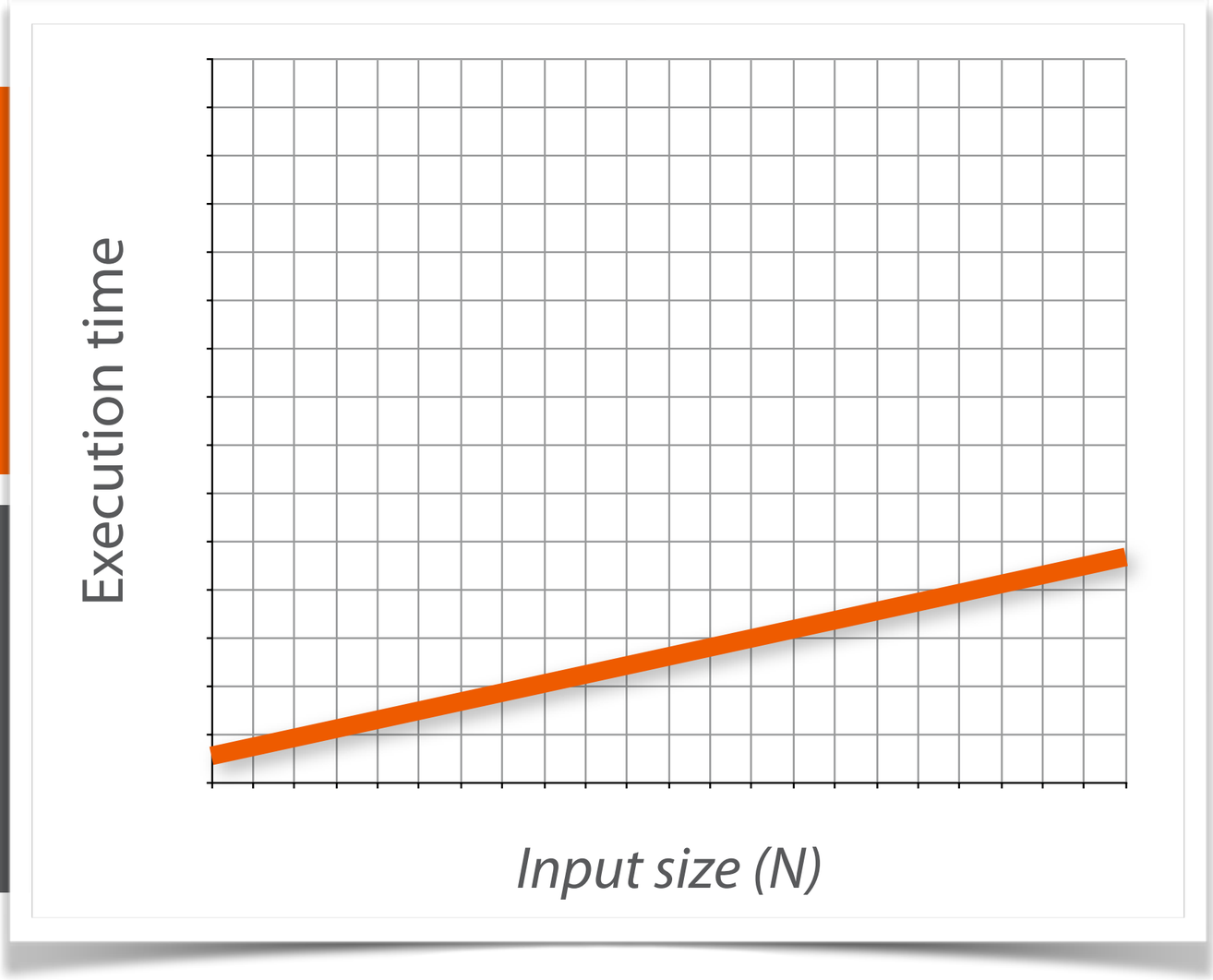
Looking at the curve



Ways of Measuring

Timing with stopwatch

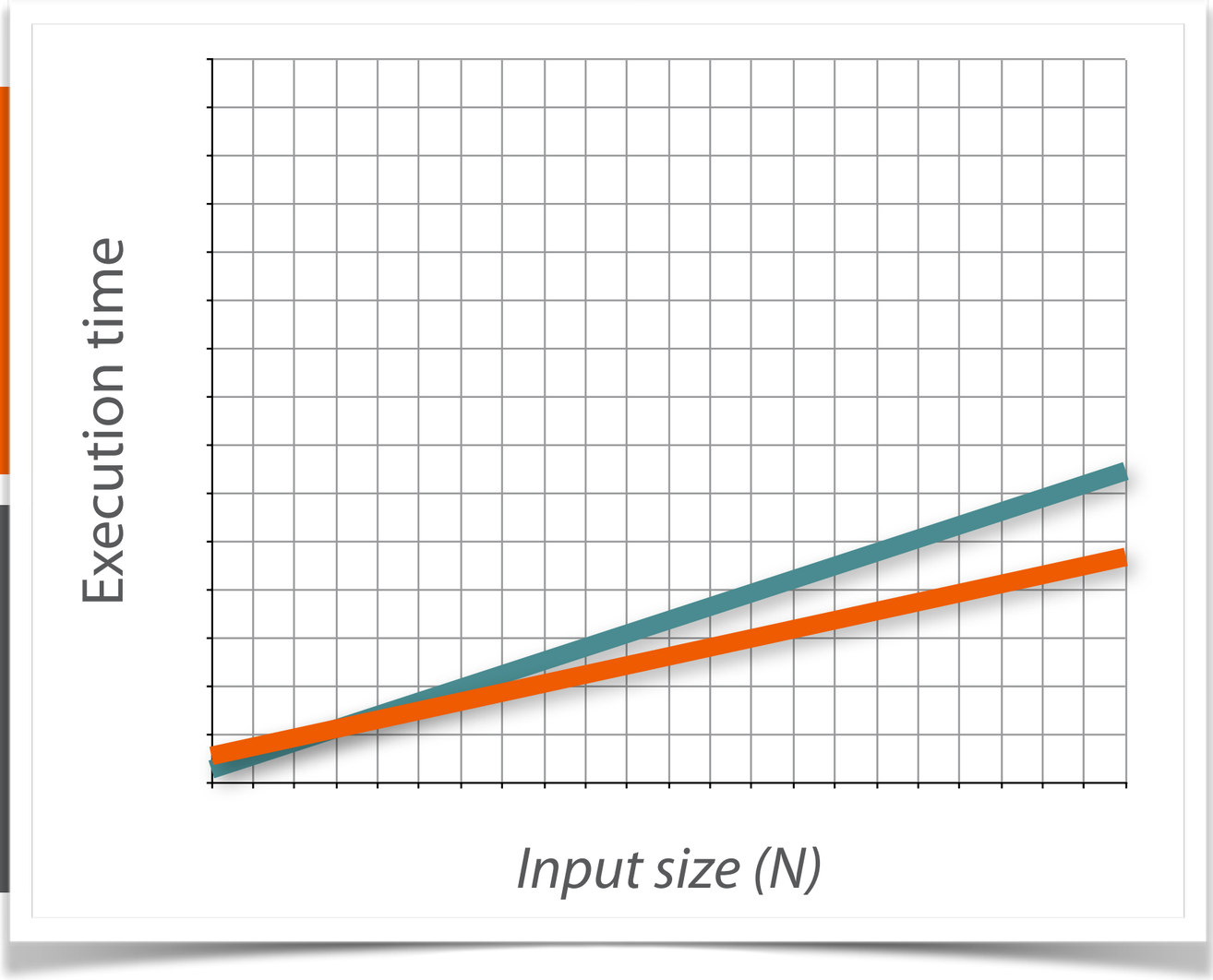
Looking at the curve



Ways of Measuring

Timing with stopwatch

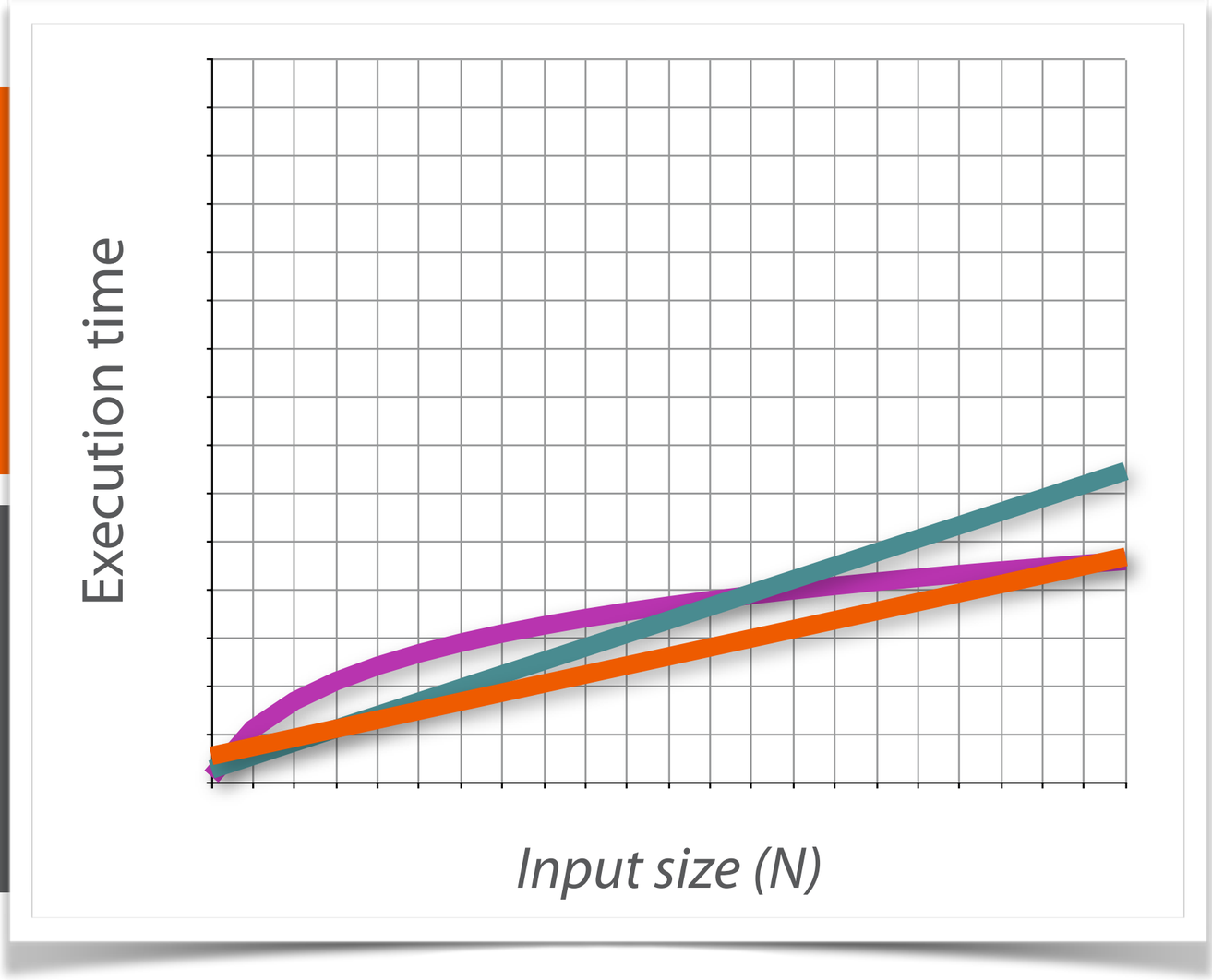
Looking at the curve



Ways of Measuring

Timing with stopwatch

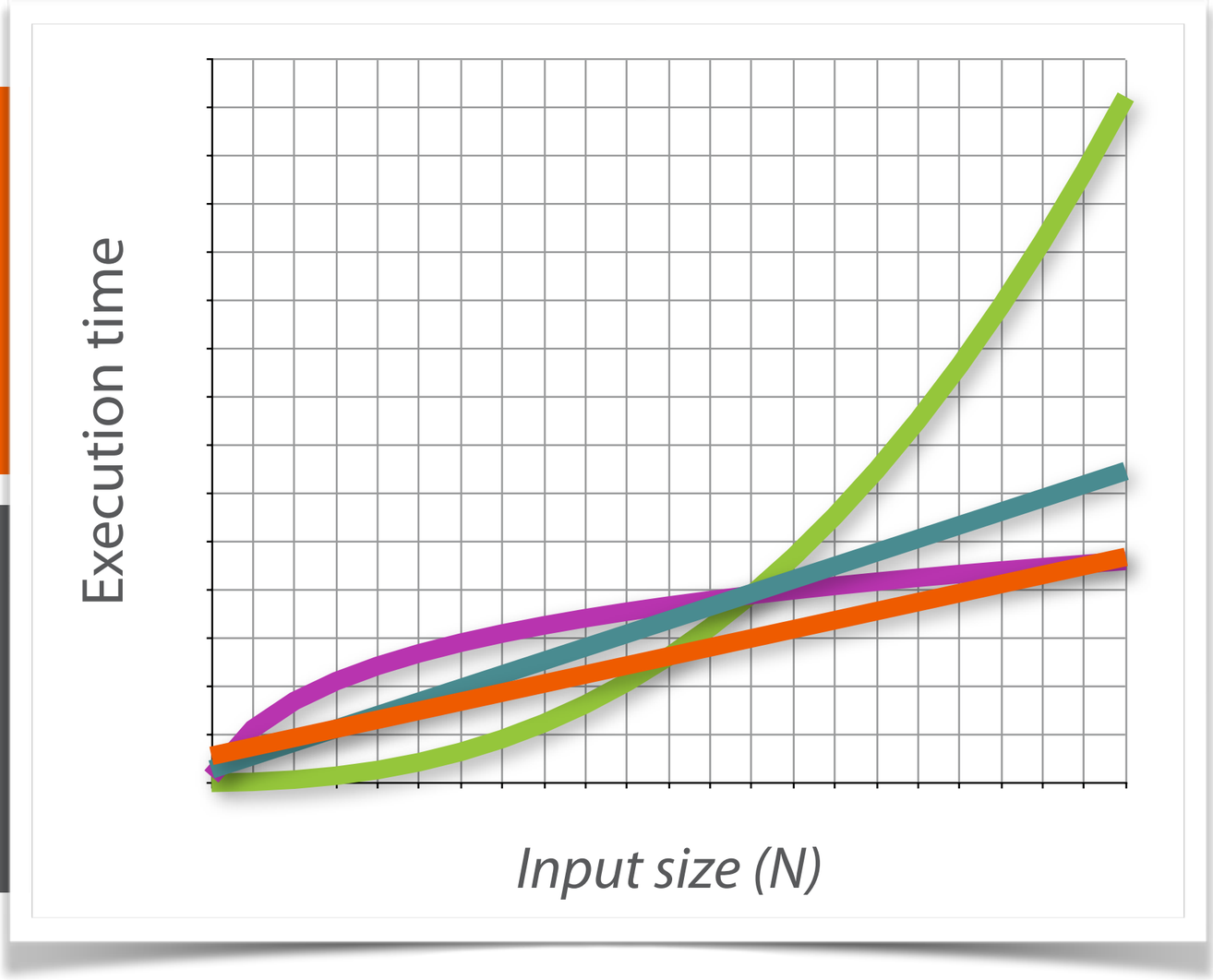
Looking at the curve



Ways of Measuring

Timing with stopwatch

Looking at the curve



Ways of Measuring

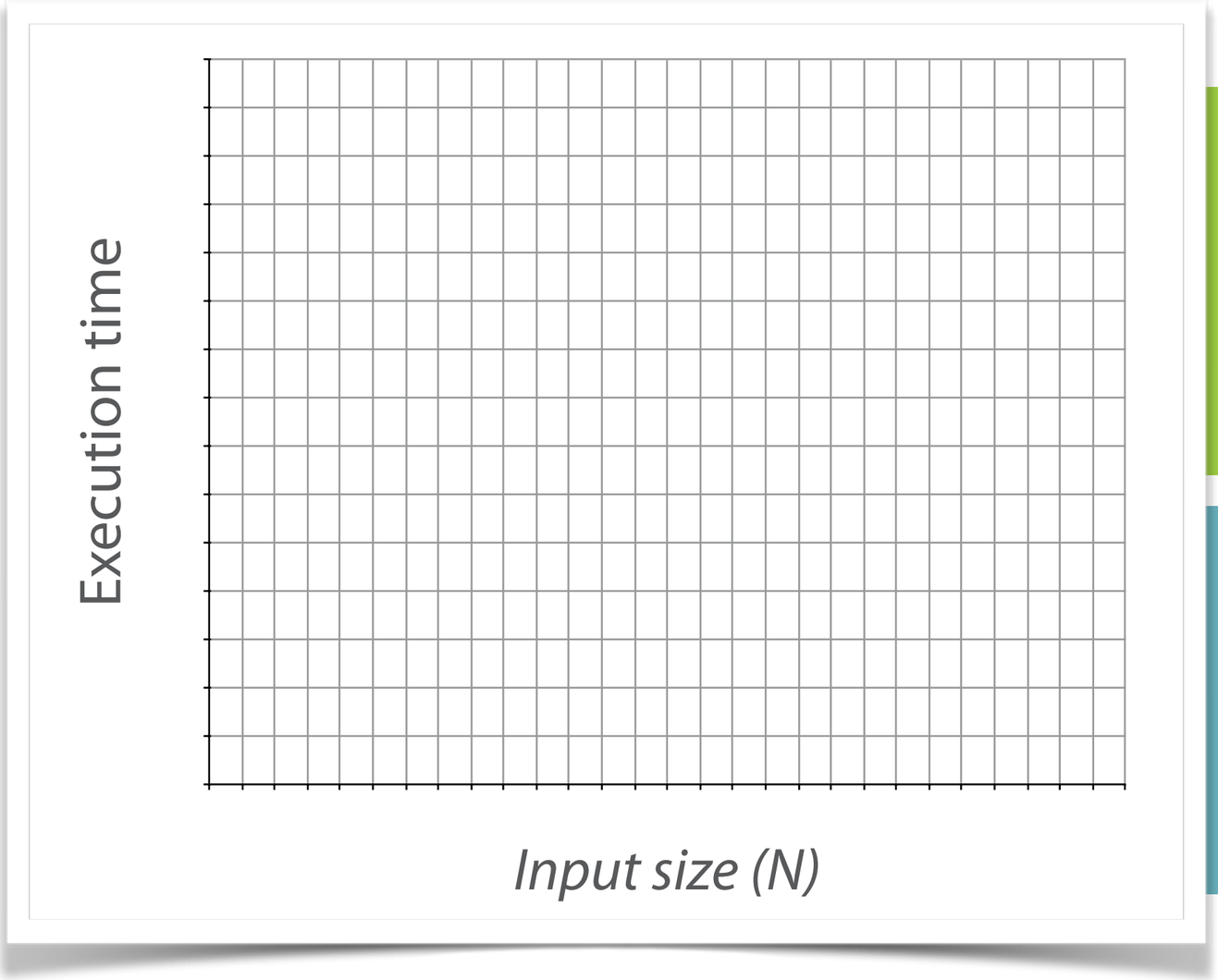
Timing with stopwatch

Counting instructions

Looking at the curve

Best case
Worst case
Average

Ways of Measuring



Counting instructions

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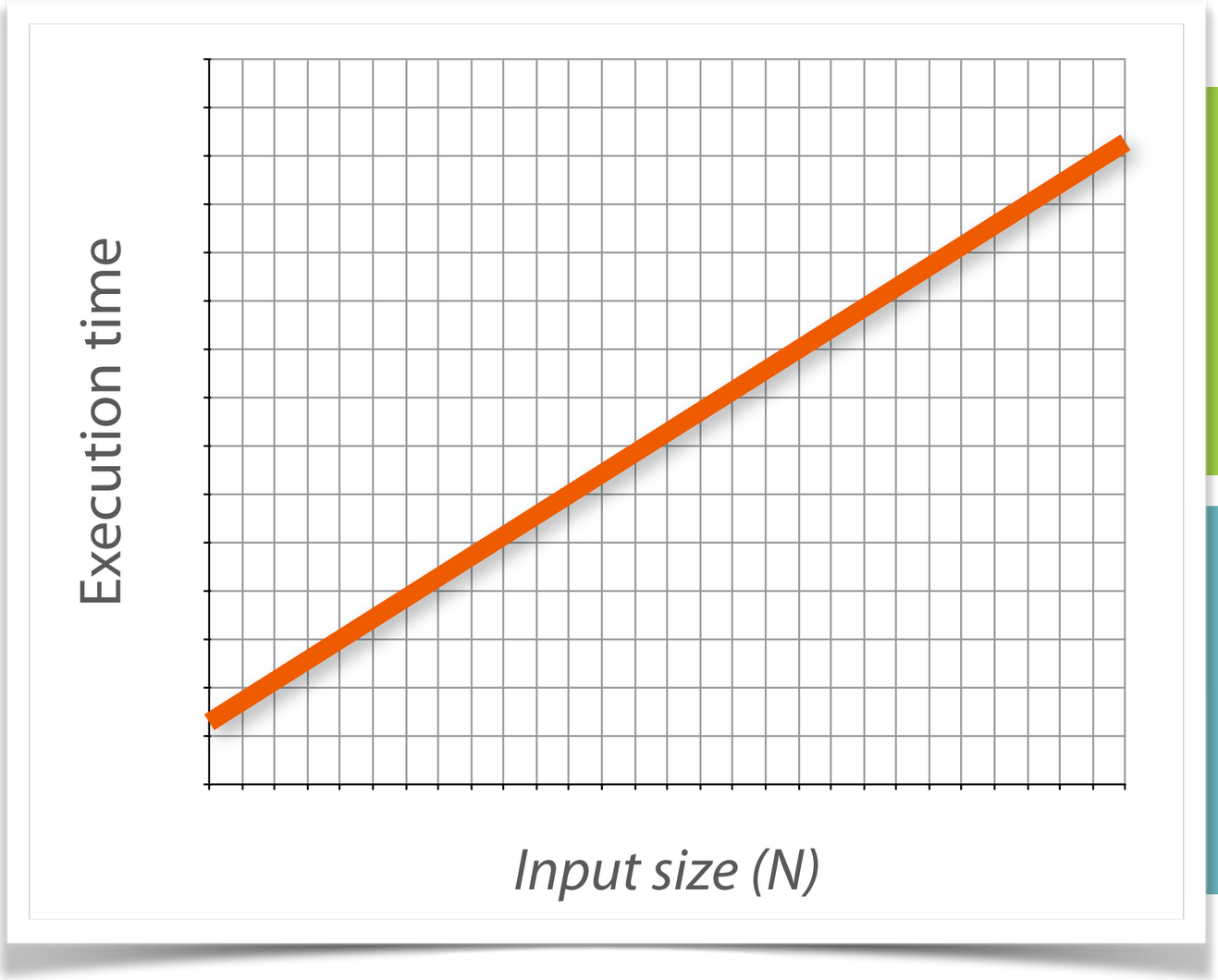
Ways of Measuring



Counting instructions

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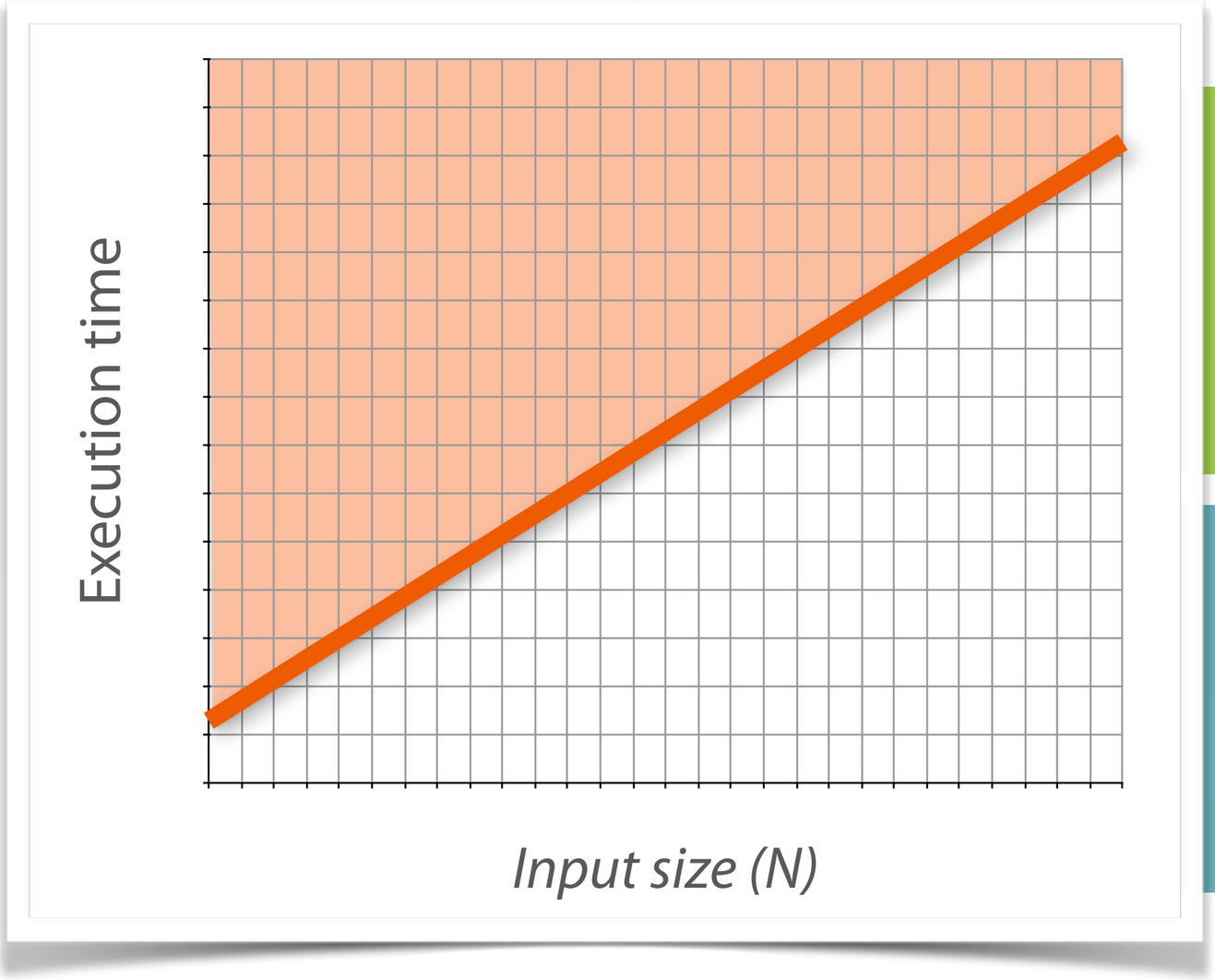
Ways of Measuring



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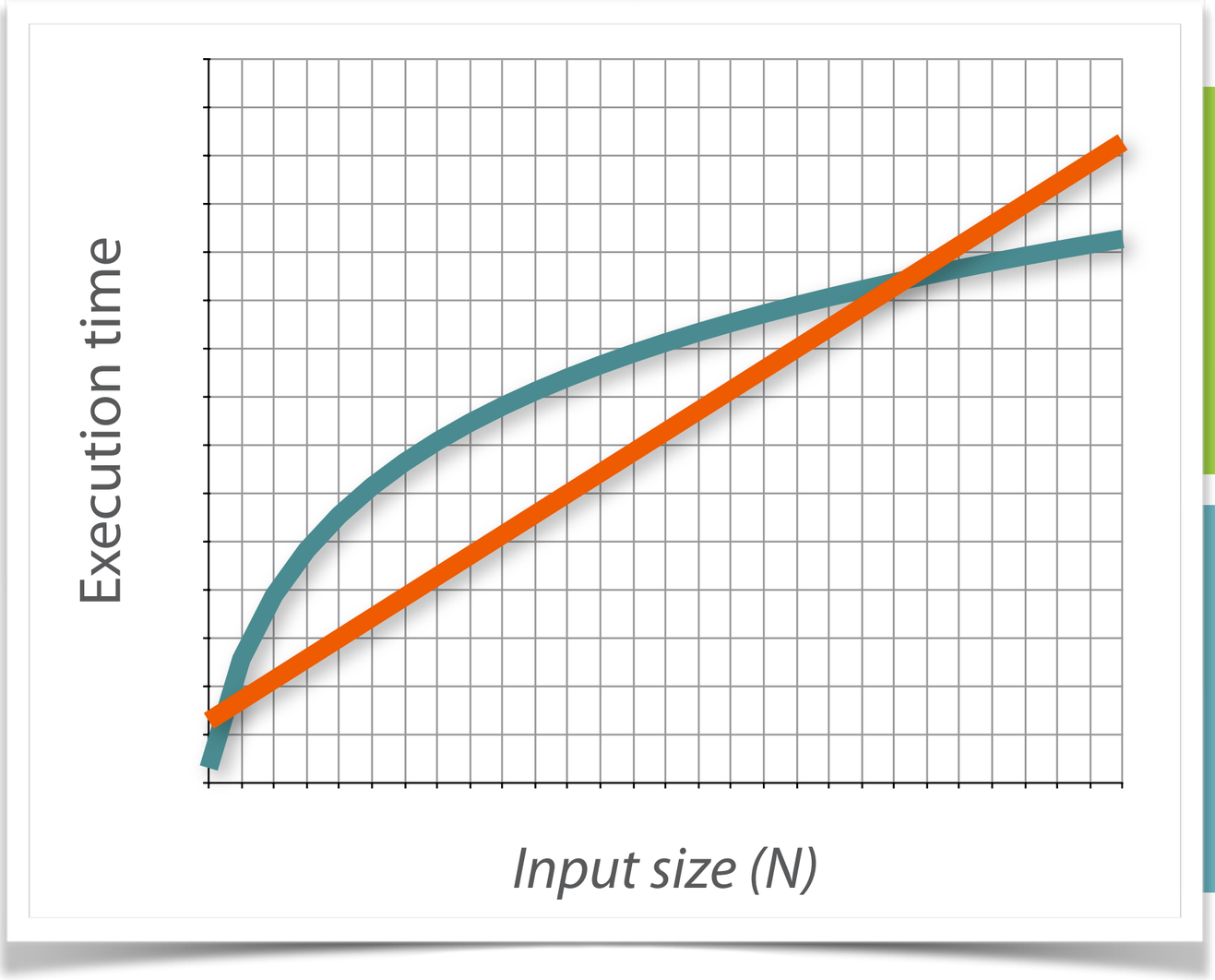

Counting instructions

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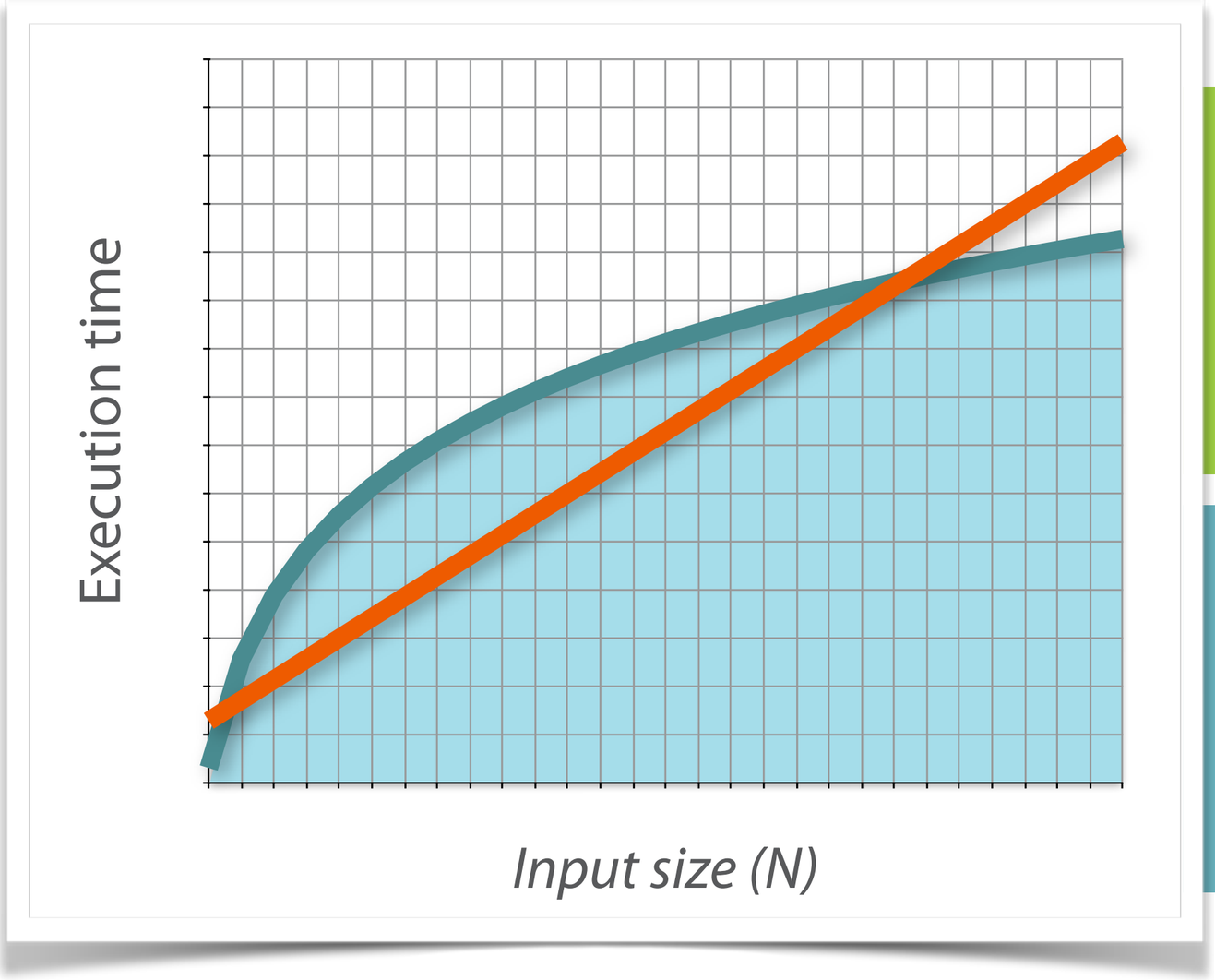

Counting instructions

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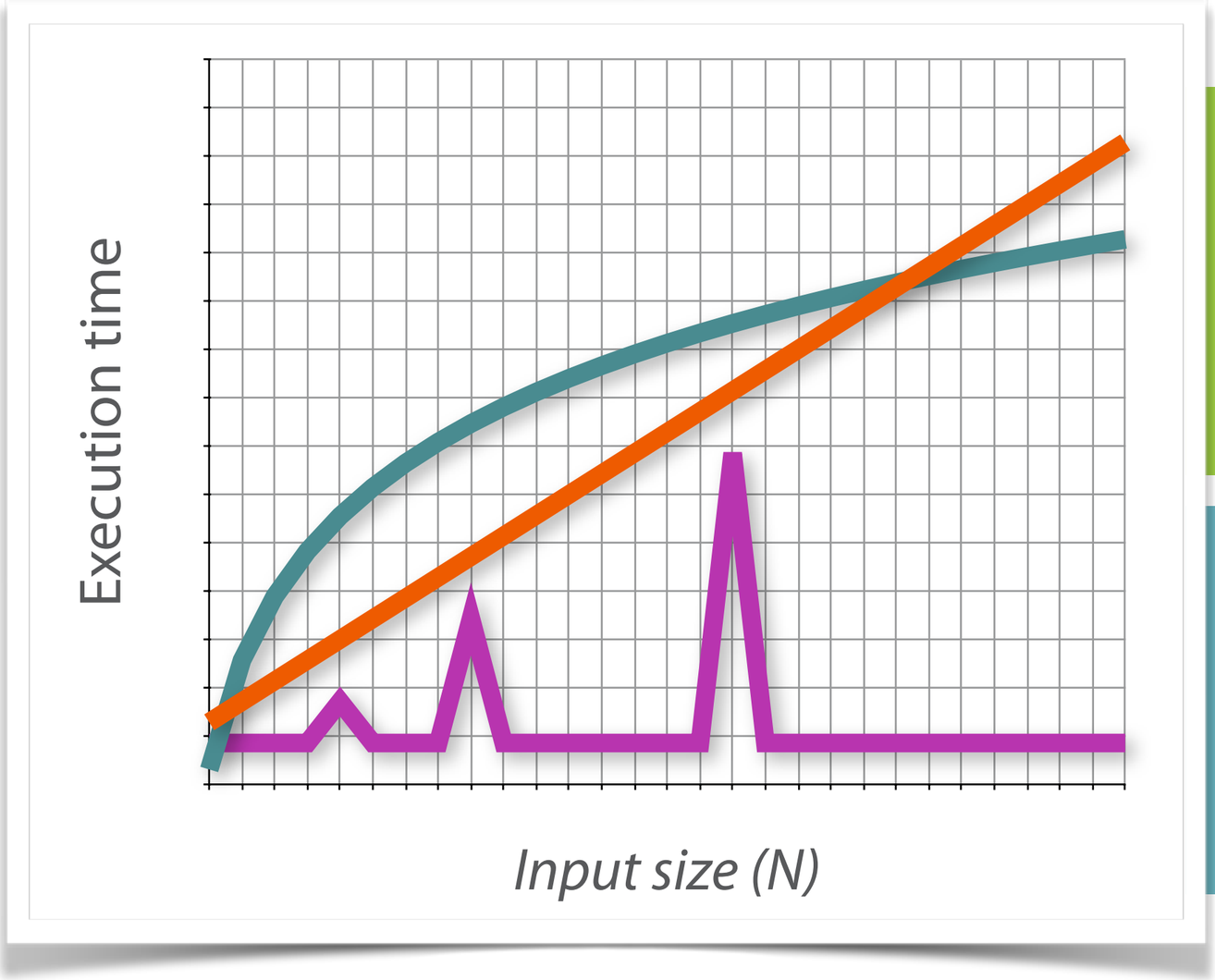

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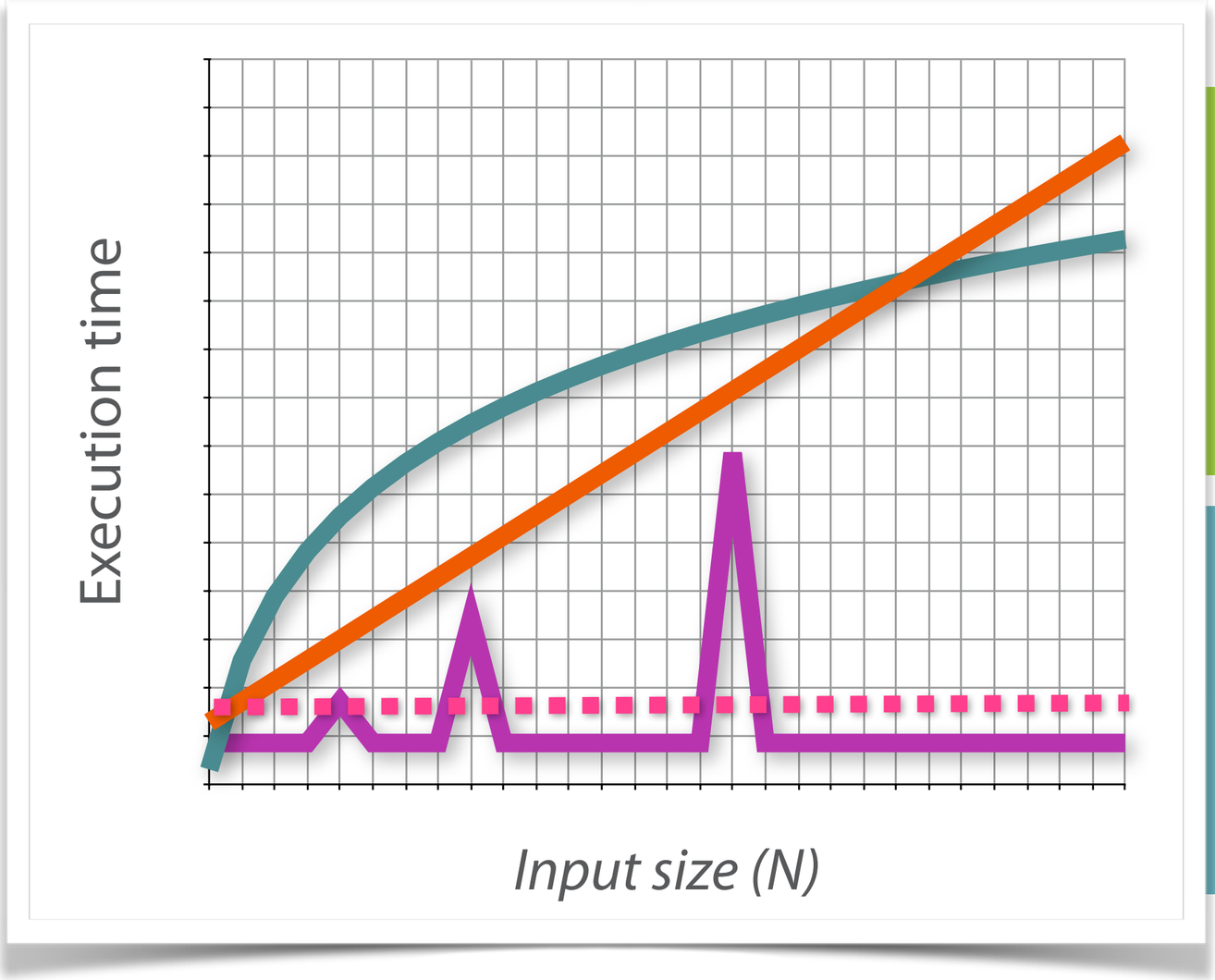

Counting instructions

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Timing with stopwatch

Counting instructions

Looking at the curve

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Worst case
Average

Asymptotic Performance

Asymptotic Performance

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

Asymptotic Performance

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

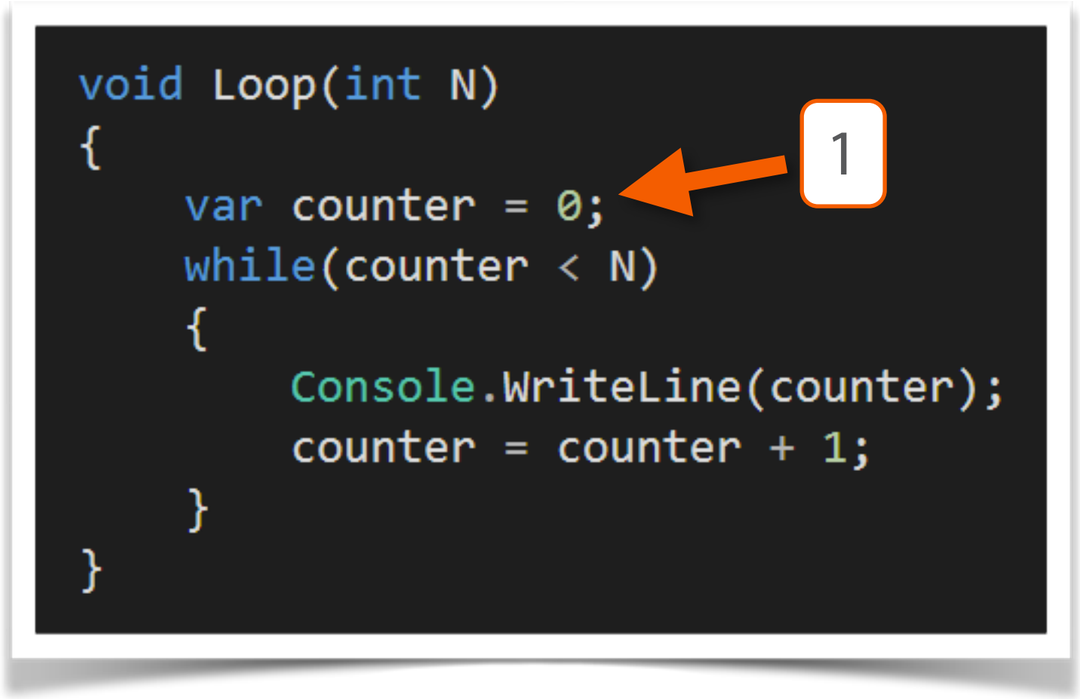
How complex?

Asymptotic Performance

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

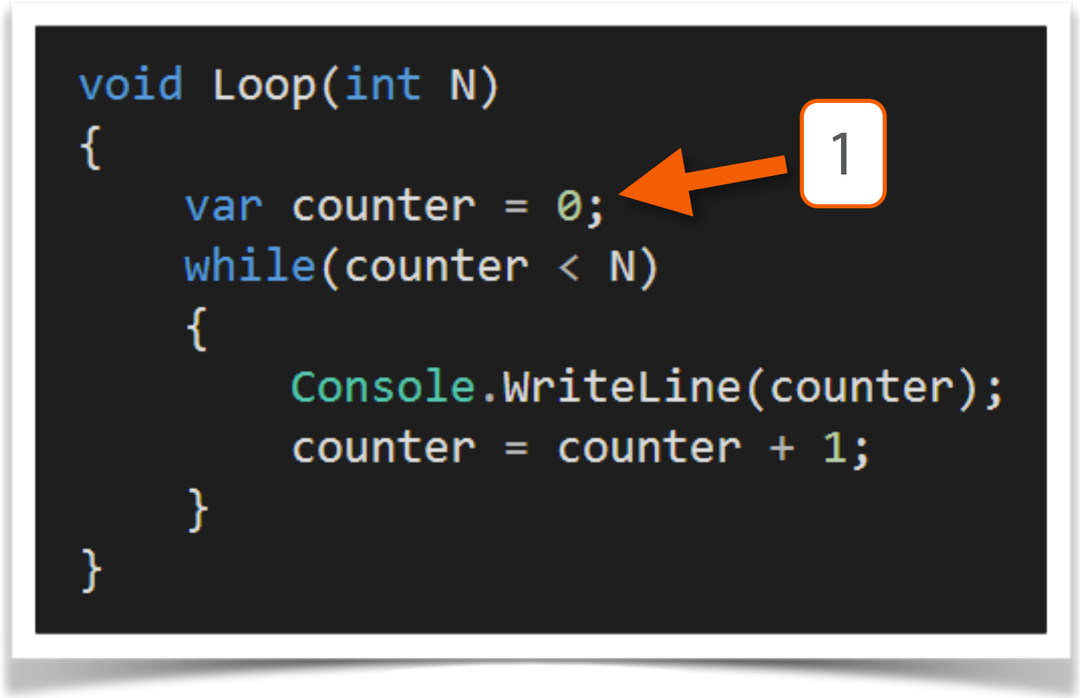
Complexity (*N*):

Asymptotic Performance



Complexity (*N*):

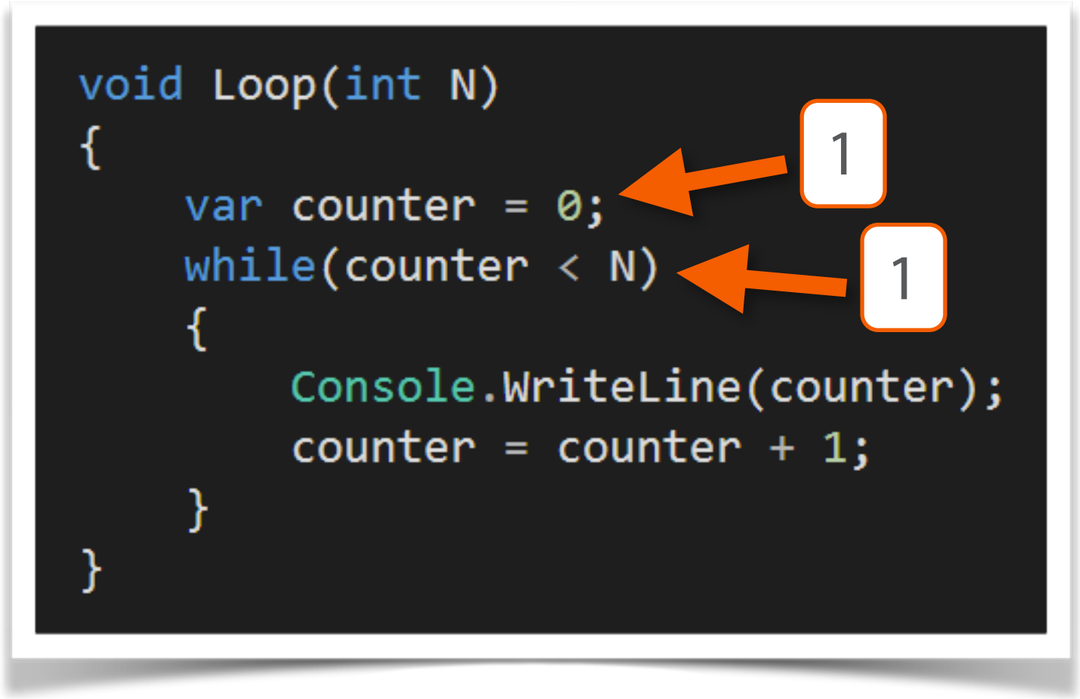
Asymptotic Performance



Complexity (*N*):

1

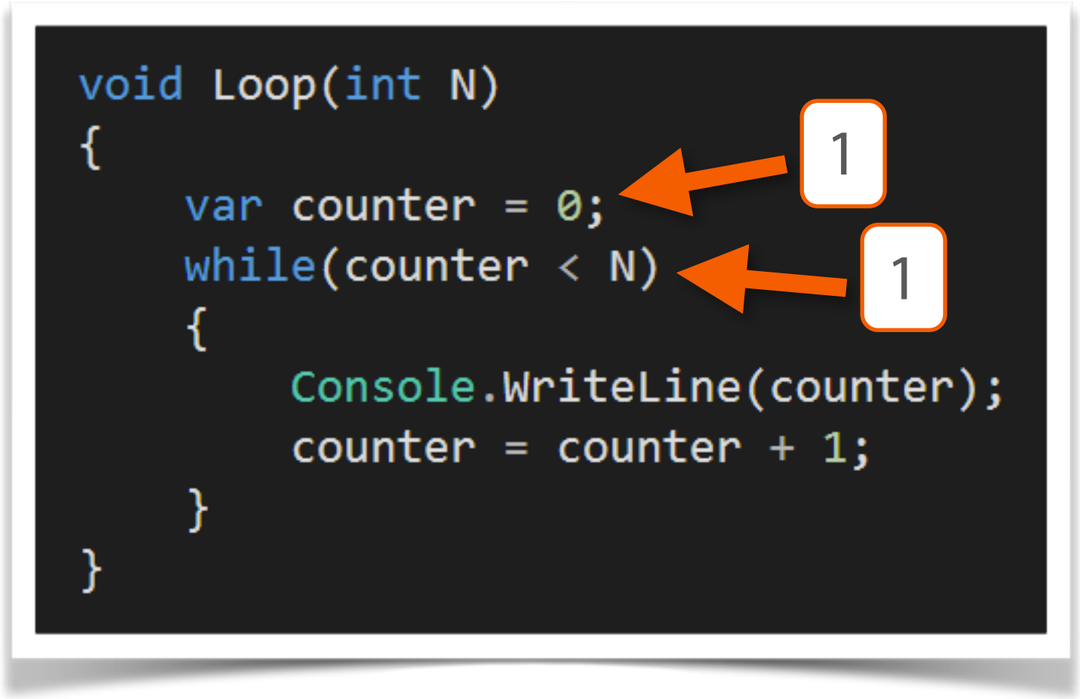
Asymptotic Performance



Complexity (*N*):

1

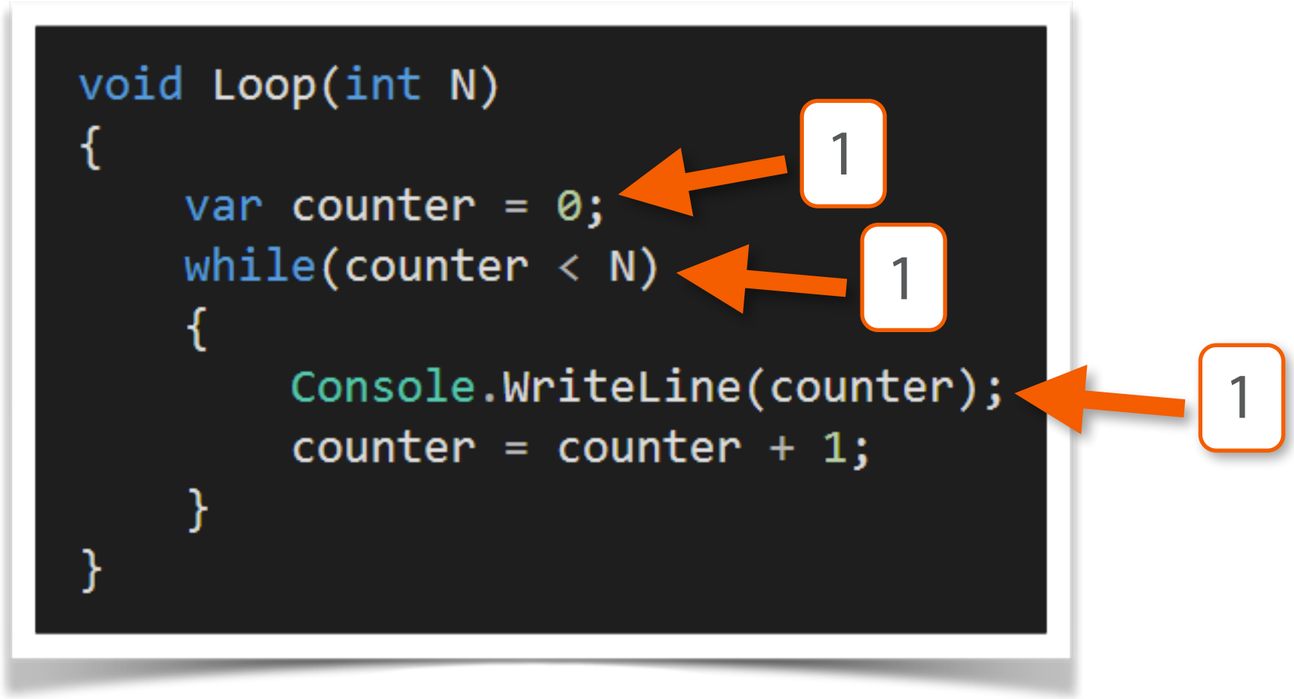
Asymptotic Performance



Complexity (*N*):

1 + 1

Asymptotic Performance



Complexity (*N*):

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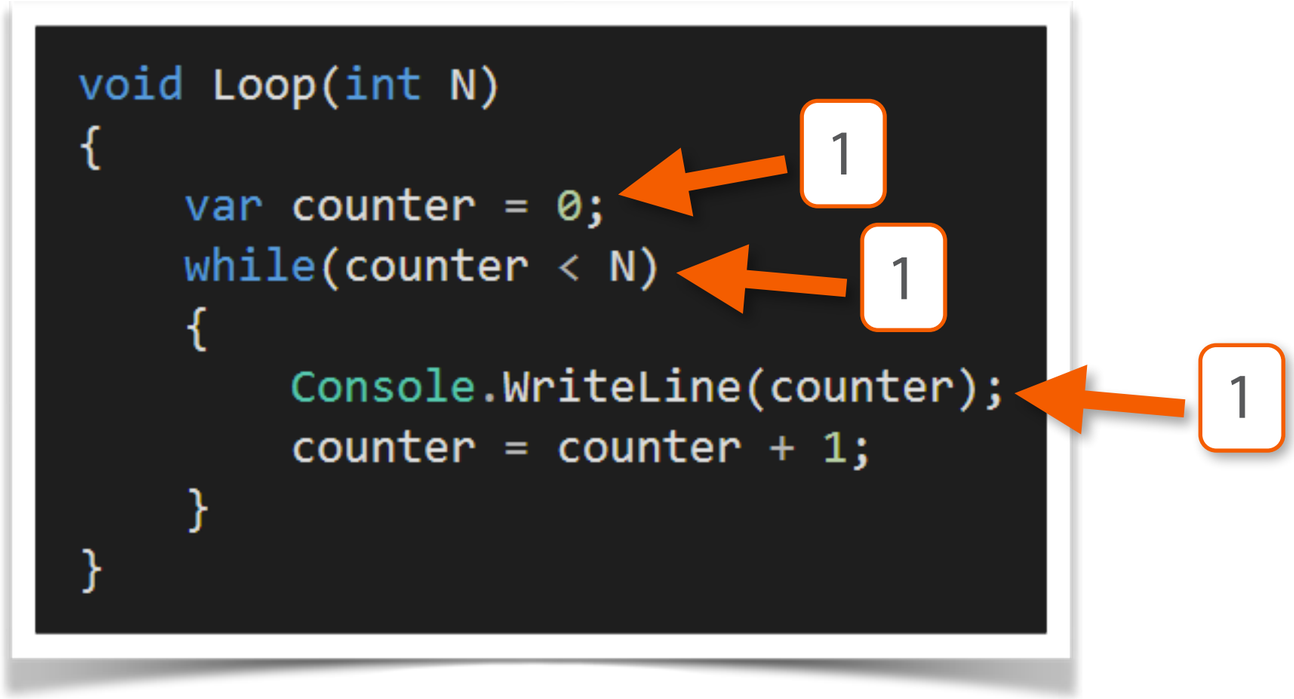
Asymptotic Performance



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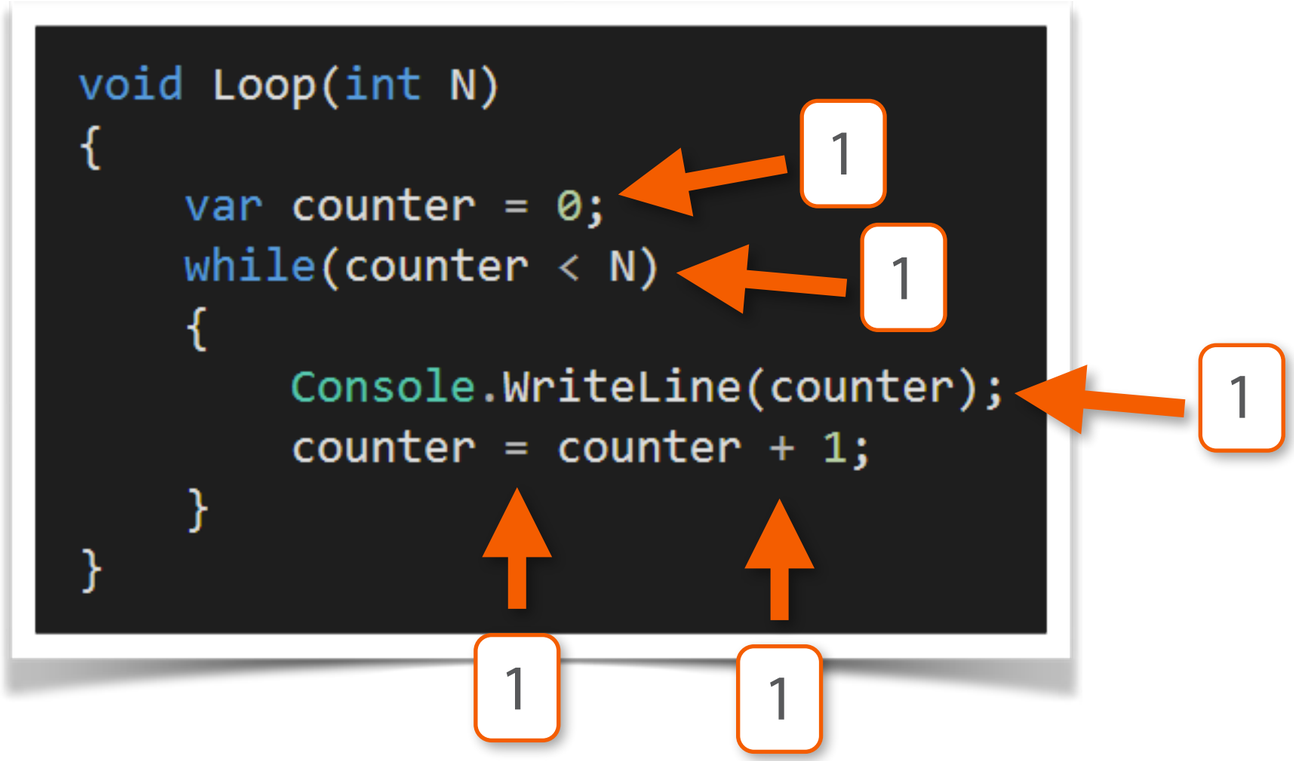
Asymptotic Performance



Complexity (*N*):

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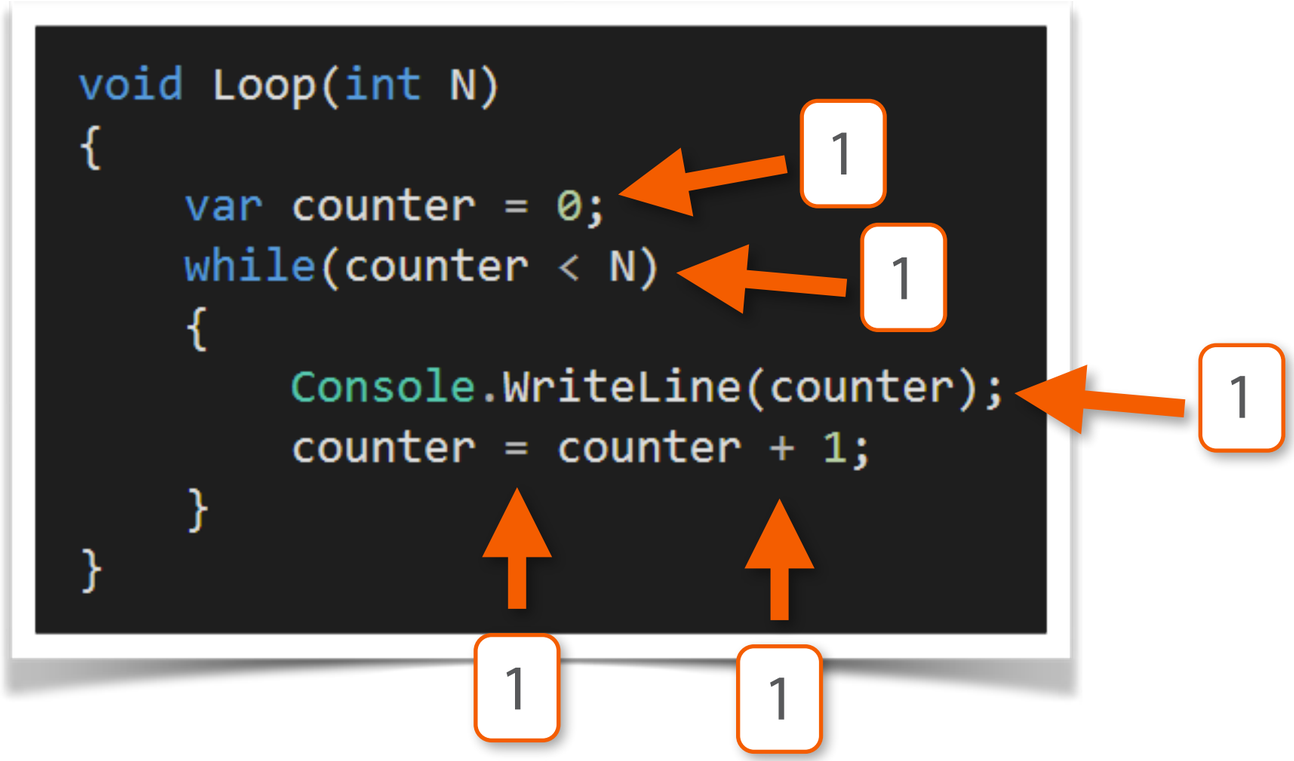
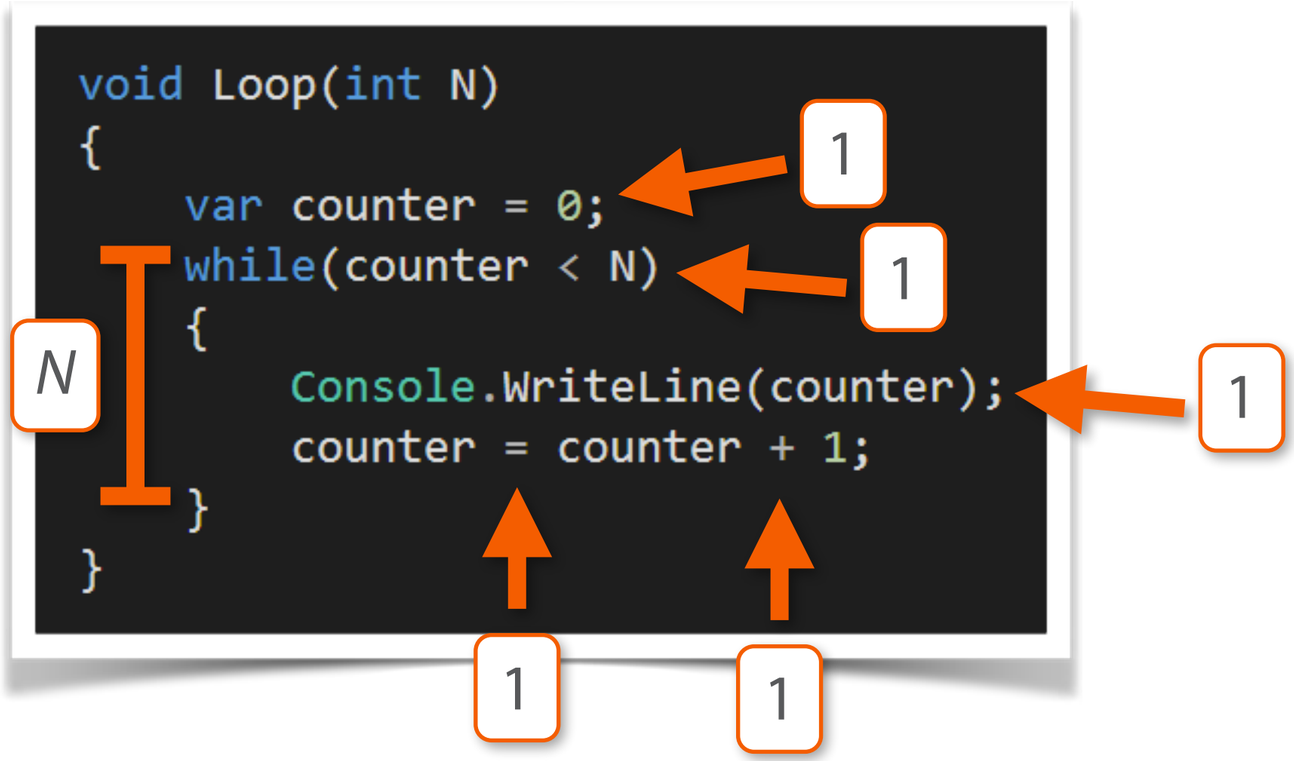
Asymptotic Performance



Complexity (*N*):

1 + 1 + 1 + 2

Asymptotic Performance



Complexity (*N*):

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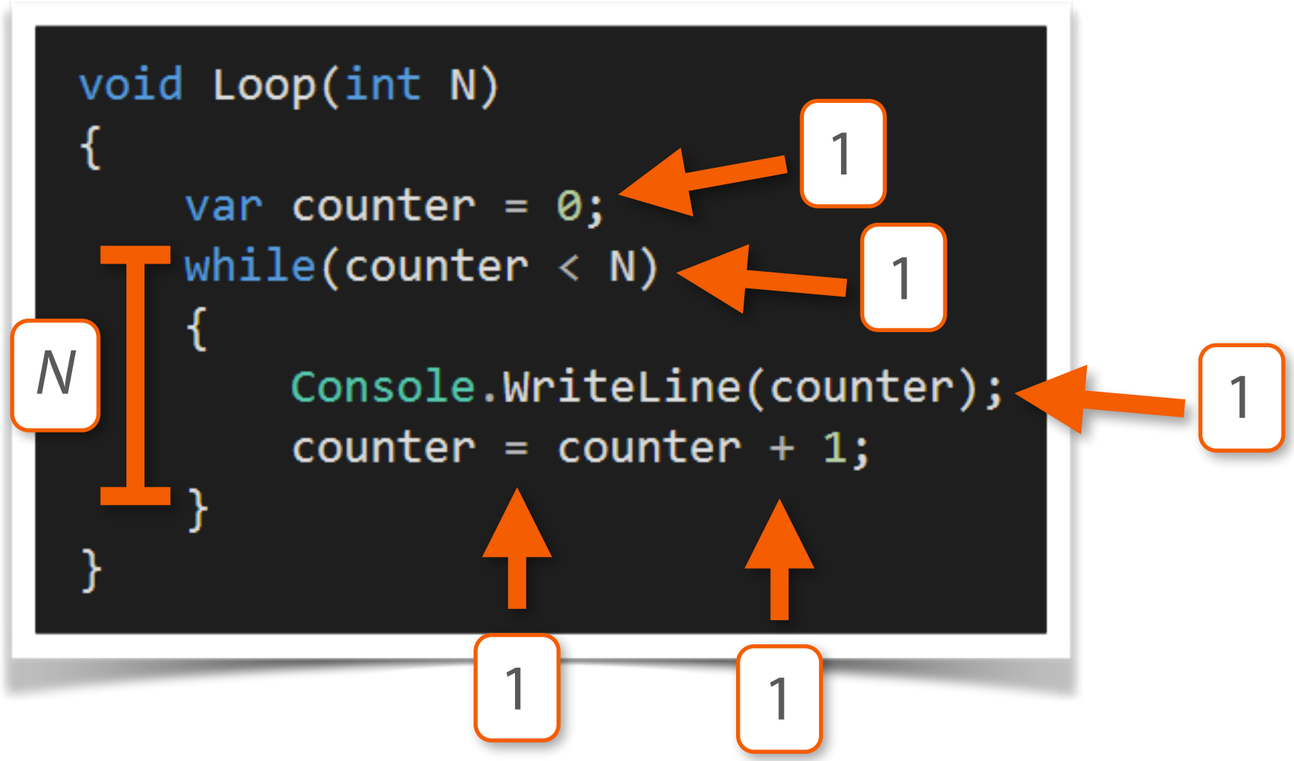
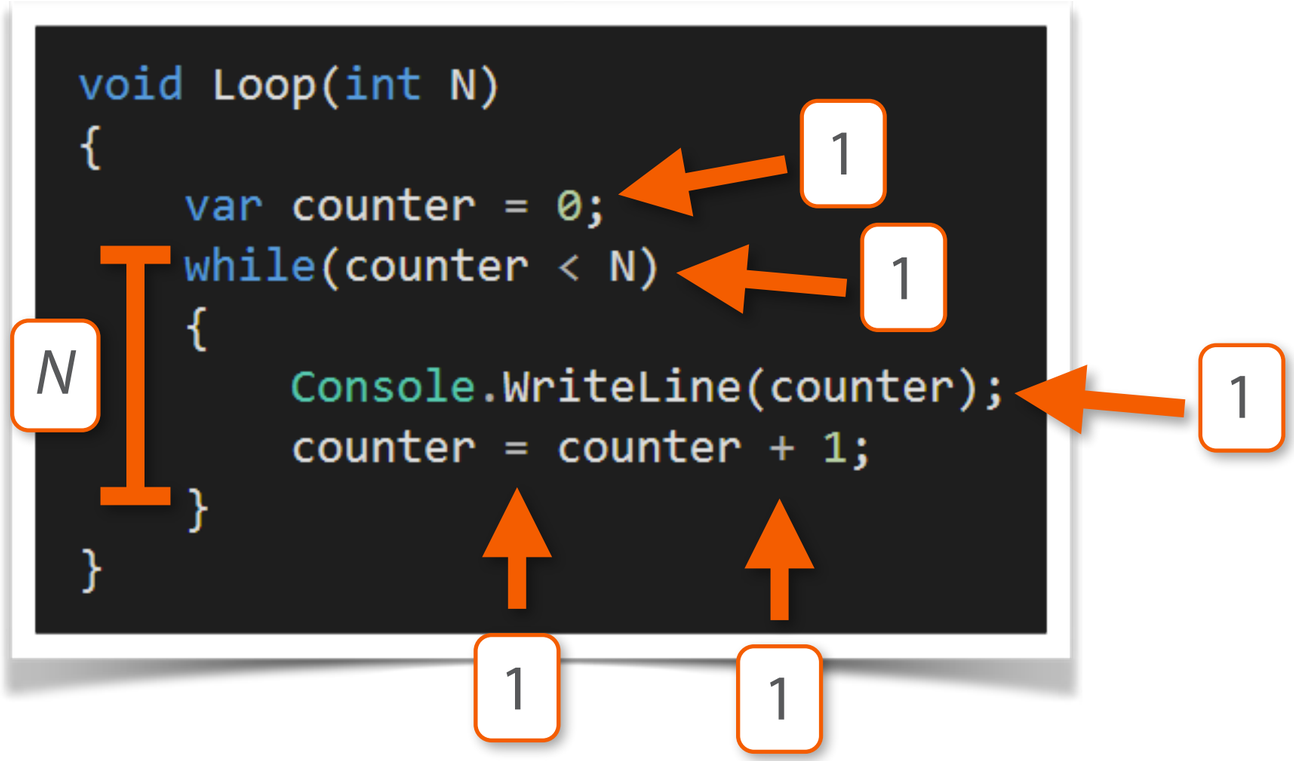
Asymptotic Performance



Complexity (*N*):

$1 + (1 + 1 + 2)\,N$

Asymptotic Performance

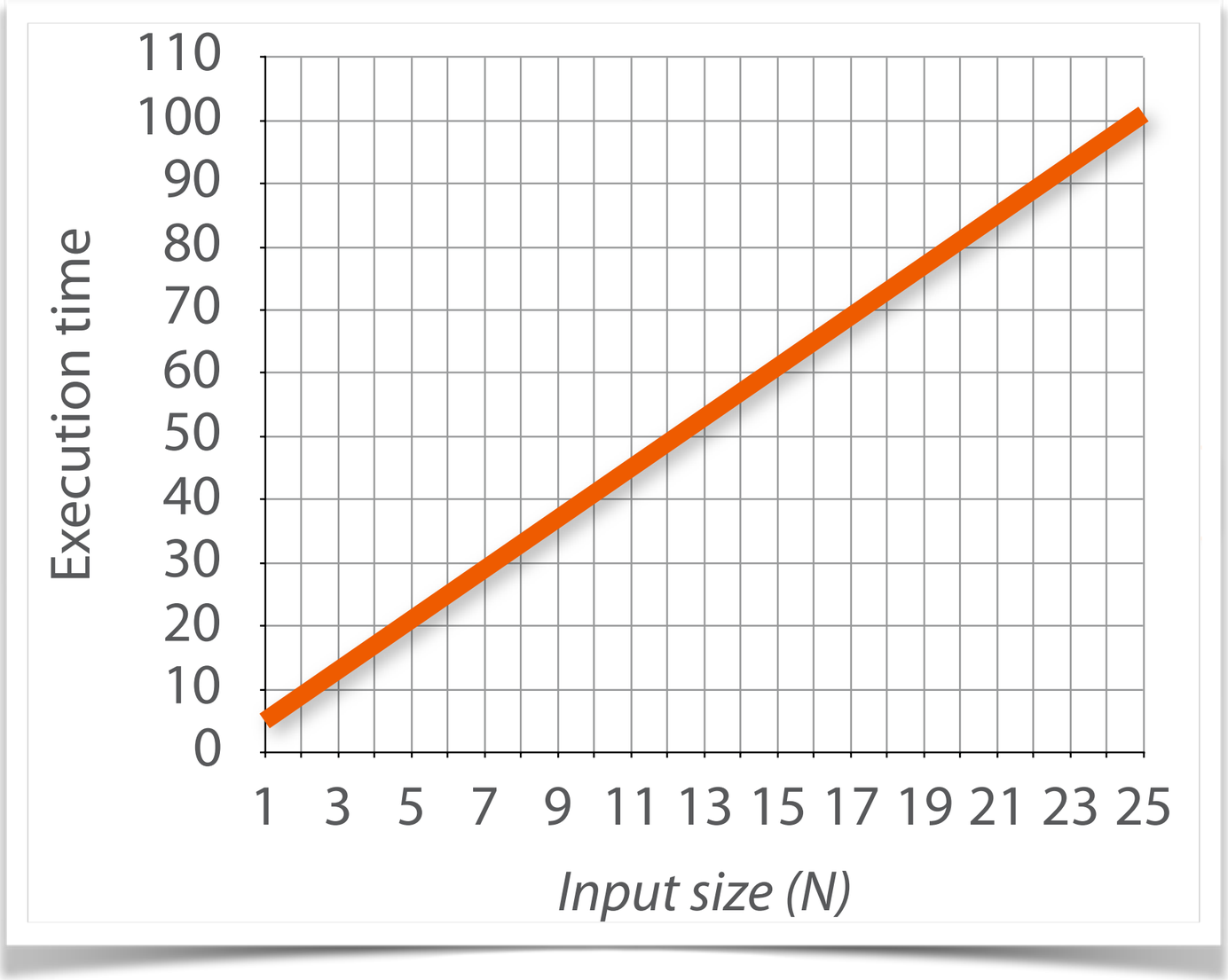


Complexity (*N*):

$$1 + (1 + 1 + 2)\,N$$

$$= 1 + 4\,N$$

Asymptotic Performance



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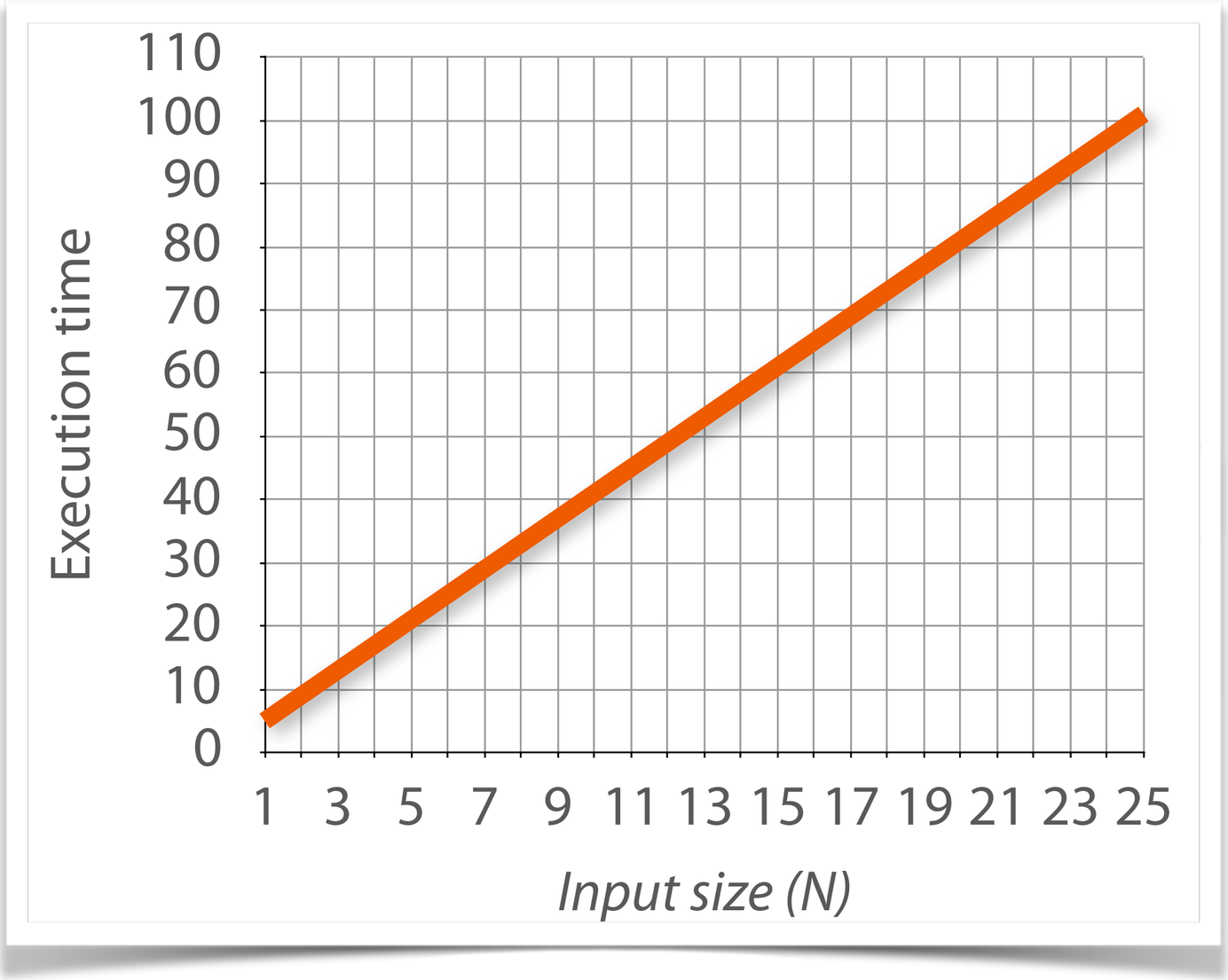
Asymptotic Performance



Complexity (*N*):

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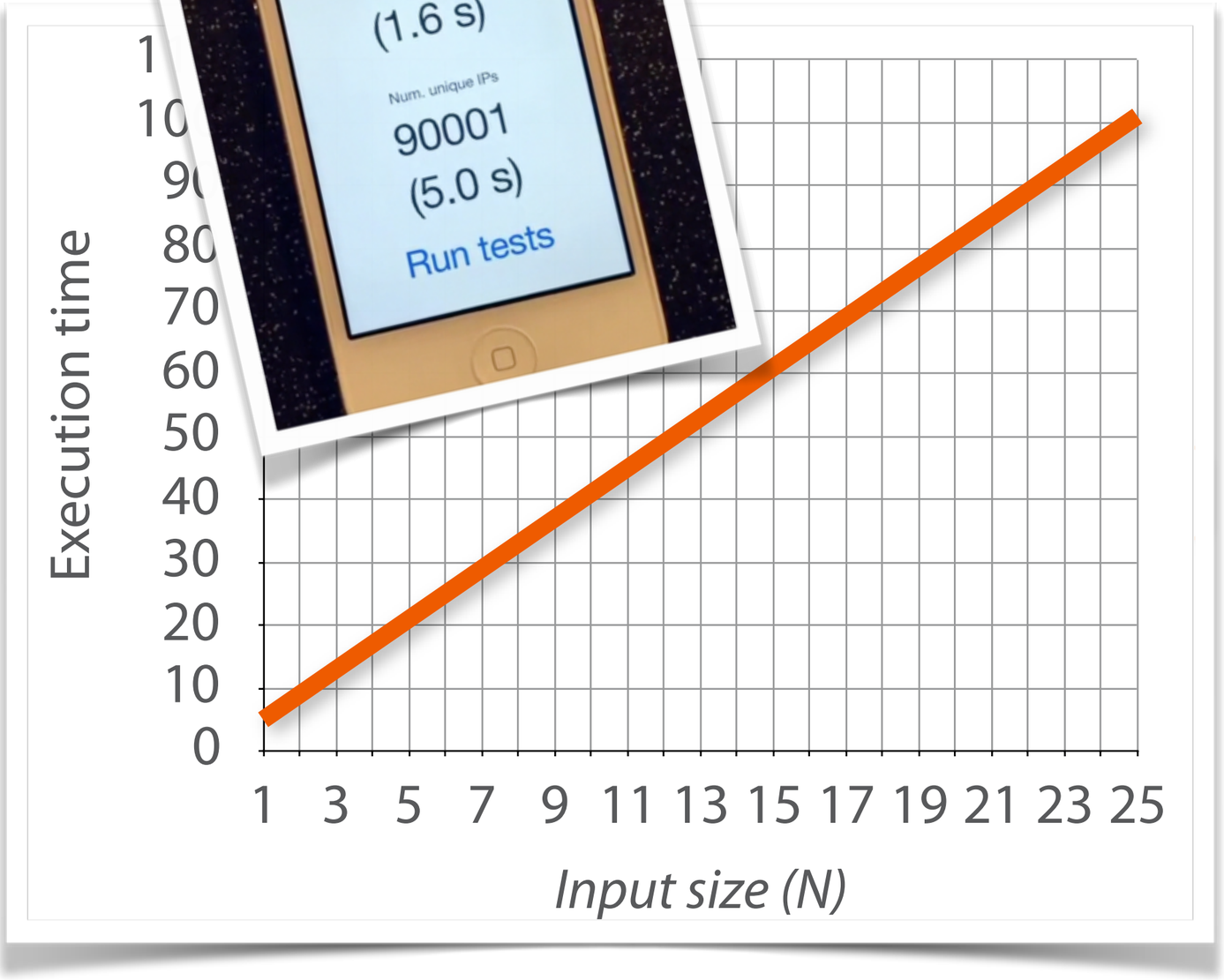
nptotic Performance



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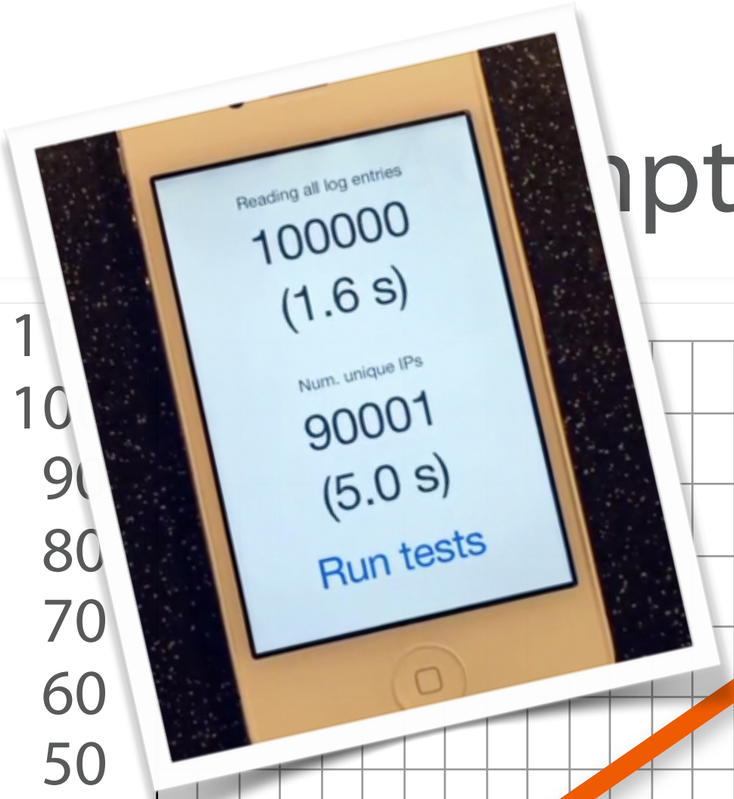
nptotic Performance



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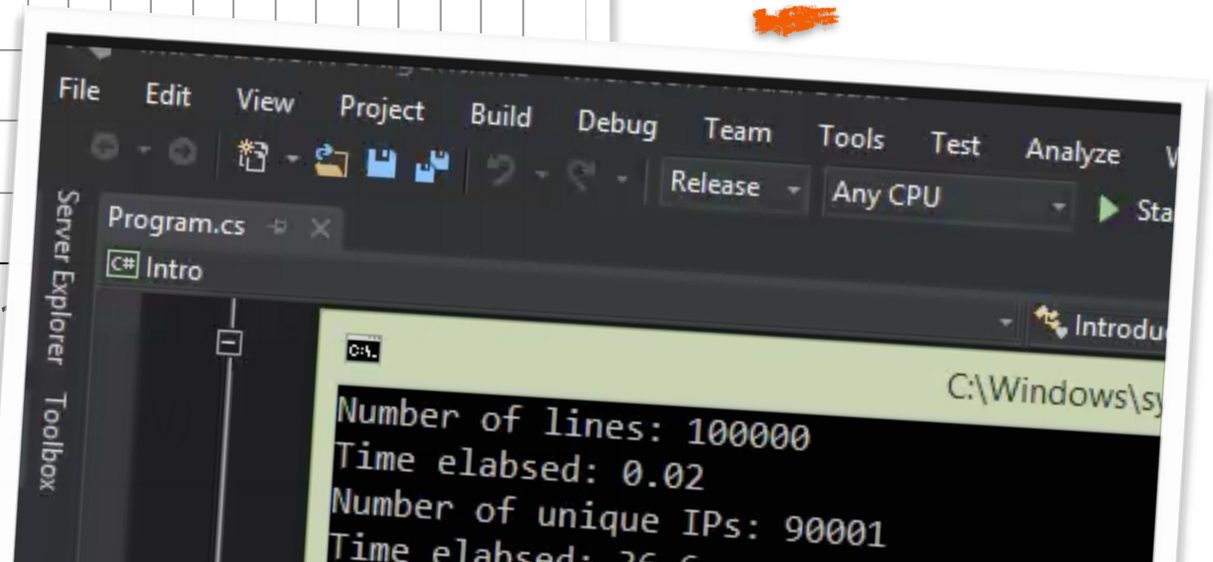
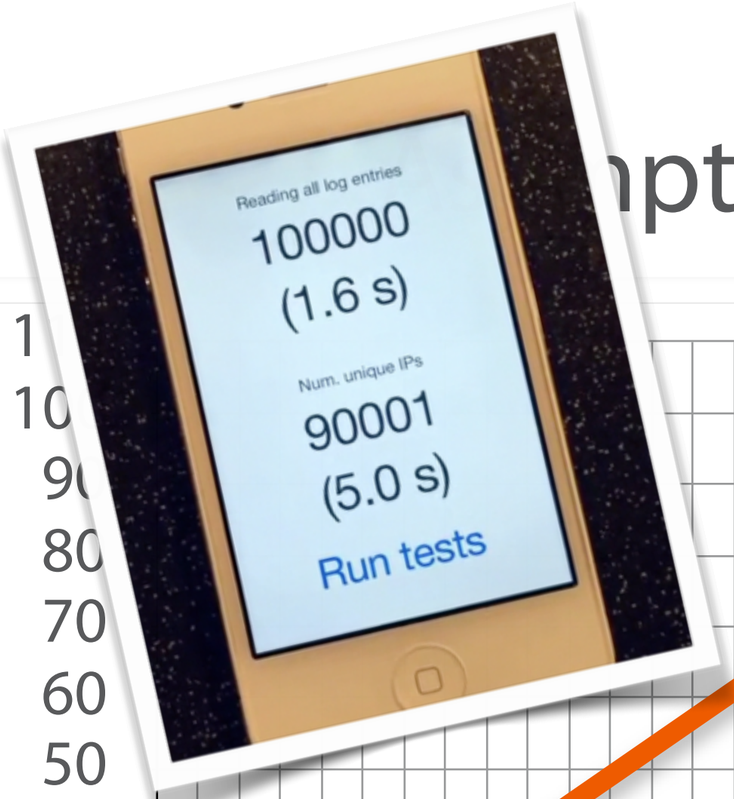


nptotic Performance



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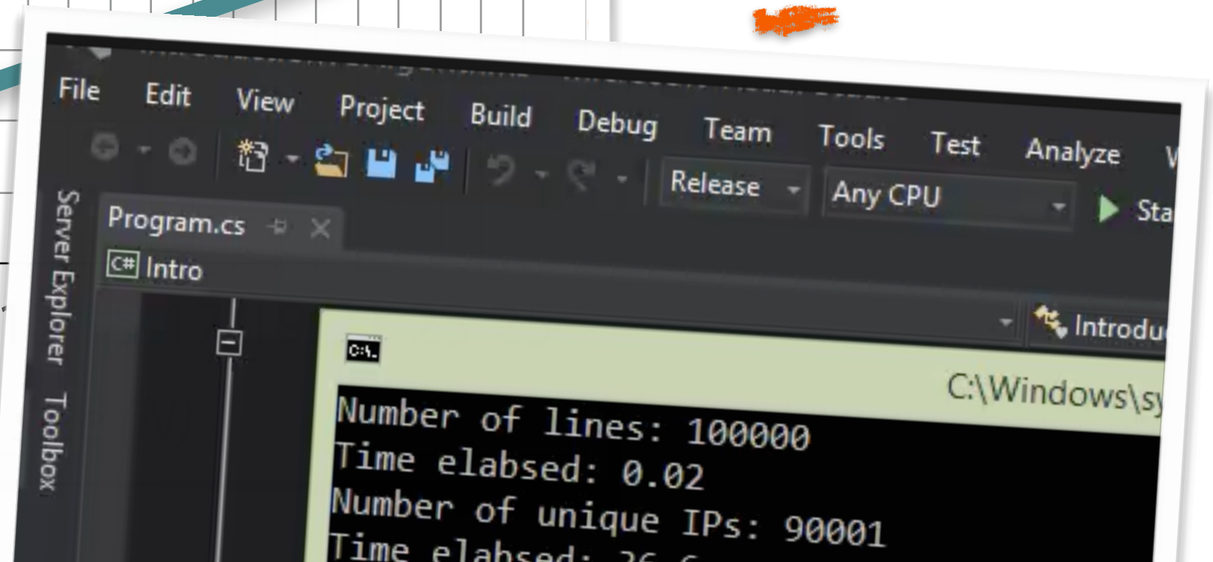
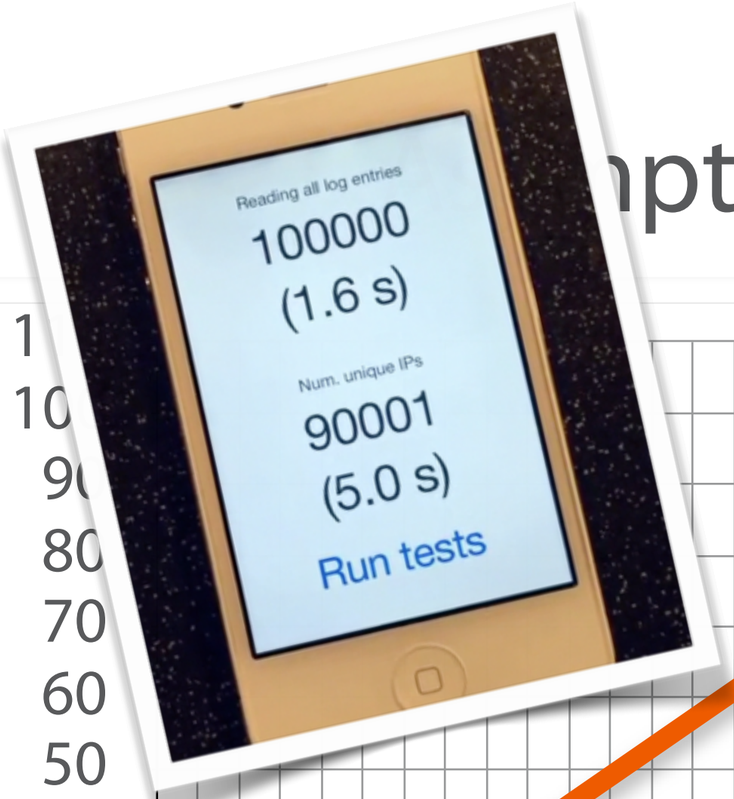


nptotic Performance



Complexity (*N*):

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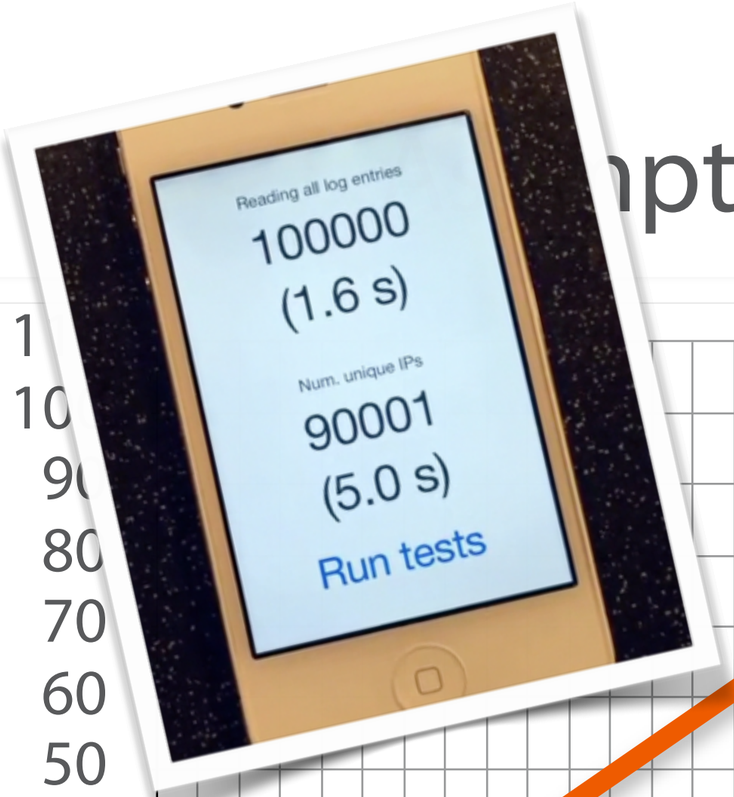
nptotic Performance



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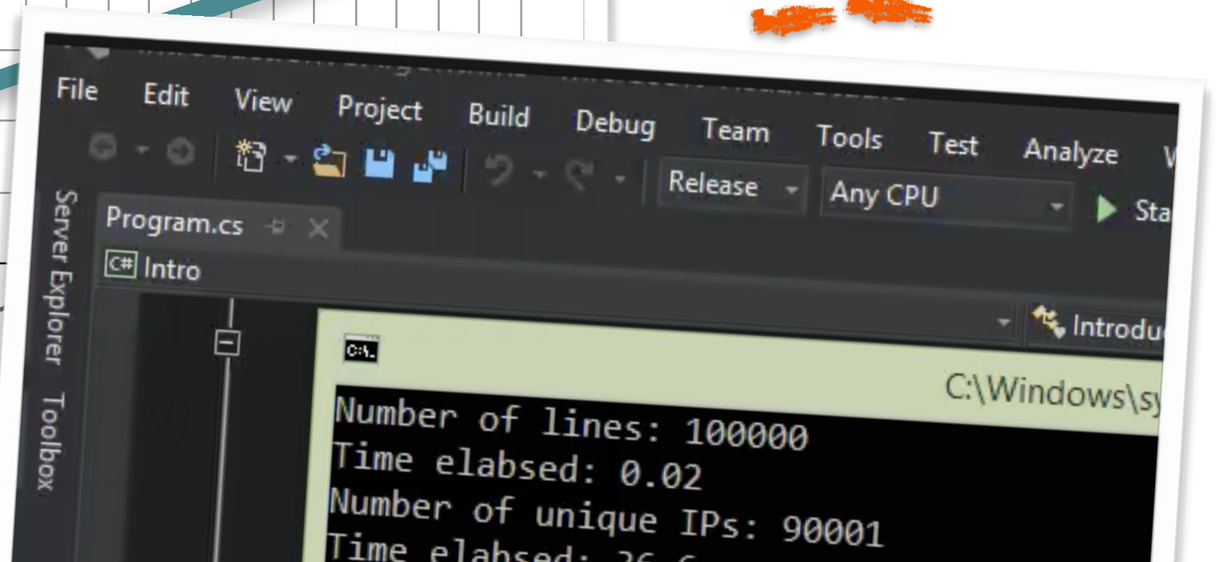
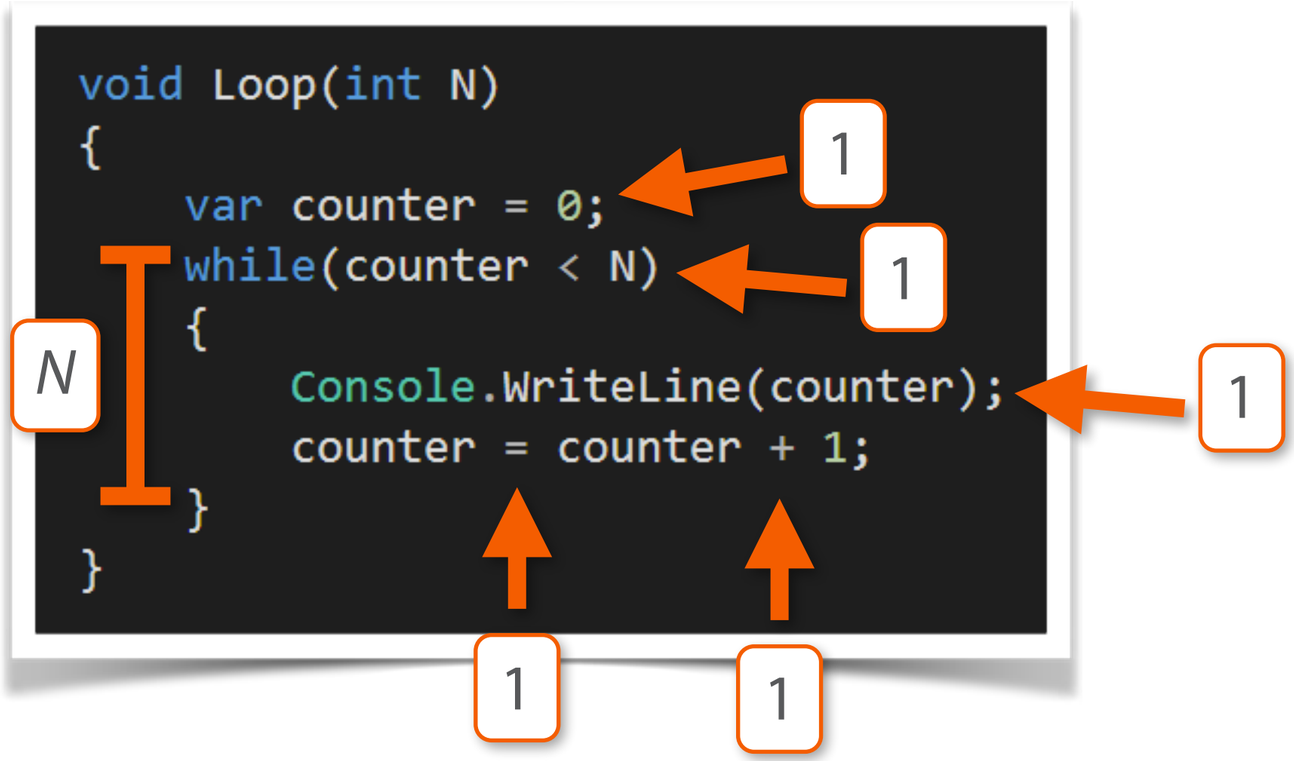


Asymptotic Performance



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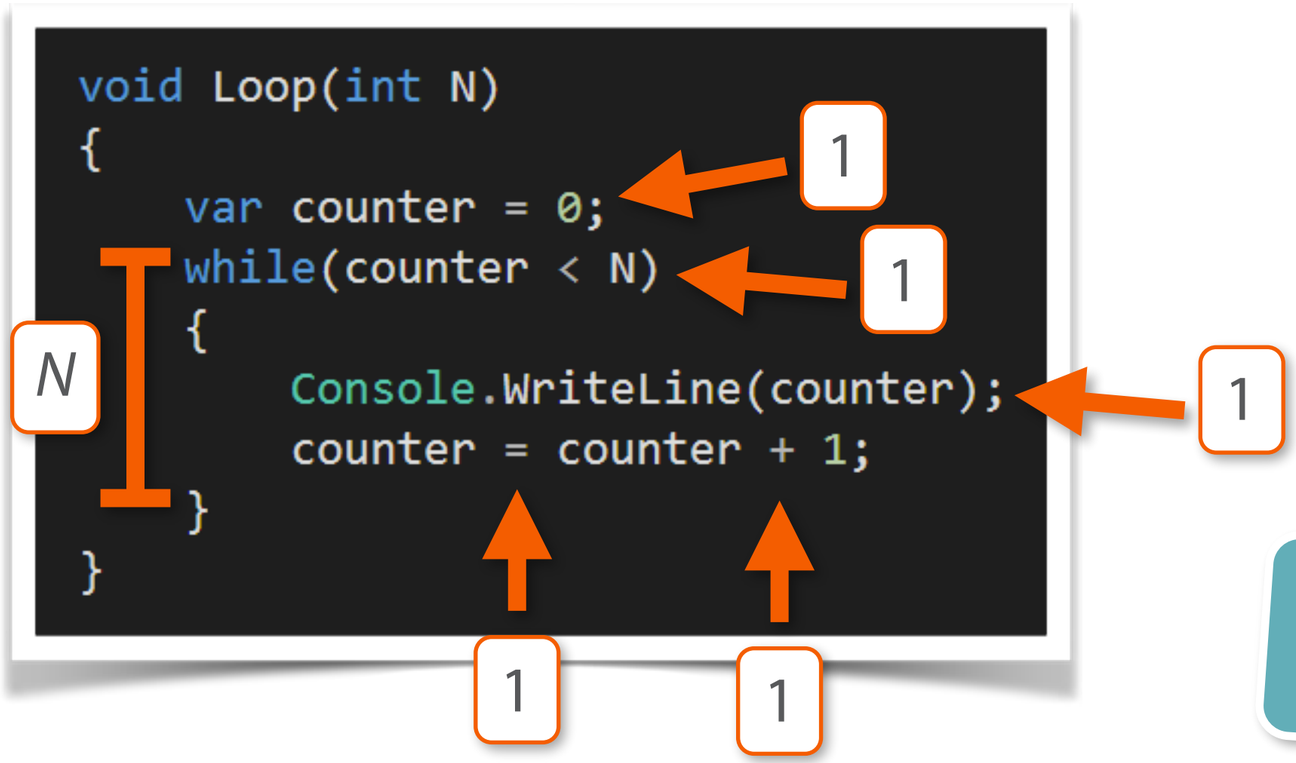

Complexity (*N*):

$1 + (1 + 1 + 2)\,N$

$= 1 + 4\,N$

$N \cdot$ (some number)

```
void CreateAllPairs(int N)
{
    var x = 0;
    var y = 0;
    while (x < N)
    {
        while (y < N)
        {
            Console.WriteLine("{0}, {1}", x, y);
            y = y + 1;
        }
        x = x + 1;
    }
}
```

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N

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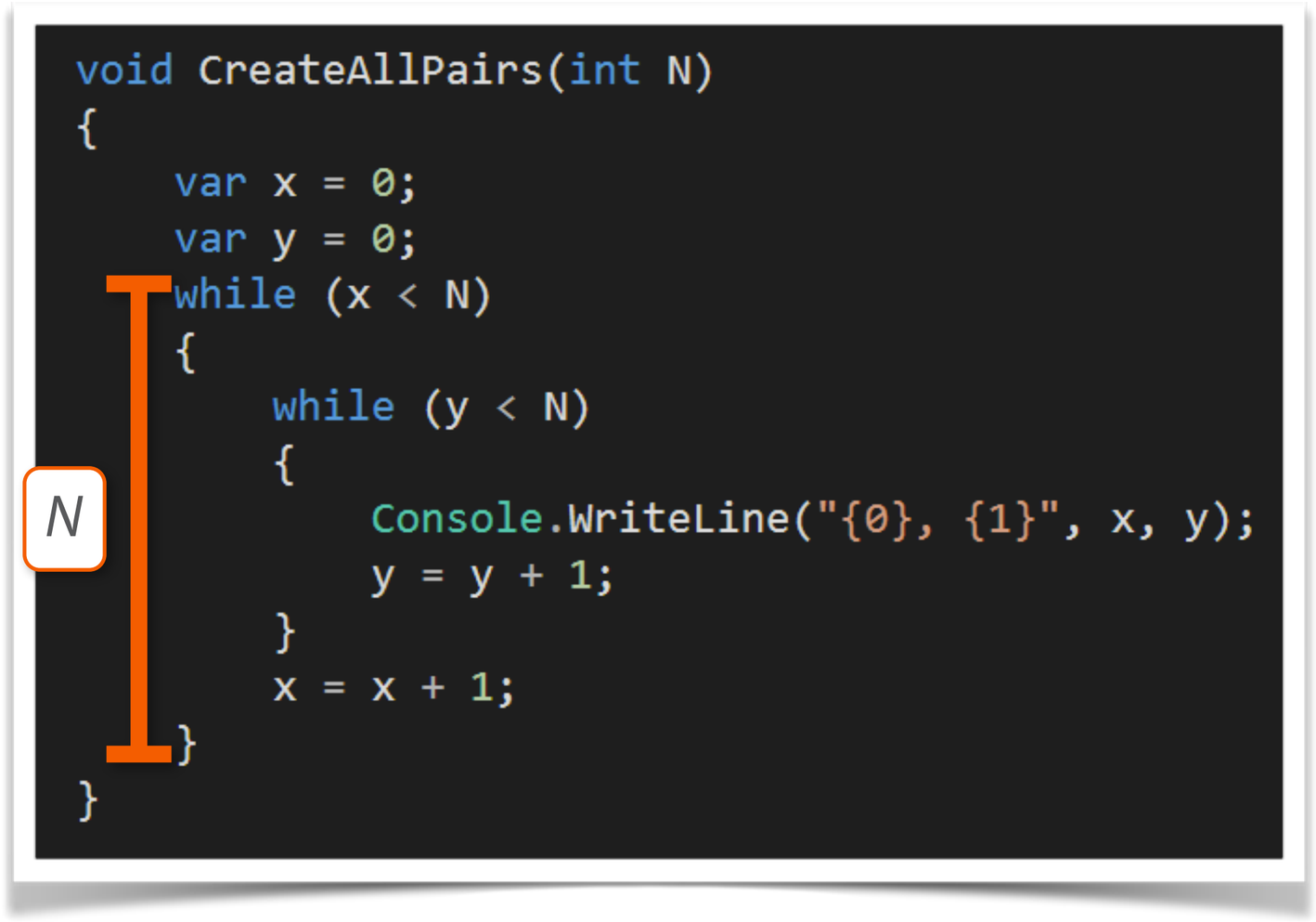
        x = x + 1;
    }
}
```

N

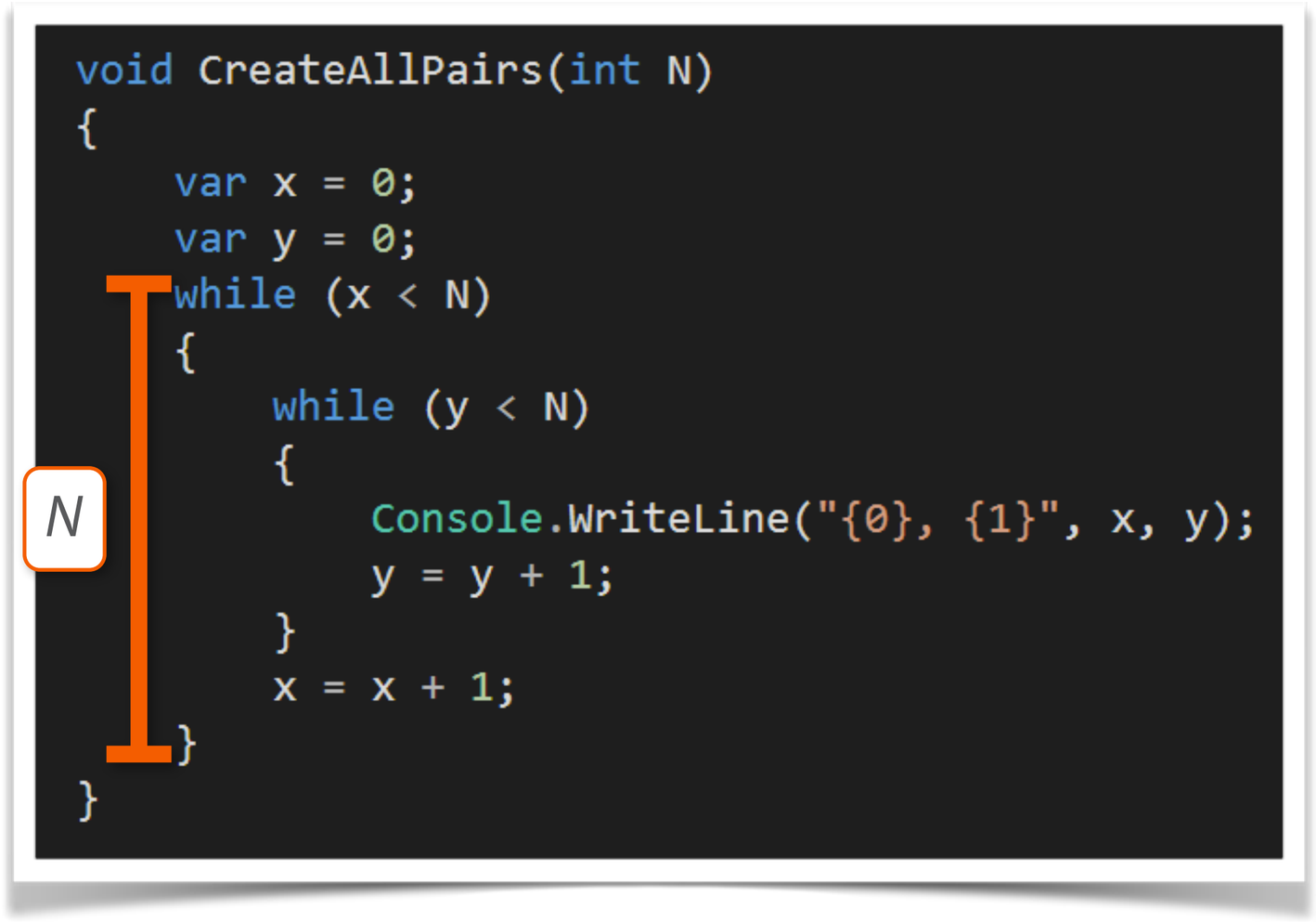
Complexity (*N*):

$$N\,(\,1+\,1\,+\,2\,)$$



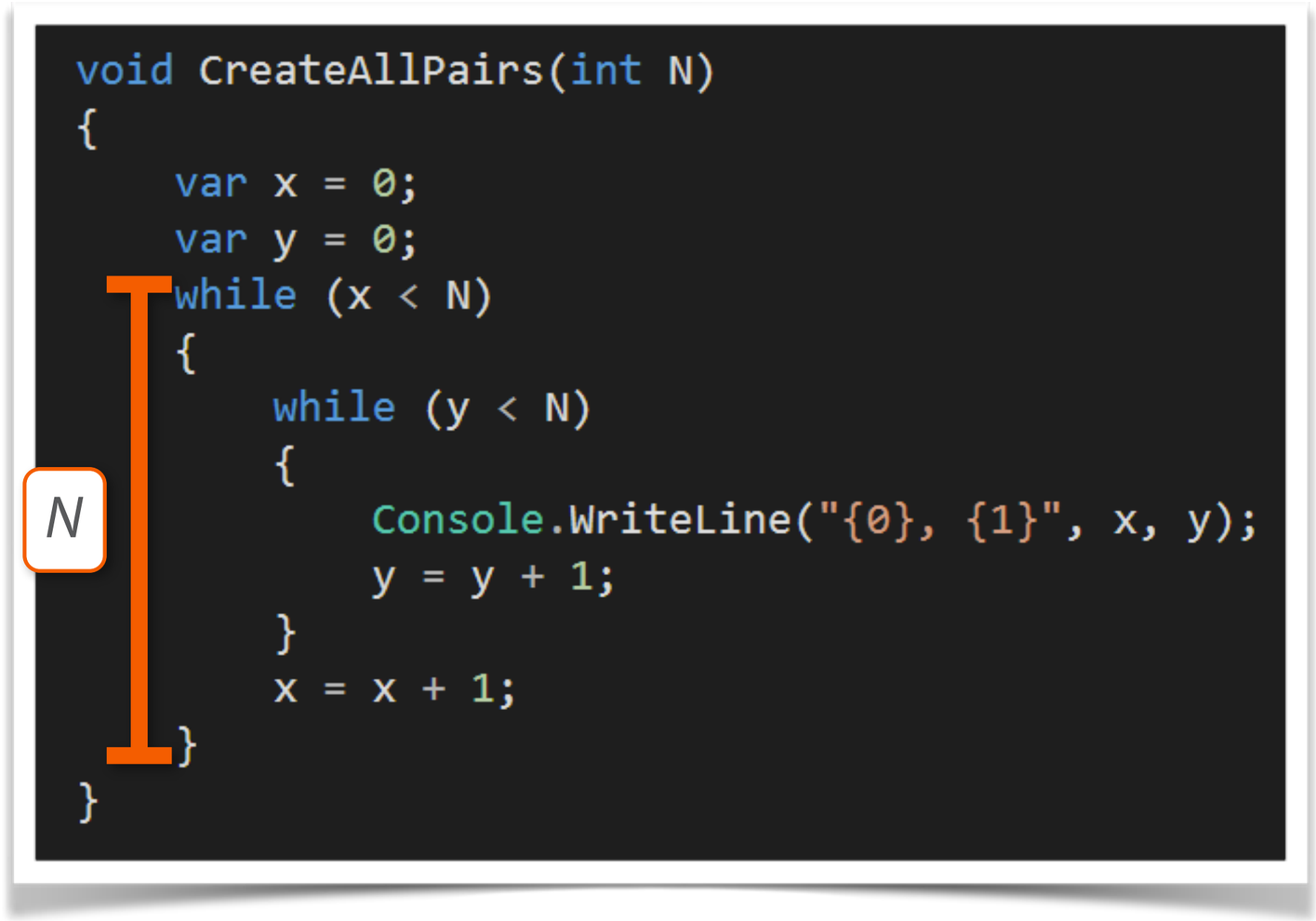
Complexity (*N*):

$$N\,(\,1+1+2\,)$$



Complexity (*N*):

$$N\,(\,1 + N\,(\,1+1+2\,)\,+2)$$



Complexity (*N*):

$$N\,(\,1 + N\,(\,1 + 1 + 2\,) + 2)$$

```
void CreateAllPairs(int N)
{
    var x = 0;
    var y = 0;
    while (x < N)
    {
        while (y < N)
        {
            Console.WriteLine("{0}, {1}", x, y);
            y = y + 1;
        }
        x = x + 1;
    }
}
```

Complexity (*N*):

$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

```
void CreateAllPairs(int N)
{
    var x = 0;
    var y = 0;
    while (x < N)
    {
        while (y < N)
        {
            Console.WriteLine("{0}, {1}", x, y);
            y = y + 1;
        }
        x = x + 1;
    }
}
```

Complexity (*N*):

$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

$$= 2 + N(3 + N(4))$$

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void CreateAllPairs(int N)
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        {
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            y = y + 1;
        }
        x = x + 1;
    }
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```

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            y = y + 1;
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}
```

Complexity (*N*):

$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

$$= 2 + N(3 + N(4))$$

$$= 2 + N(3 + 4N)$$

```
void CreateAllPairs(int N)
{
    var x = 0;
    var y = 0;
    while (x < N)
    {
        while (y < N)
        {
            Console.WriteLine("{0}, {1}", x, y);
            y = y + 1;
        }
        x = x + 1;
    }
}
```

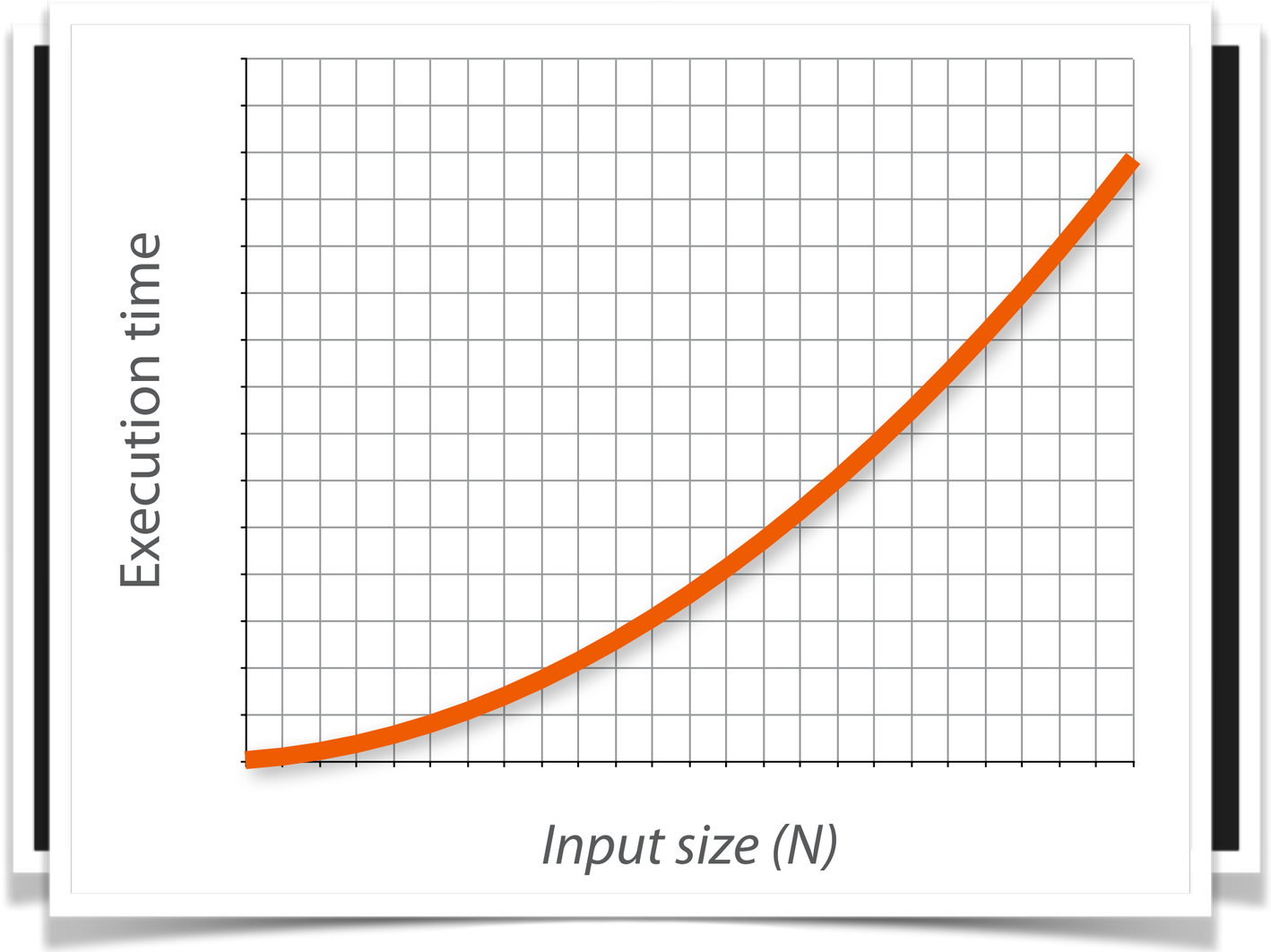
Complexity (*N*):

$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

$$= 2 + N(3 + N(4))$$

$$= 2 + N(3 + 4N)$$

$$= 2 + 3N + 4N^2$$



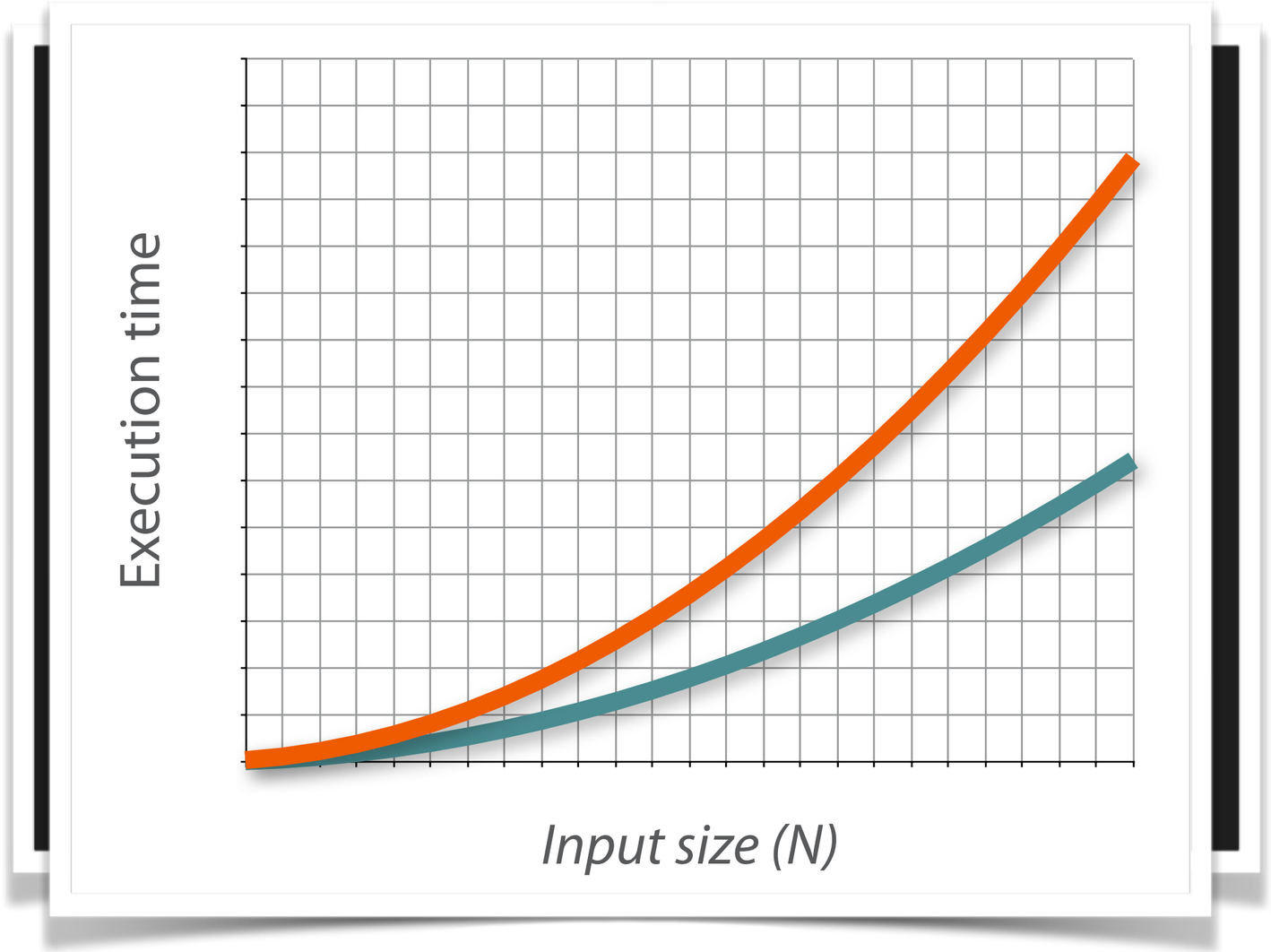
Complexity (*N*):

$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

$$= 2 + N(3 + N(4))$$

$$= 2 + N(3 + 4N)$$

$$= 2 + 3N + 4N^2$$



Complexity (*N*):

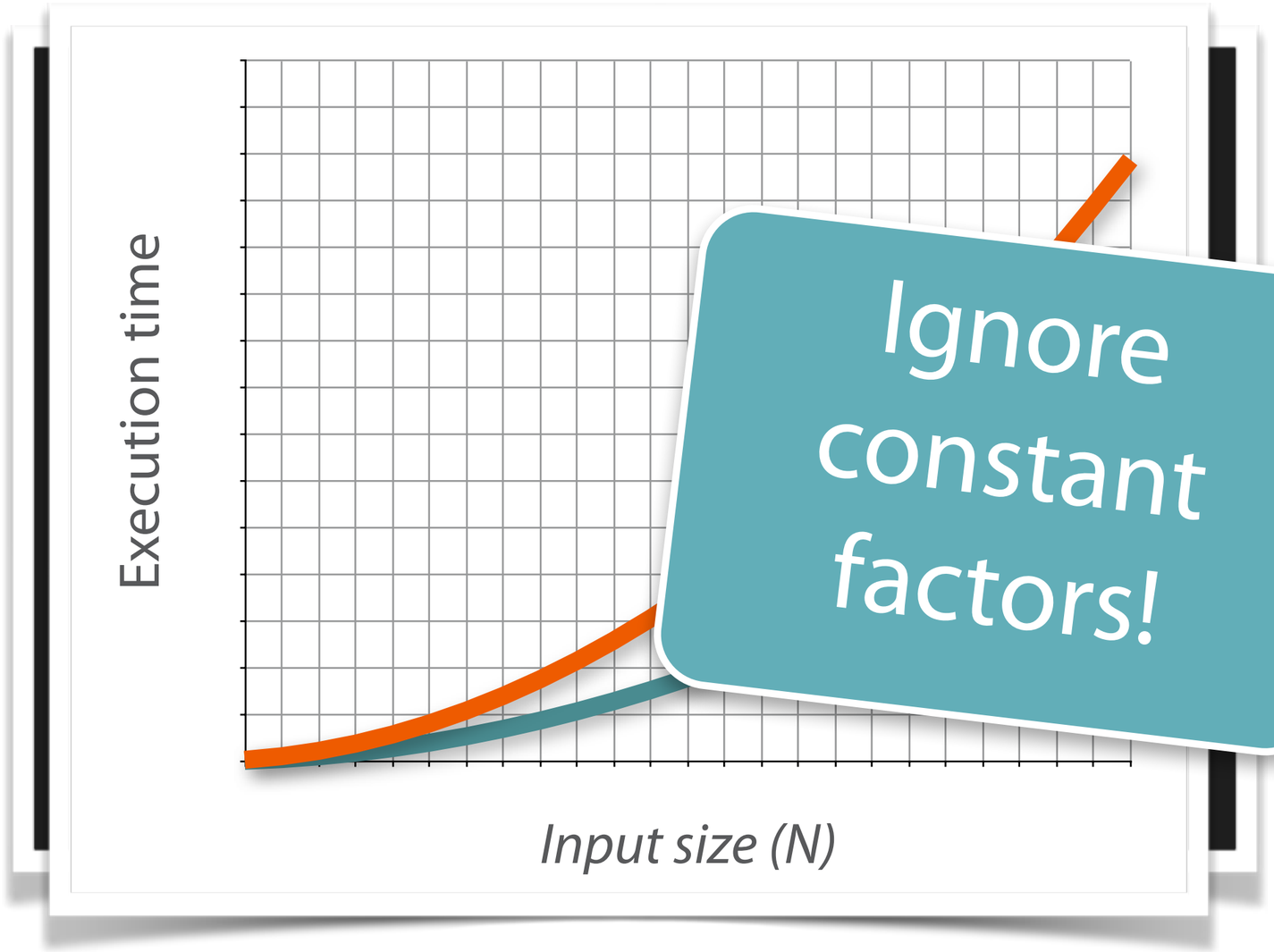
$$1 + 1 + N(1 + N(1 + 1 + 2) + 2)$$

$$= 2 + N(3 + N(4))$$

$$= 2 + N(3 + 4N)$$

$$= 2 + 3N + 4N^2$$

$$(2 + 3N + 4N^2) \cdot 0.5$$



Complexity (*N*):

$$+ 1 + \; N (1 + N (1 + 1 + 2) + 2)$$

$$= 2 \quad + N (3 + N (\quad 4 \quad) \quad)$$

$$= 2 + N (3 + 4N)$$

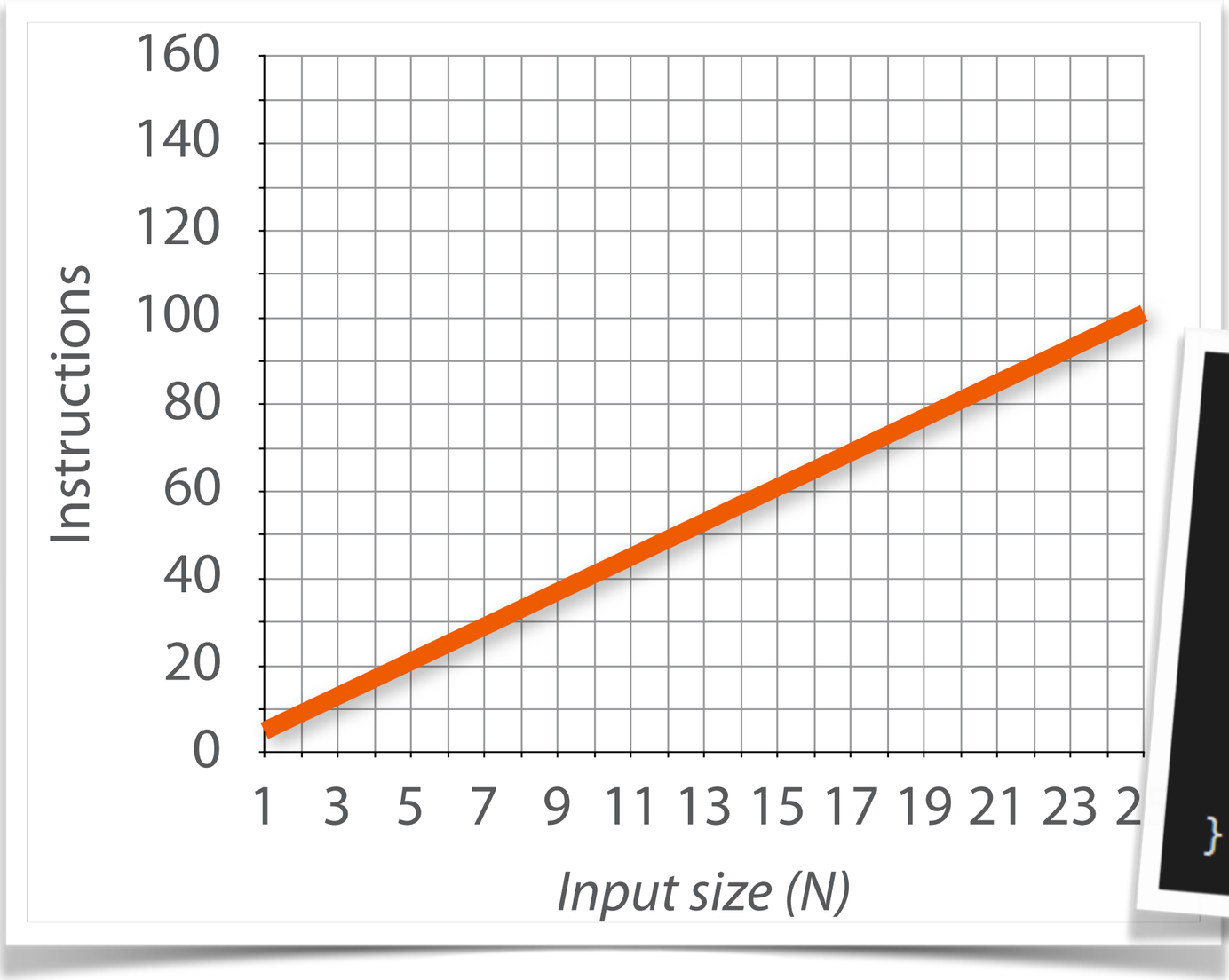
$$= 2 + 3N + 4N^2$$

$$(2 + 3N + 4N^2) \cdot 0.5$$

Big Theta (Θ)

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

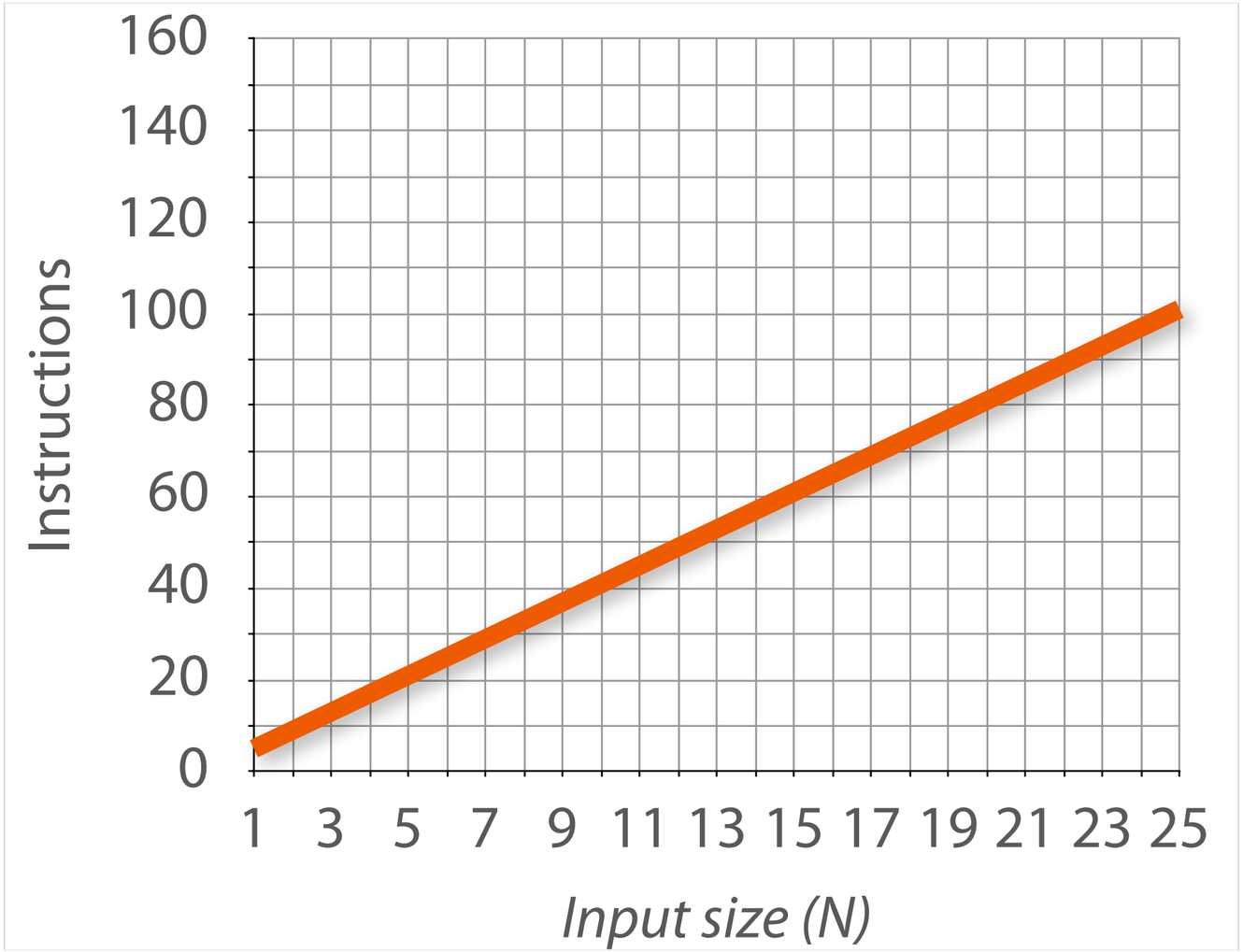
Big Theta (Θ)



$f(N) = 4N + 1$

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

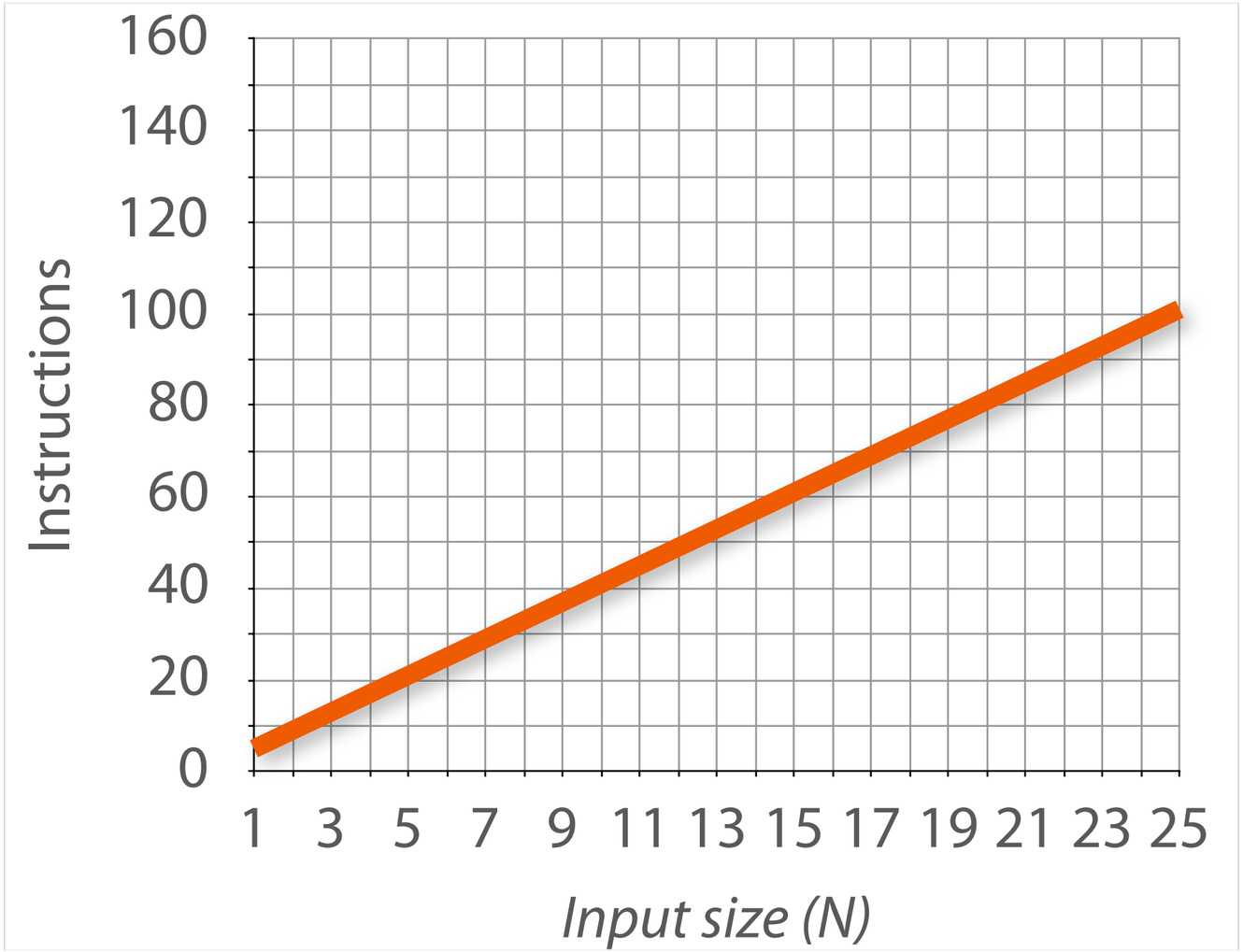
Big Theta (Θ)



$f(N) = 4N + 1$

$g(N) = N$

Big Theta (Θ)

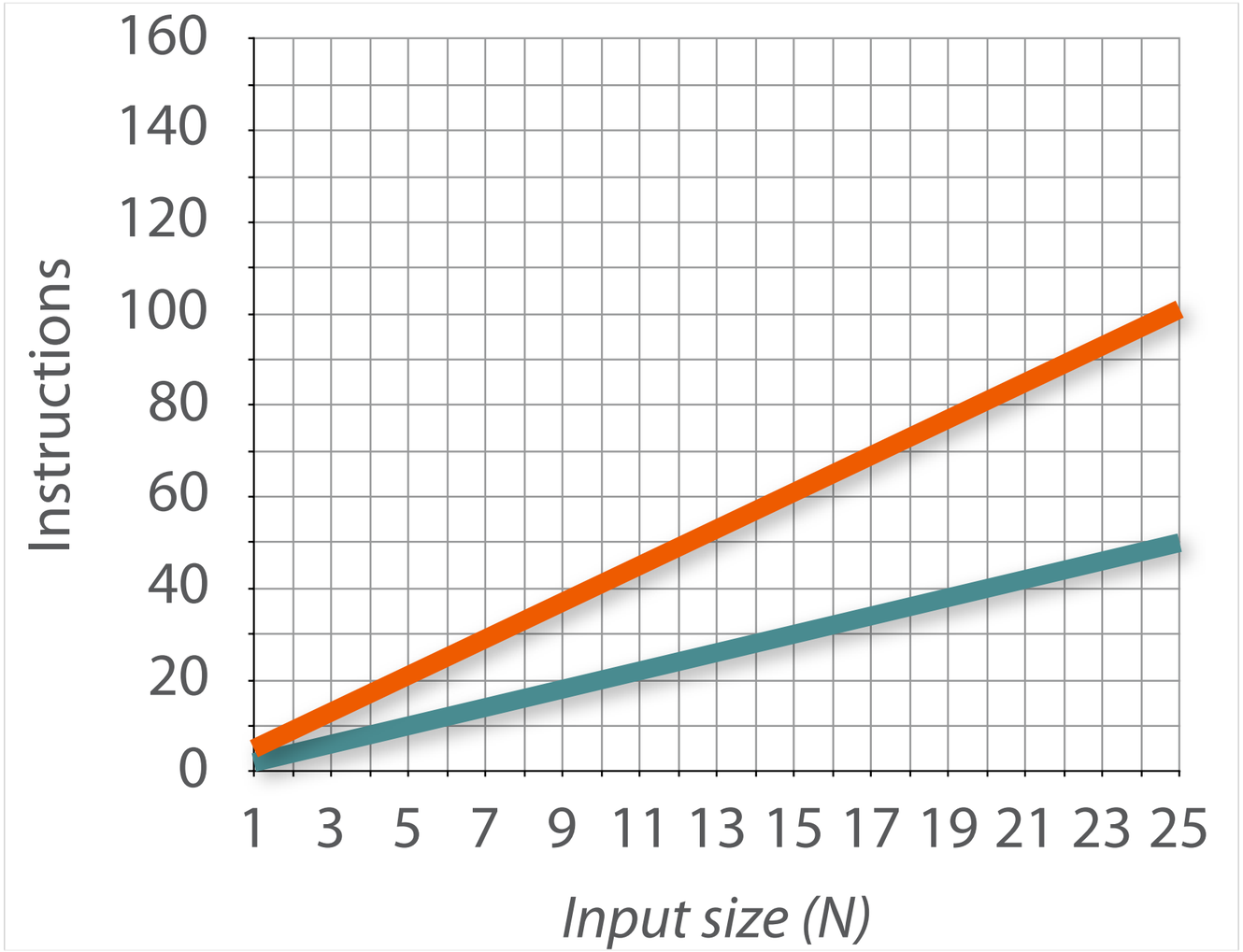


$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)

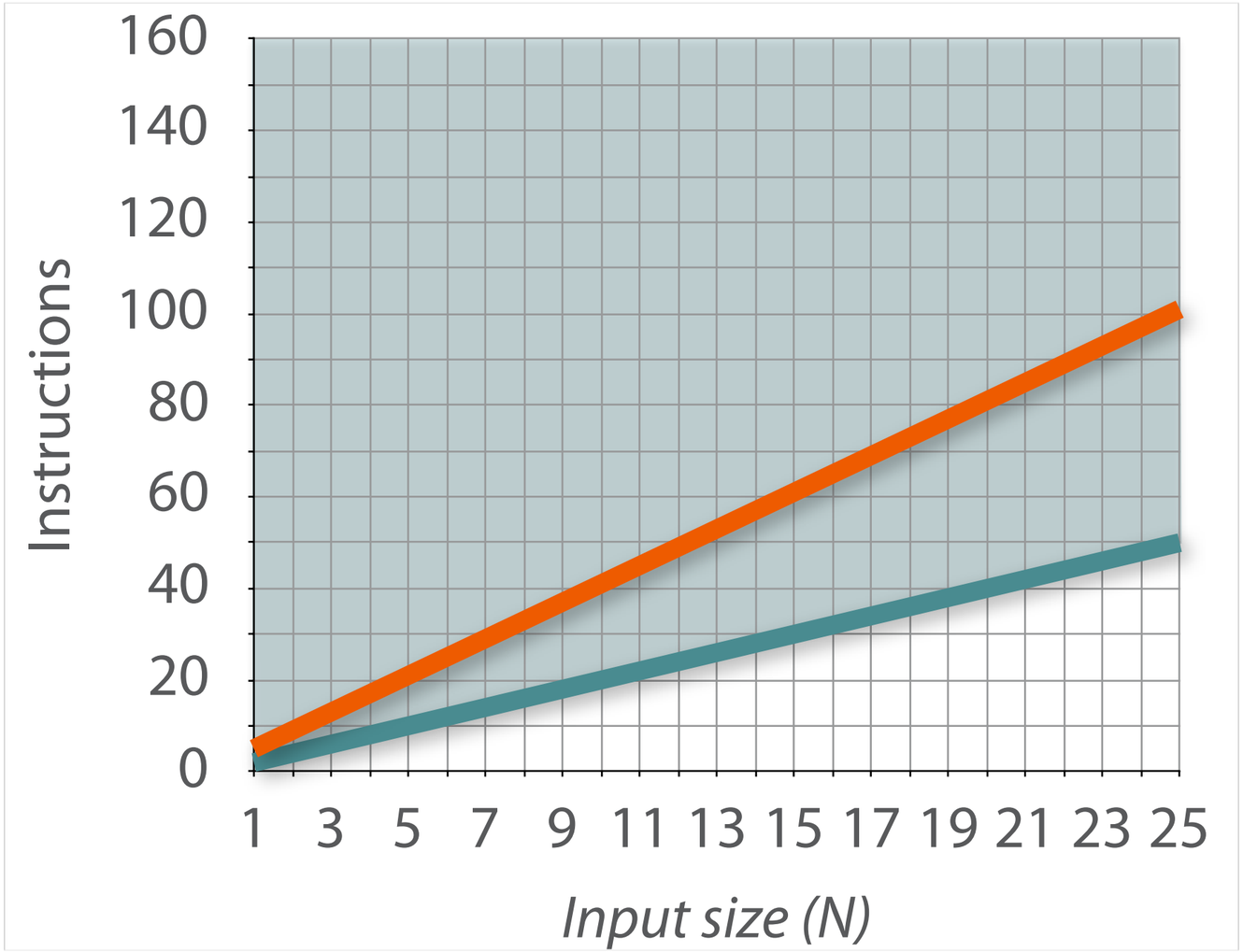


$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)

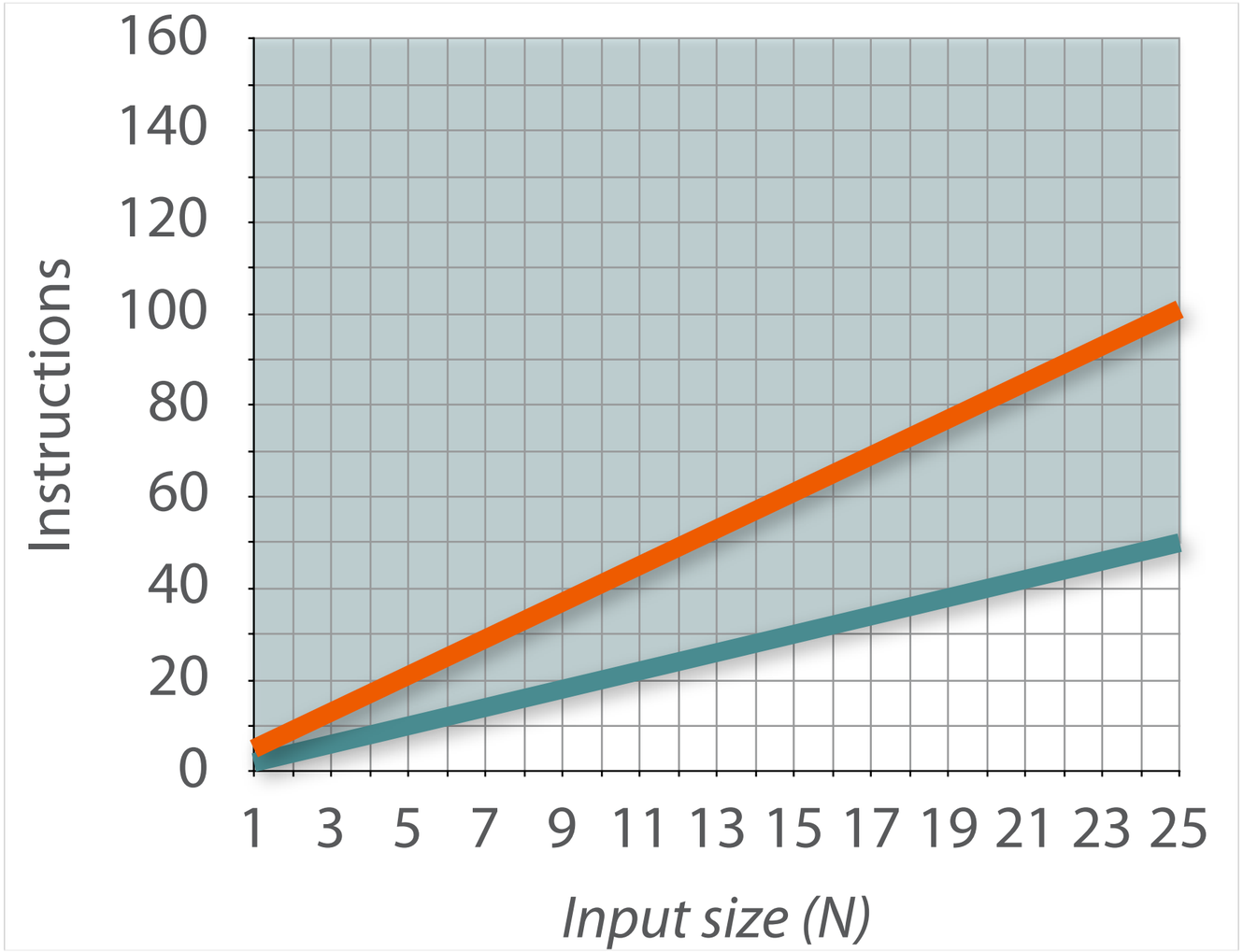


$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)



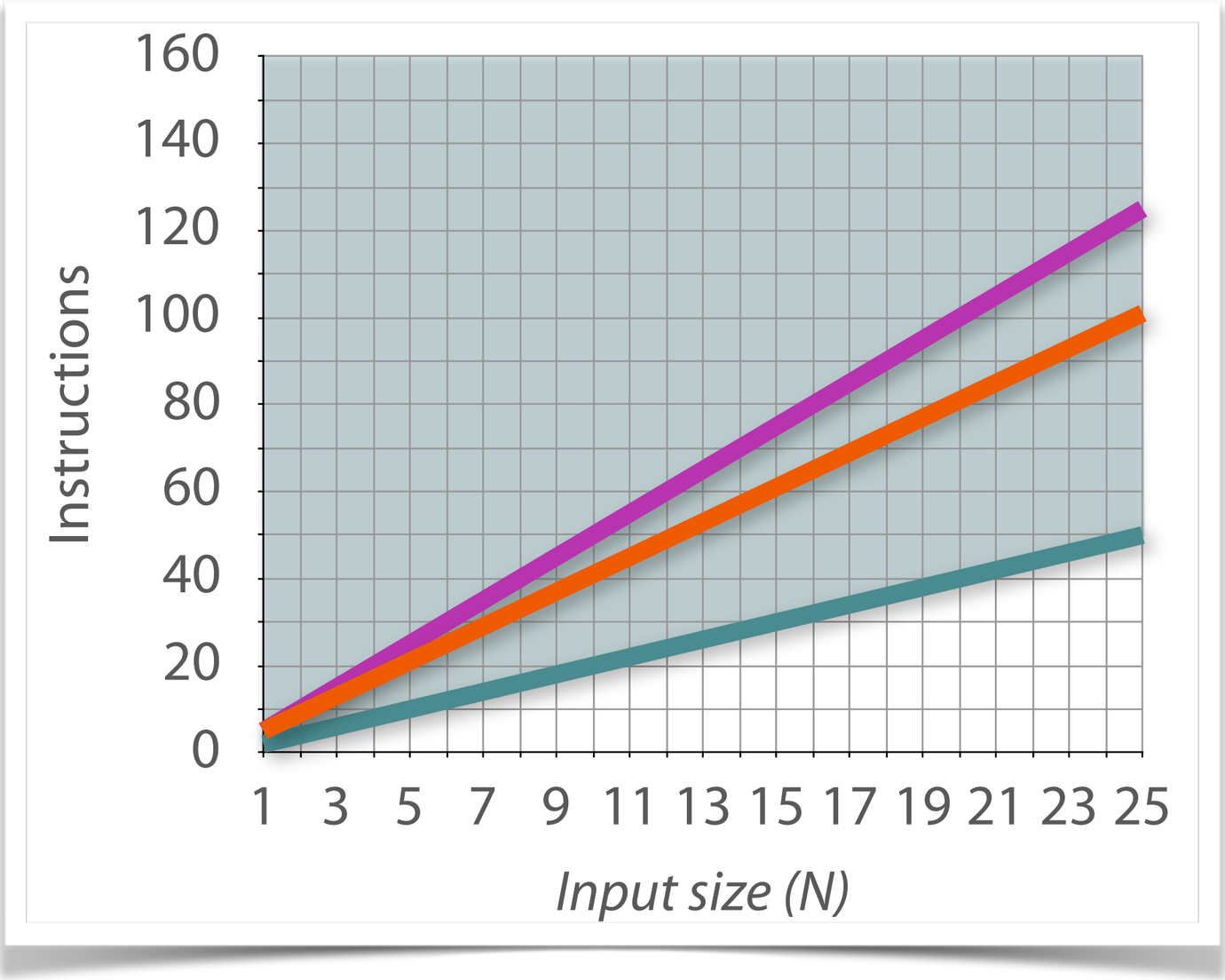
$5g(N) = 5N$

$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)



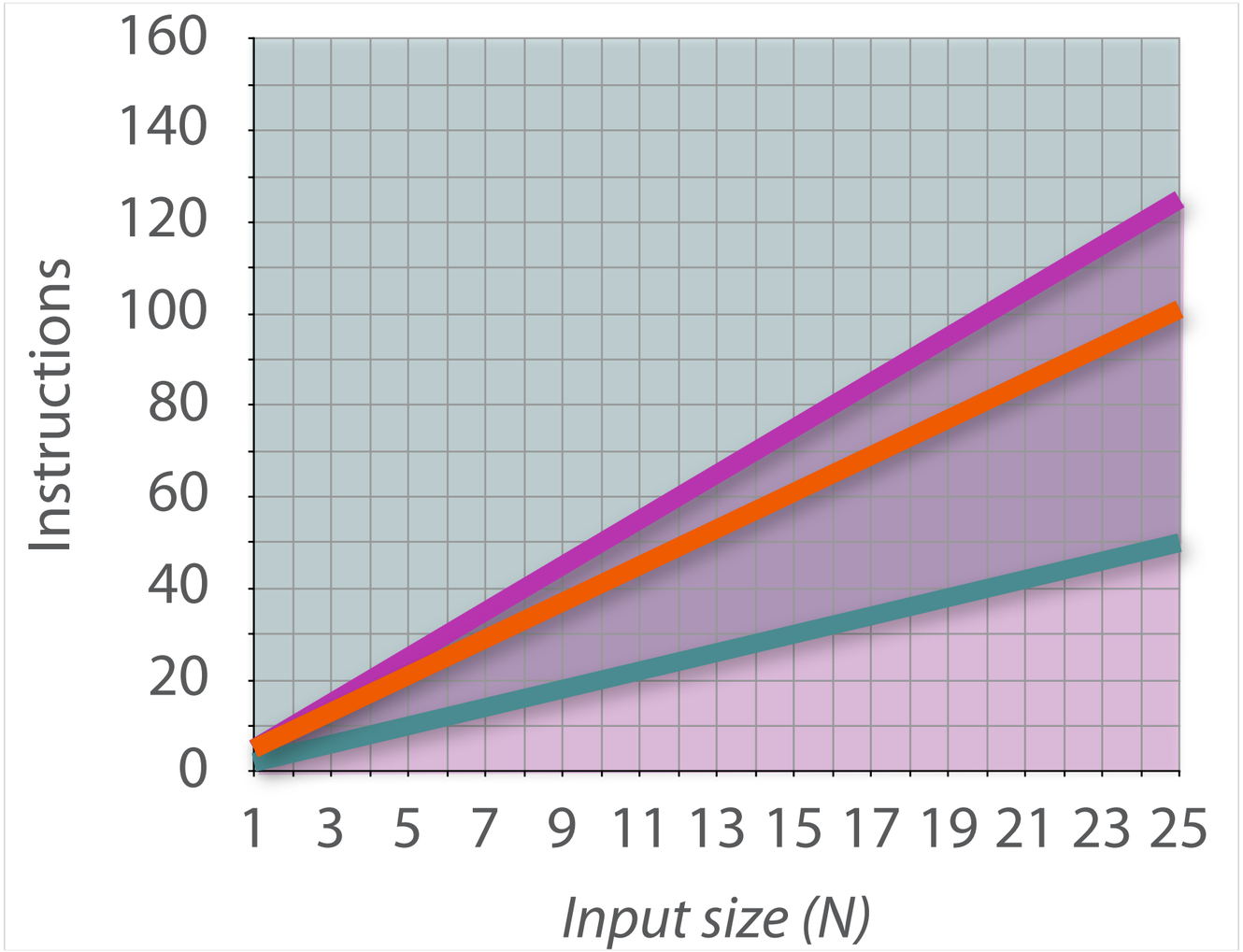
$5g(N) = 5N$

$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)



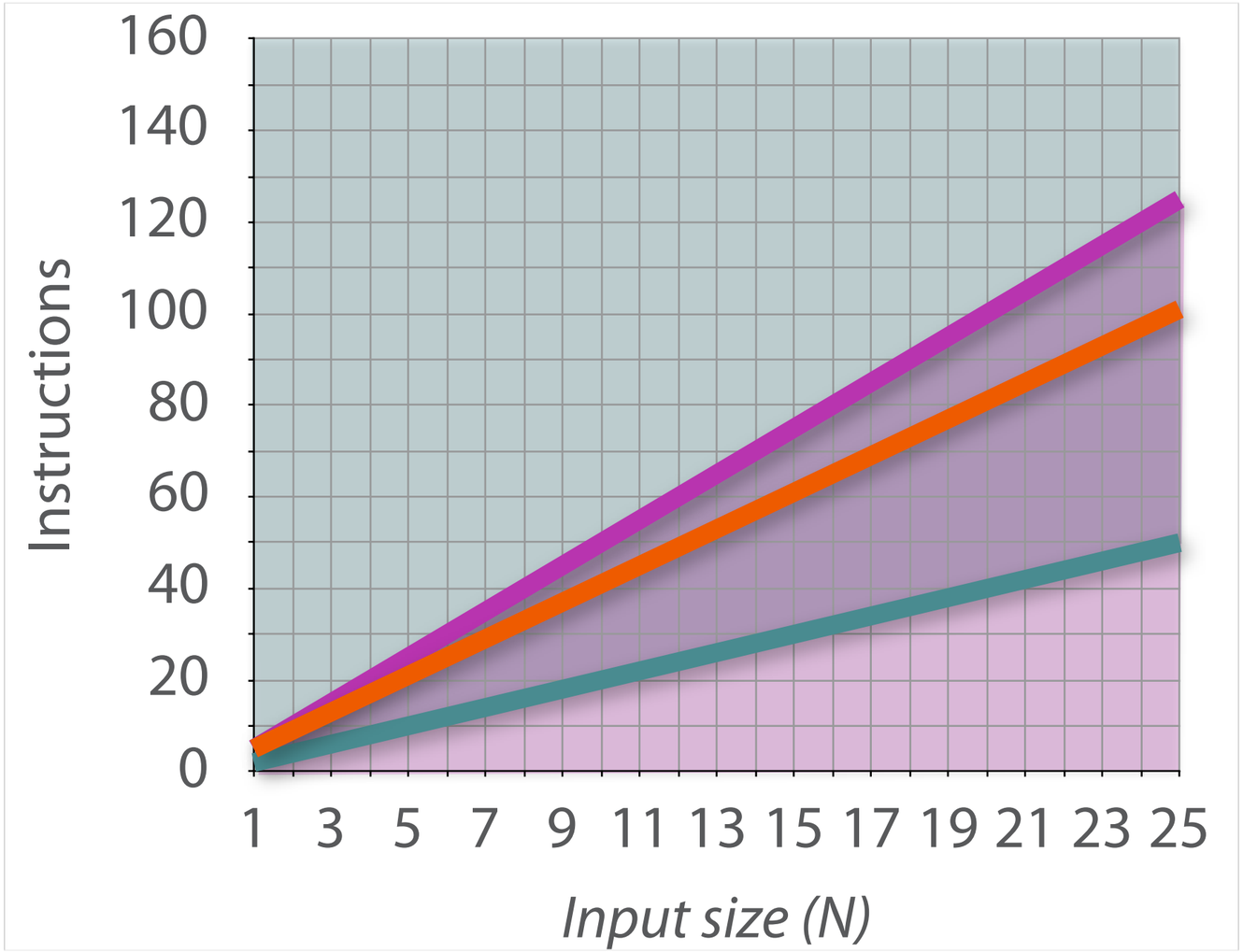
$5g(N) = 5N$

$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

Big Theta (Θ)



$5g(N) = 5N$

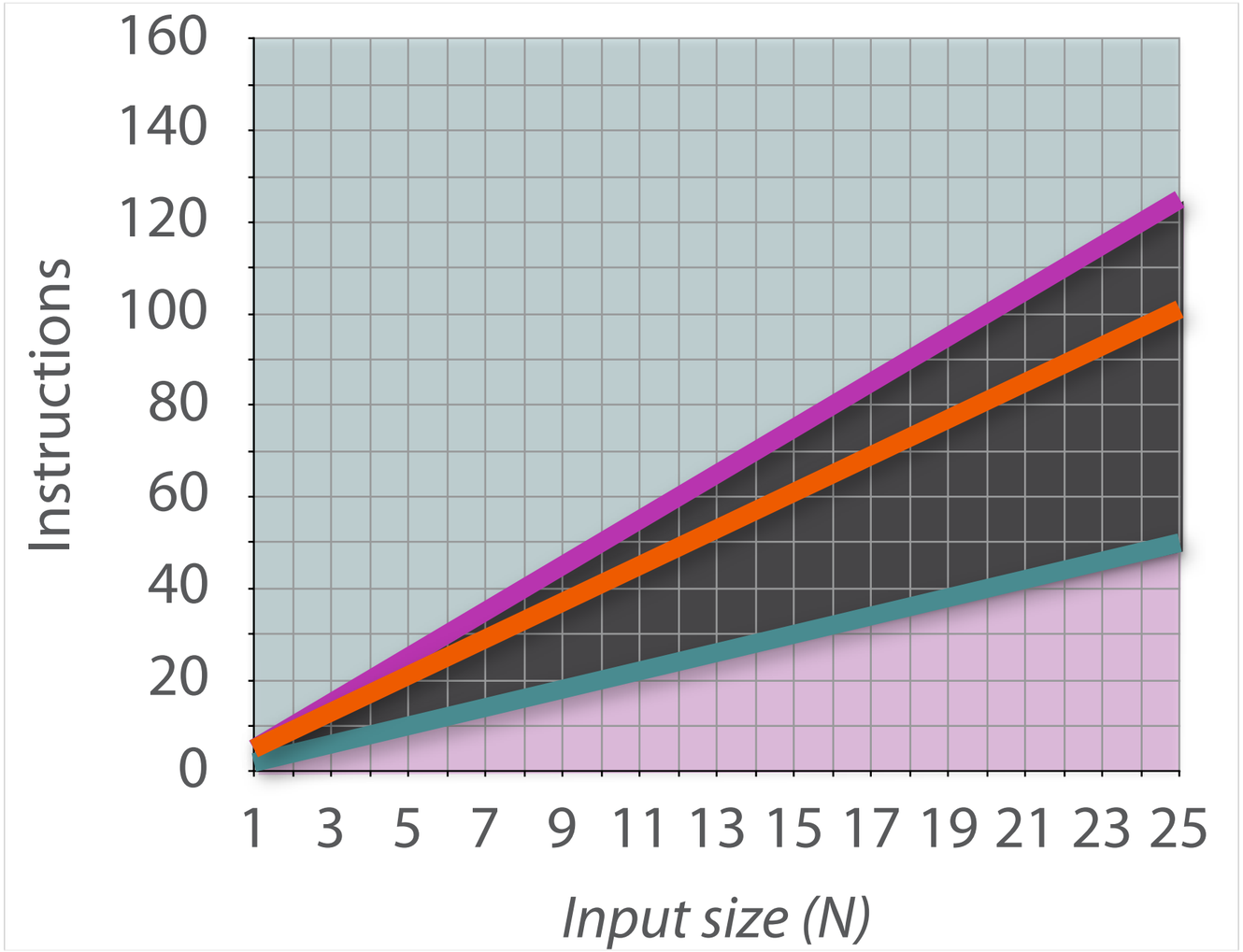
$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

$3g(N) < f(N) < 5g(N)$

Big Theta (Θ)



$5g(N) = 5N$

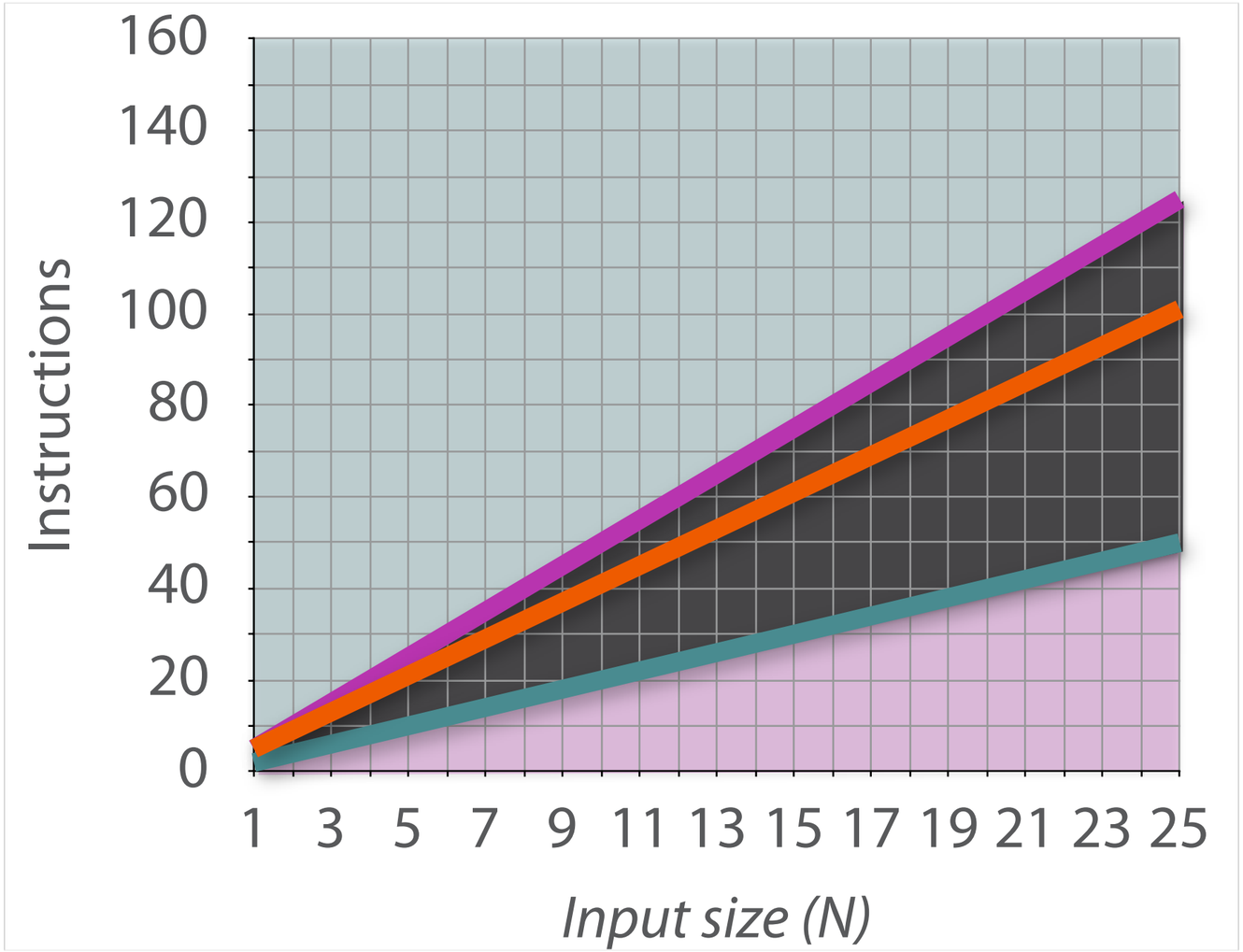
$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

$3g(N) < f(N) < 5g(N)$

Big Theta (Θ)



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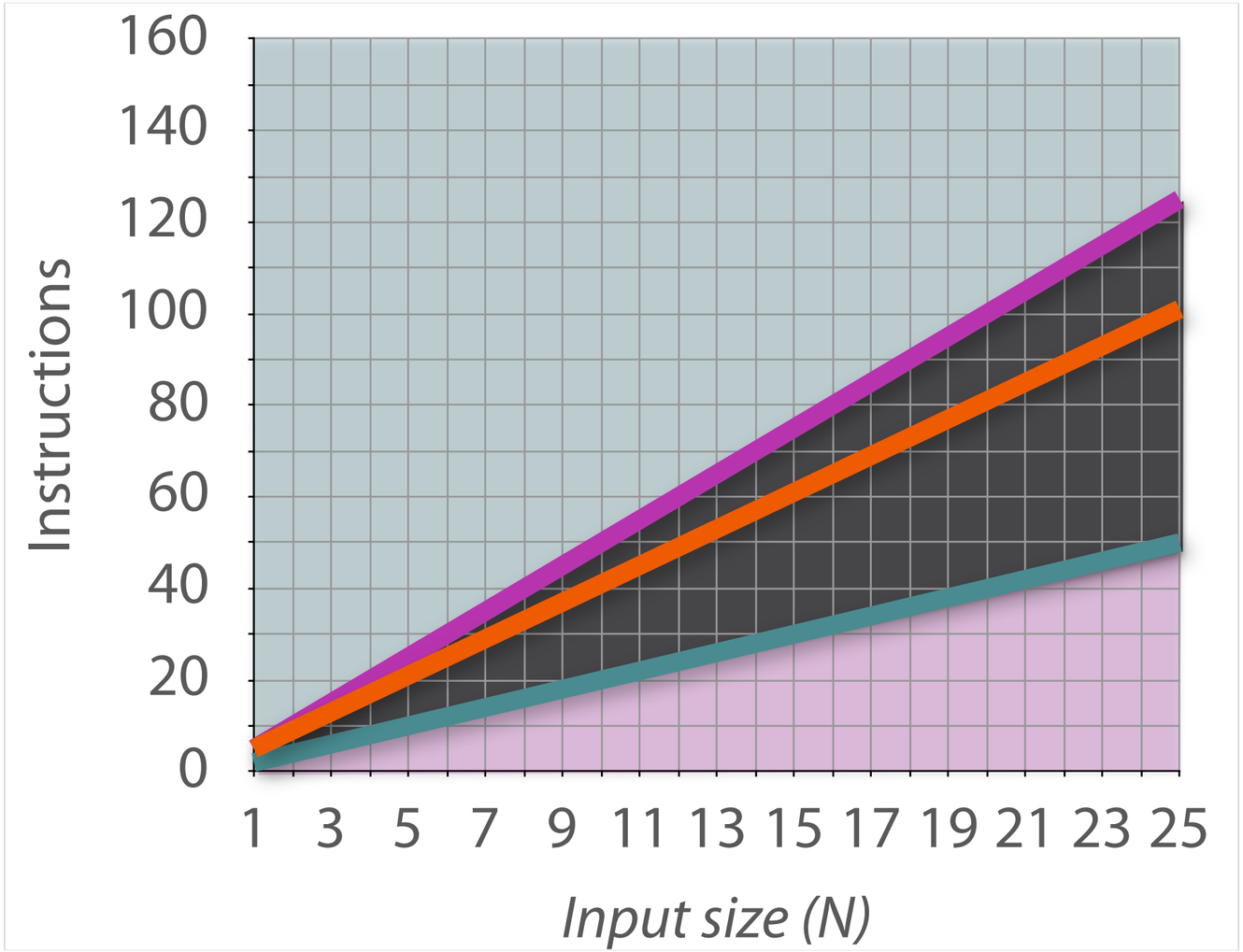
Big Theta (Θ)



$5g(N) = 5N$

$f(N) = 4N + 1$

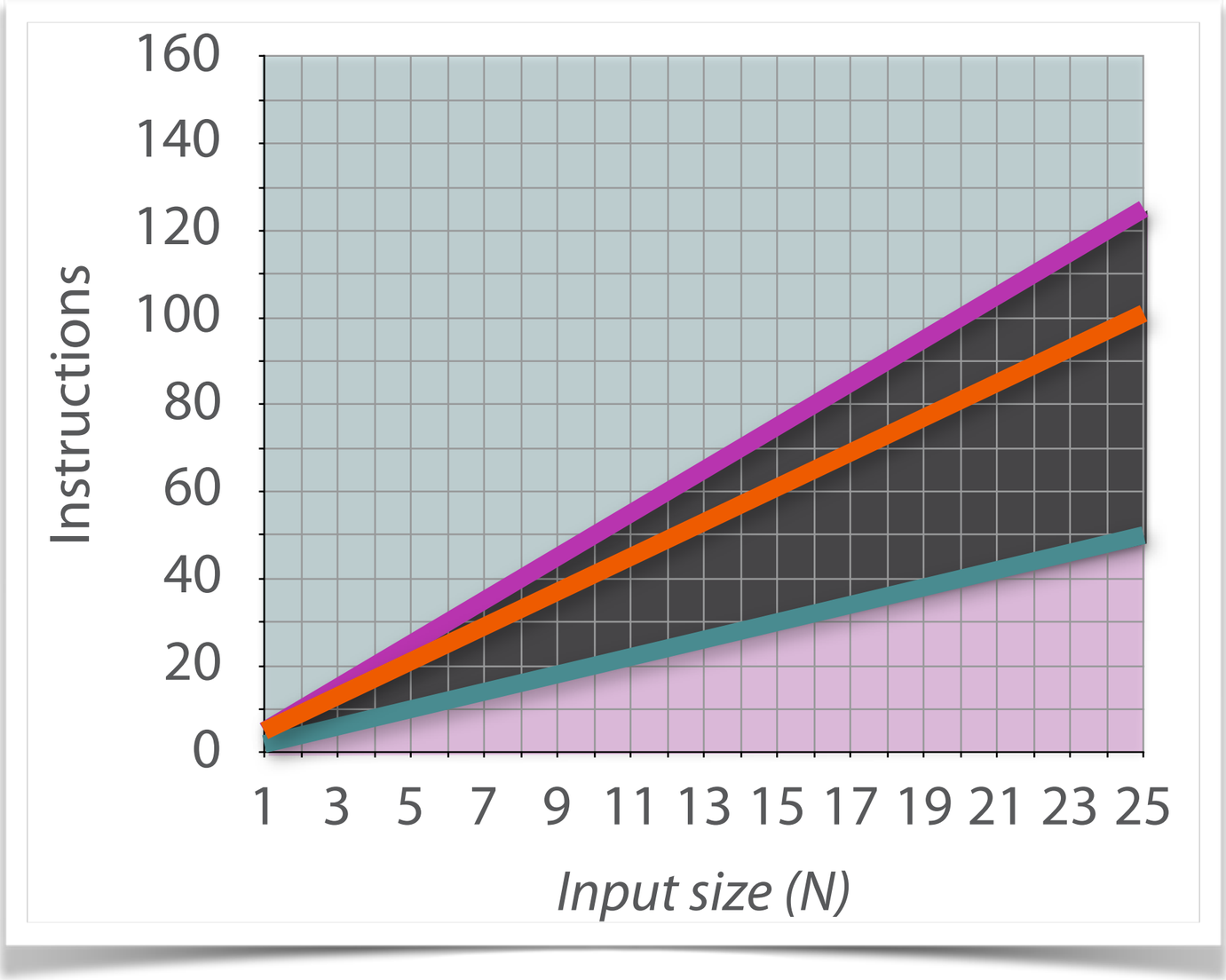
$g(N) = N$

$3g(N) = 3N$

$3g(N) < f(N) < 5g(N)$

$\Theta(g(N))$

Big Theta (Θ)



$5g(N) = 5N$

$f(N) = 4N + 1$

$g(N) = N$

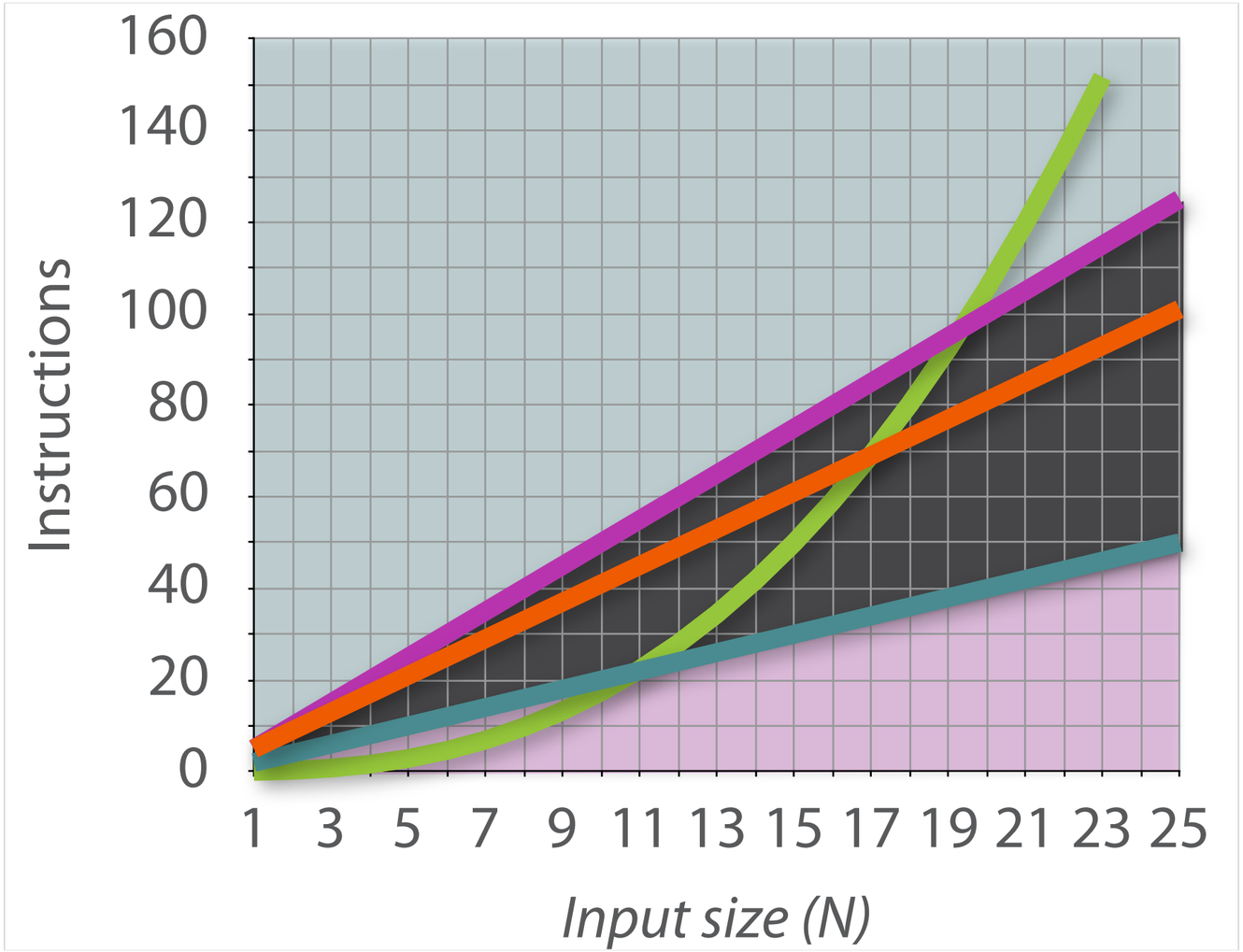
$3g(N) = 3N$

$3g(N) < f(N) < 5g(N)$

$\Theta(g(N))$

$\Theta(N)$

Big Theta (Θ)



$5g(N) = 5N$

$f(N) = 4N + 1$

$g(N) = N$

$3g(N) = 3N$

$3g(N) < f(N) < 5g(N)$

$\Theta(g(N))$

$\Theta(N)$

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    {
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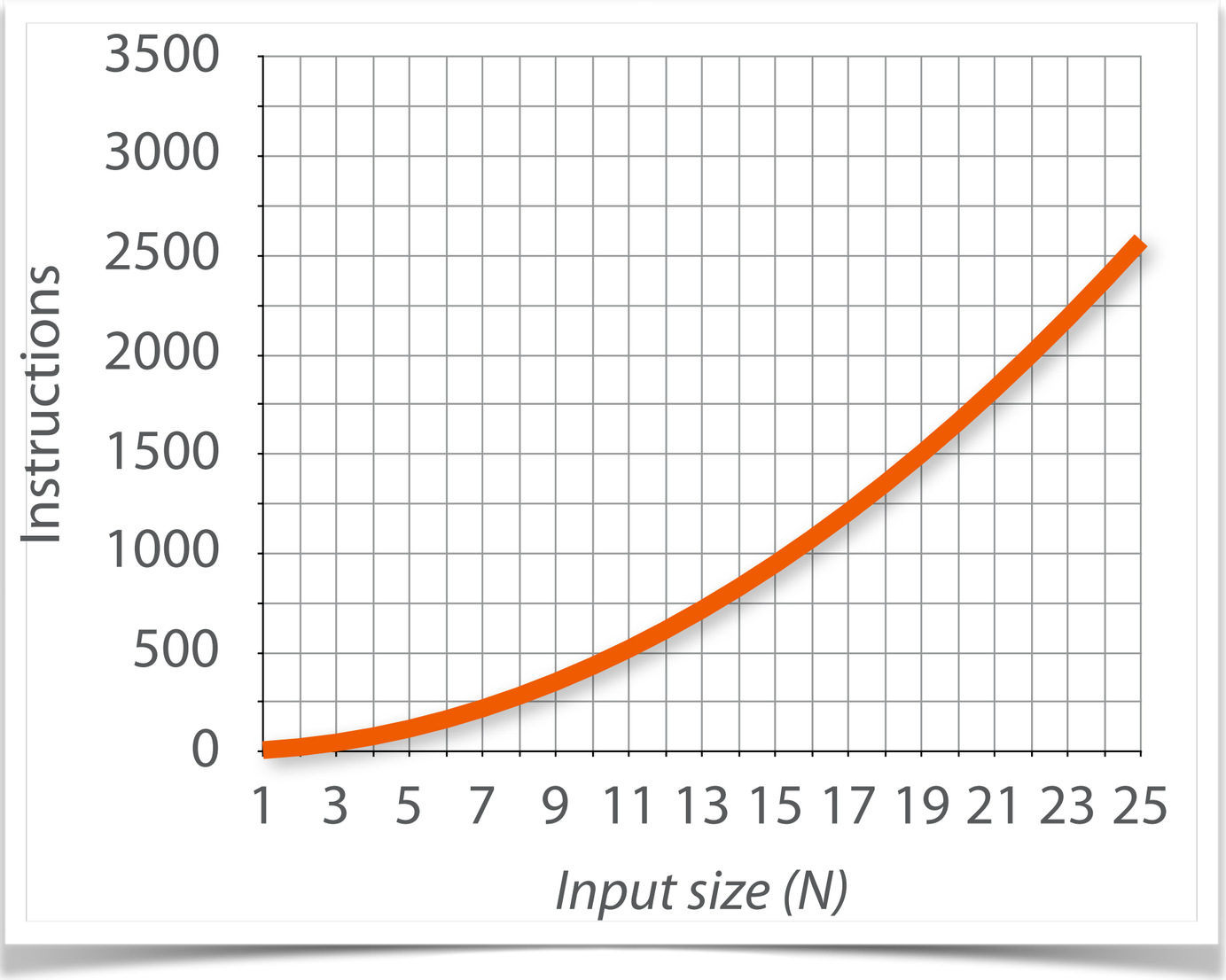
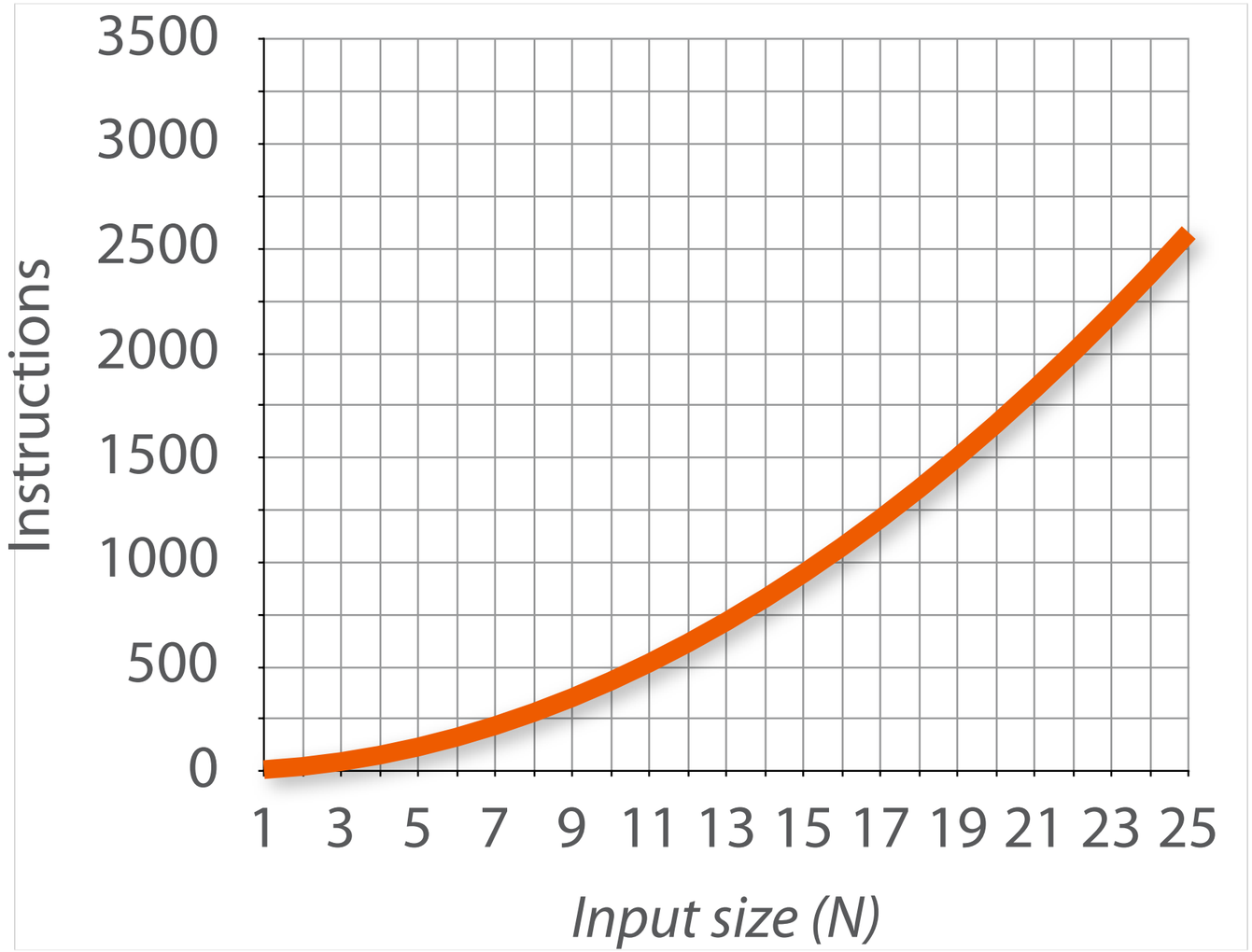
            y = y + 1;
        }
        x = x + 1;
    }
}
```

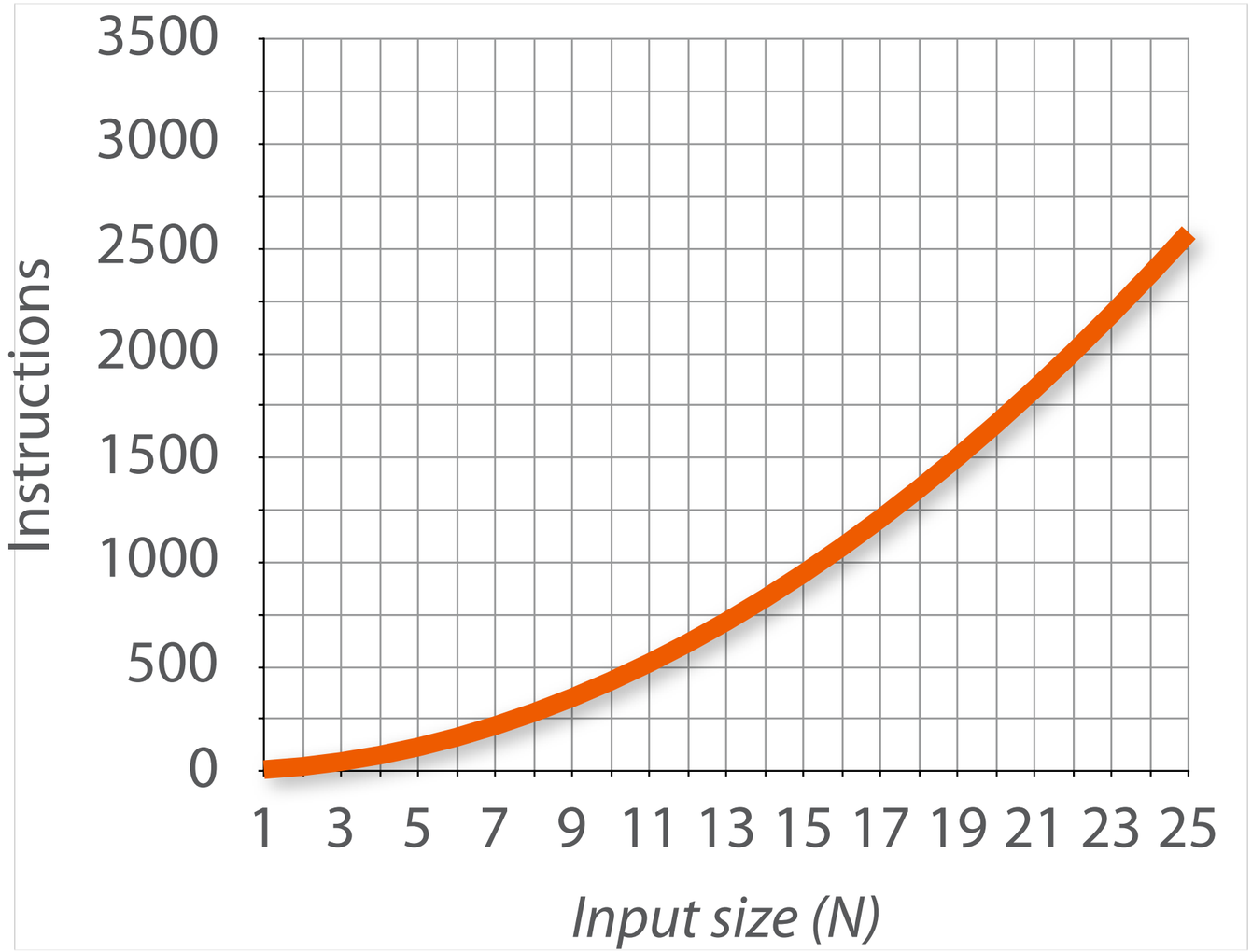

3500
3000
2500
2000
1500
1000
500
0
Instructions
1 3 5 7 9 11 13 15 17 19 21 23 25
Input size (N)

$f(N) = 2 + 3N + 4N^2$



$f(N) = 2 + 3N + 4N^2$

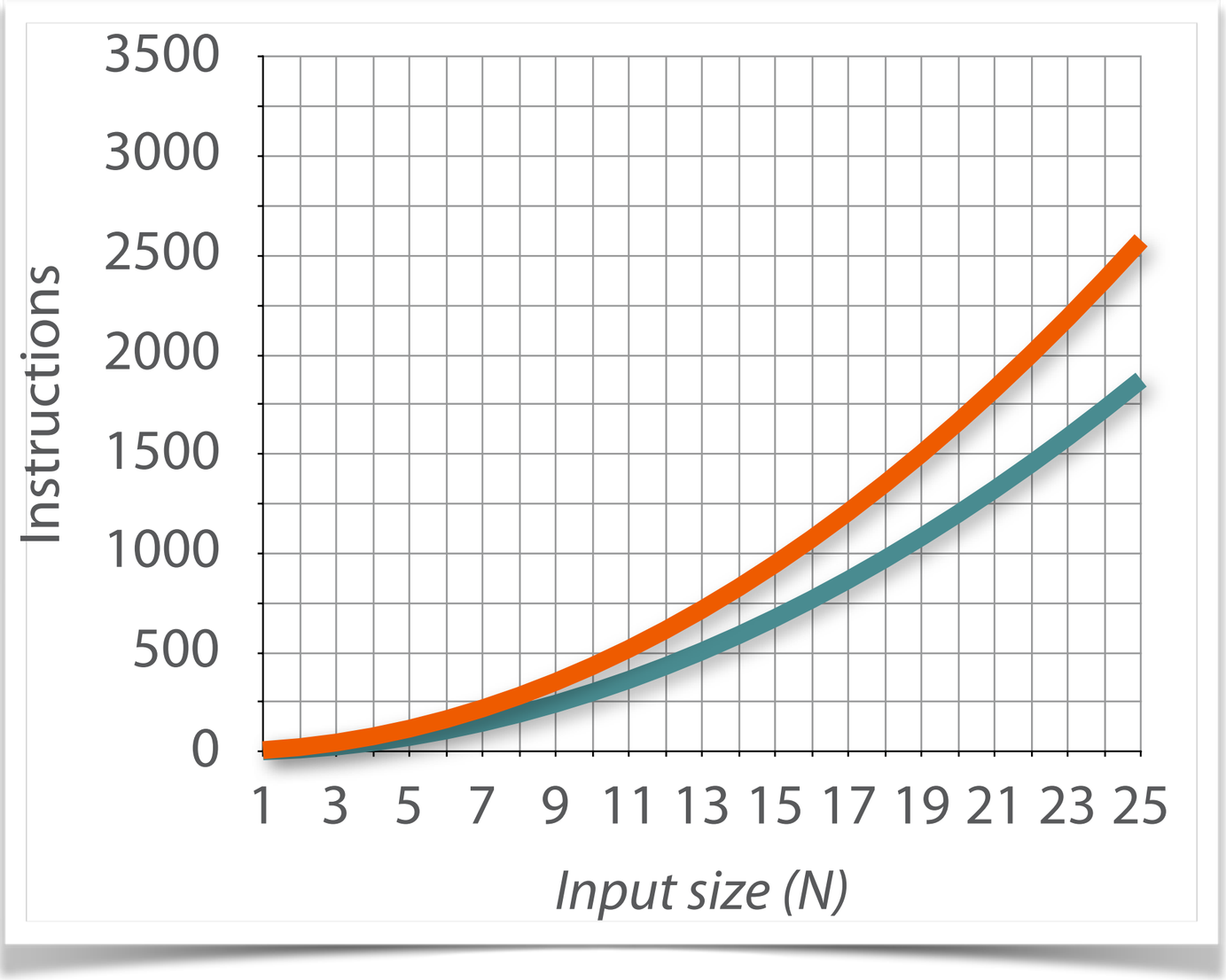
$g(N) = N^2$



$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

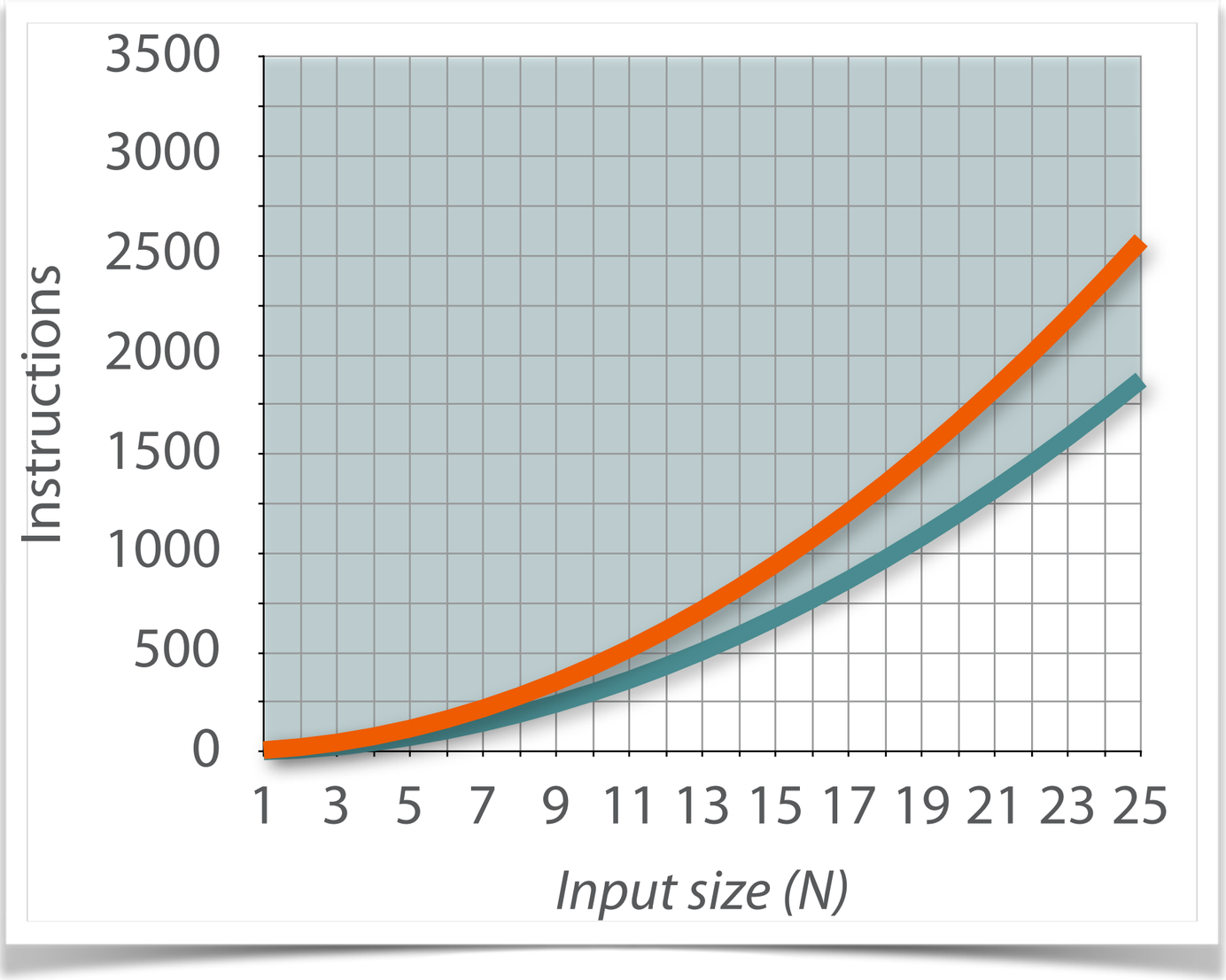
$3g(N) = 3N^2$



$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

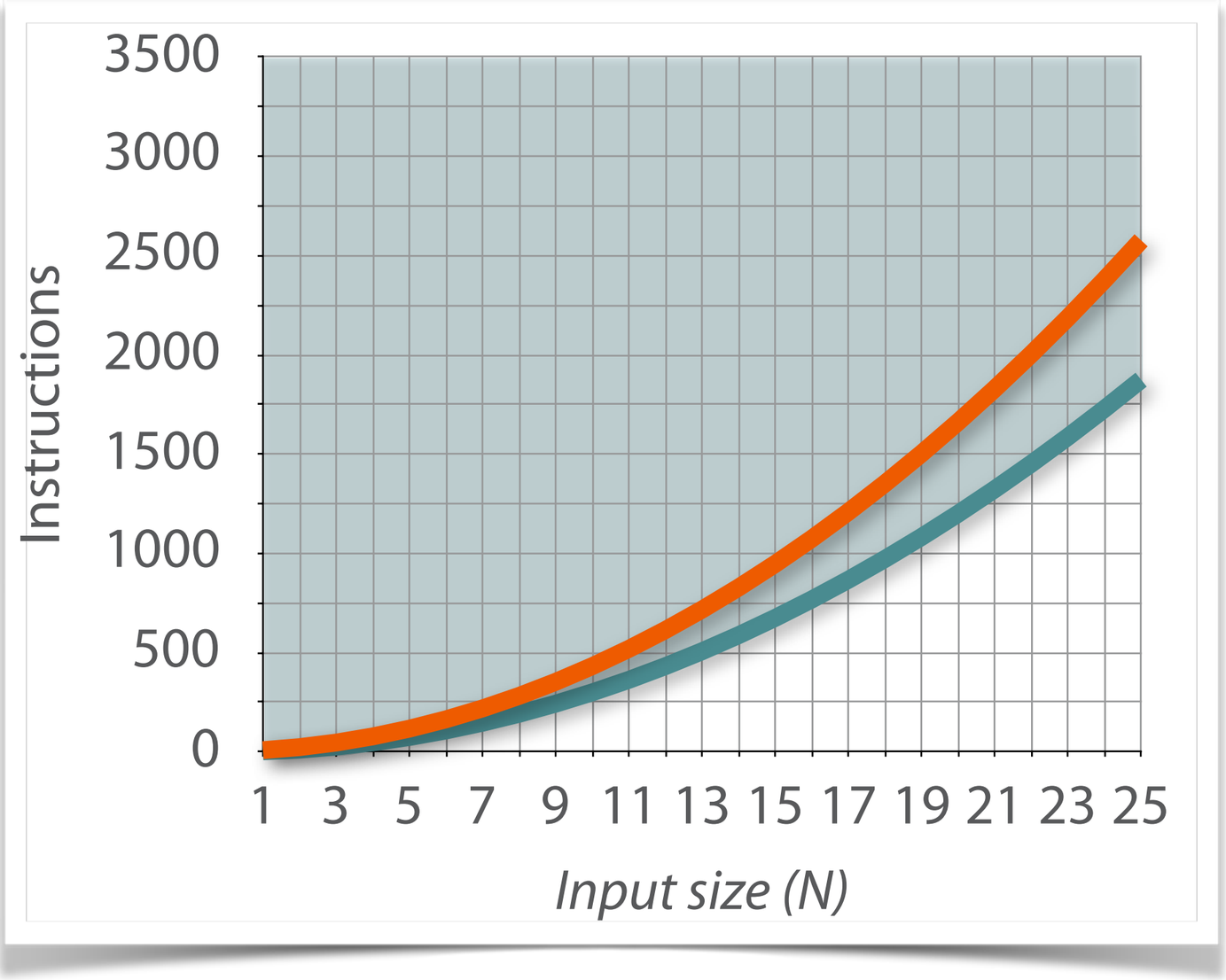
$3g(N) = 3N^2$



$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$

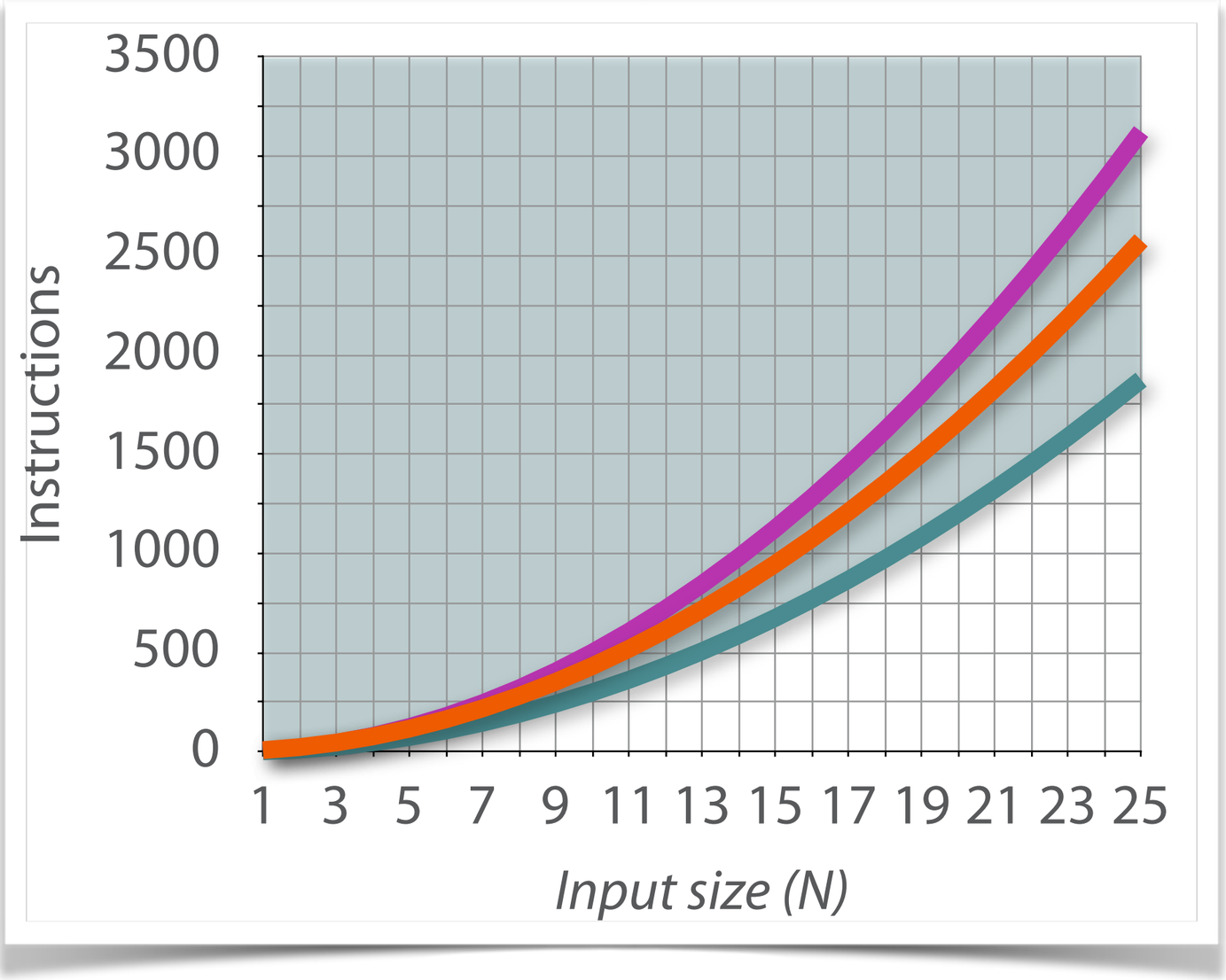


$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$

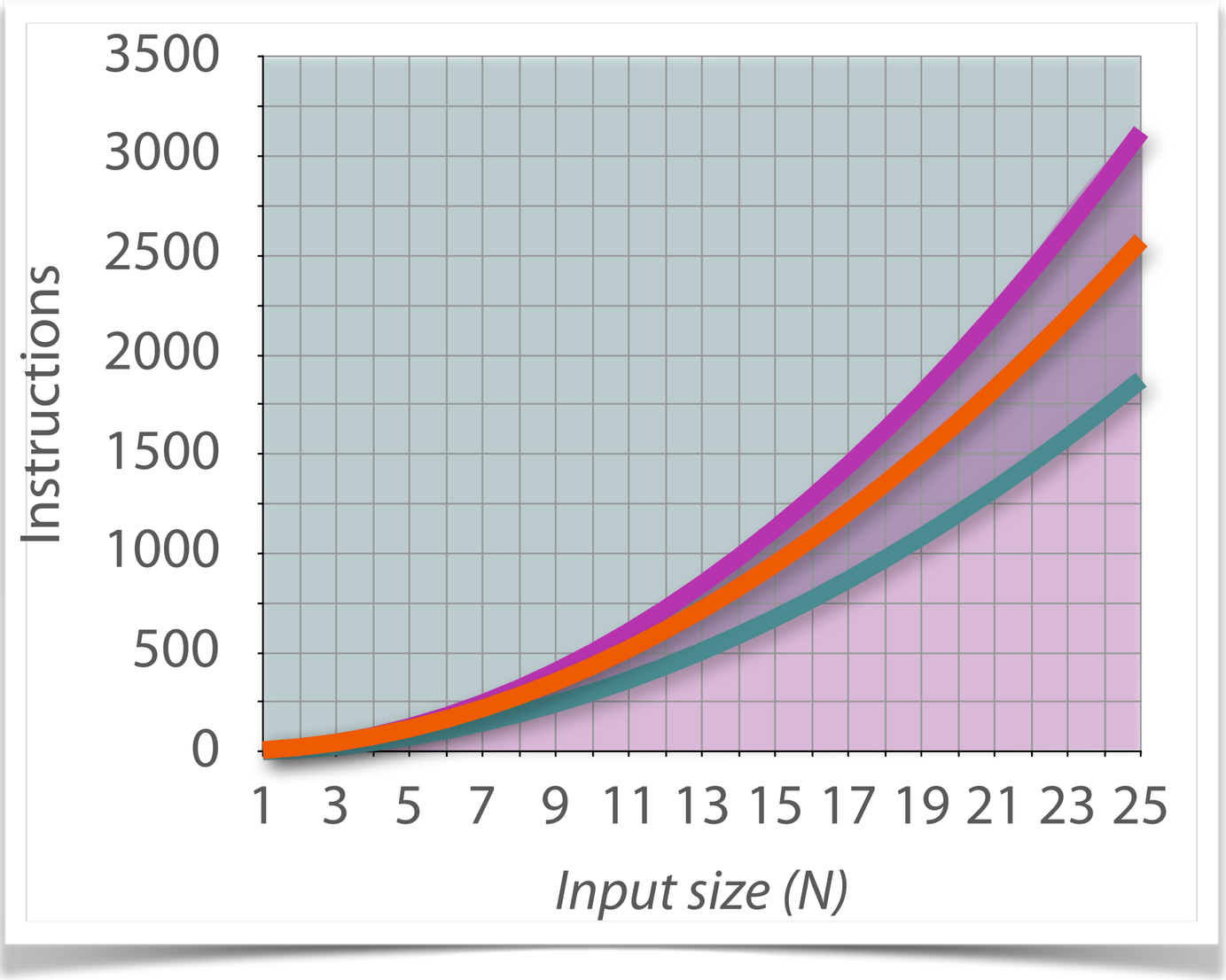


$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$

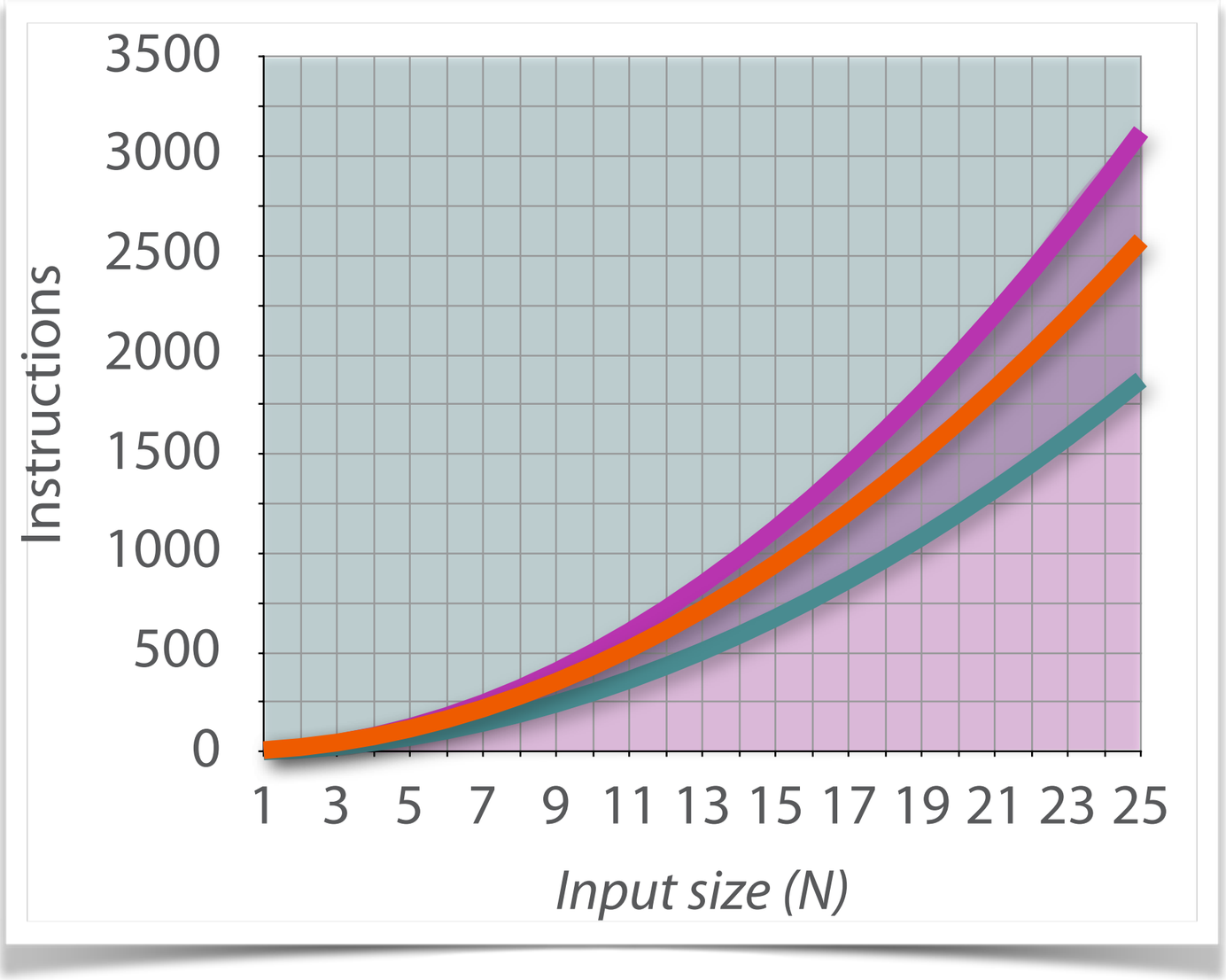


$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$



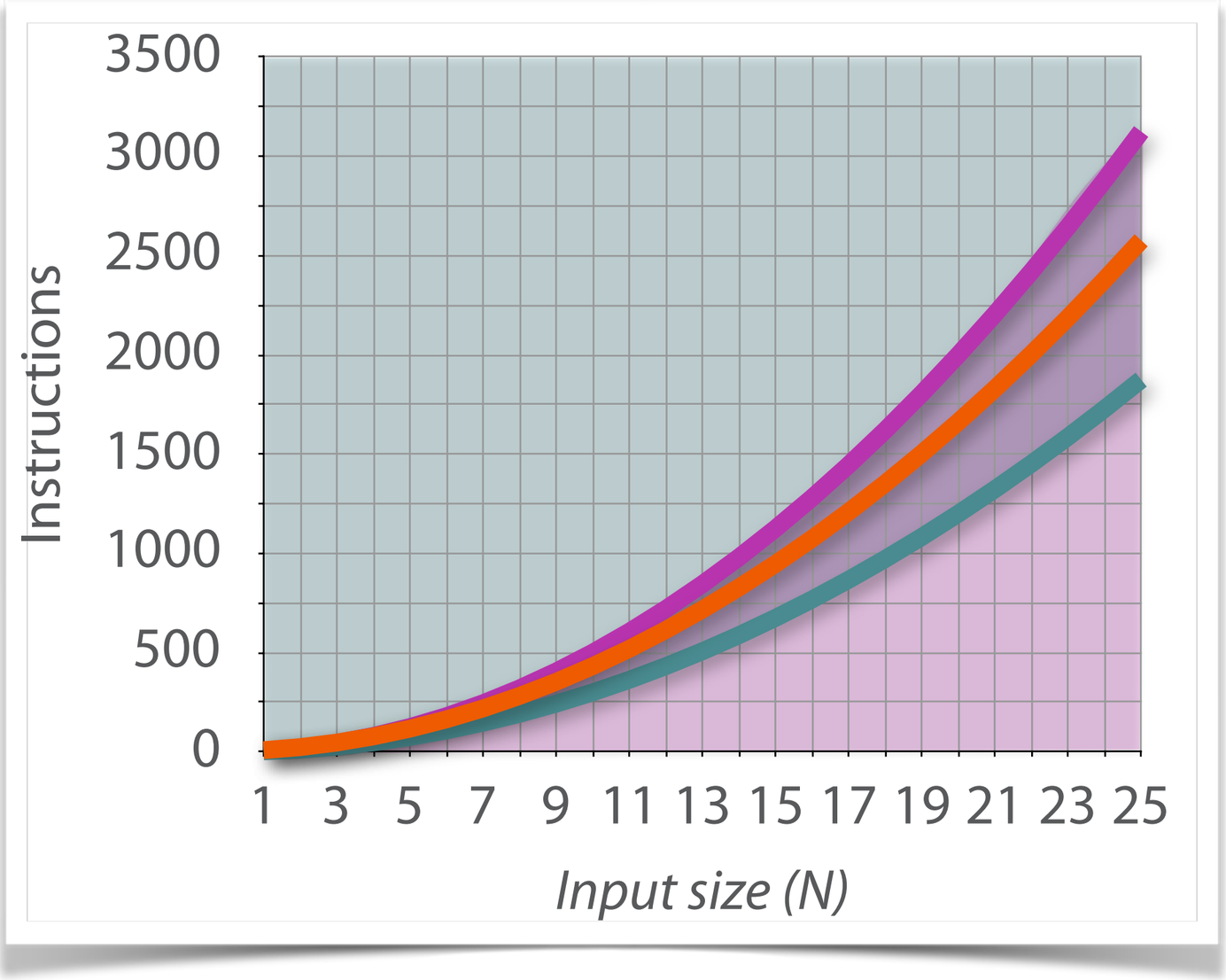
N	1	2	3	4
f(N)	9	24	47	78

$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$



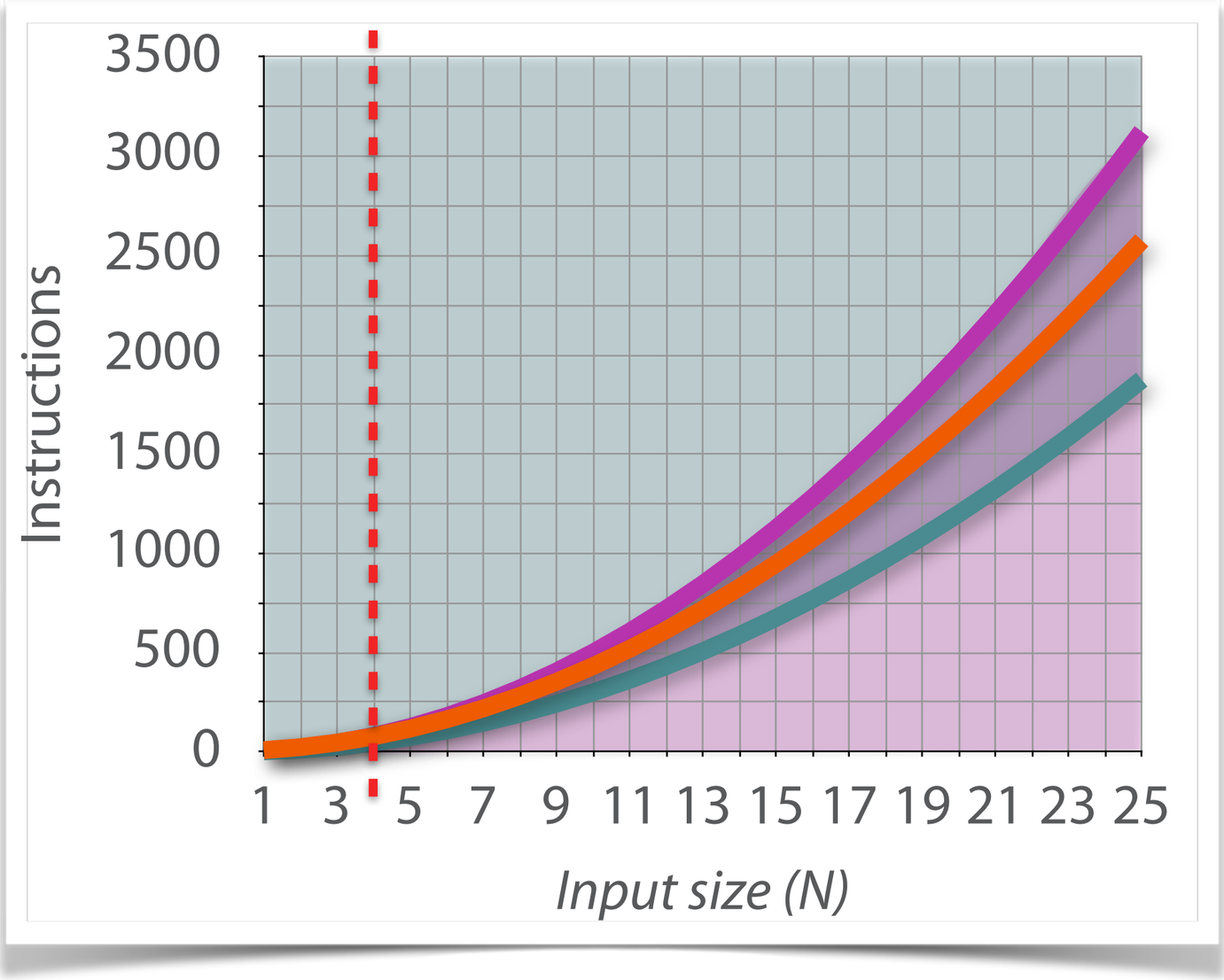
N	1	2	3	4
f(N)	9	24	47	78
5g(N)	5	20	45	80

$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$



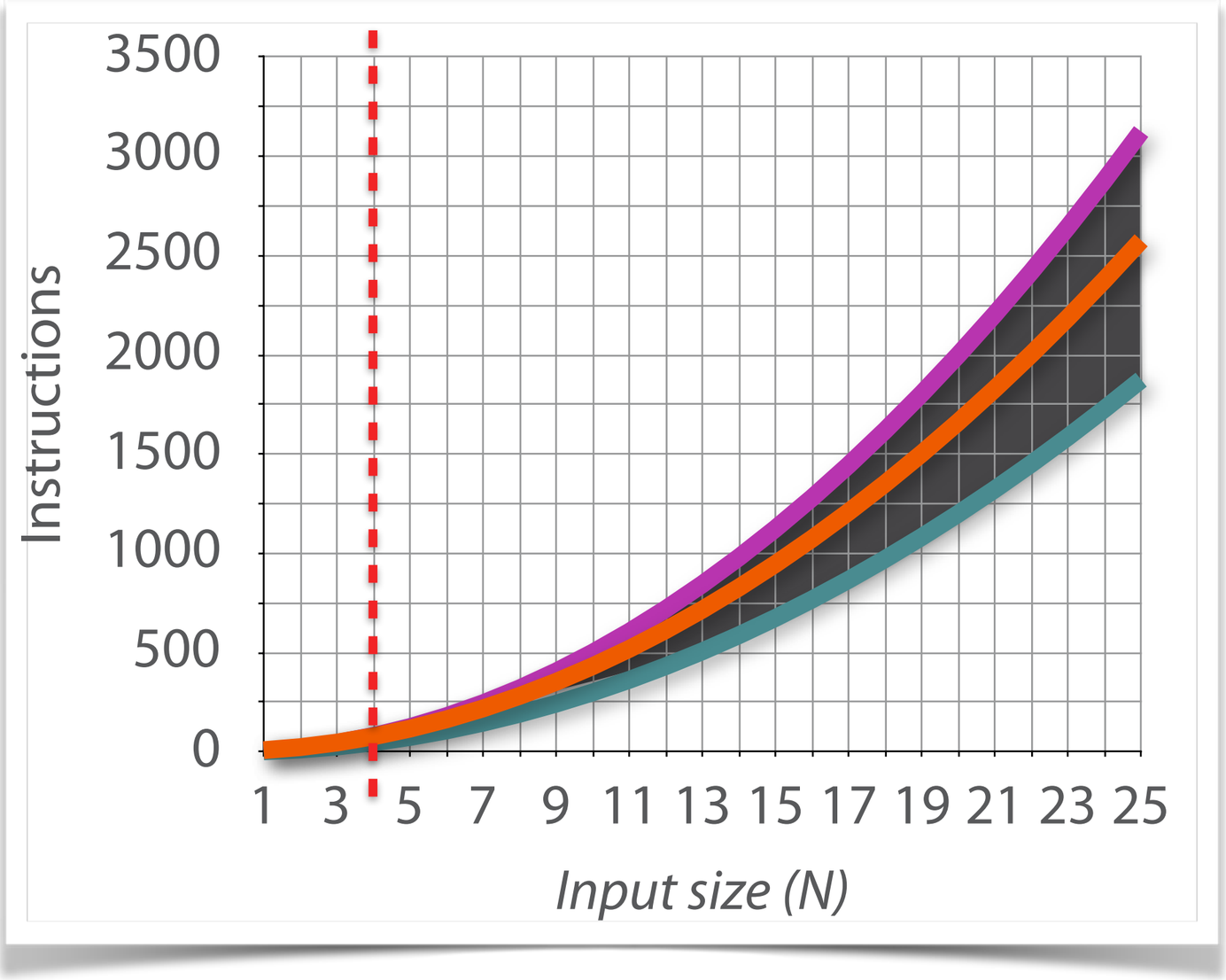
N	1	2	3	4
f(N)	9	24	47	78
5g(N)	5	20	45	80

$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$



N	1	2	3	4
f(N)	9	24	47	78
5g(N)	5	20	45	80

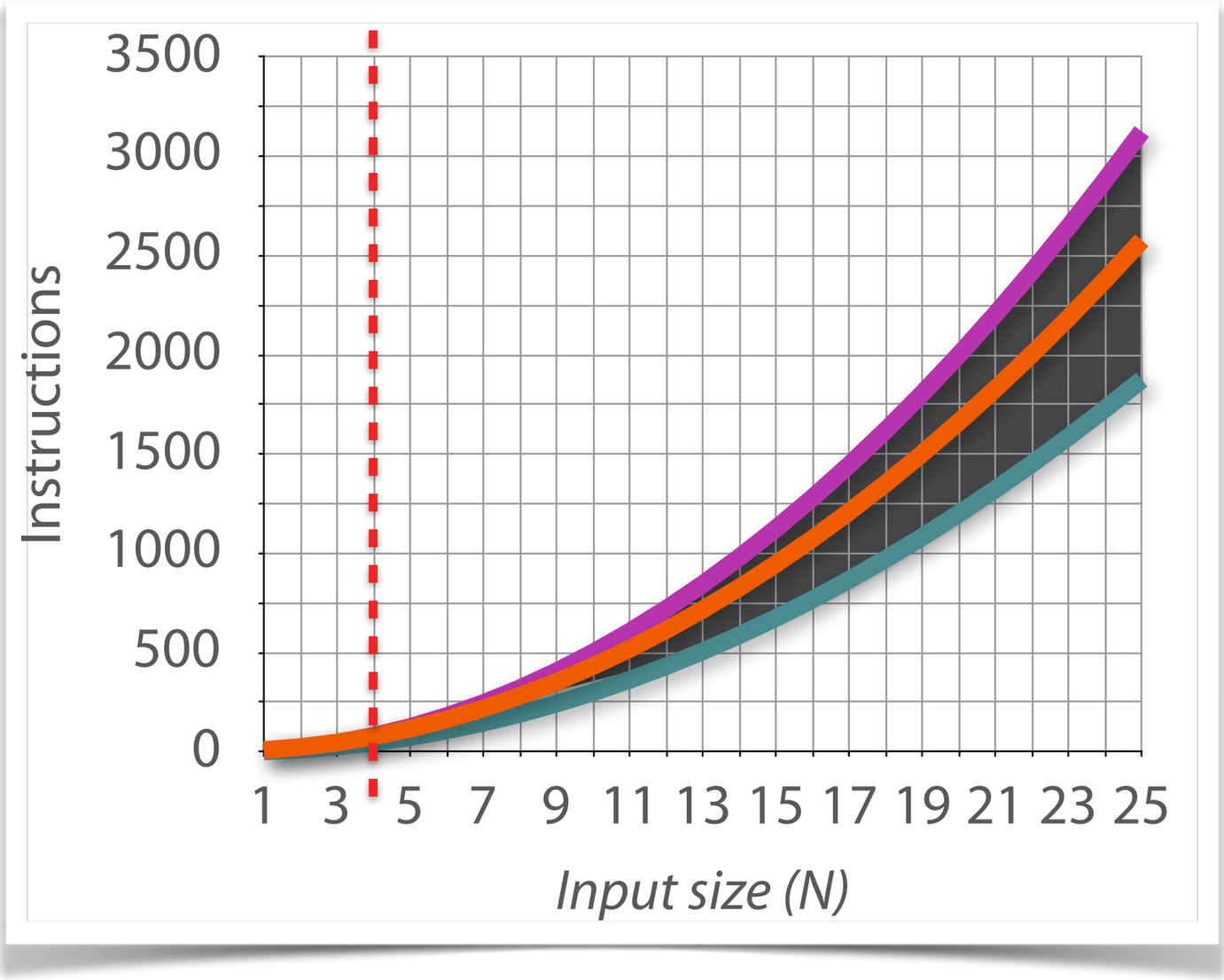
$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$

$$3g(N) < f(N) < 5g(N)$$



N	1	2	3	4
f(N)	9	24	47	78
5g(N)	5	20	45	80

$5g(N) = 5N^2$

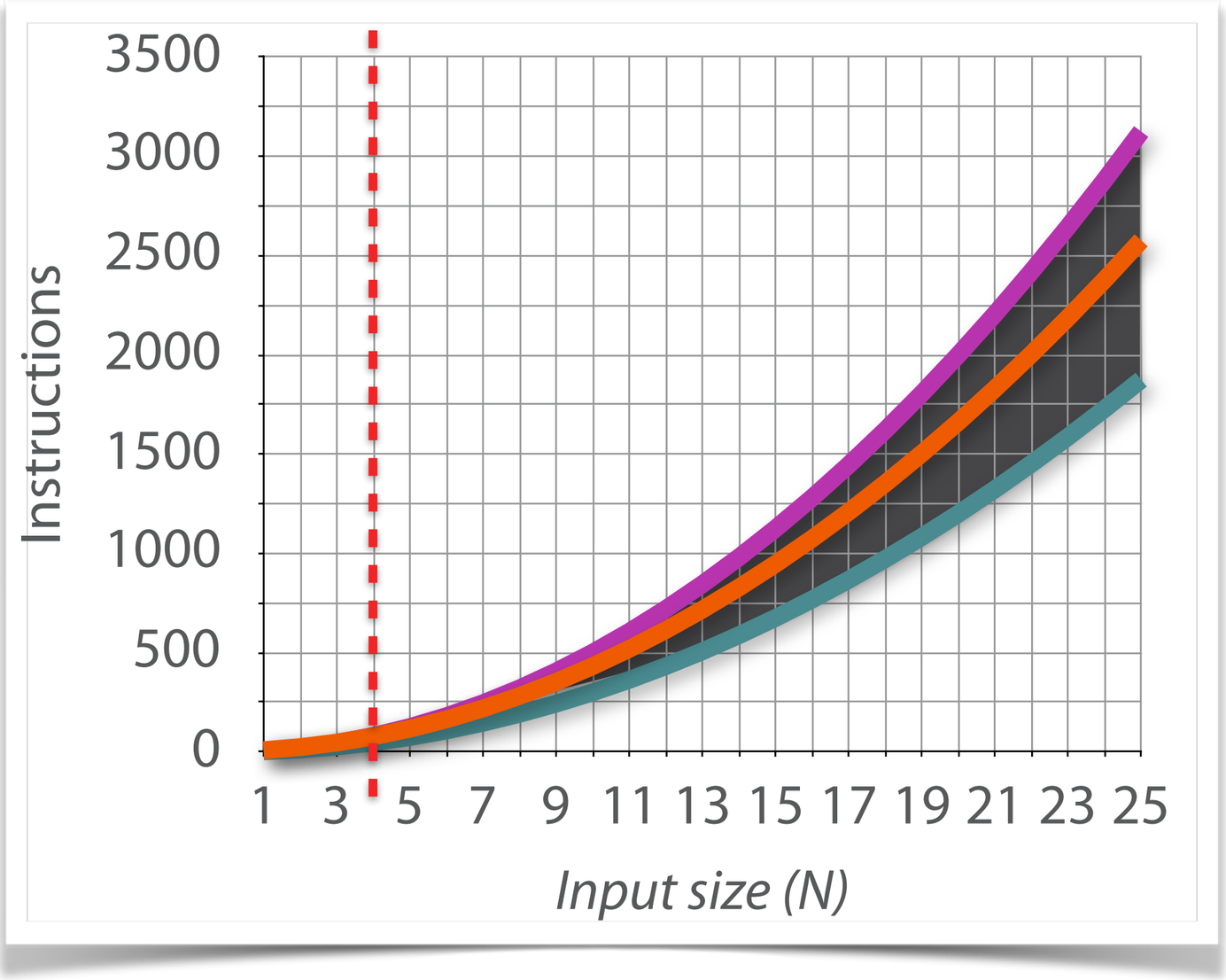
$f(N) = 2 + 3N + 4N^2$

$g(N) = N^2$

$3g(N) = 3N^2$

$3g(N) < f(N) < 5g(N)$

when $N > 3$



N	1	2	3	4
f(N)	9	24	47	78
5g(N)	5	20	45	80

$5g(N) = 5N^2$

$f(N) = 2 + 3N + 4N^2$

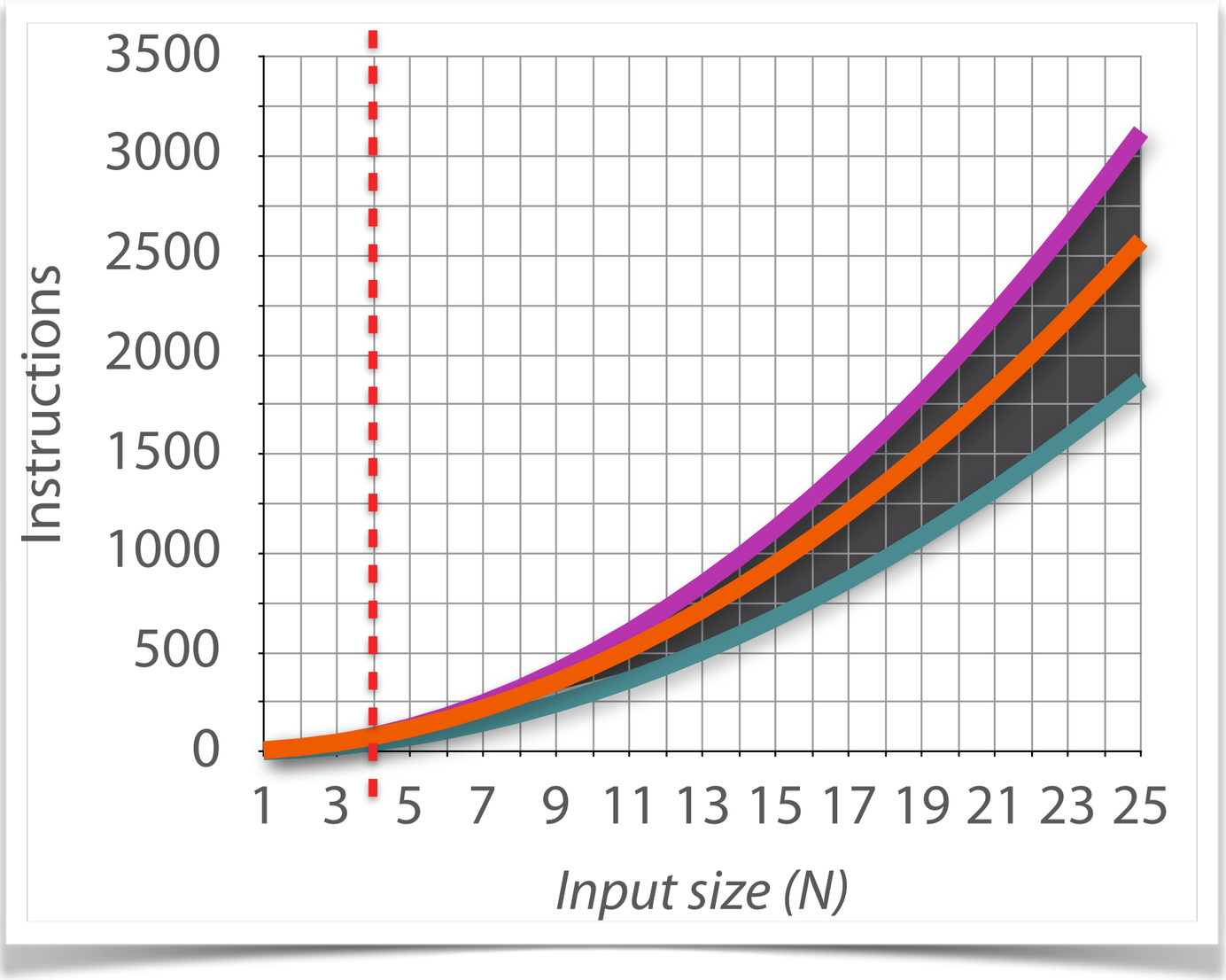
$g(N) = N^2$

$3g(N) = 3N^2$

$3g(N) < f(N) < 5g(N)$

when $N > 3$

$\Theta(g(N))$



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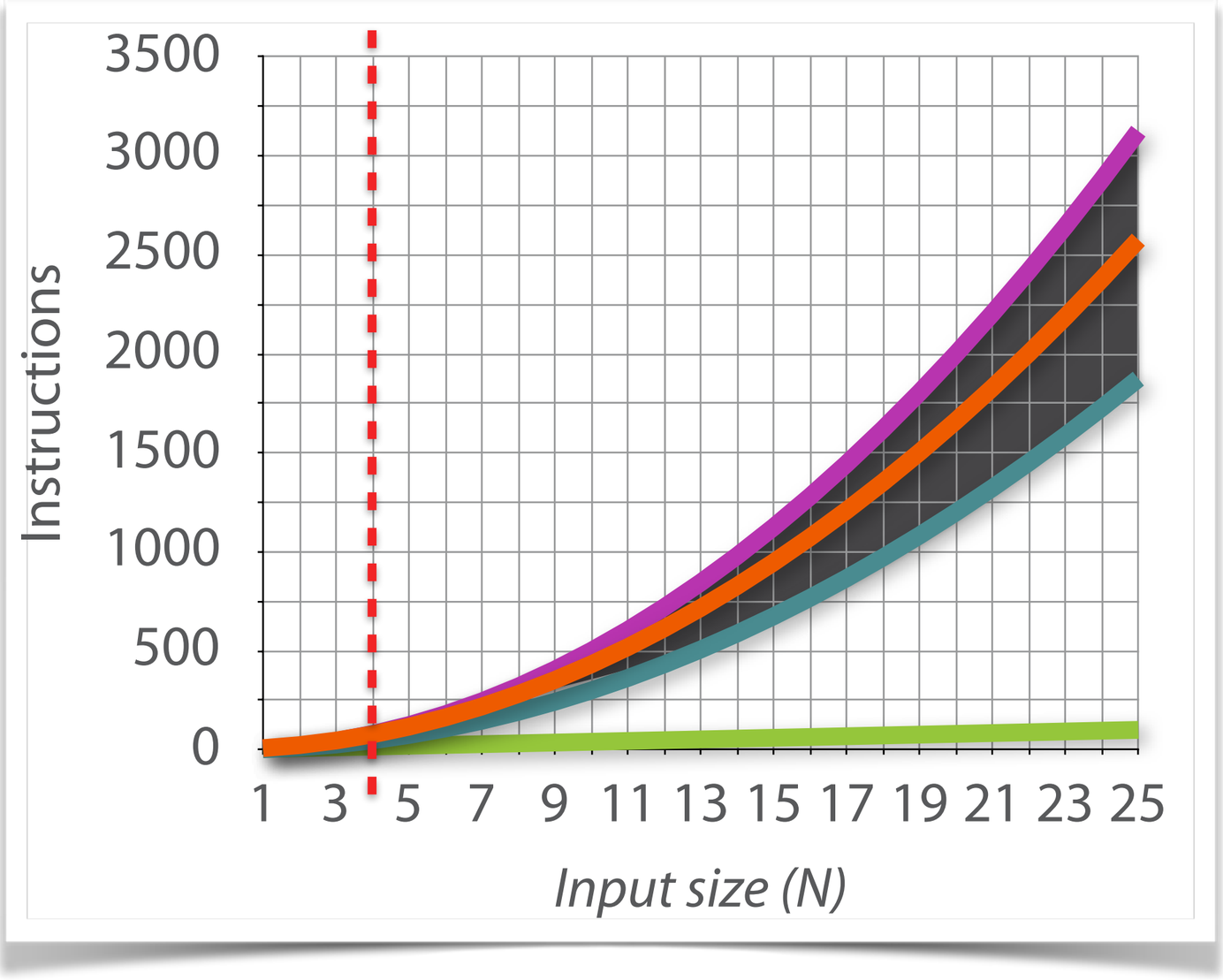
$\Theta(g(N))$

$\Theta(N^2)$



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$3g(N) < f(N) < 5g(N)$

when $N > 3$

$\Theta(g(N))$

$\Theta(N^2)$

Consider a function, $f(N)$…

$$f(N) = 2 + 3N + 4N^2$$

Consider a function, $f(N)$…

$f(N) = 2 + 3N + 4N^2$

If there exist:

Consider a function, $f(N)$… $f(N) = 2 + 3N + 4N^2$

If there exist:

- a function, $g(N)$ $g(N) = N^2$

Consider a function, $f(N)$…

$f(N) = 2 + 3N + 4N^2$

If there exist:

- a function, $g(N)$ $g(N) = N^2$
- two constants, say c_1 and c_2 $3g(N) < f(N) < 5g(N)$

Consider a function, $f(N)$… $f(N) = 2 + 3N + 4N^2$

If there exist:

- a function, $g(N)$ $g(N) = N^2$
- two constants, say c_1 and c_2 $3g(N) < f(N) < 5g(N)$
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If there exist:

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- another constant, n when $N > 3$

so that $c_1 \cdot g(N) < f(N) < c_2 \cdot g(N)$ when $N > n$

Consider a function, $f(N)$… $f(N) = 2 + 3N + 4N^2$

If there exist:

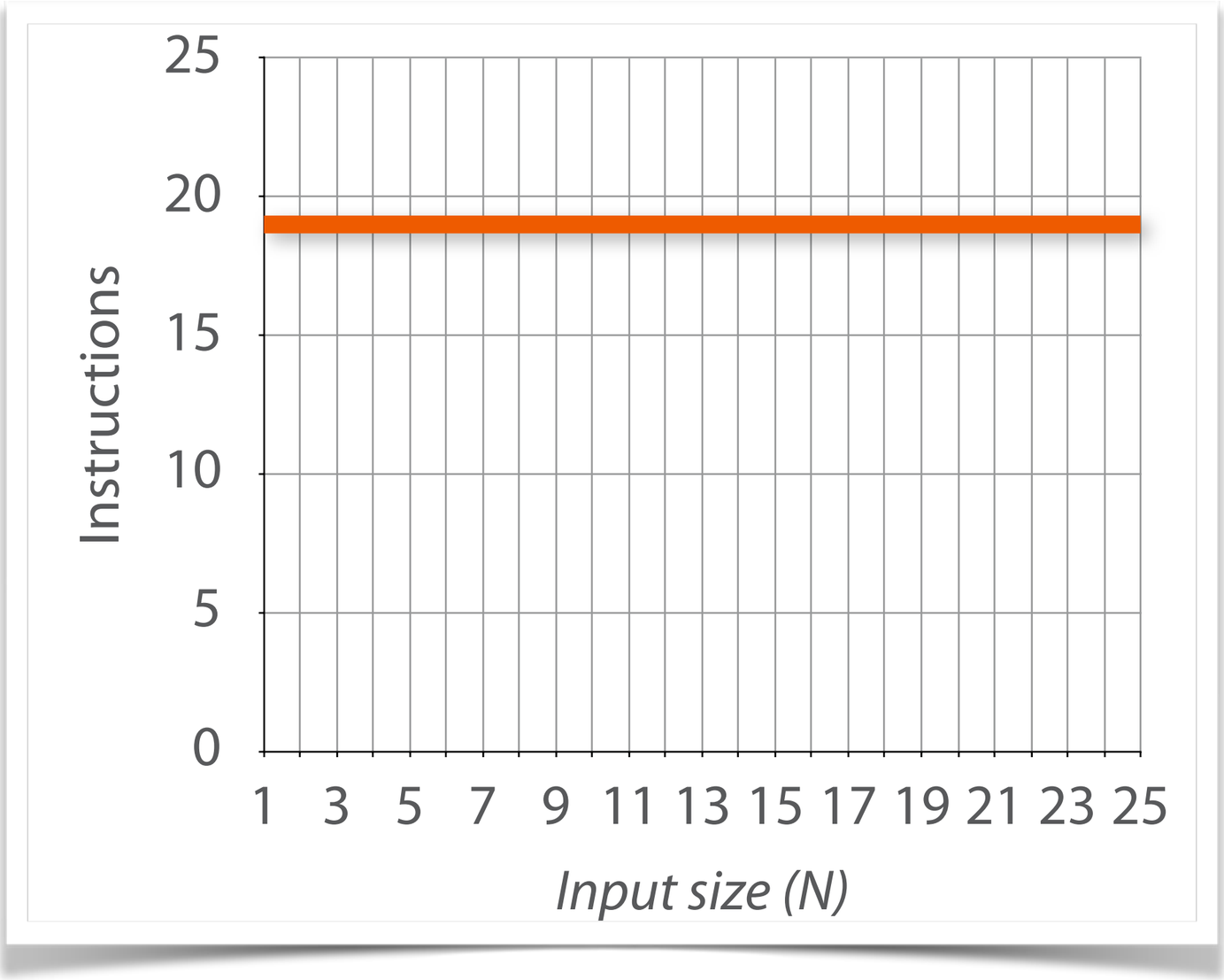
- a function, $g(N)$ $g(N) = N^2$
- two constants, say c_1 and c_2 $3g(N) < f(N) < 5g(N)$
- another constant, n when $N > 3$

so that $c_1 \cdot g(N) < f(N) < c_2 \cdot g(N)$ when $N > n$

…then $f(N)$ is $\Theta(g(N))$

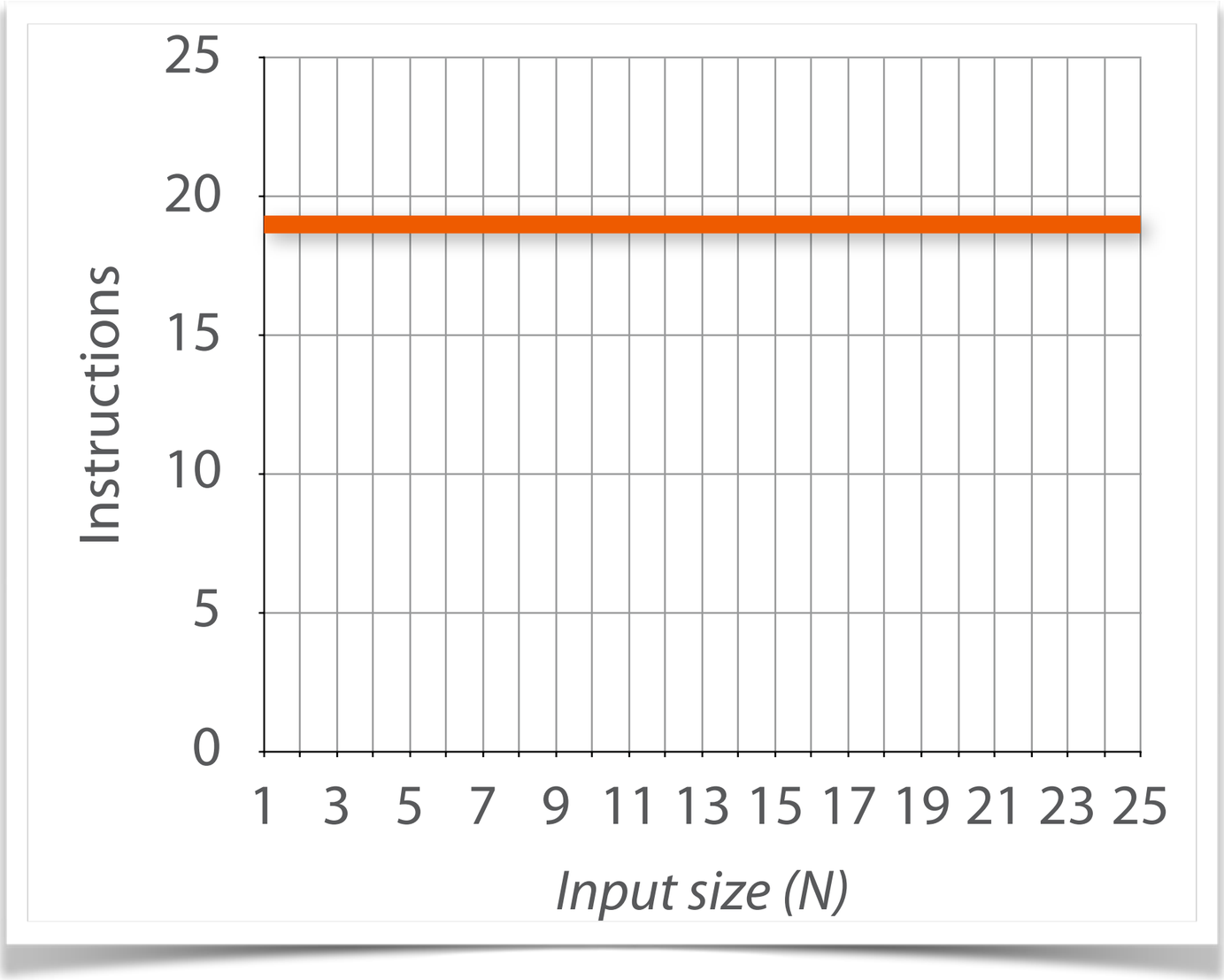
Constant Complexity

Constant Complexity



$f(N) = 19$

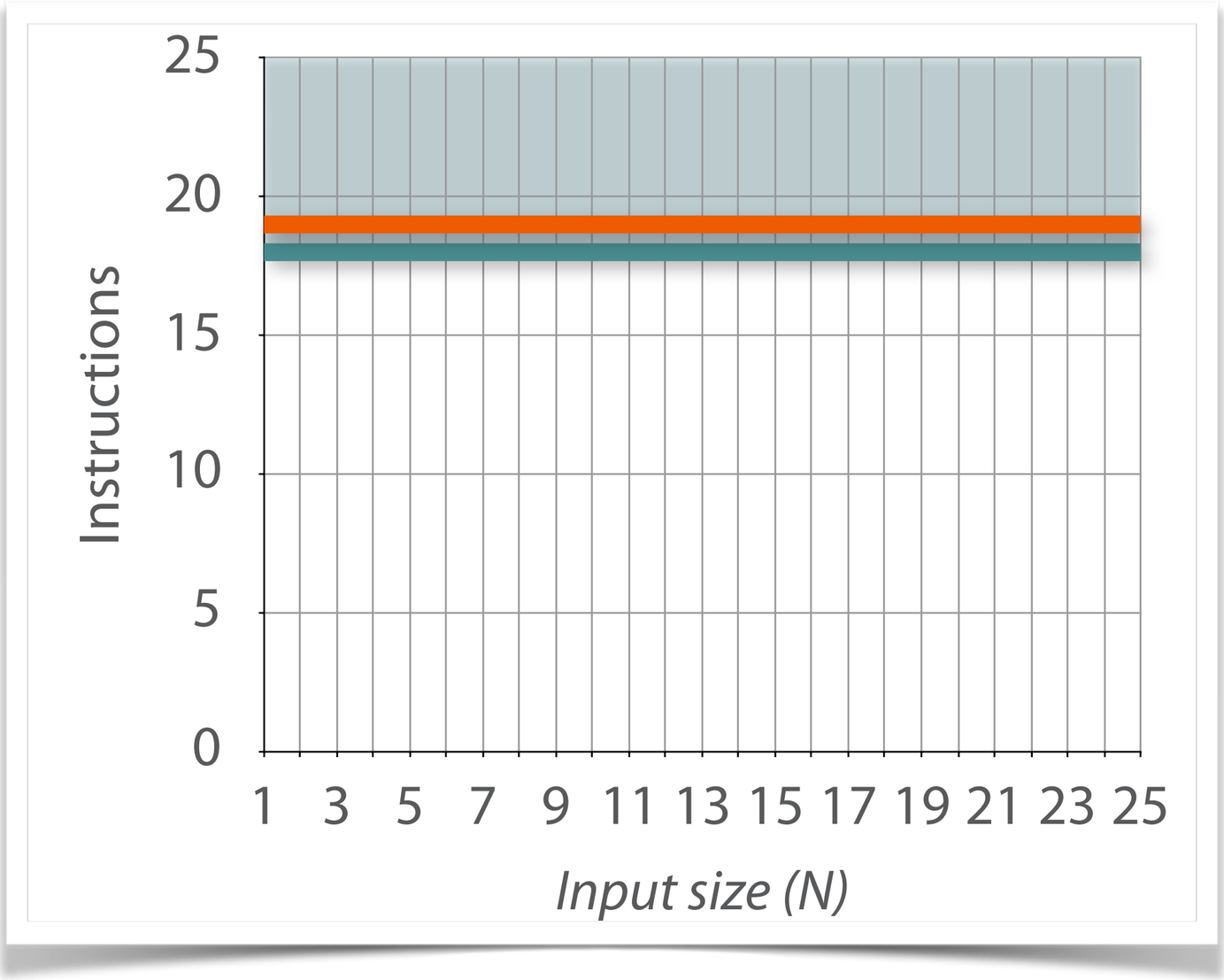
Constant Complexity



$f(N) = 19$

$g(N) = 1$

Constant Complexity

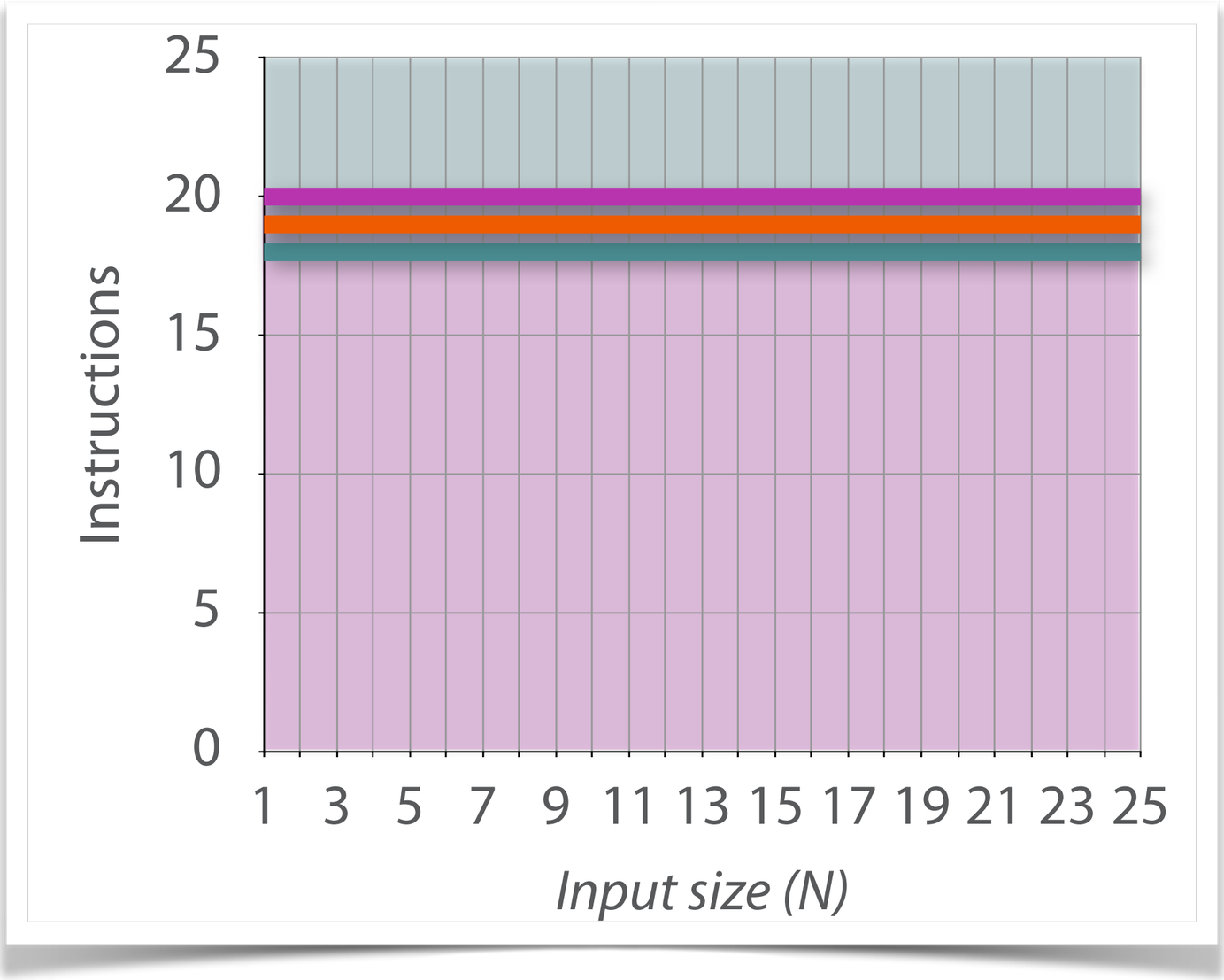


$f(N) = 19$

$g(N) = 1$

$18.999\ldots\ g(N) = 18.999\ldots \cdot 1$

Constant Complexity



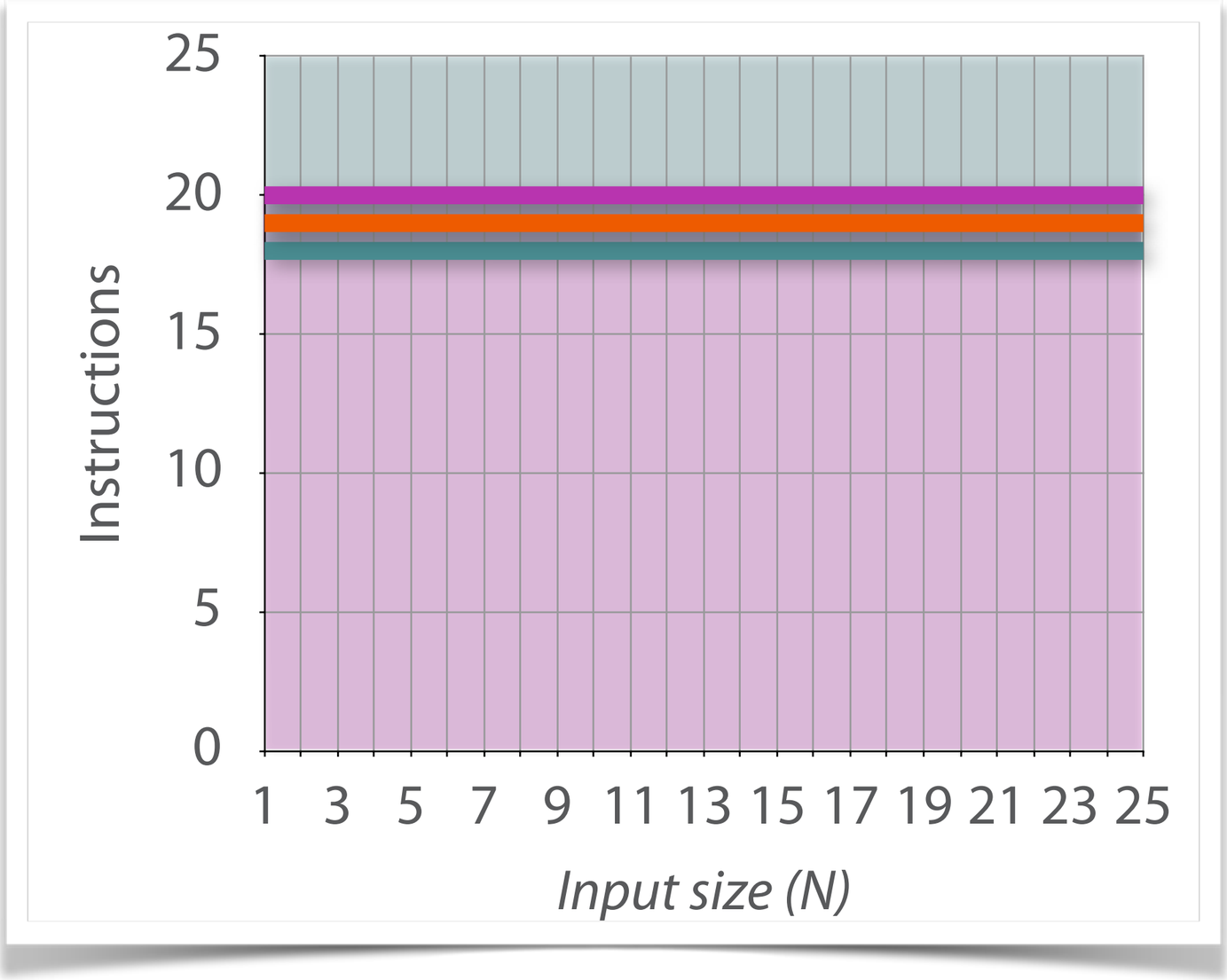
19.00…1 $g(N) = 19.00…1 \cdot 1$

$f(N) = 19$

$g(N) = 1$

18.999… $g(N) = 18.999… \cdot 1$

Constant Complexity



$19.00\ldots1\ g(N) = 19.00\ldots1 \cdot 1$

$f(N) = 19$

$g(N) = 1$

$18.999\ldots\ g(N) = 18.999\ldots \cdot 1$

$\Theta(1)$

Is $\ominus$ Cheating?

Is Θ Cheating?

$\Theta(N)$

Is Θ Cheating?

$\Theta(N)$ $\Theta(N^2)$

Is Θ Cheating?

$\Theta(N)$

$c_1 = 900$
$c_2 = 903$

$\Theta(N^2)$

$c_1 = 1$
$c_2 = 3$

Is Θ Cheating?

$\Theta(N)$

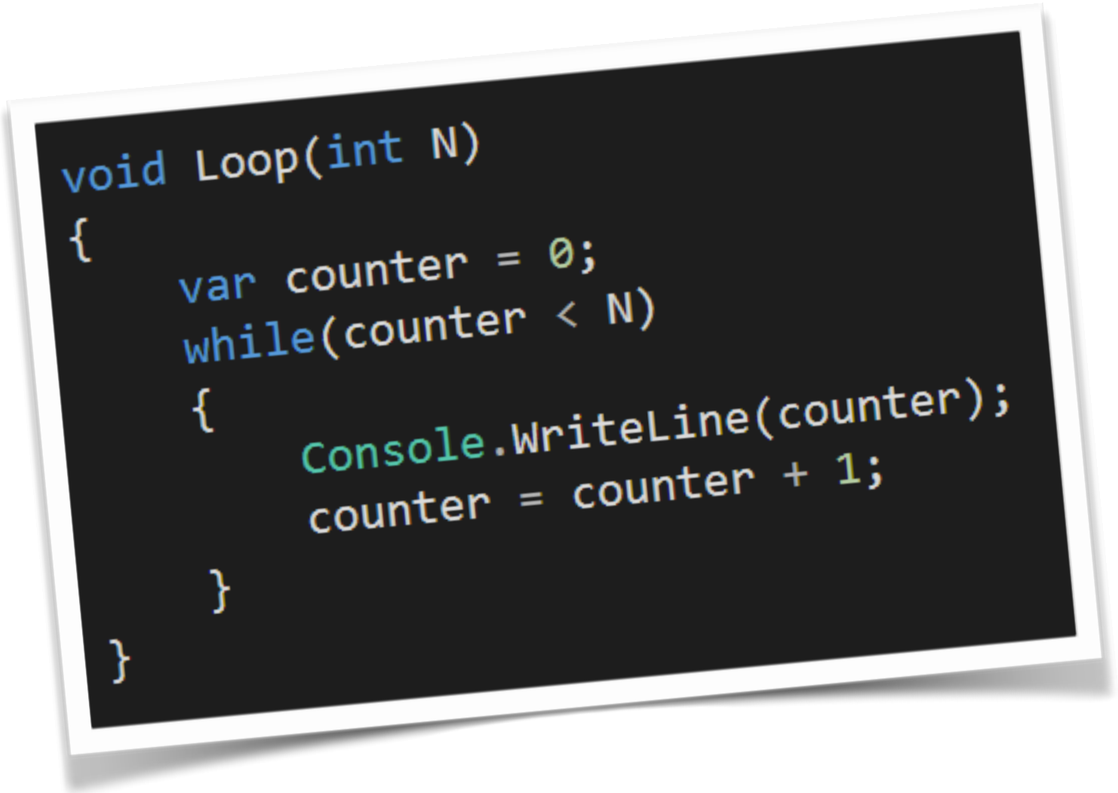
$c_1 = 900$
$c_2 = 903$

$\Theta(N^2)$

$c_1 = 1$
$c_2 = 3$

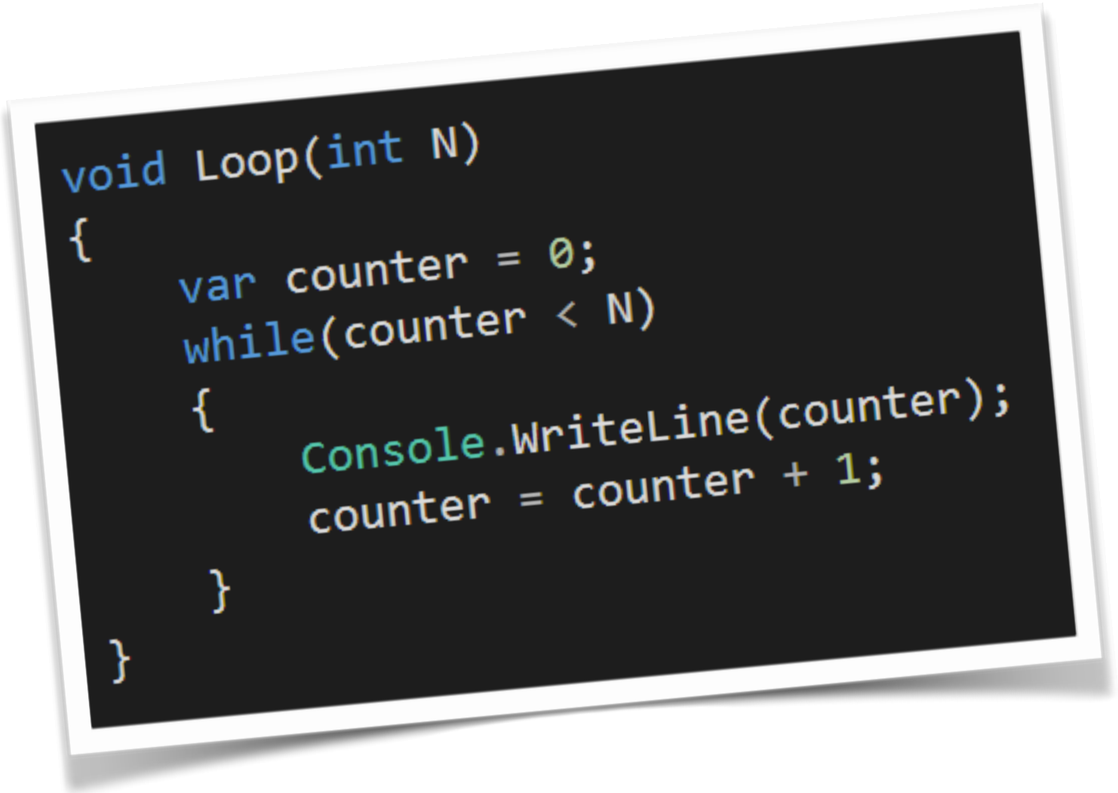
$\Theta(N^2)$ wins until $N > 900$

Big *O*


void Loop(int N)
{
var counter = 0;
while(counter < N)
{
Console.WriteLine(counter);
counter = counter + 1;
}
}

Big *O*

Consistent



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Big *O*

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    }
}
```

```
bool ContainsNeedle(int needle, List<int> haystack)
{
    foreach (var sample in haystack)
    {
        if (sample == needle)
            return true;
    }
    return false;
}
```

Big *O*

Consistent

```
void Loop(int N)
{
    var counter = 0;
    while(counter < N)
    {
        Console.WriteLine(counter);
        counter = counter + 1;
    }
}
```

Depends on input content

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Big *O*

Consistent

Depends on input content

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```

```
bool Contain
t needle, List<int> haystack)
e in haystack)
eedle)
return true;
}
return false;
}
```

Worst-case is interesting

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bool ContainsNeedle(int needle, List<int> haystack)
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Complexity (*N*):

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At most N.

Complexity (*N*):

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    {
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            return true;
    }
    return false;
}
```

At most $N \cdot 1$

Complexity (*N*):

```
bool ContainsNeedle(int needle, List<int> haystack)
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    foreach (var sample in haystack)
    {
        if (sample == needle)
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    }
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}
```

At most $N \cdot 1 + 1$

Complexity (*N*):

```
bool ContainsNeedle(int needle, List<int> haystack)
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    foreach (var sample in haystack)
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            return true;
    }
    return false;
}
```

At most $N \cdot 1 + 1$

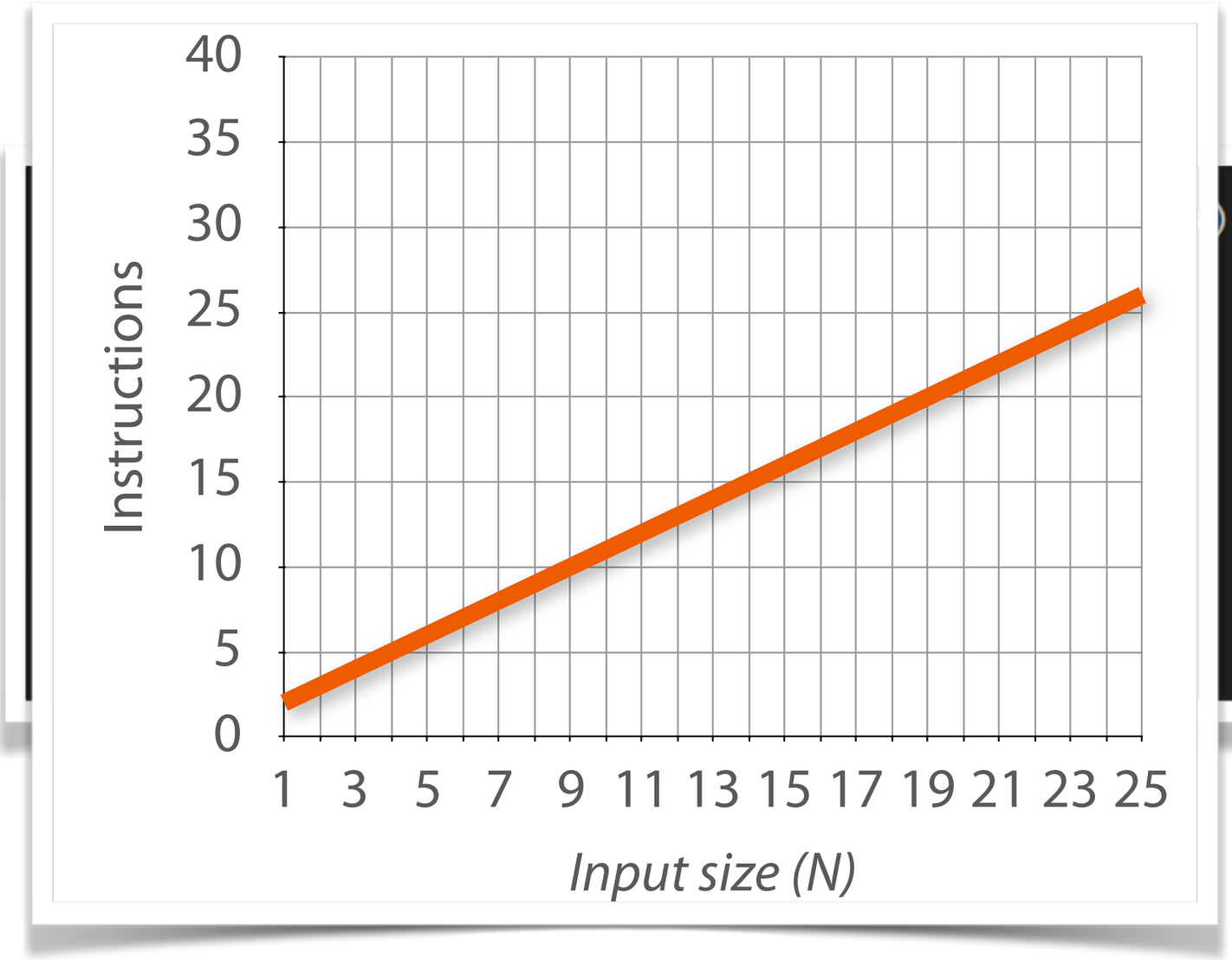
$= N + 1$

Complexity (*N*):

```
bool ContainsNeedle(int needle, List<int> haystack)
{
    foreach (var sample in haystack)
    {
        if (sample == needle)
            return true;
    }
    return false;
}
```

At most $N \cdot 1 + 1$

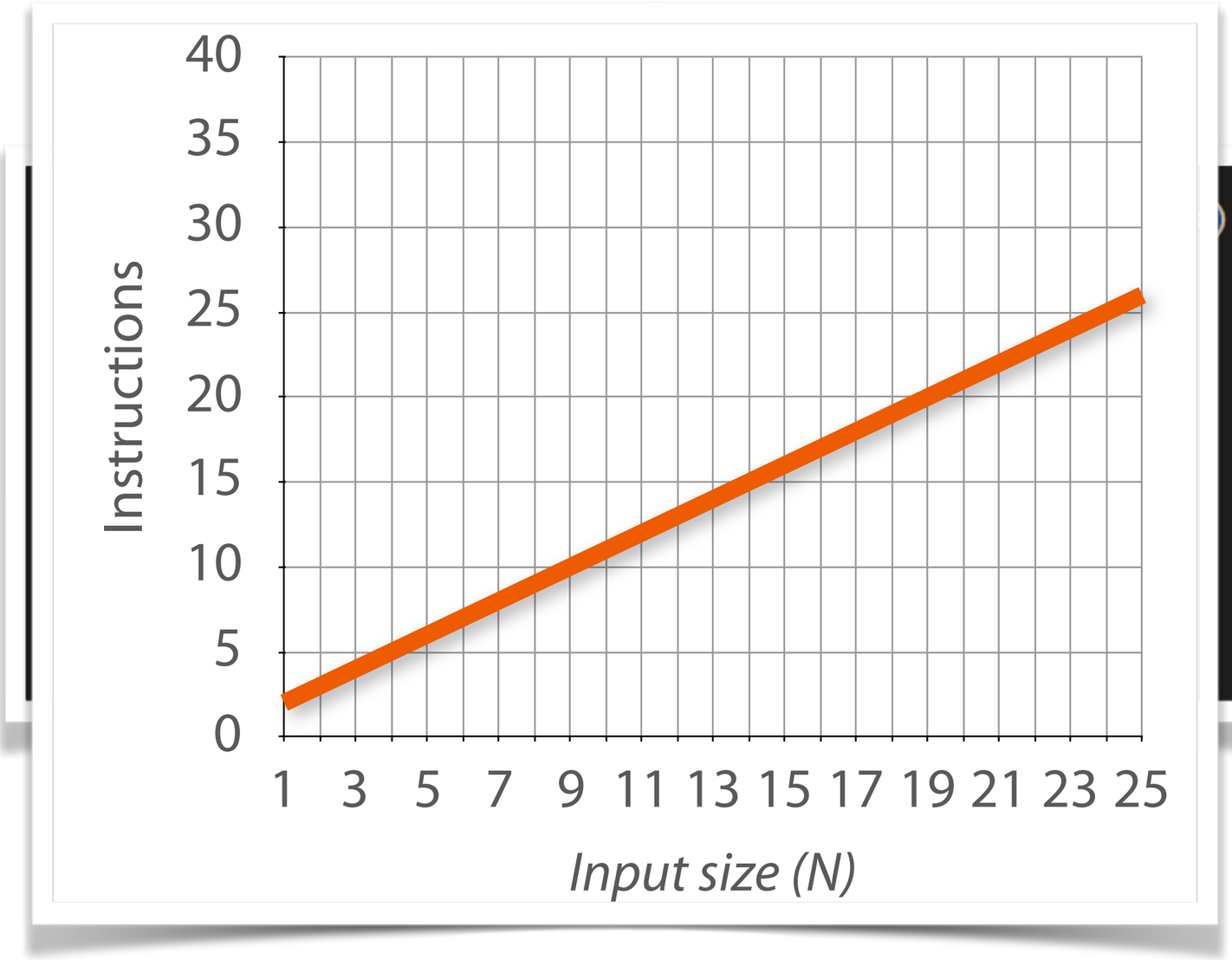
$f(N) = N + 1$



Complexity (*N*):

At most $N \cdot 1 + 1$

$f(N) = N + 1$

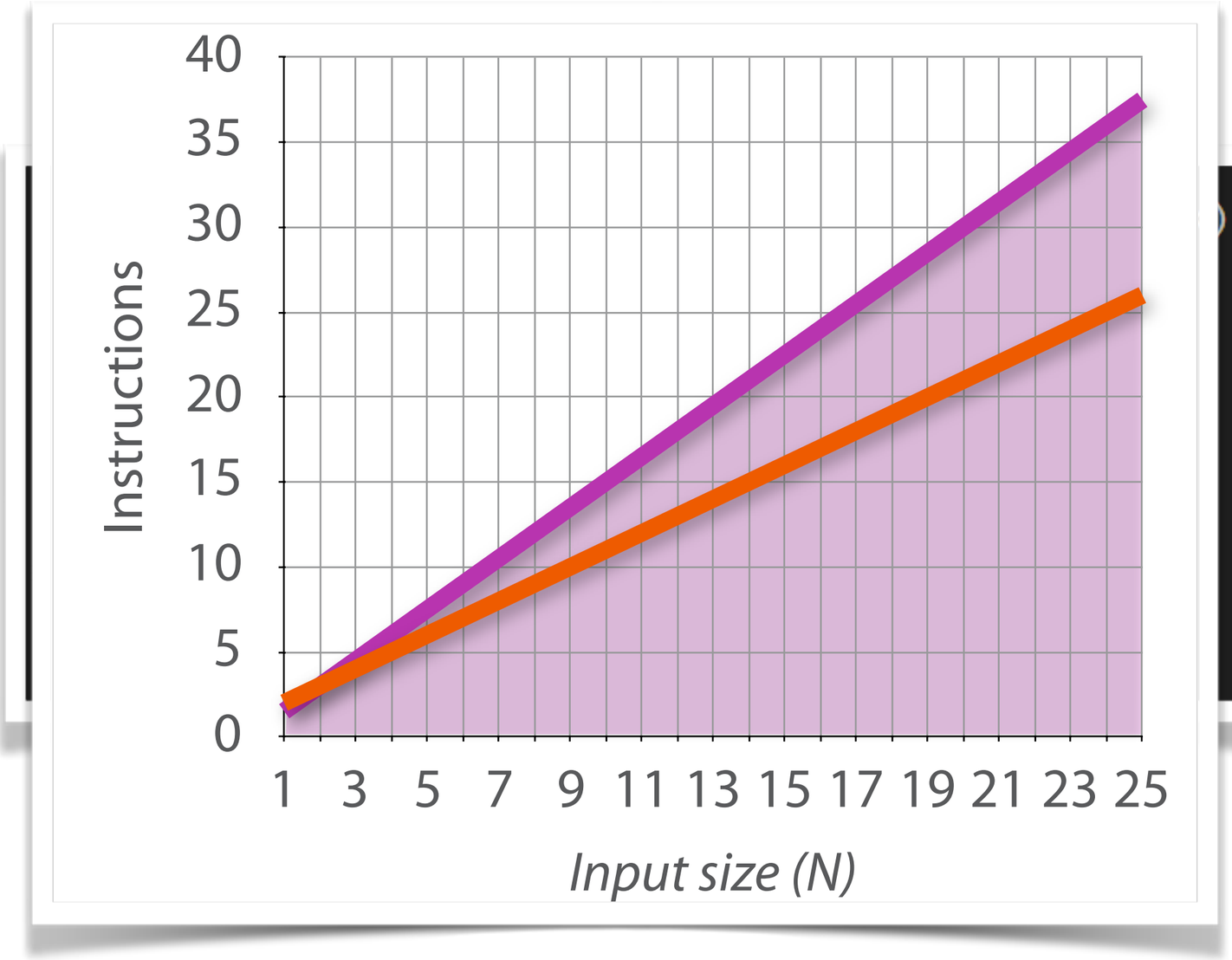


Complexity (*N*):

At most $N \cdot 1 + 1$

$f(N) = N + 1$

$g(N) = N$



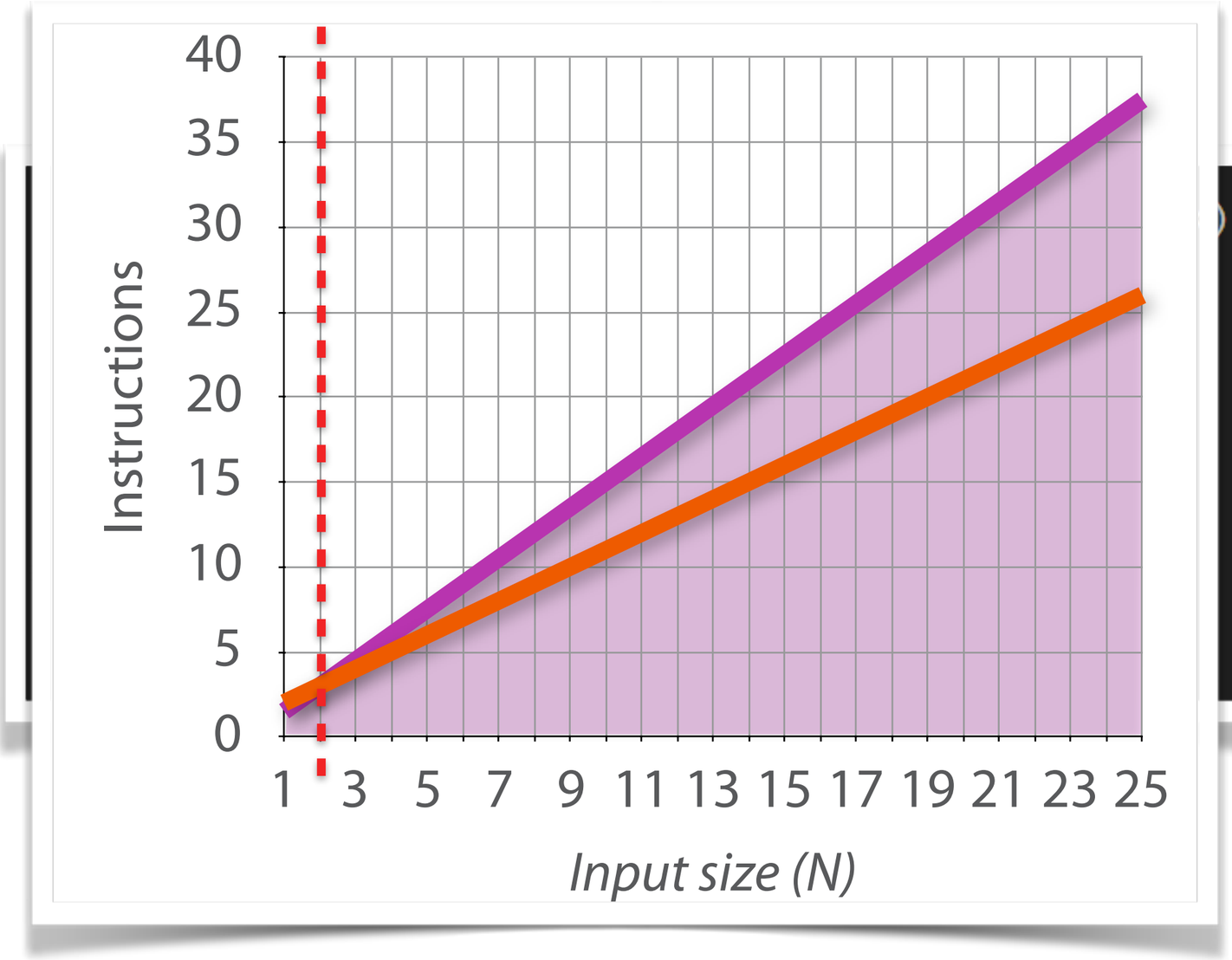
Complexity (*N*):

1.5 $g(N) = 1.5\ N$

At most $N \cdot 1 + 1$

$f(N) = N + 1$

$g(N) = N$



Complexity (*N*):

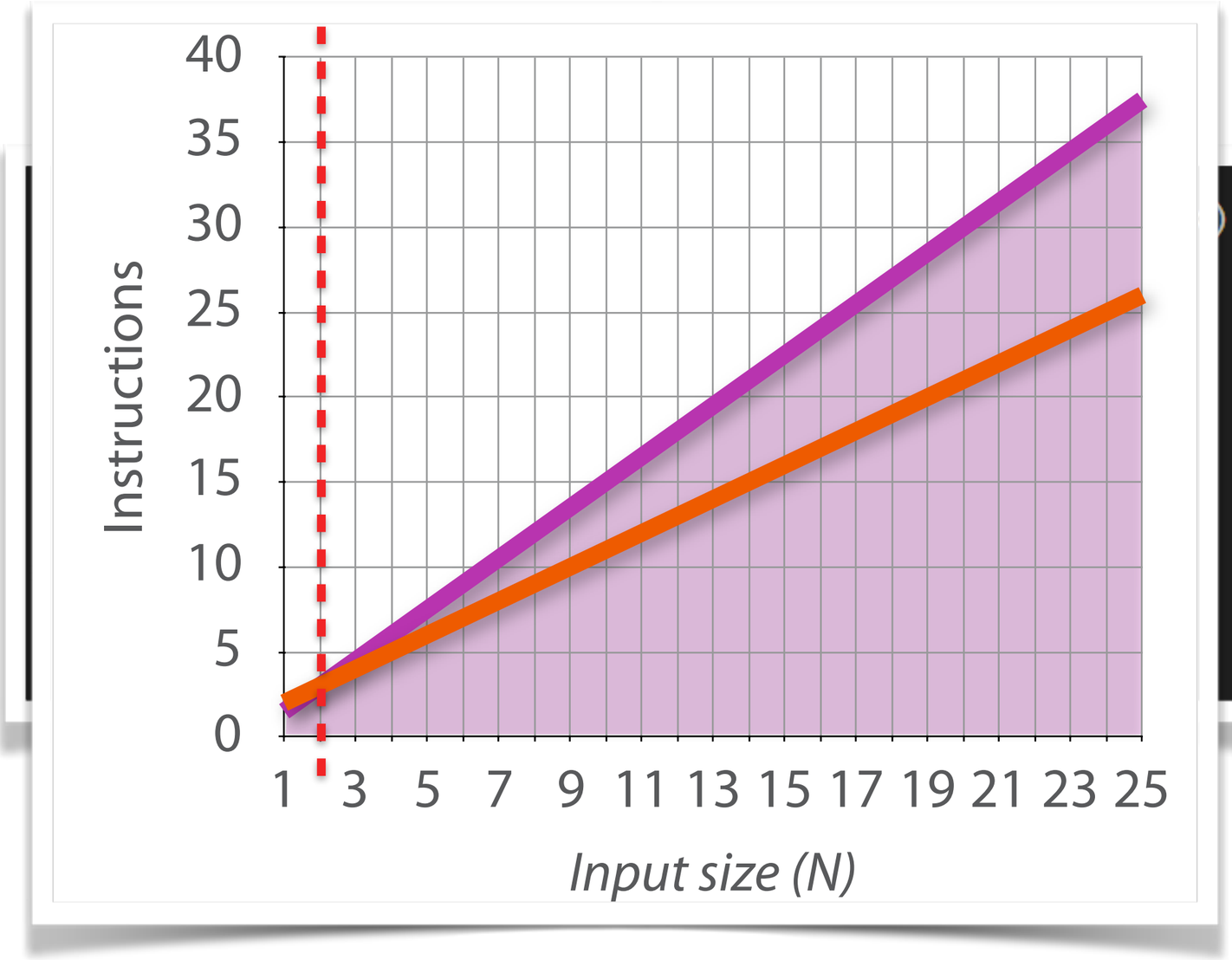
$1.5\ g(N) = 1.5\ N$

At most $N \cdot 1 + 1$

$f(N) = N + 1$

$g(N) = N$

$f(N) < 1.5\ g(N)$
when $N > 1$



Complexity (*N*):

$1.5\ g(N) = 1.5\ N$

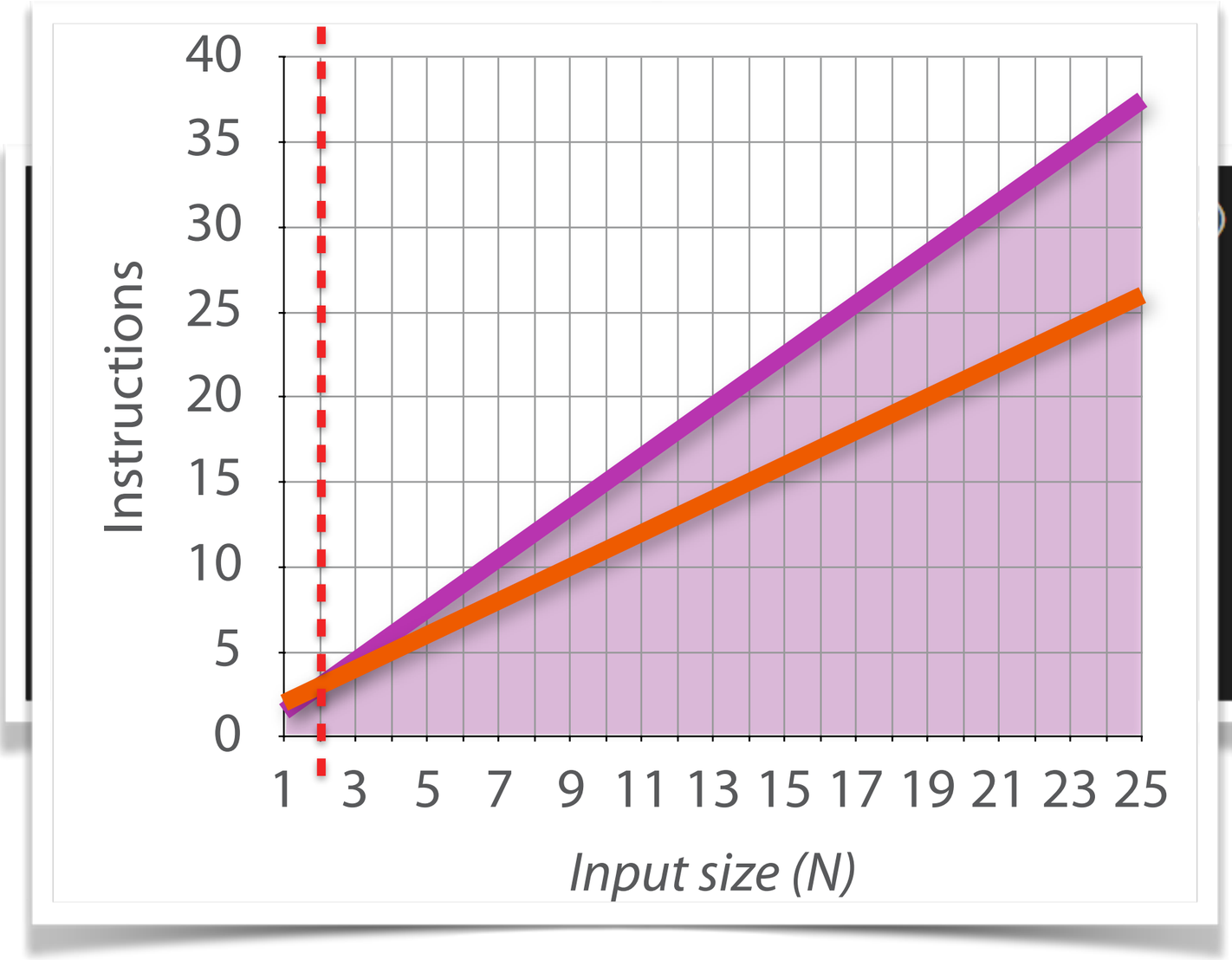
At most $N \cdot 1 + 1$

$f(N) = N + 1$

$g(N) = N$

$f(N) < 1.5\ g(N)$
when $N > 1$

$O(g(N))$



Complexity (*N*):

$1.5\ g(N) = 1.5\ N$

At most $N \cdot 1 + 1$

$f(N) = N + 1$

$g(N) = N$

$f(N) < 1.5\ g(N)$
when $N > 1$

$O(g(N))$

$O(N)$

Consider a function, $f(N)$…

$f(N) = N + 1$

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

Consider a function, $f(N)$… $f(N) = N + 1$

If there exist:

- a function, $g(N)$ $g(N) = N$

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

- a function, $g(N)$
- a constant, say c

$g(N) = N$

$f(N) < 1.5\ g(N)$

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

- a function, $g(N)$ — $g(N) = N$
- a constant, say c — $f(N) < 1.5\ g(N)$
- another constant, n — when $N > 1$

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

- a function, $g(N)$ — $g(N) = N$
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- another constant, n — when $N > 1$

so that $f(N) < c \cdot g(N)$ when $N > n$

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

- a function, $g(N)$ — $g(N) = N$
- a constant, say c — $f(N) < 1.5\ g(N)$
- another constant, n — when $N > 1$

so that $f(N) < c \cdot g(N)$ when $N > n$

…then $f(N)$ is $O(g(N))$

Binary Search

Binary Search

1	*3*	*3*	*7*	*10*	*11*	*16*	*17*	*20*	*21*	*25*	*26*	*30*	*32*	*34*	*35*

Binary Search

Needle: 26

1	***3***	***3***	***7***	***10***	***11***	***16***	***17***	***20***	***21***	***25***	***26***	***30***	***32***	***34***	***35***

Binary Search

Needle: 26

1	***3***	***3***	***7***	***10***	***11***	***16***	***17***	***20***	***21***	***25***	***26***	***30***	***32***	***34***	***35***

Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

1	***3***	***3***	***7***	***10***	***11***	***16***	***17***	***20***	***21***	***25***	***26***	***30***	***32***	***34***	***35***

Binary Search

Needle: 26

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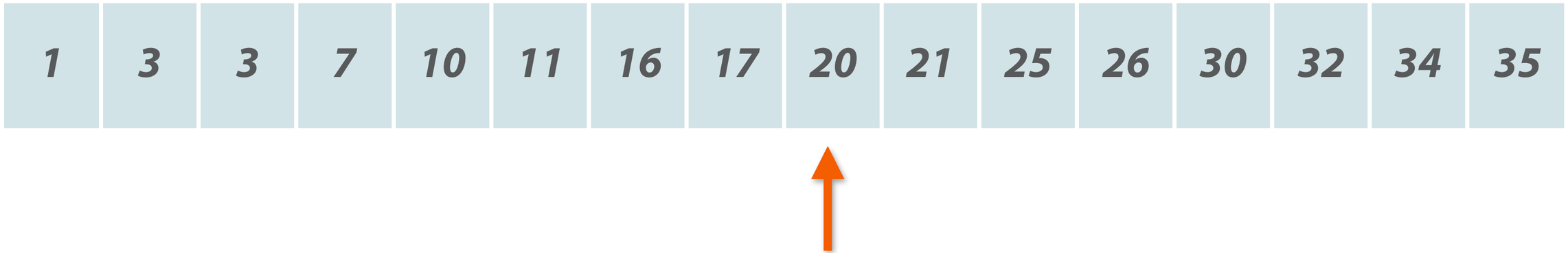

Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

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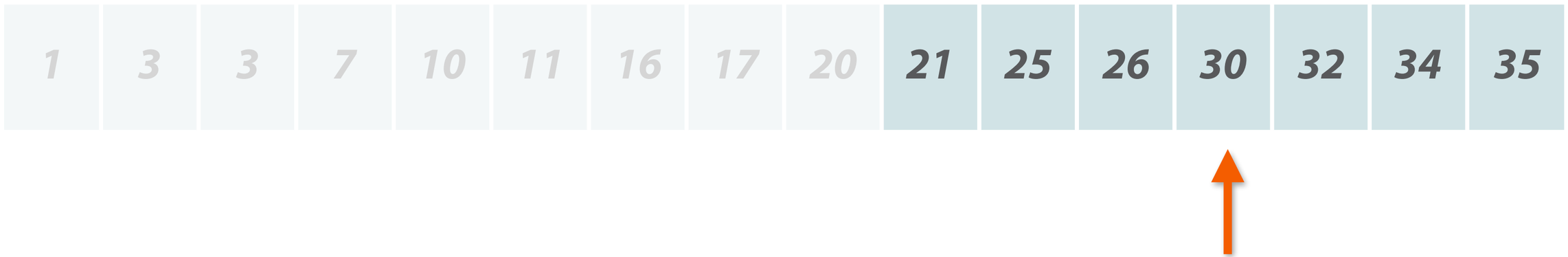
Needle: 26

Linear search: *O*(*N*)

N entries in haystack

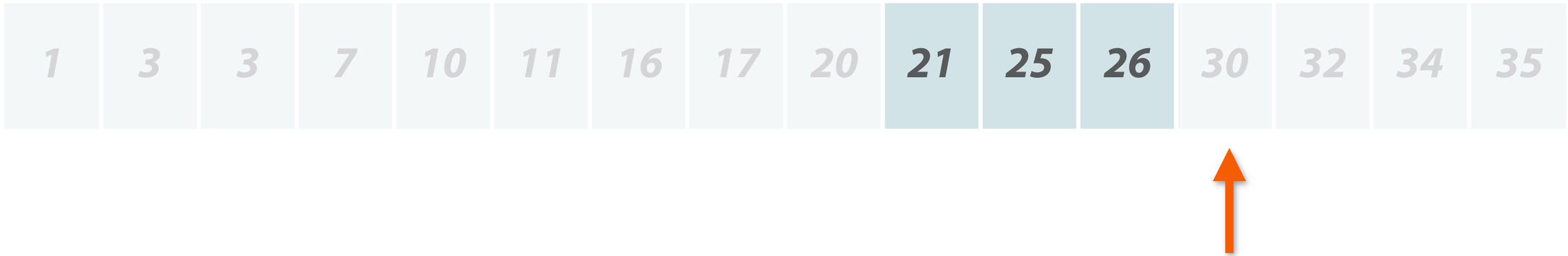


Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

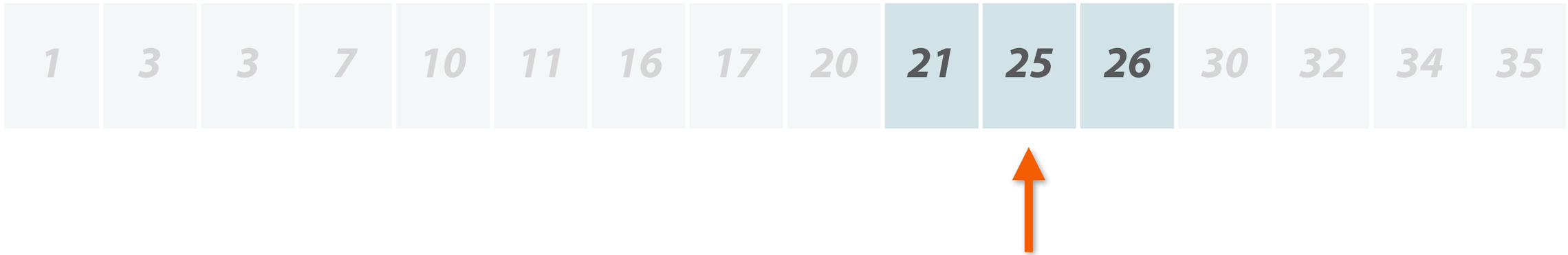


Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

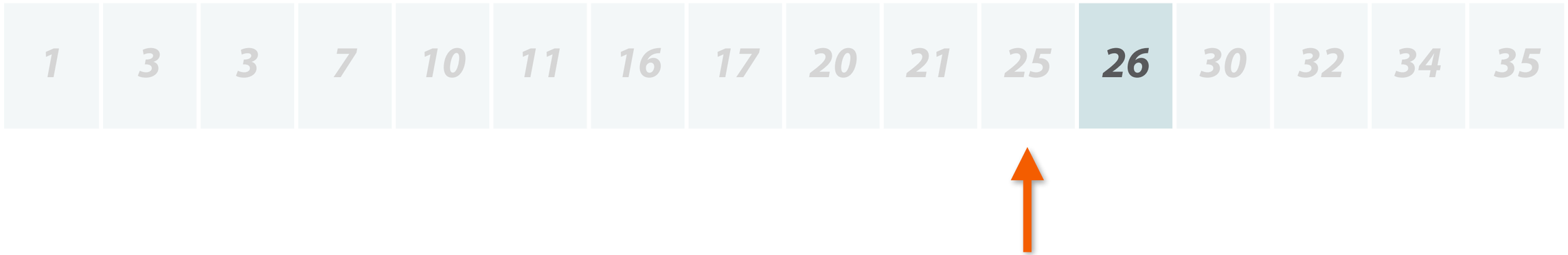


Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

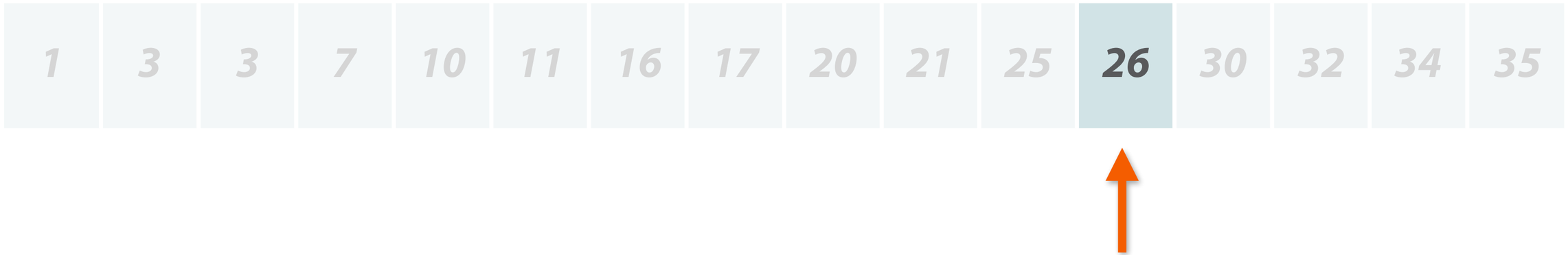


Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack



Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

1	3	3	7	10	11	16	17	20	21	25	**26**	30	32	34	35

Number of comparisons =

Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

Number of comparisons = Number of halvings of *N*

Logarithms

Logarithms

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$$

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9 = 512$$

Logarithms

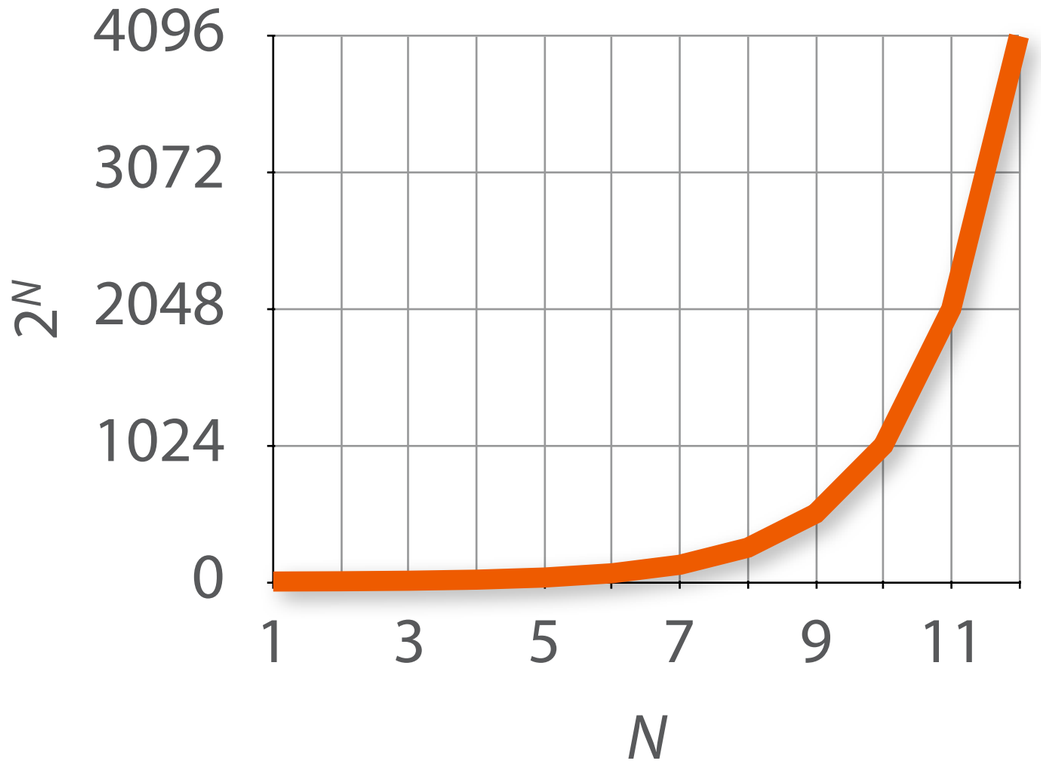
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9 = 512$$

$$2^N$$

Exponential function

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$
$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9 = 512$$



2^N

Exponential function

Logarithms

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8 = 256$$

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^9 = 512$$

$$2^N = 4096$$

Logarithms

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$$2^N = 4096$$

What should N be?

Logarithms

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$$2^N = 4096$$

$$\log_2 4096$$

Logarithms

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$$2^N = 4096$$

$$\log_2 4096$$

$$\log_2 4096 = 12$$

$$2^{12} = 4096$$

Logarithms

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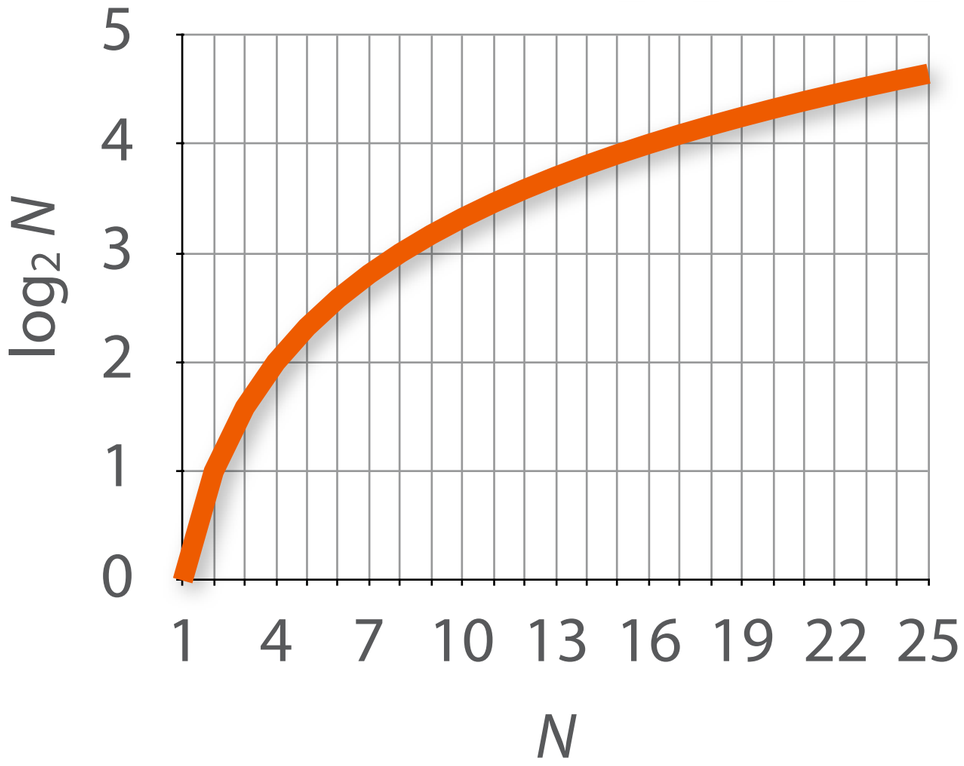

$2^N = 4096$

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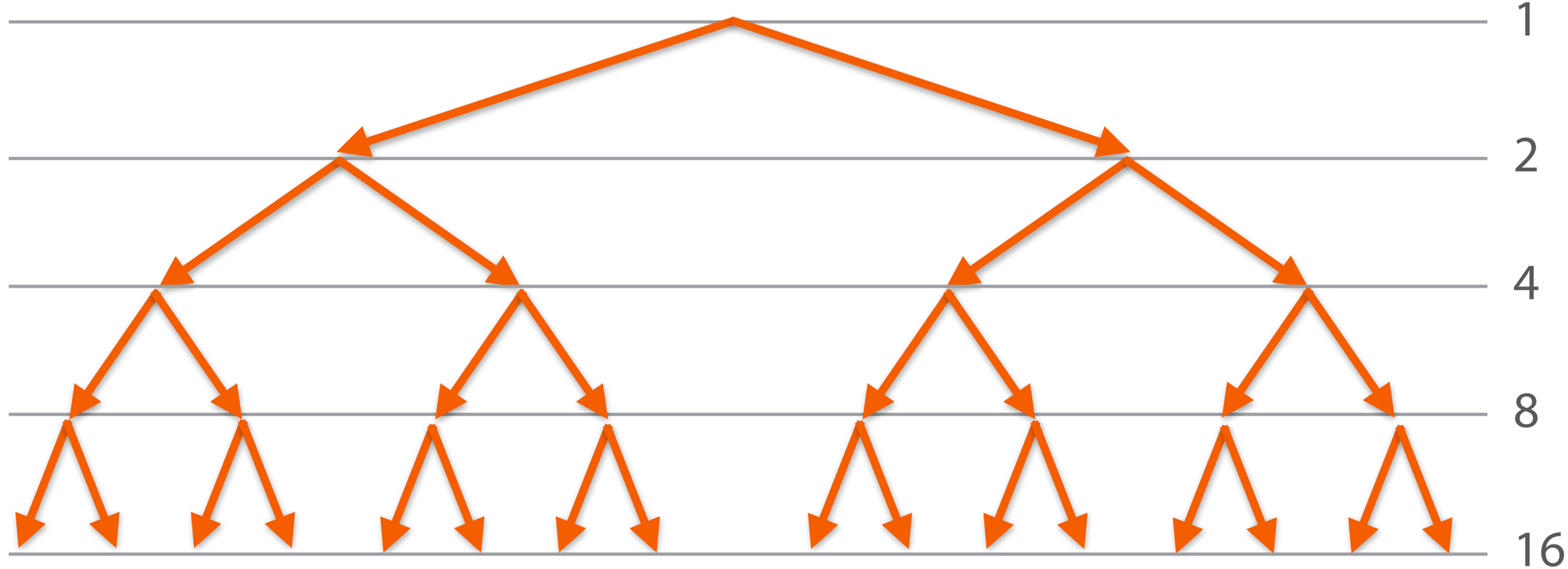
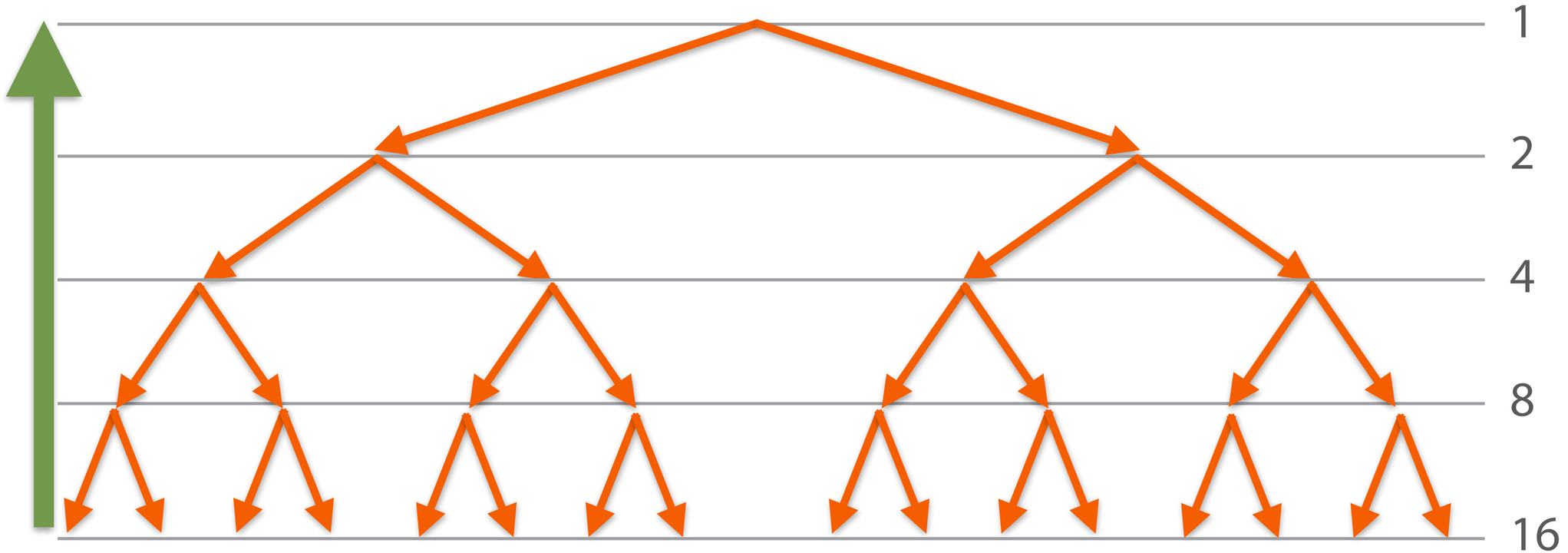
Logarithms

Logarithms



Total number after 4 doublings: exponential function ($2^4 = 16$)

Logarithms



Total number after 4 doublings: exponential function ($2^4 = 16$)

Total number of *halvings* of 16: $\log_2 16 = 4$

Binary Search

Needle: 26

Linear search: *O*(*N*)

N entries in haystack

Number of comparisons = Number of halvings of *N*

Binary Search

Needle: 26

Linear search: $O(N)$

N entries in haystack

Number of comparisons = Number of halvings of N = $\log_2 N$

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Needle: 26

Linear search: $O(N)$

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$\log_2 N$

Binary Search

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Linear search: $O(N)$

N entries in haystack

1	3	3	7	10	11	16	17	20	21	25	**26**	30	32	34	35

Number of comparisons = Number of halvings of N $= \log_2 N$

$$c \cdot \log_2 N$$

Binary Search

Needle: 26

Linear search: $O(N)$

N entries in haystack

Number of comparisons = Number of halvings of N = $\log_2 N$

Complexity: $O(\log_2 N)$

Binary Search

Needle: 26

Linear search: $O(N)$

N entries in haystack

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Number of comparisons = Number of halvings of

10,000 entries: 14 comparisons

Complexity: $O(\log_2 N)$

Binary Search

Needle: 26

Linear search: $O(N)$

N entries in haystack

1	3	3	7	10	11	16	17	20	21	25	**26**	30	32	34	35

Number of comparisons = Number of halvings of

Complexity: $O(\log_2 N)$

10,000 entries: 14 comparisons

1,000,000 entries: 20 comparisons

A Few Points About Big *O*

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$O(999^N)$ rules almost everything

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Find the *lowest possible* worst-case

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$N^2 + N$ is $O(N^3)$

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A Few Points About Big *O*

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Find the *lowest possible* worst-case

Tight bound

$N^2 + N$ is $O(N^3)$

$O(N^2)$ is more interesting

Big Omega (Ω)

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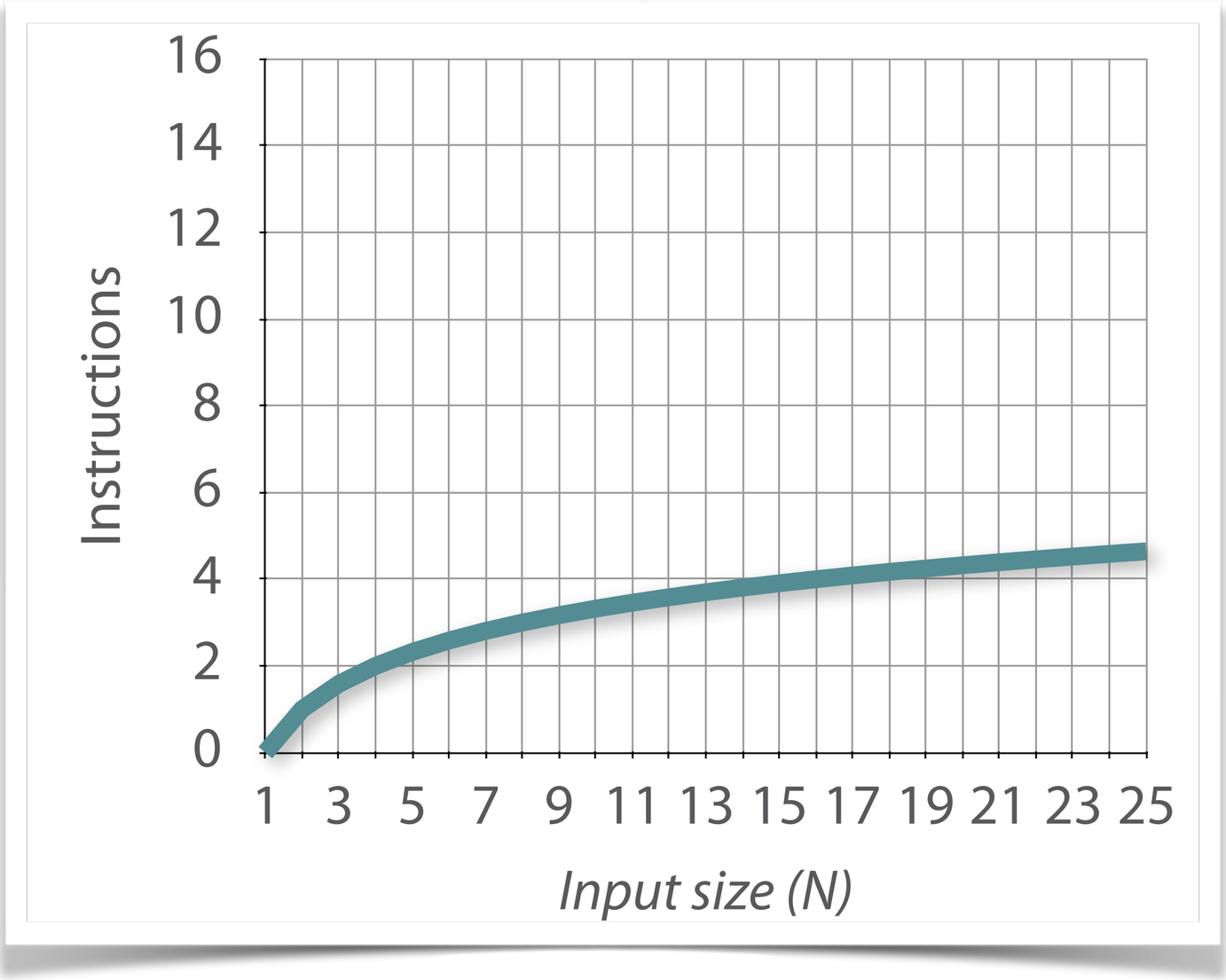
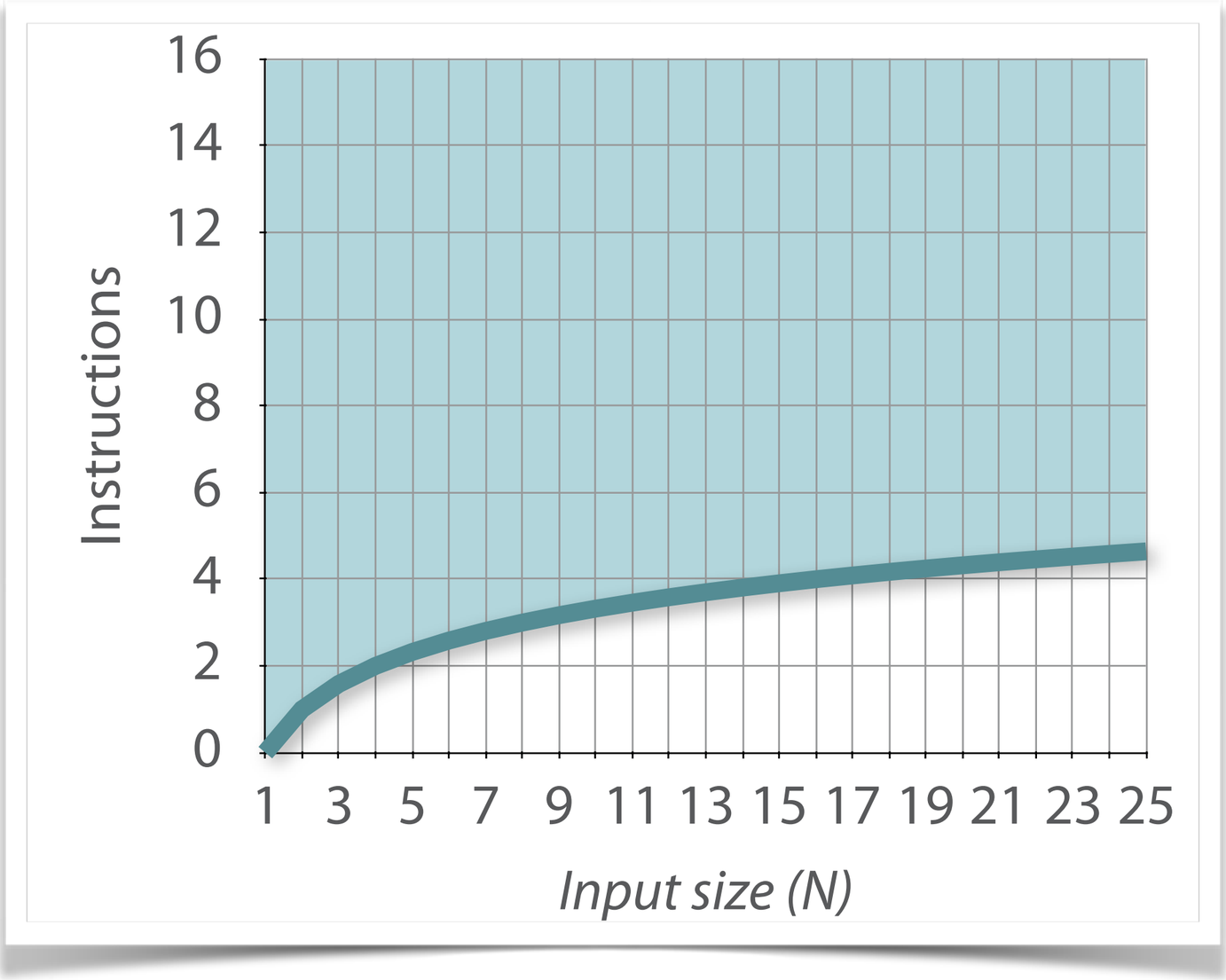

16
14
12
10
8
6
4
2
0
Instructions
1 3 5 7 9 11 13 15 17 19 21 23 25
Input size (N)

Big Omega (Ω)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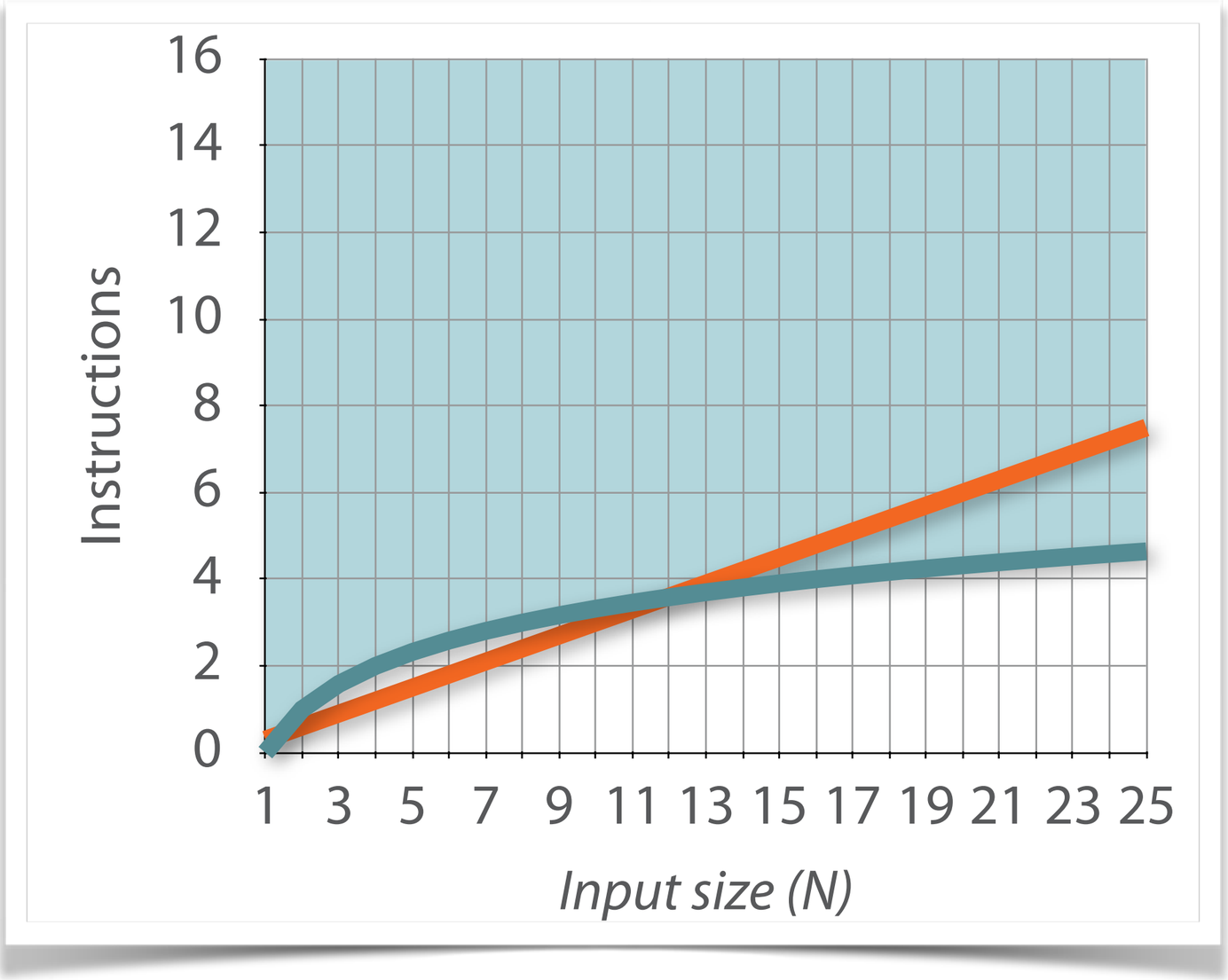
16
14
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0
Instructions
1 3 5 7 9 11 13 15 17 19 21 23 25
Input size (N)

Consider a function, $f(N)$…

$f(N) = N + 1$

If there exist:

- a function, $g(N)$ $g(N) = N$
- a constant, say c $f(N) < 1.5\ g(N)$
- another constant, n when $N > 1$

so that $f(N) < c \cdot g(N)$ when $N > n$

…then $f(N)$ is $O(g(N))$

§

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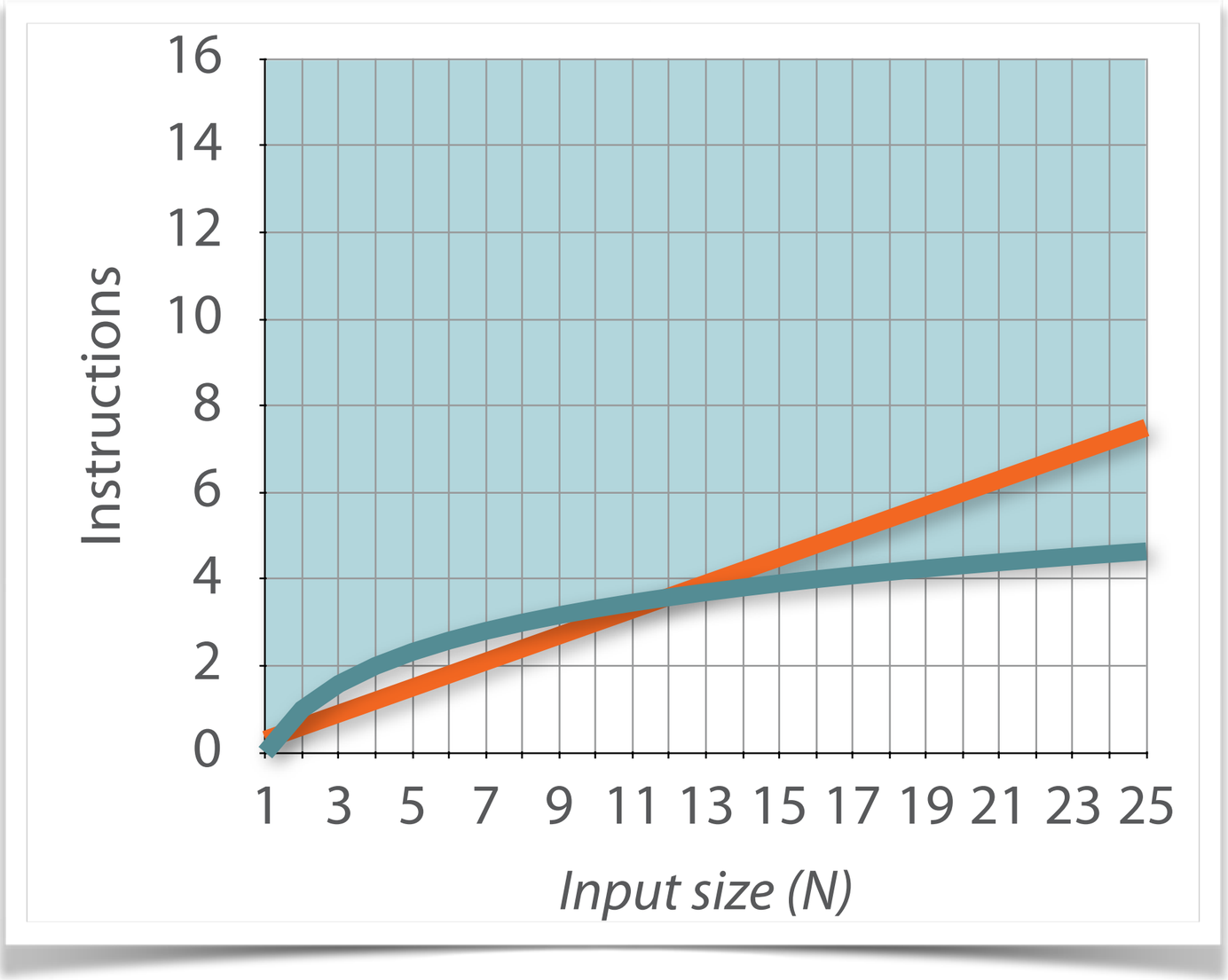
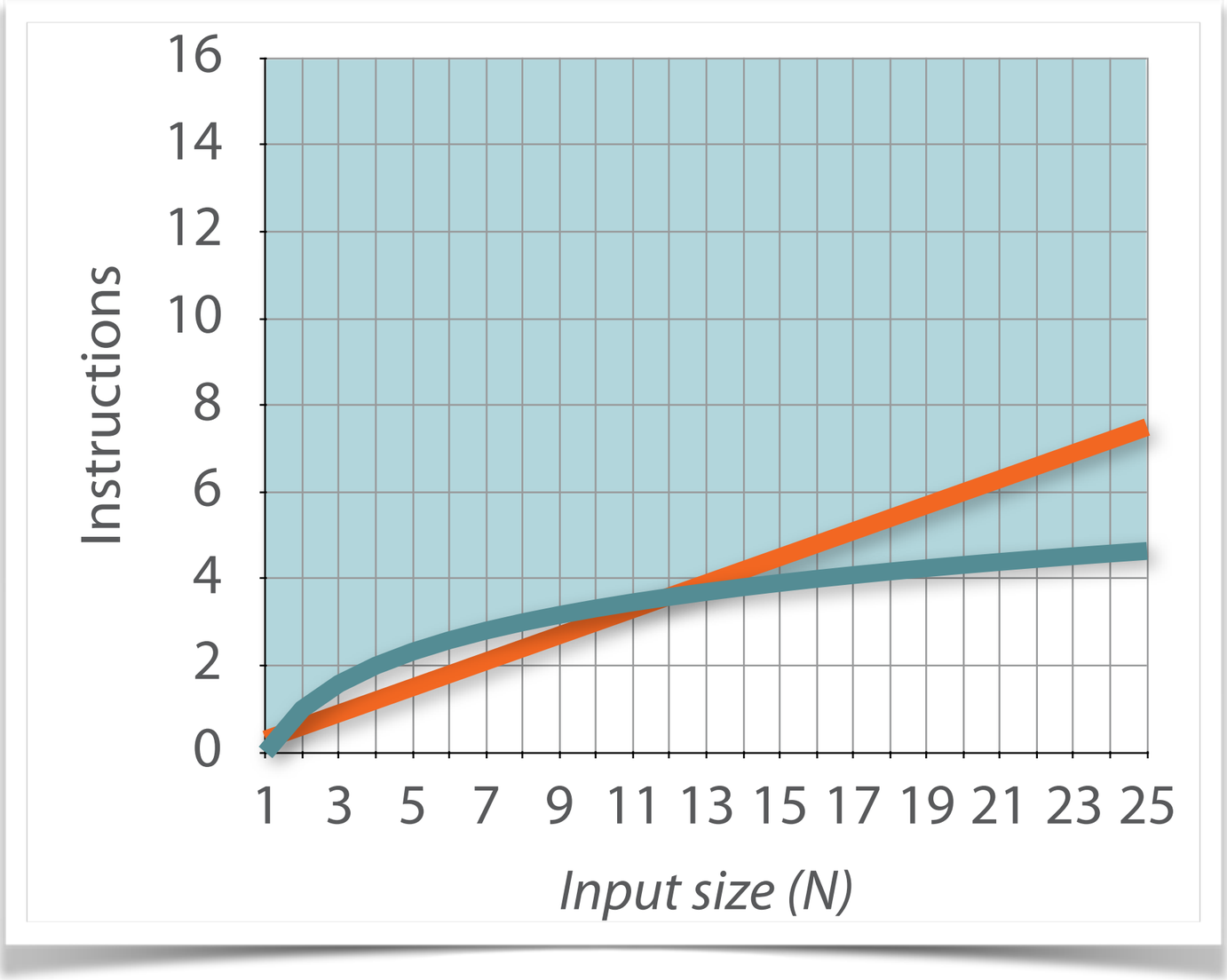
so that $f(N) > c \cdot g(N)$ when $N > n$

…then $f(N)$ is $\Omega(g(N))$

Big Omega (Ω)

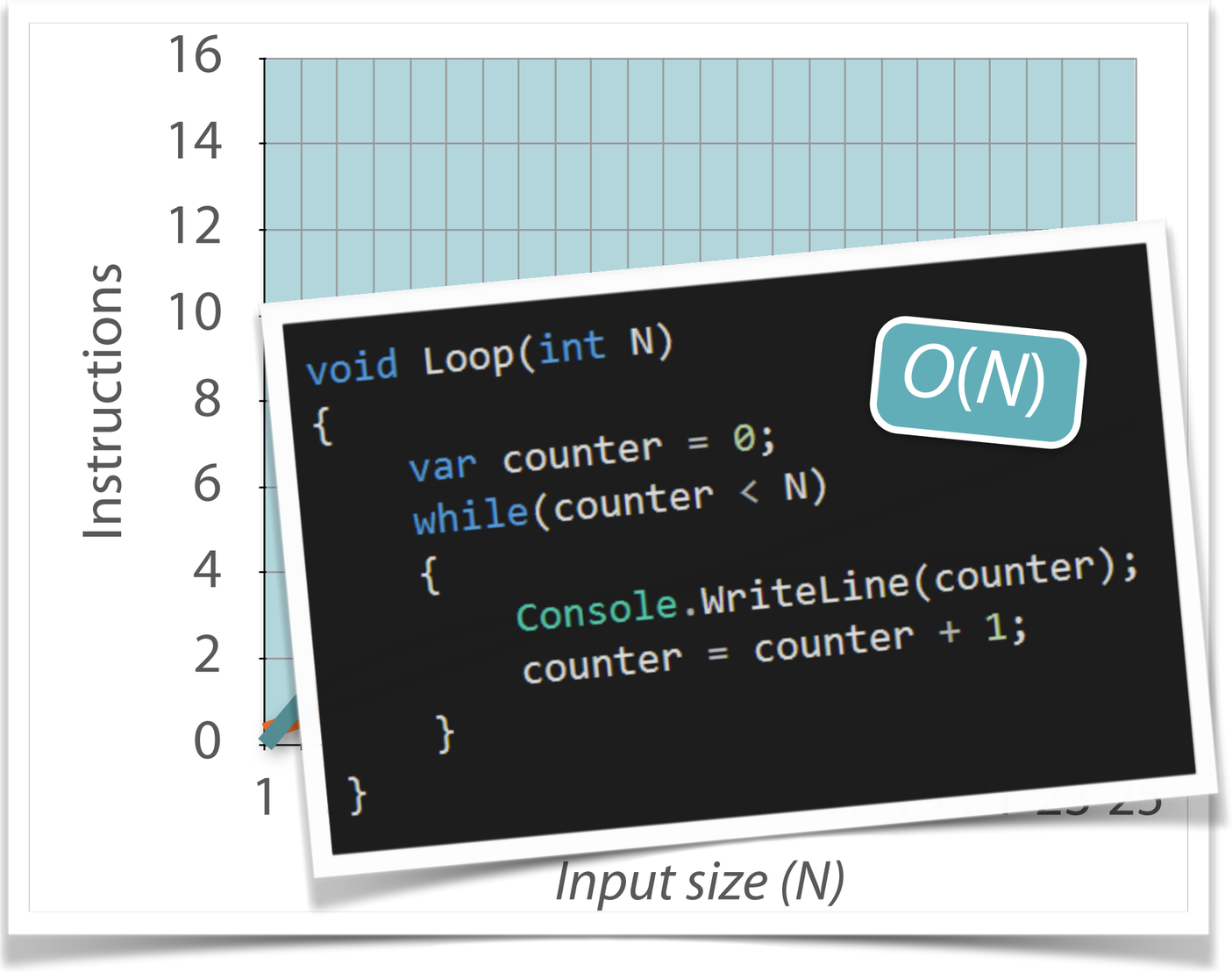

16
14
12
10
8
6
4
2
0
Instructions
1 3 5 7 9 11 13 15 17 19 21 23 25
Input size (N)

Big Omega (Ω)



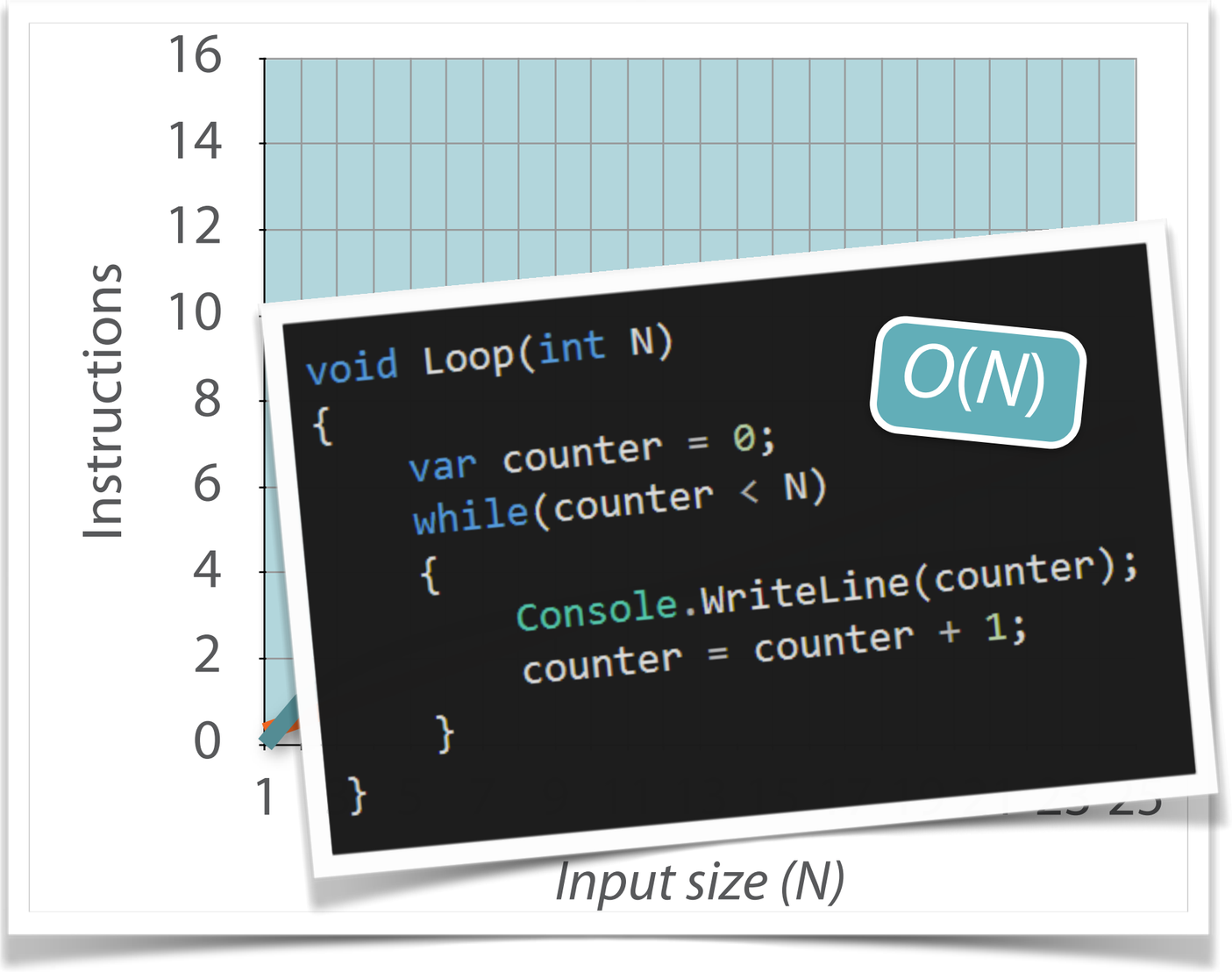
Search algorithms: Ω(1)

Big Omega (Ω)



Search algorithms: Ω(1)

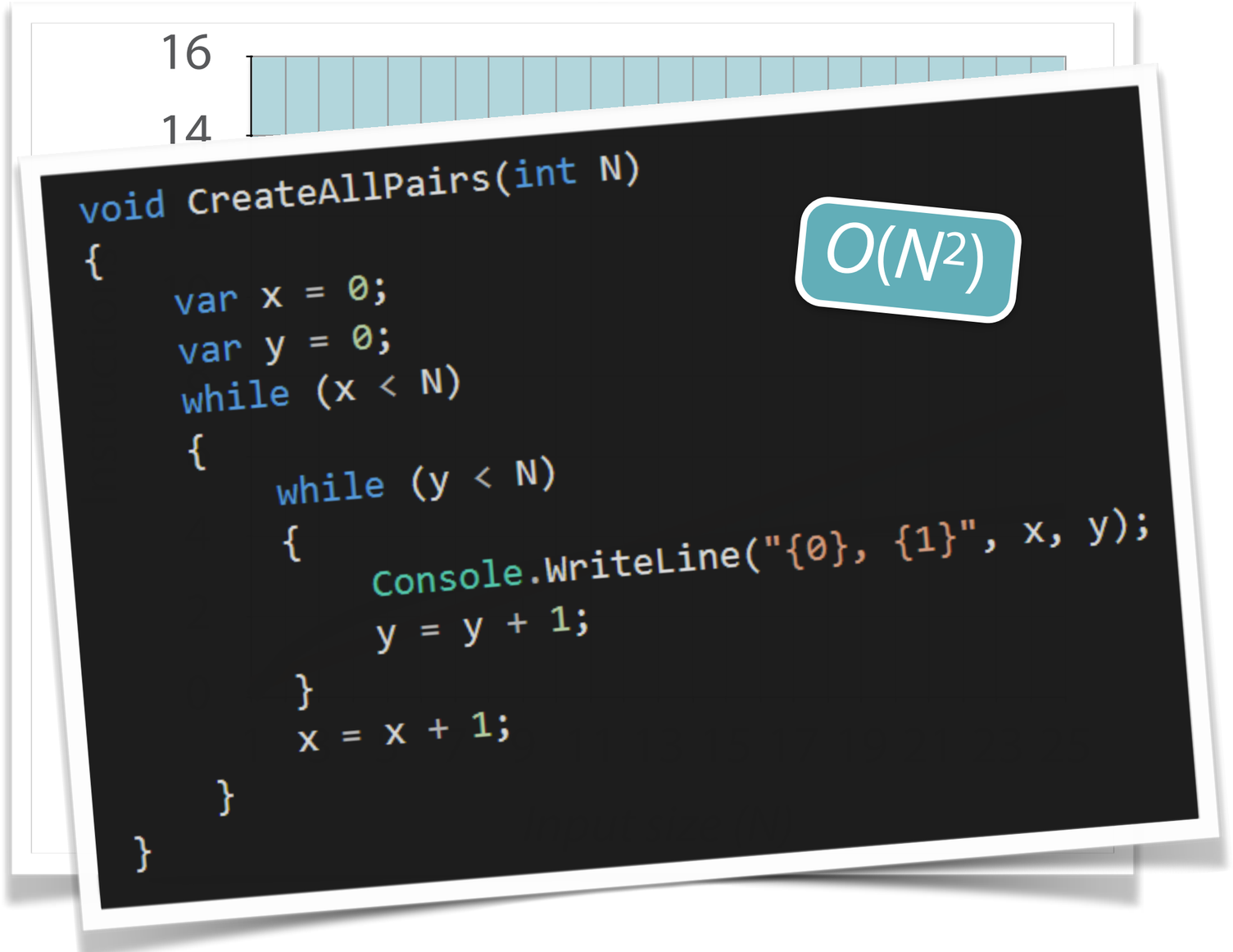
Big Omega (Ω)



Search algorithms: Ω(1)

Simple loop: Ω(*N*)

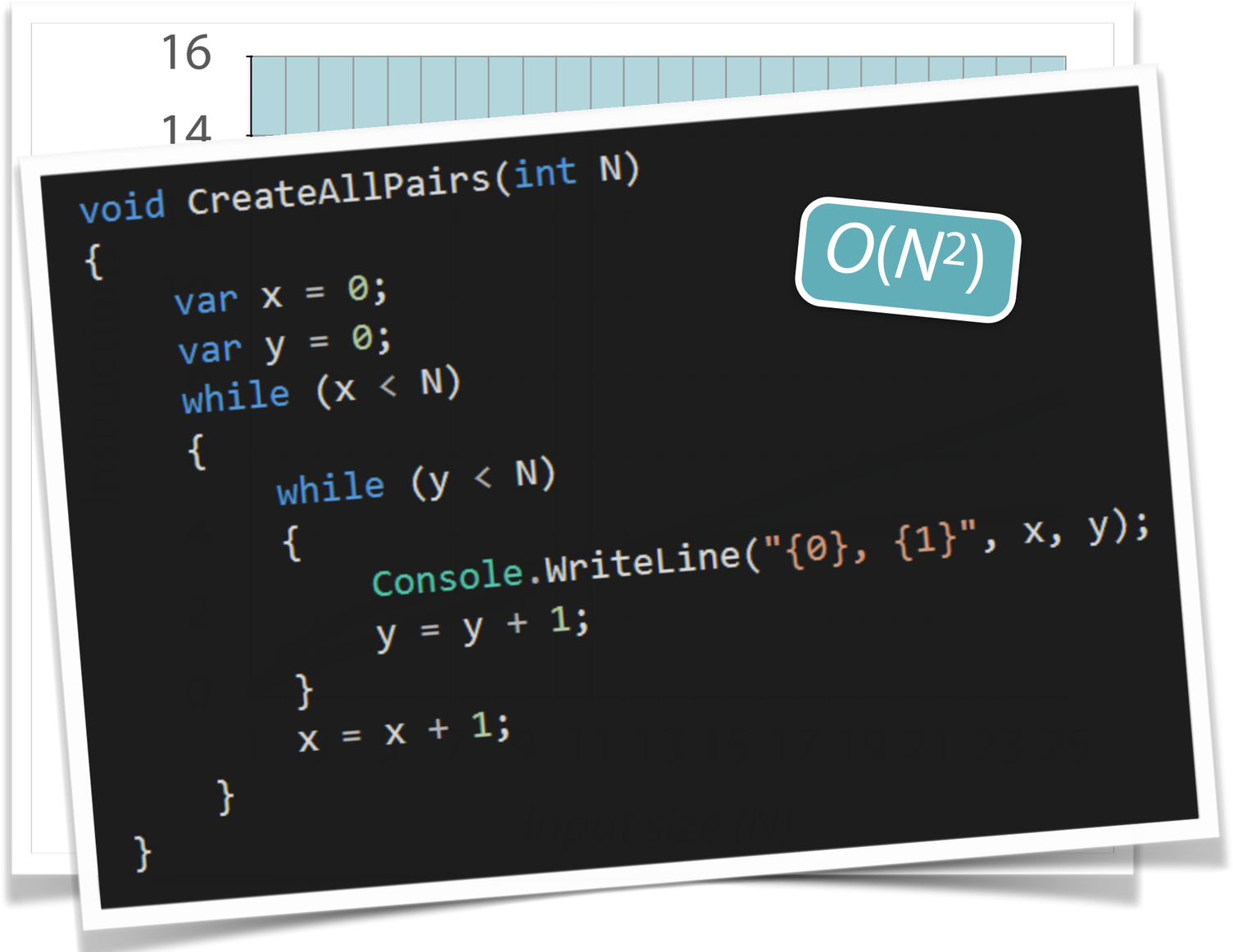
Big Omega (Ω)



Search algorithms: Ω(1)

Simple loop: Ω(*N*)

Big Omega (Ω)



Search algorithms: Ω(1)

Simple loop: Ω(*N*)

Pair generation: Ω(N^2)

Recursive Methods

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```
int Faculty(int n)
{
    if (n == 1)
        return 1;
    return n * Faculty(n - 1);
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$$Faculty(8) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

Recursive Methods

Faculty(N) = N · Faculty(N-1)

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Recursive Methods

Faculty(N) = N · Faculty(N-1)

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…

2 · Faculty(1)

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Recursive Methods

Faculty(N) = N · Faculty(N-1)

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N recursion steps

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Faculty(N) = N · Faculty(N-1)

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N recursion steps

For each step:
only a *constant* amount
of instructions, say c

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Complexity(N) = $c \cdot N$

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Recursive Methods

Faculty(N) = N · Faculty(N-1)

(N-1) · Faculty(N-2)

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Upper bound constant: $c + 1$

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

Recursive Methods

Faculty(N) = N · Faculty(N-1)

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Upper bound constant: $c + 1$

Lower bound constant: $c - 1$

Recursive Methods

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$\Theta(N)$

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Upper bound constant: $c + 1$

Lower bound constant: $c - 1$

```
int Fibonacci(int n)
{
    if (n <= 1) // throw error if n < 0
        return 1;
    return Fibonacci(n - 1) + Fibonacci(n - 2);
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int Fibonacci(int n)
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1 1

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1 1 2

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1 1 2 3

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1 1 2 3 5

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int Fibonacci(int n)
{
    if (n <= 1) // throw error if n < 0
        return 1;
    return Fibonacci(n - 1) + Fibonacci(n - 2);
}
```

1 1 2 3 5 8 13 21 34 55

```
int Fibonacci(int n)
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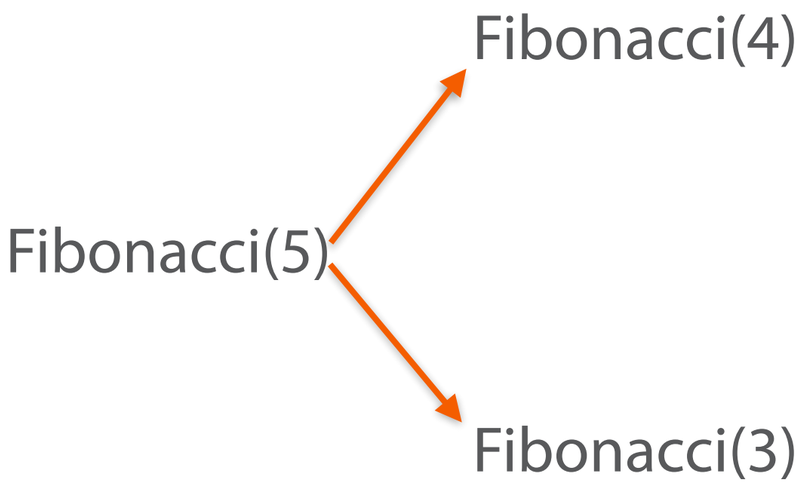
    if (n <= 1) // throw error if n < 0
        return 1;
    return Fibonacci(n - 1) + Fibonacci(n - 2);
}
```

1 1 2 3 5 8 13 21 34 55

Fibonacci(5)

```
int Fibonacci(int n)
{
    if (n <= 1) // throw error if n < 0
        return 1;
    return Fibonacci(n - 1) + Fibonacci(n - 2);
}
```

1 1 2 3 5 8 13 21 34 55



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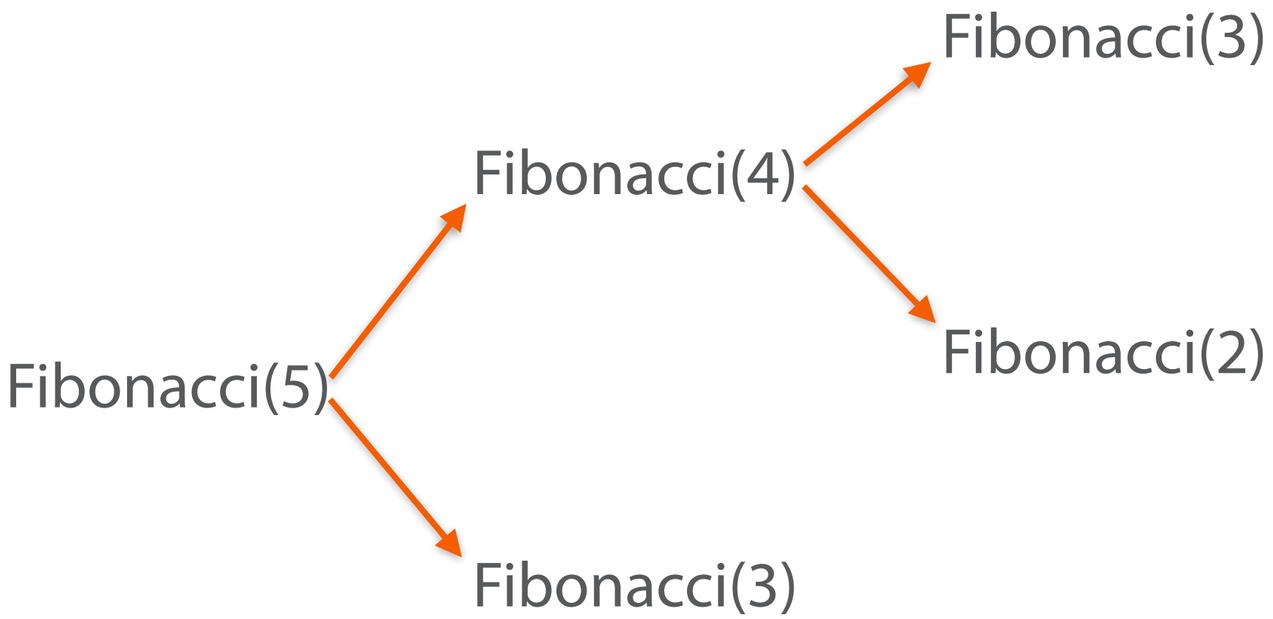
1 1 2 3 5 8 13 21 34 55



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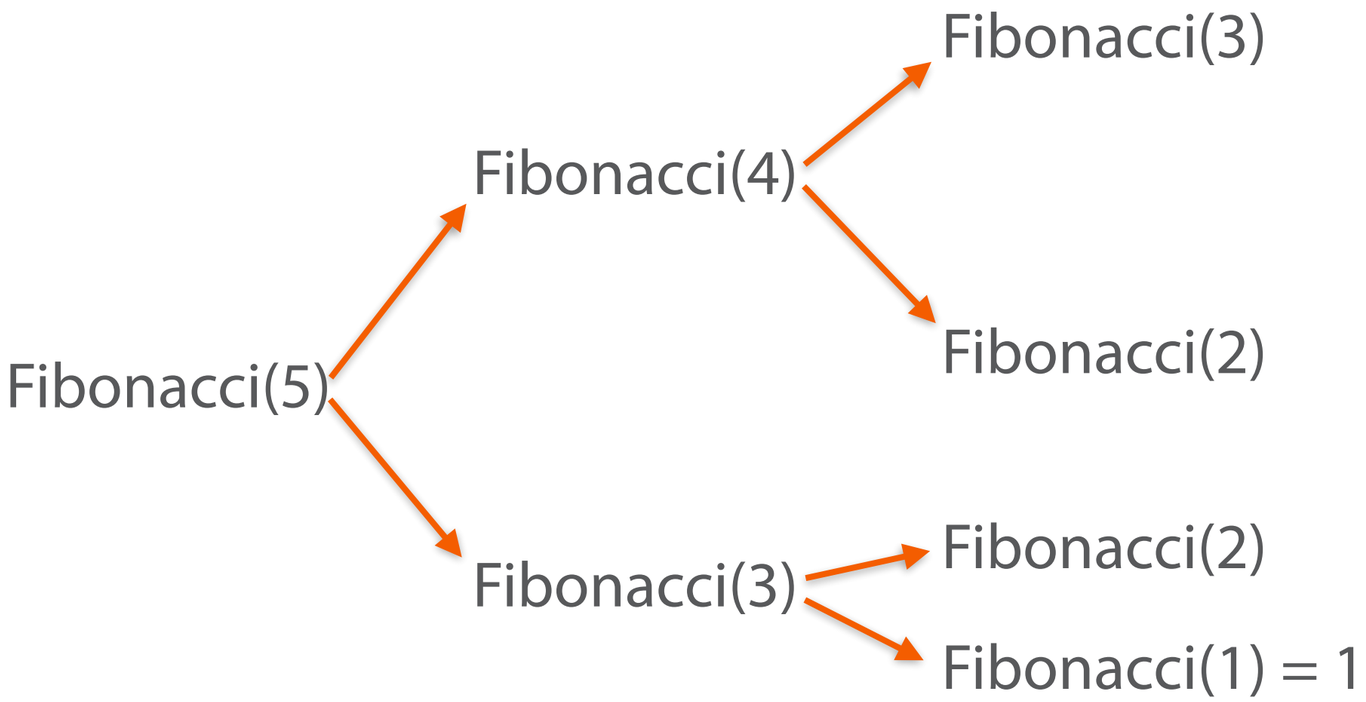

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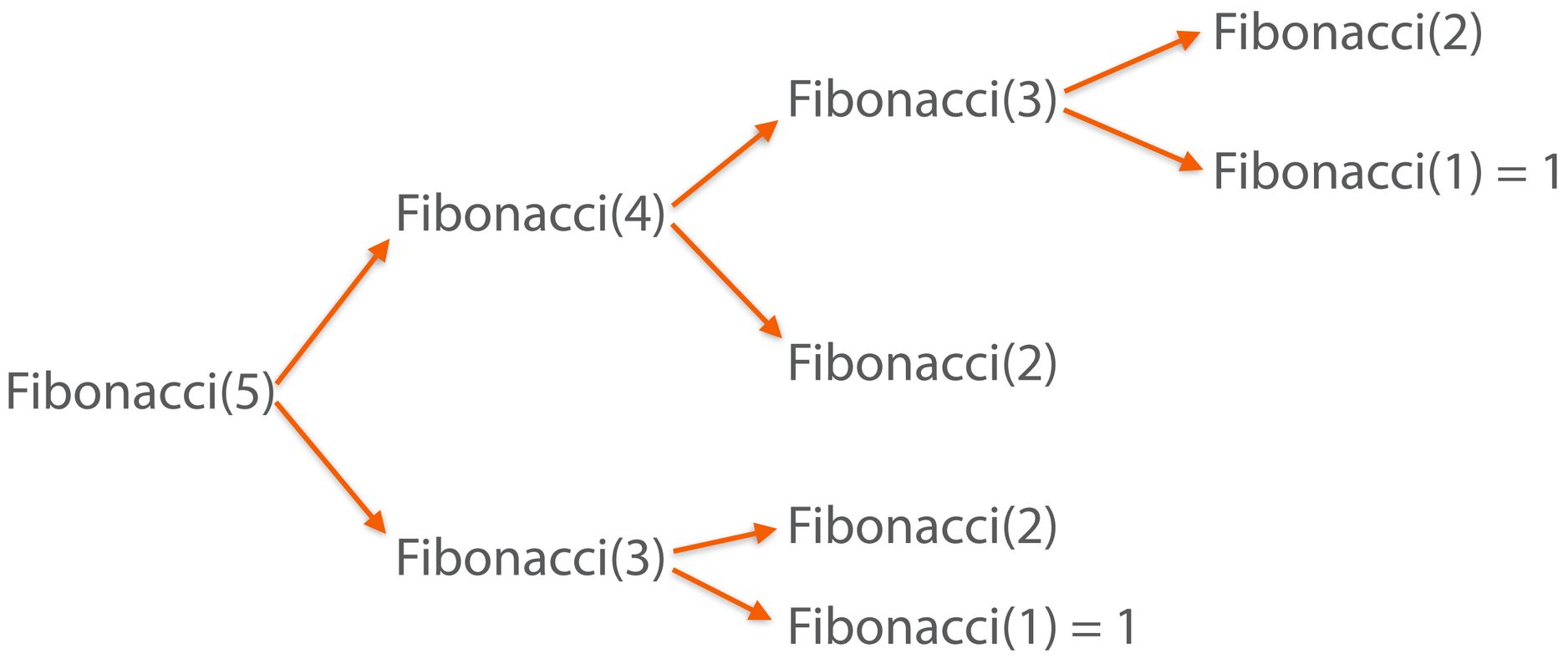

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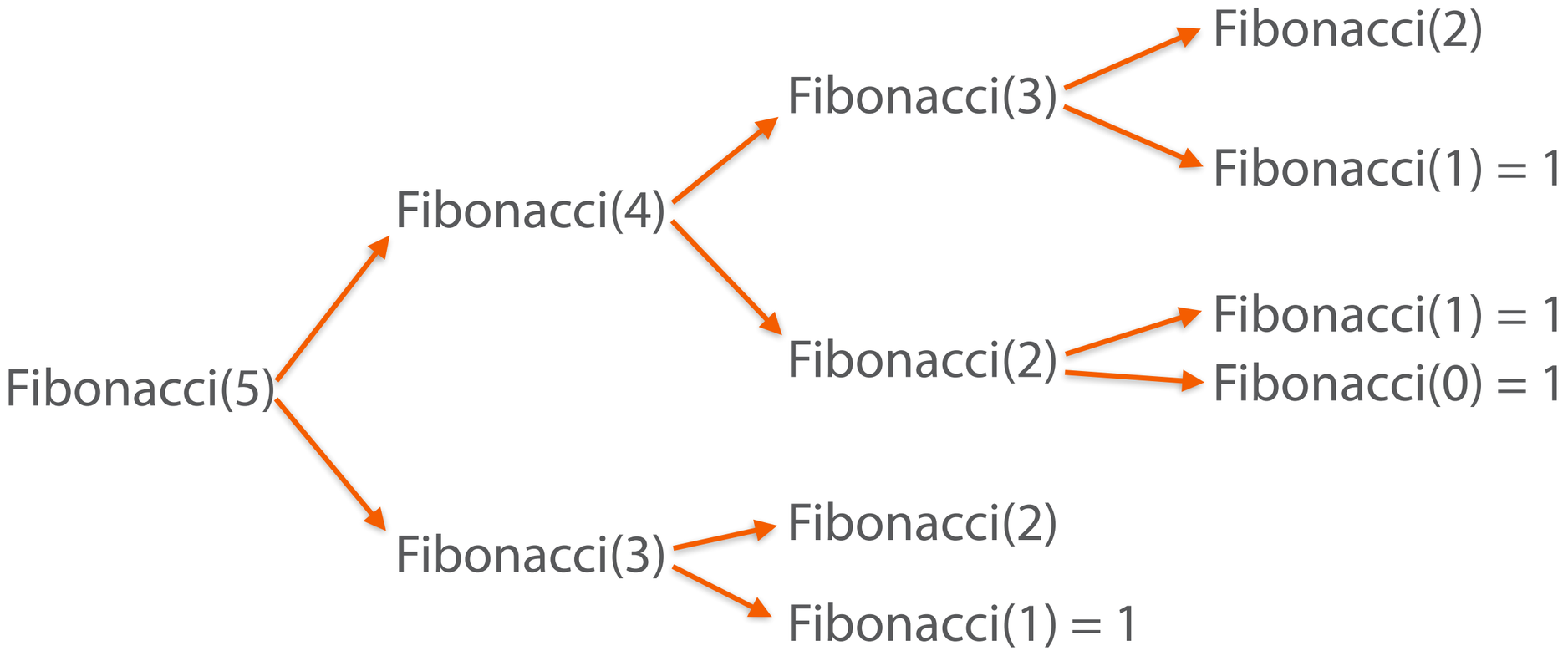

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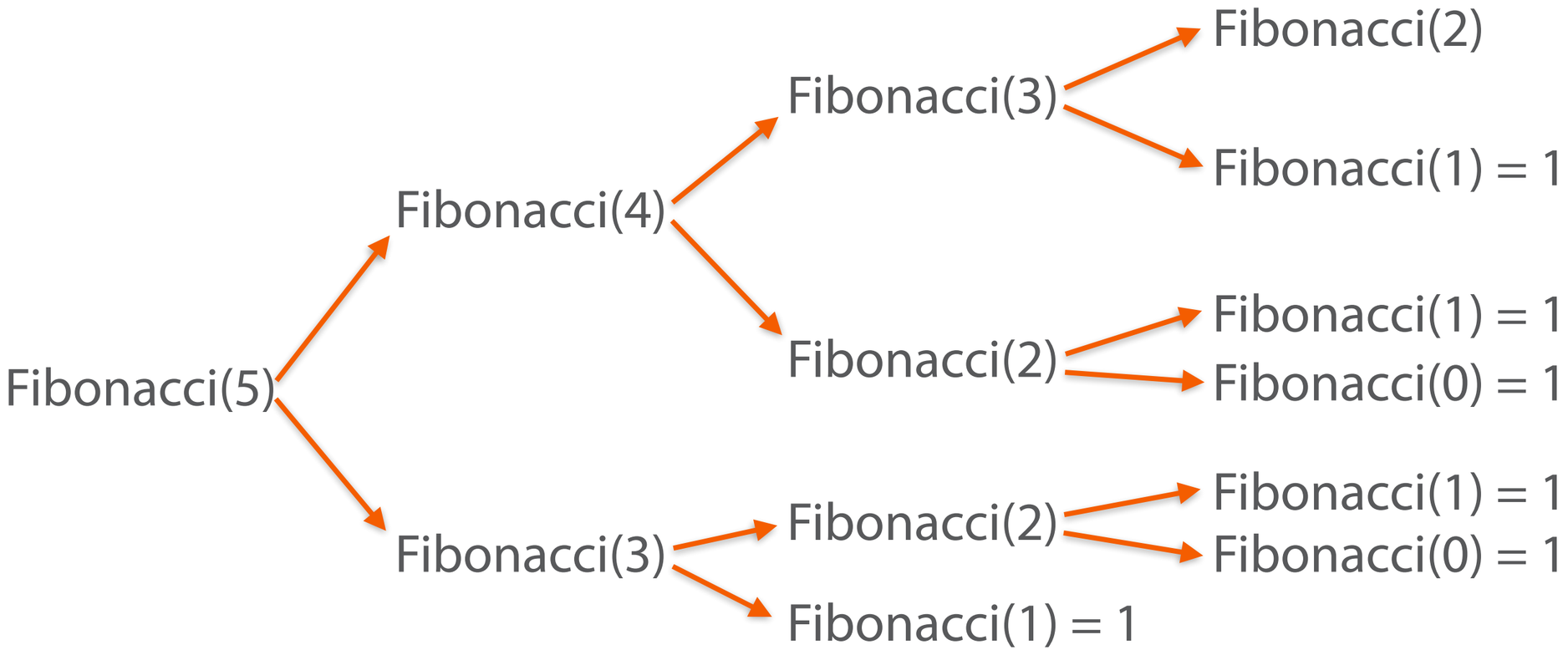

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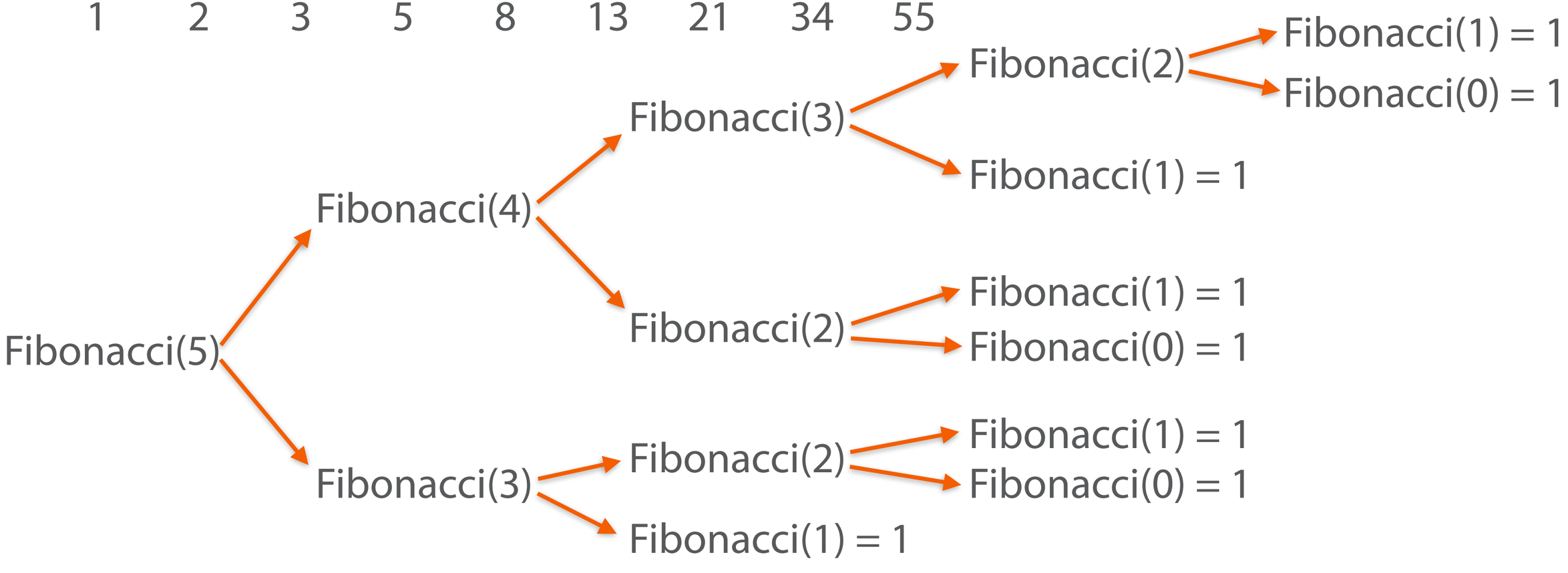

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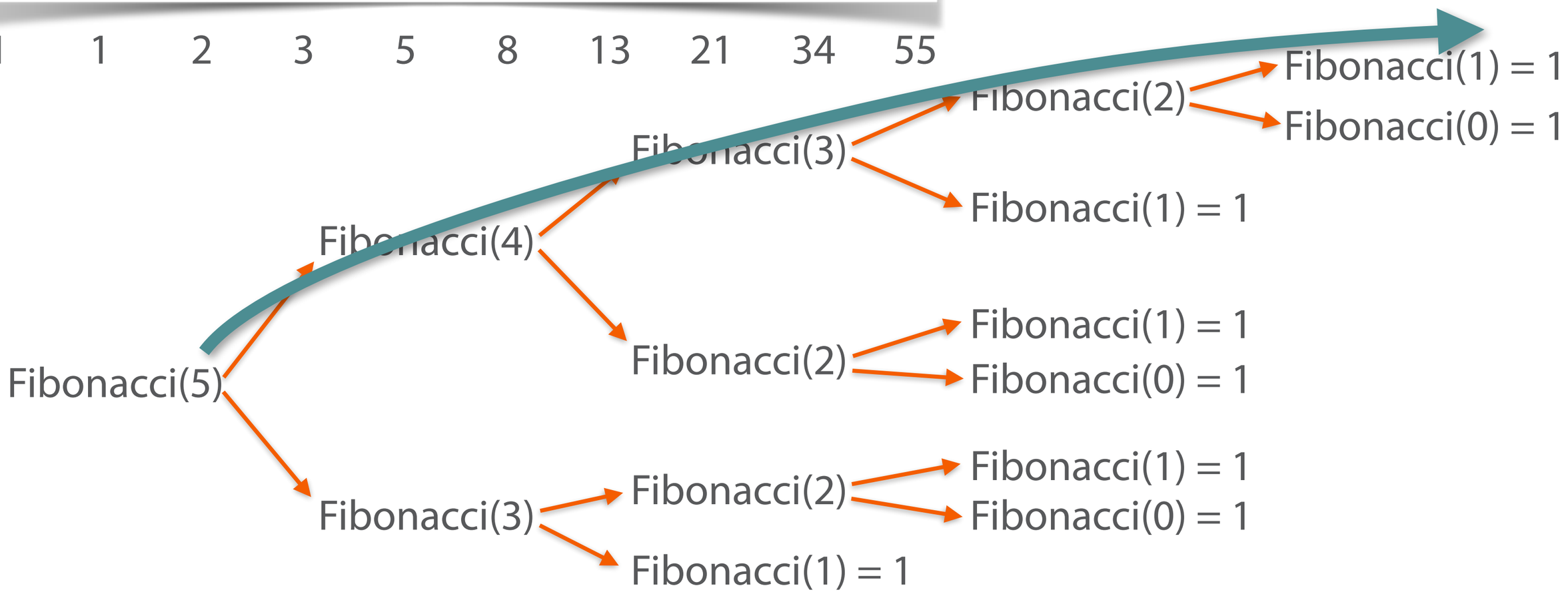

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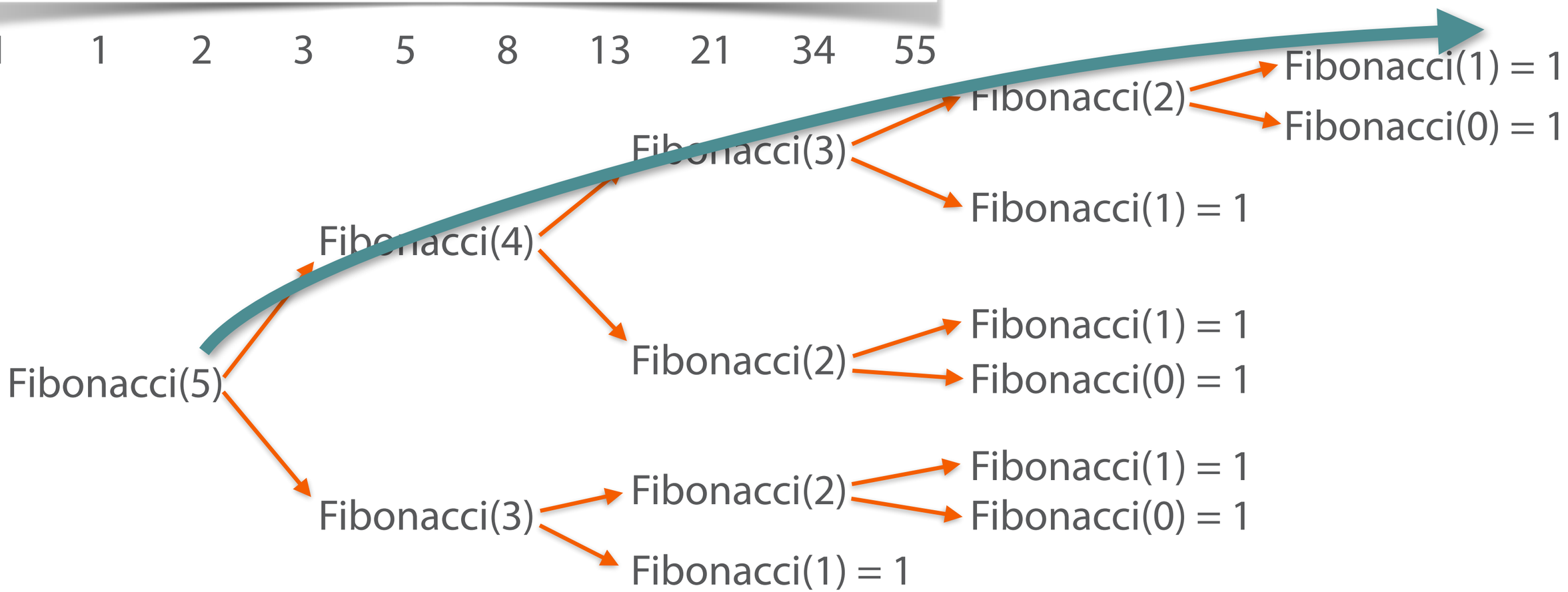
Fibonacci(*N*): *N* recursion steps

1 1 2 3 5 8 13 21 34 55



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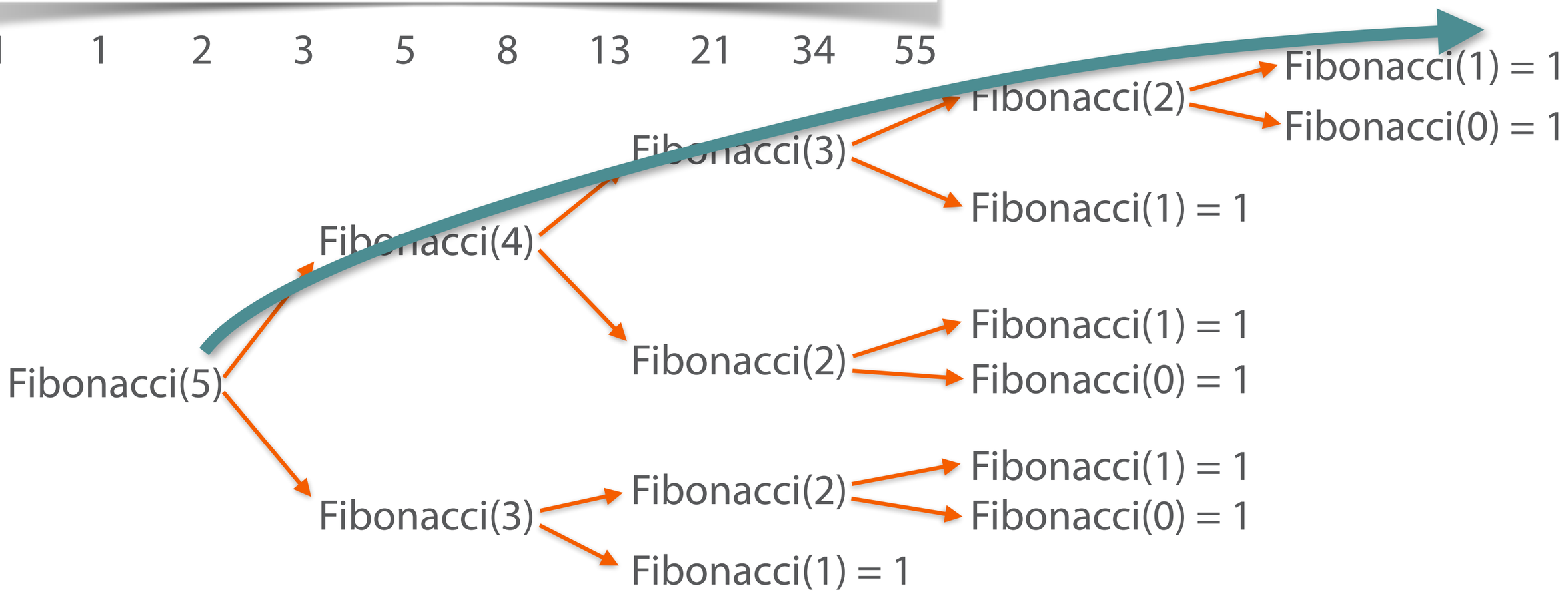
Fibonacci(*N*): *N* recursion steps

Each step *at most* doubles

1 1 2 3 5 8 13 21 34 55



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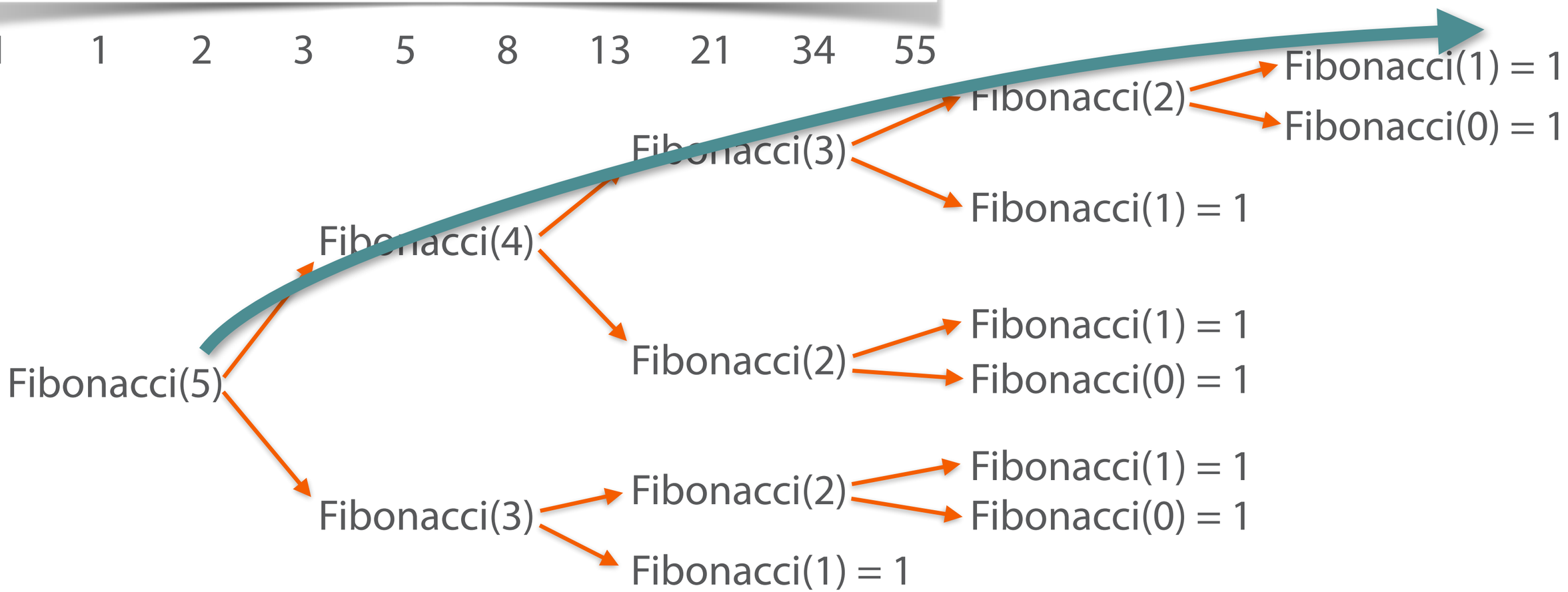
Fibonacci(*N*): *N* recursion steps

Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ \ldots \ \cdot 2 = c \cdot 2^{N-1}$

1 1 2 3 5 8 13 21 34 55



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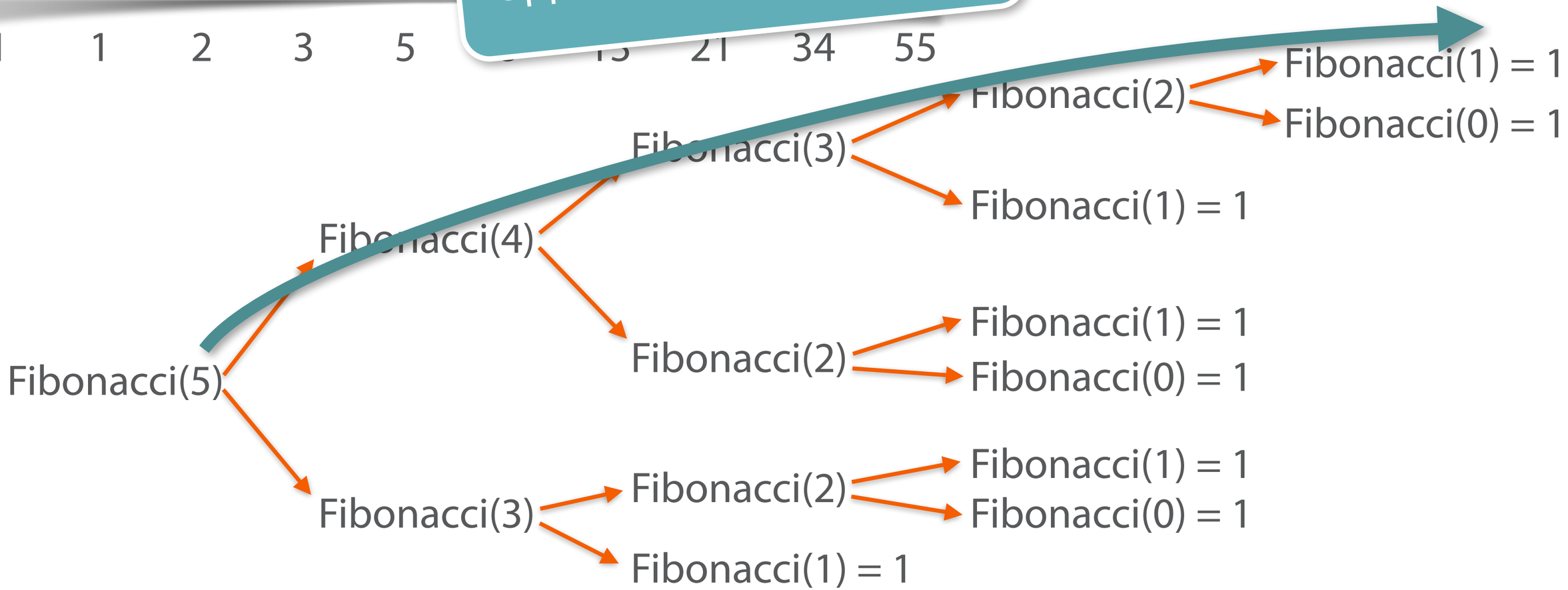
Fibonacci(*N*): *N* recursion steps

Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = c \cdot 2^{N-1}$

Upper bound constant: $c \cdot 2$

1 1 2 3 5 ... 13 21 34 55



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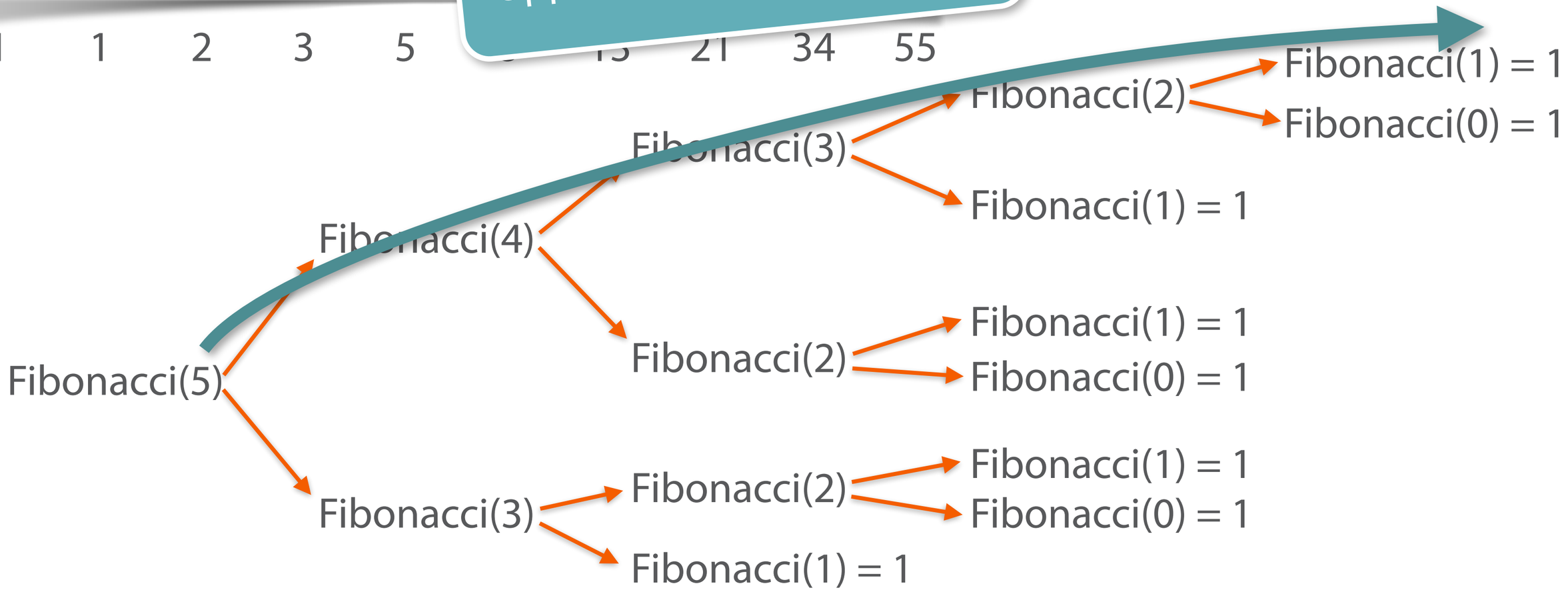
Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = c \cdot 2^{N-1}$

Upper bound: $c \cdot 2 \cdot 2^{N-1}$

Upper bound constant: $c \cdot 2$

1 1 2 3 5 ... 13 21 34 55



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}
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Fibonacci(*N*): *N* recursion steps

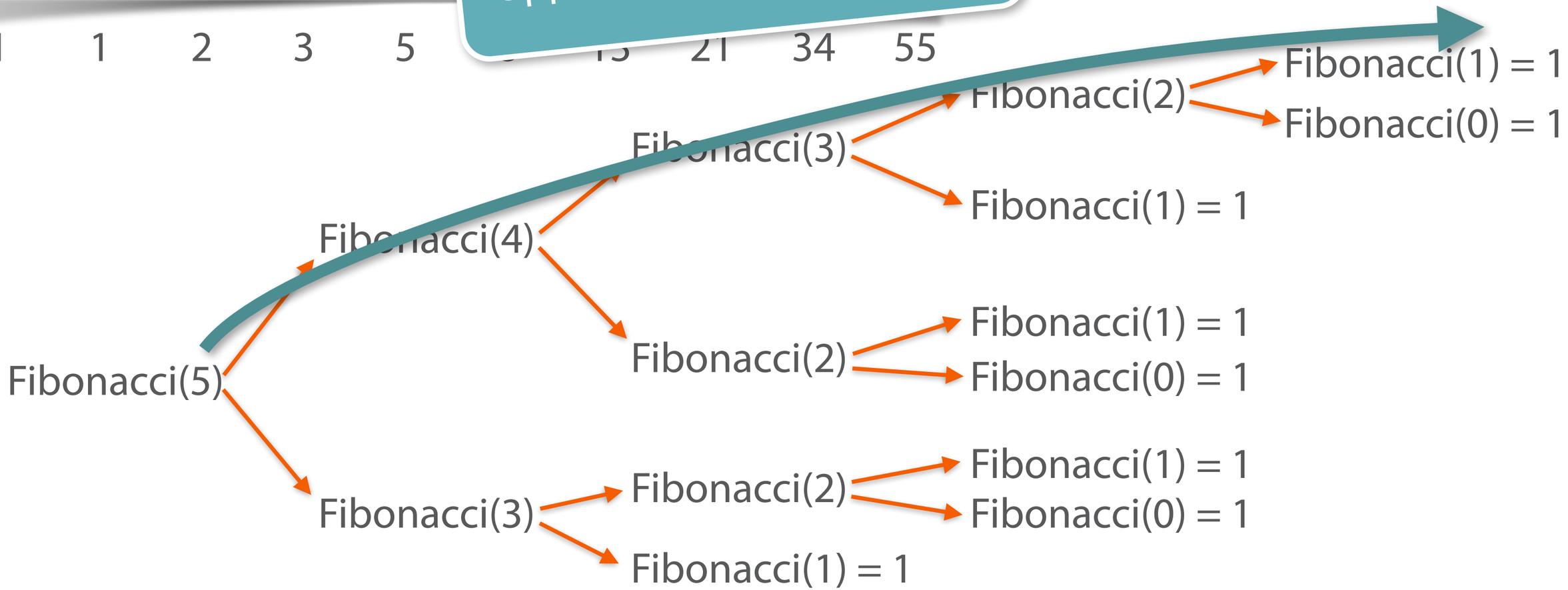
Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = c \cdot 2^{N-1}$

Upper bound: $c \cdot 2 \cdot 2^{N-1} = c \cdot 2^{N}$

Upper bound constant: $c \cdot 2$

1 1 2 3 5 ... 13 21 34 55



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Fibonacci(*N*): *N* recursion steps

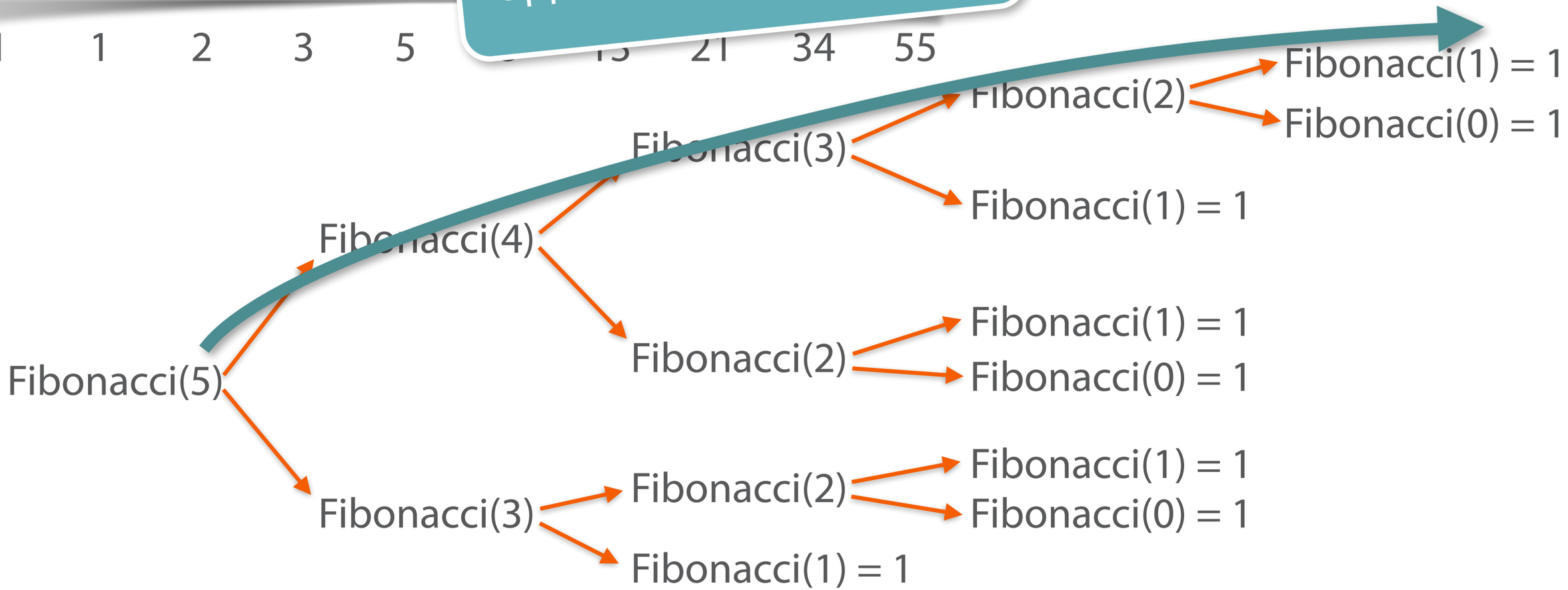
Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = c \cdot 2^{N-1}$

Upper bound: $c \cdot 2 \cdot 2^{N-1} = c \cdot 2^{N}$

$O(2^N)$

1 1 2 3 5 ... 13 21 34 55



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Each step *at most* doubles

$c \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = c \cdot 2^{N-1}$

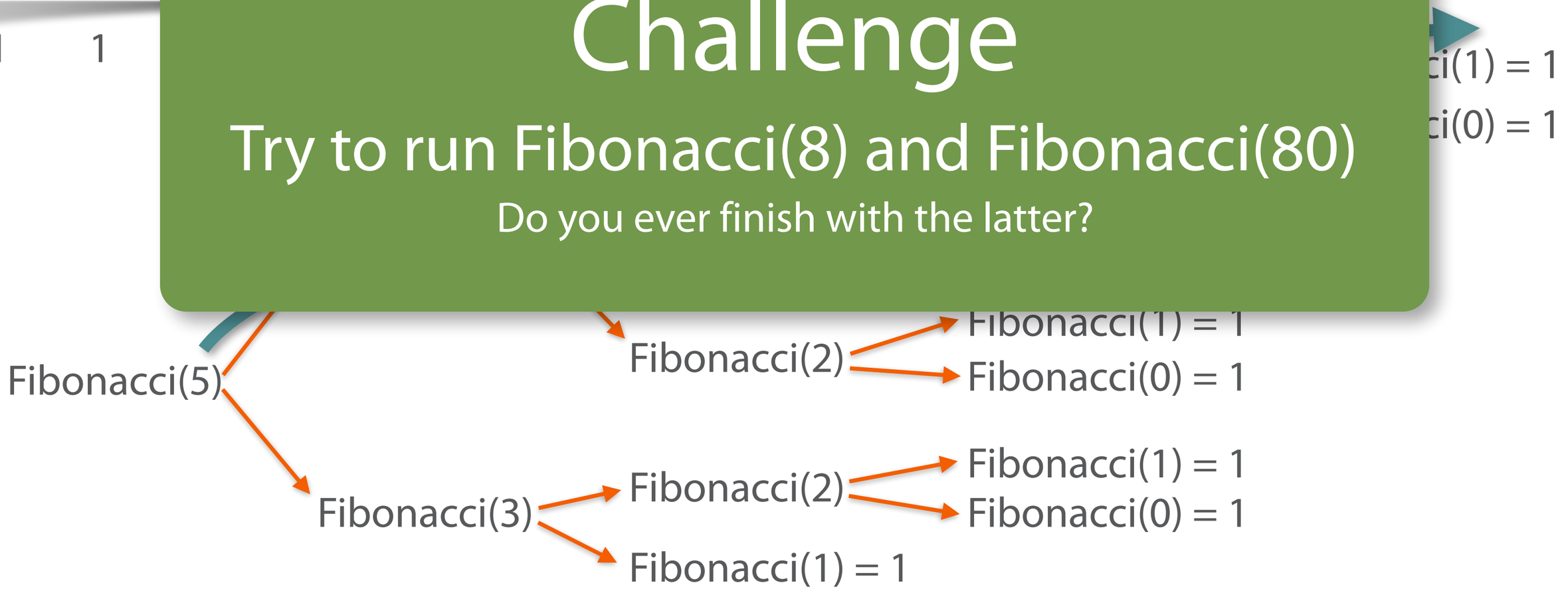
Upper bound: $c \cdot 2 \cdot 2^{N-1} = c \cdot 2^{N}$

1 1

Challenge

Try to run Fibonacci(8) and Fibonacci(80)

Do you ever finish with the latter?



```
internal int Fibonacci2(int n)
{
    int low = 1;
    int high = 1;
    for (int i = 0; i < n; i++)
    {
        var oldHigh = high;
        high = low + high;
        low = oldHigh;
    }
    return low;
}
```

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internal int Fibonacci2(int n)
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Complexity(*N*):

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Complexity(*N*):

c_1 +

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Complexity(*N*):

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Complexity(*N*):

$$c_1 + N \cdot c_2$$

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        high = low + high;
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    }
    return low;
}
```

Upper bound constant: $c_2 + 1$

Complexity(N):

$$c_1 + N \cdot c_2$$

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internal int Fibonacci2(int n)
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Upper bound constant: $c_2 + 1$

Complexity(N):

$$c_1 + N \cdot c_2$$

$$O(N)$$

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Upper bound constant: $c_2 + 1$

Lower bound constant: $c_2 - 1$

Complexity(N):

$$c_1 + N \cdot c_2$$

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Complexity(N):

$$c_1 + N \cdot c_2$$

$O(N)$

$\Theta(N)$

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int? BinarySearch(List<int> haystack, int needle, int min, int max)
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    var midpoint = (max + min) / 2;
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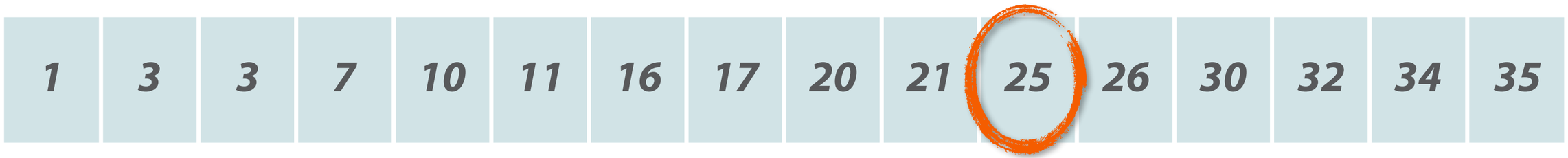


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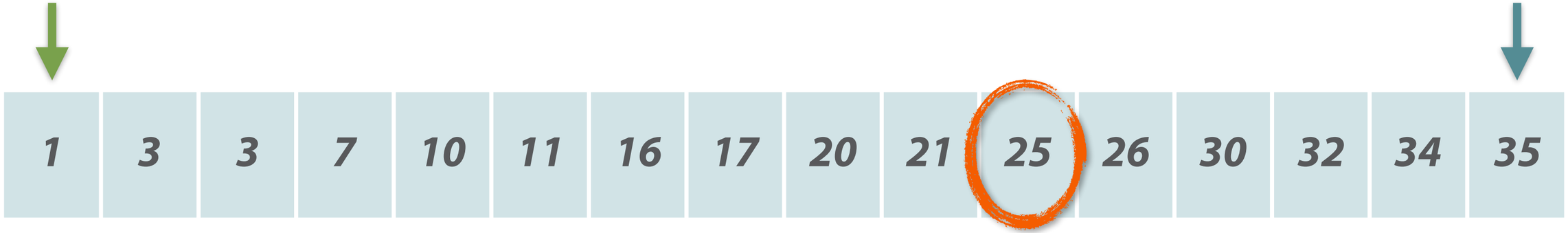


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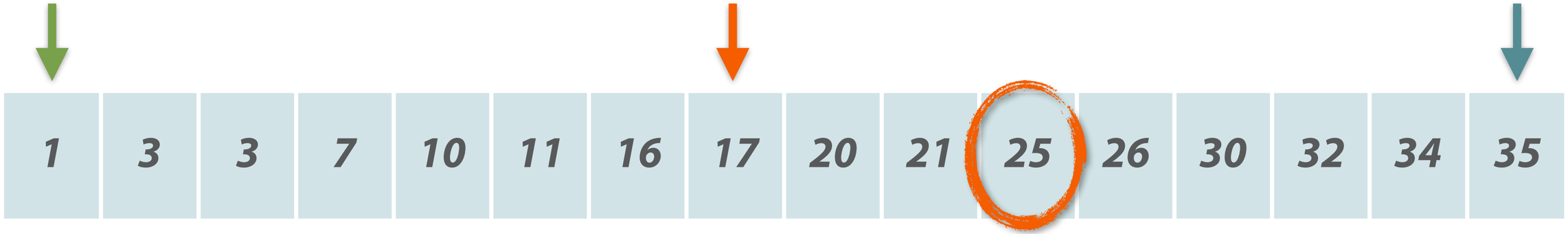


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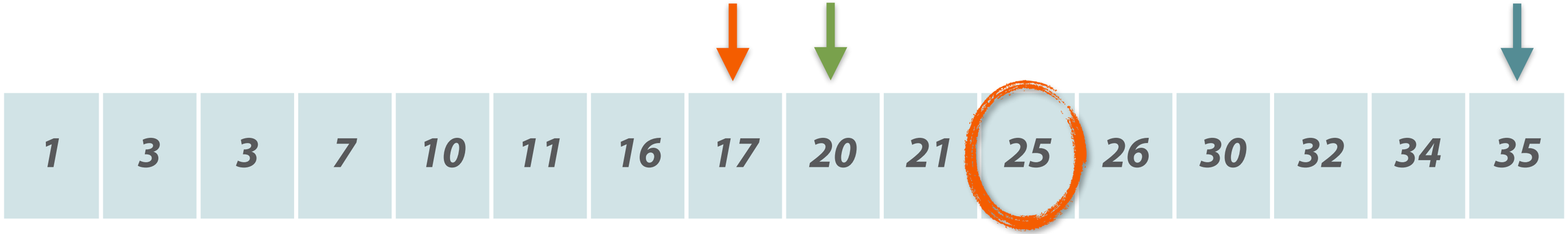


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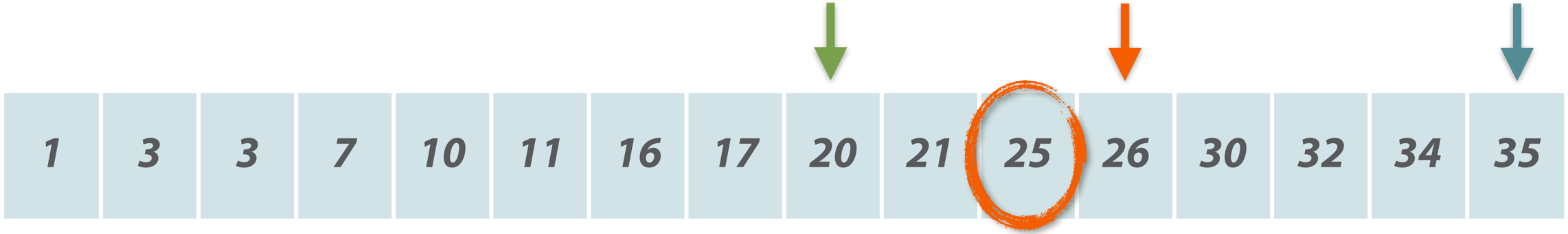


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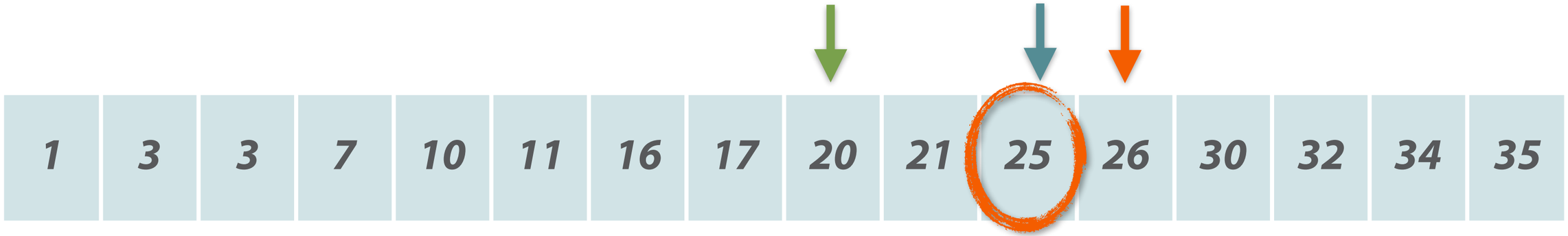


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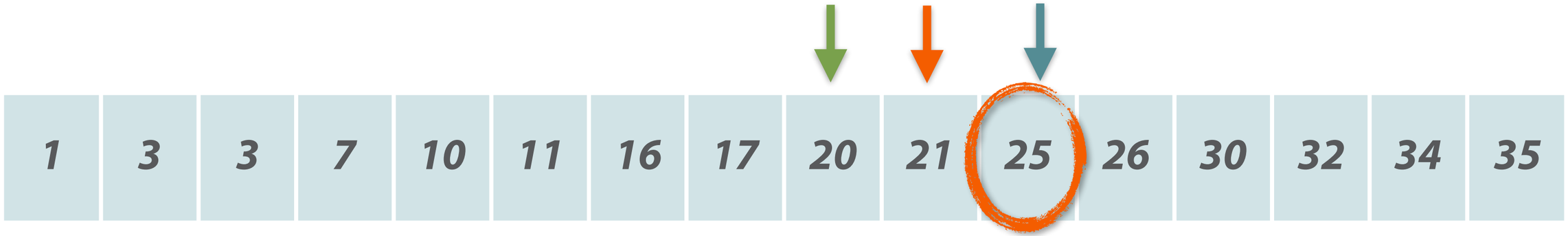


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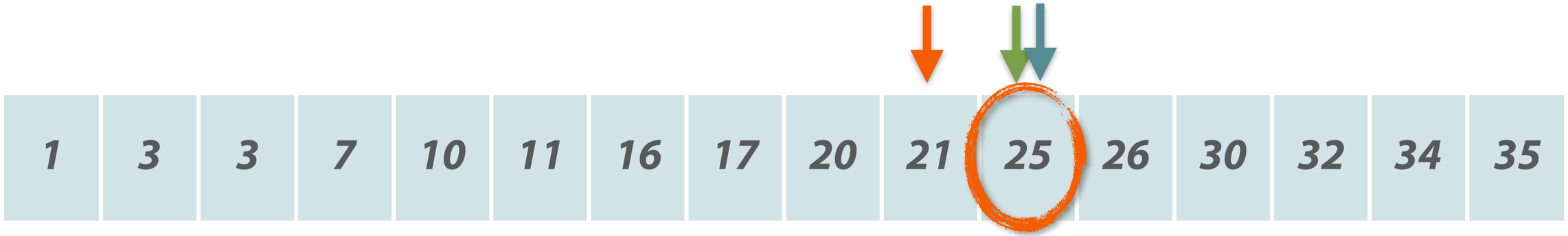


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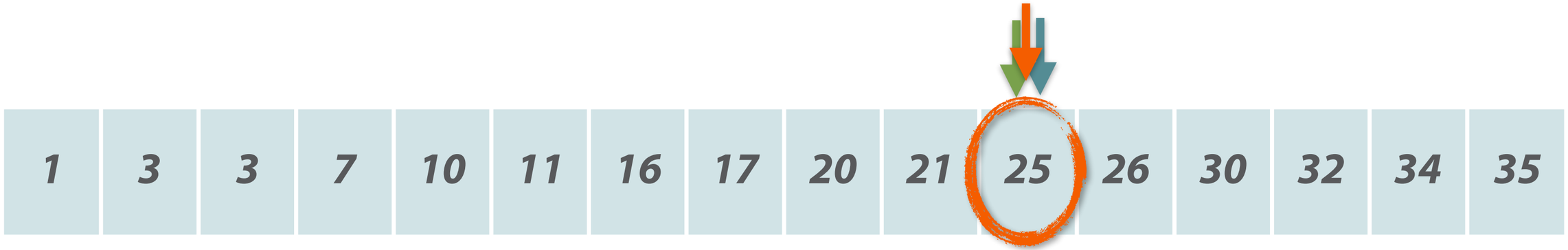


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1	3	3	7	10	11	16	17	20	21	25	26	30	32	34	35

Haystack size: *N*

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Worst-case:

1	3	3	7	10	11	16	17	20	21	25	26	30	32	34	35

Haystack size: *N*

```
int? BinarySearch(List<int> haystack, int needle, int min, int max)
{
    var midpoint = (max + min) / 2;
    if (haystack.Count > 0 && haystack[midpoint] == needle)
        return midpoint;
    if (min >= max)
        return null;
    if (haystack[midpoint] > needle)
        return BinarySearch(haystack, needle, min, midpoint - 1);
    return BinarySearch(haystack, needle, midpoint + 1, max);
}
```

Worst-case: $\log_2 N$ recursion steps

1	3	3	7	10	11	16	17	20	21	25	26	30	32	34	35

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Worst-case: $\log_2 N$ recursion steps max c instructions per step

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Worst-case: $\log_2 N$ recursion steps max c instructions per step

$c \cdot \log_2 N$ instructions in total

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Worst-case: $\log_2 N$ recursion steps max c instructions per step

$c \cdot \log_2 N$ instructions in total

$O(\log_2 N)$

Haystack size: N

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}
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Worst-case: $\log_2 N$ recursion steps max c instructions per step

$c \cdot \log_2 N$ instructions in total

$O(\log_2 N)$

Haystack size: N

General strategy

1: Fold out and count steps

2: Count size per step

```
i                                                    x)
{
    return null;
    if (haystack[midpoint] > needle)
        return BinarySearch(haystack, needle, min, midpoint - 1);
    return BinarySearch(haystack, needle, midpoint + 1, max);
}
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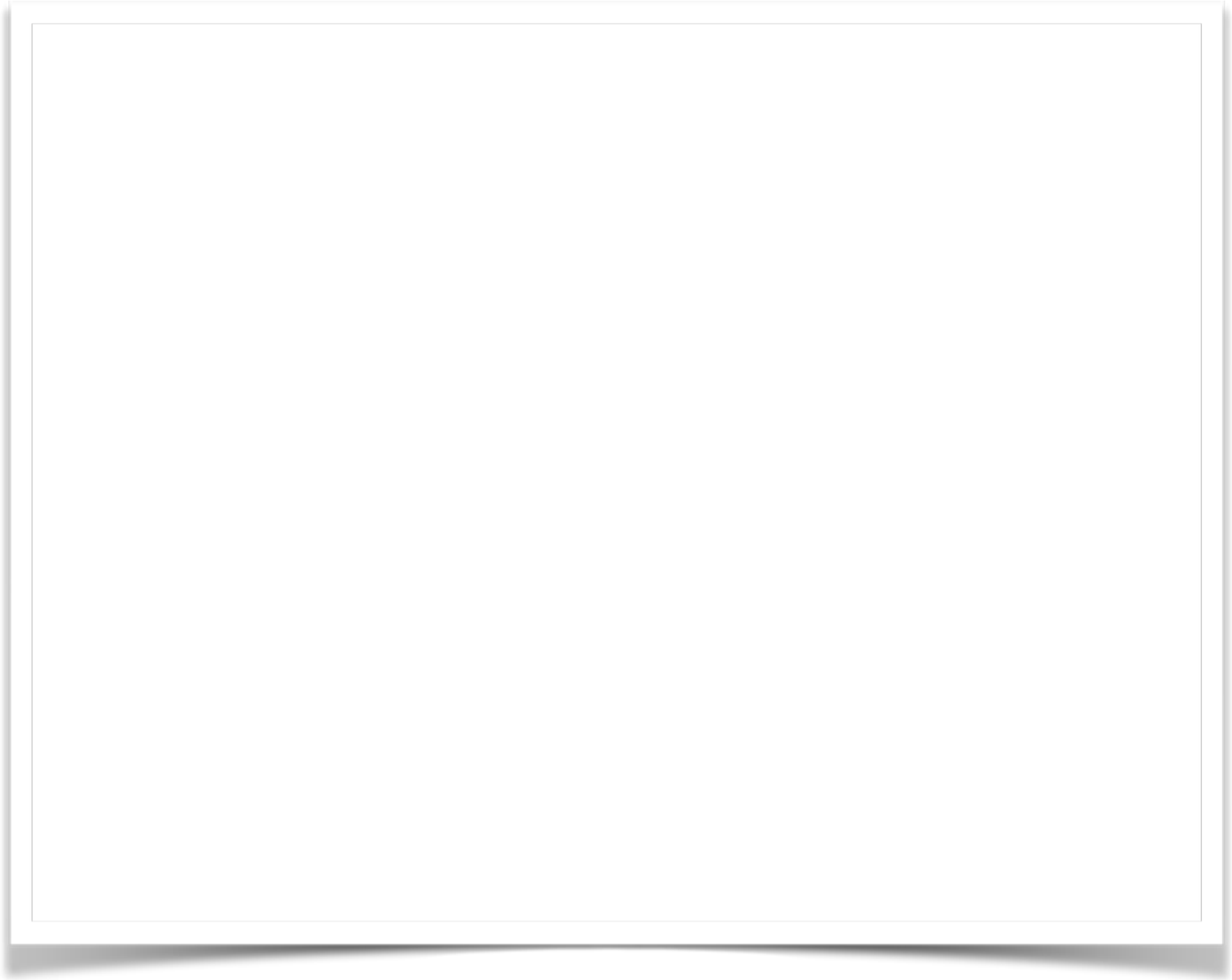
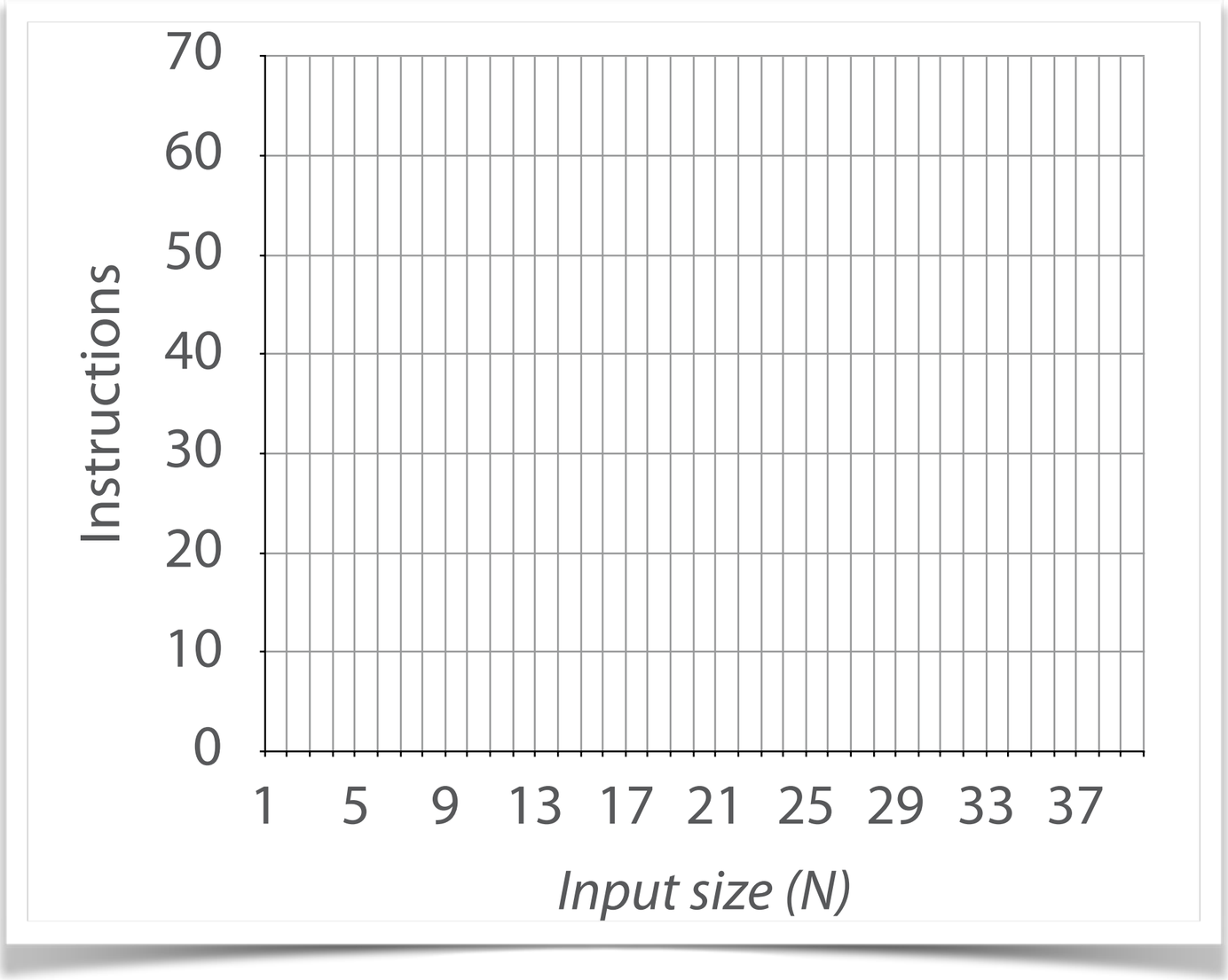
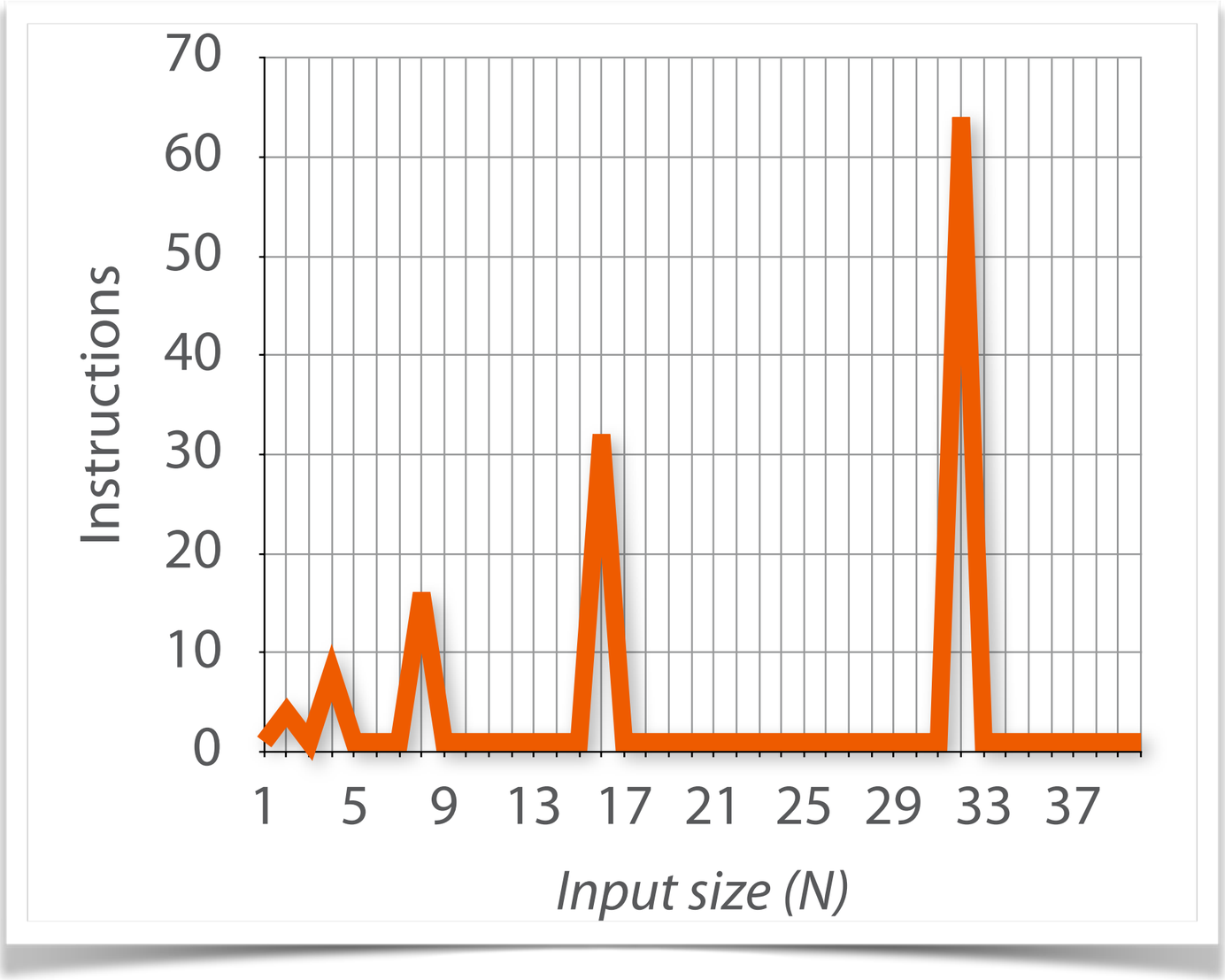
Amortized Complexity



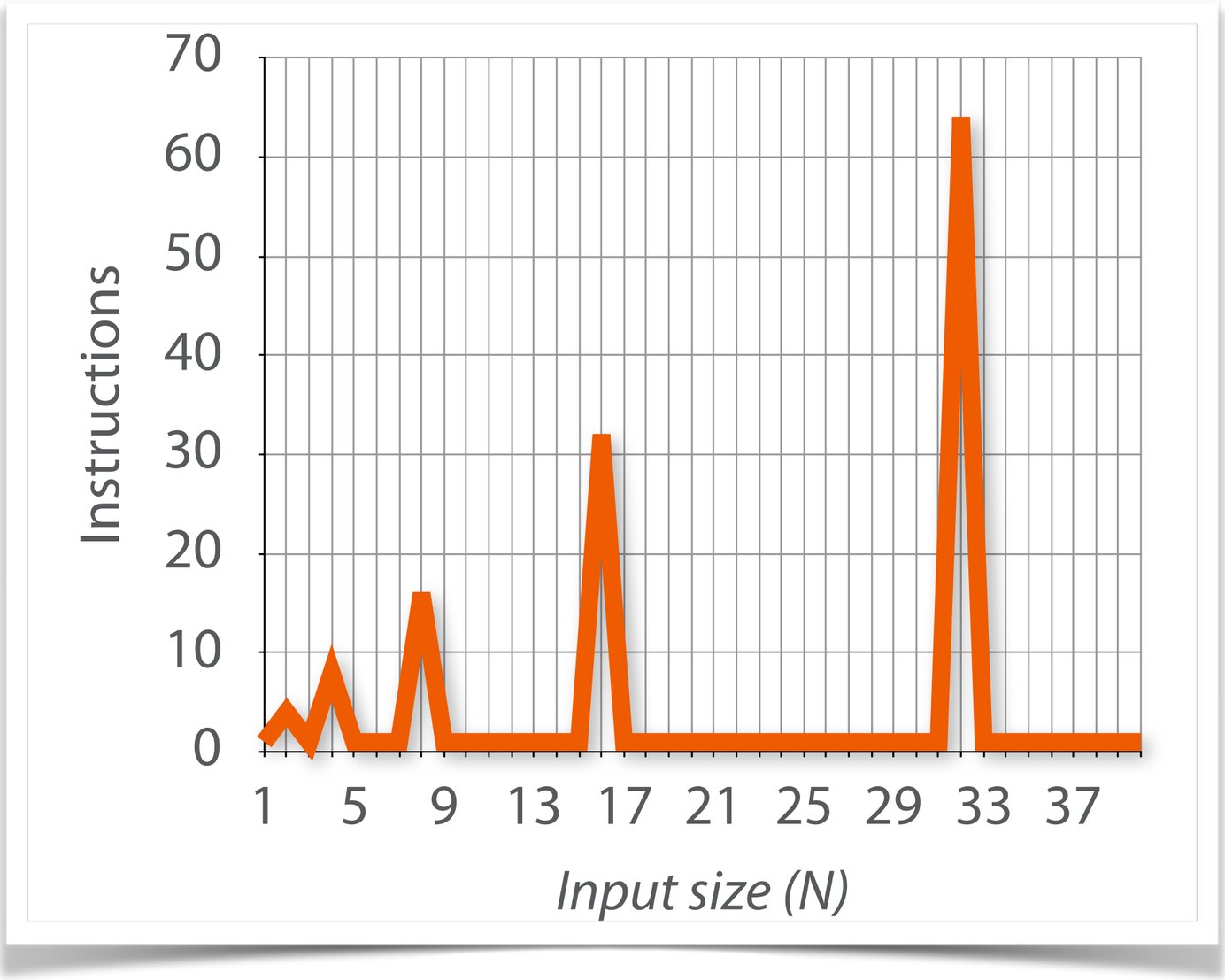
Amortized Complexity



Amortized Complexity

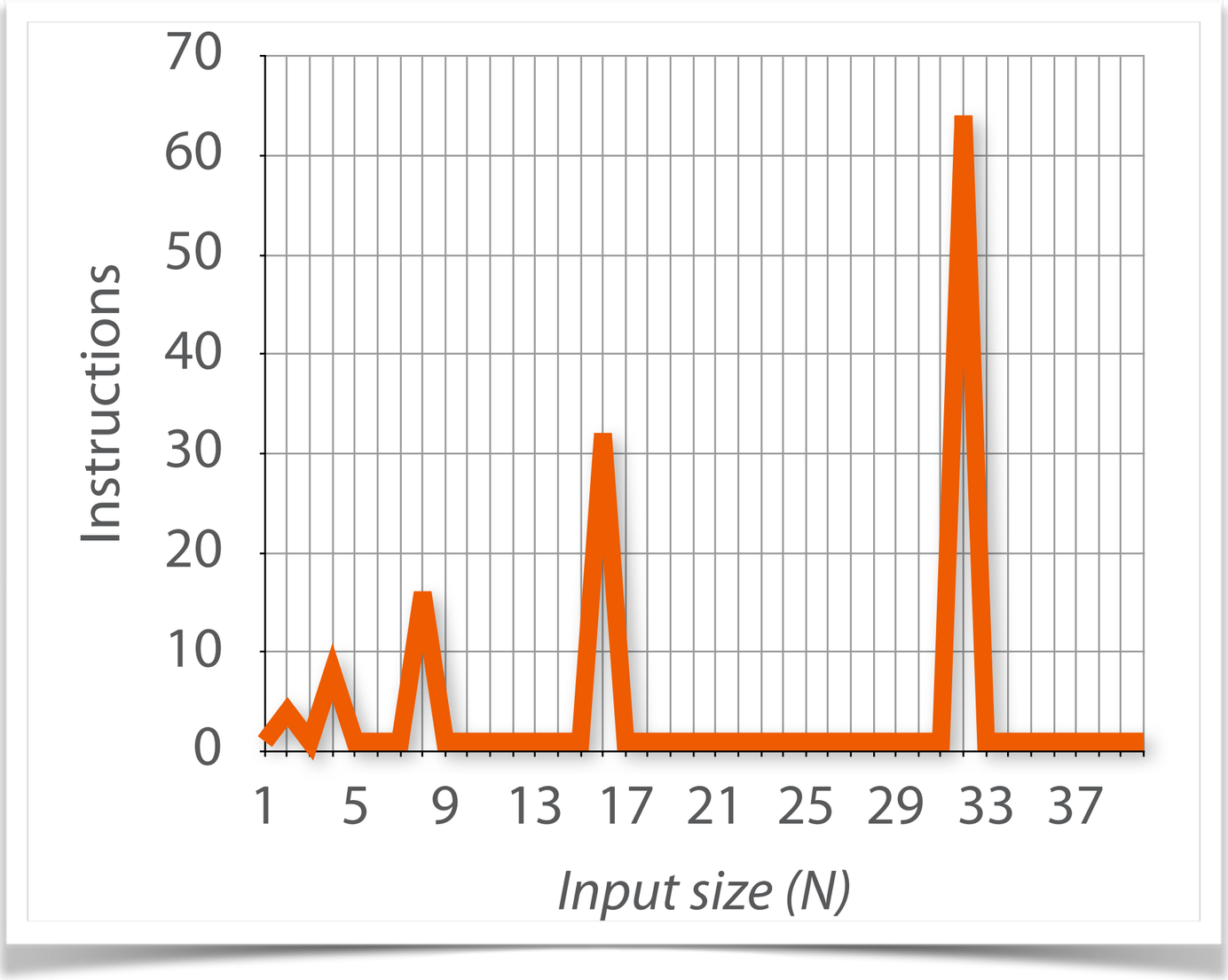


Amortized Complexity



Complexity of multiple calls

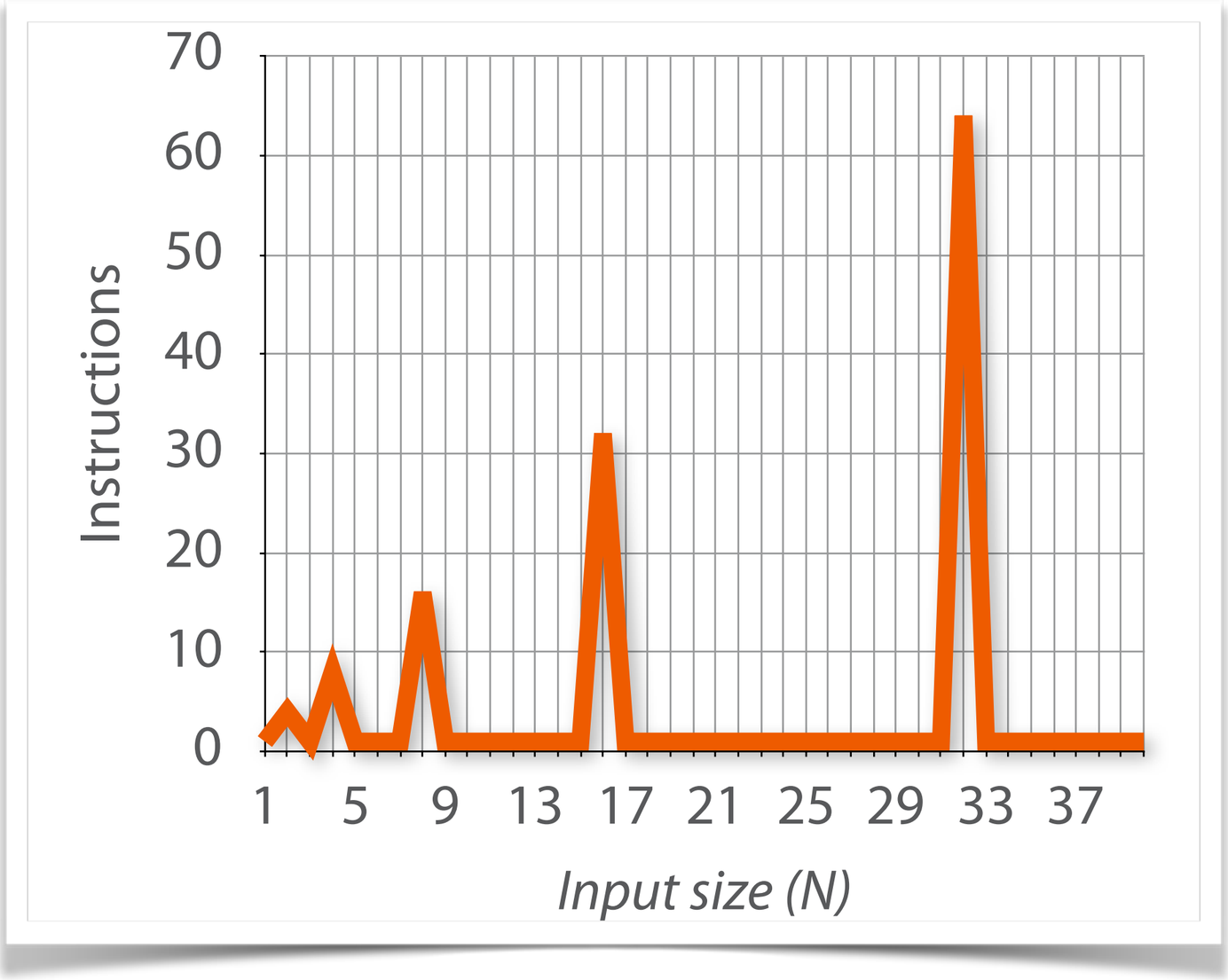
Amortized Complexity



Complexity of multiple calls

Typically: $O(1)$ per call

Amortized Complexity

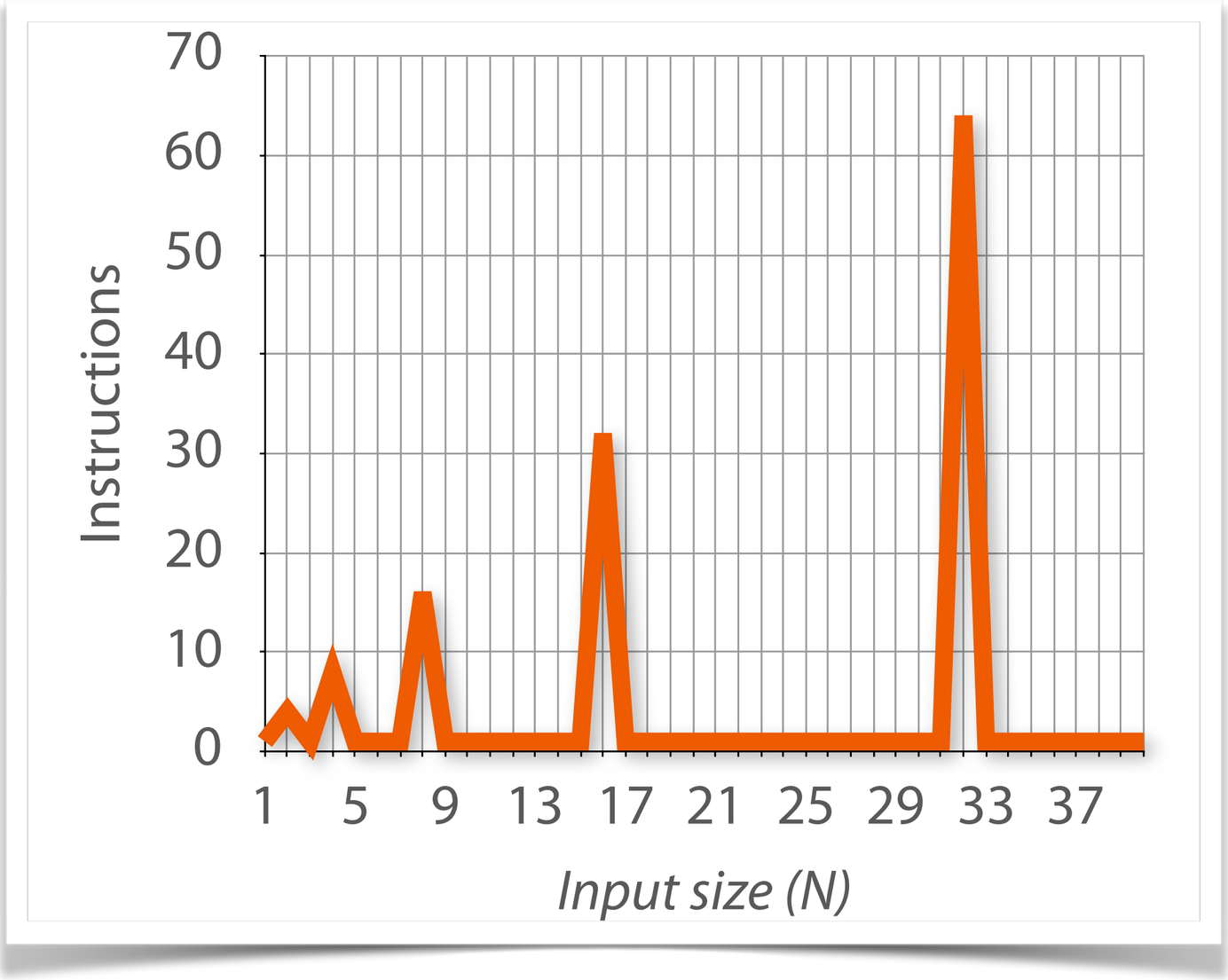


Complexity of multiple calls

Typically: $O(1)$ per call

Housekeeping: $O(N)$

Amortized Complexity



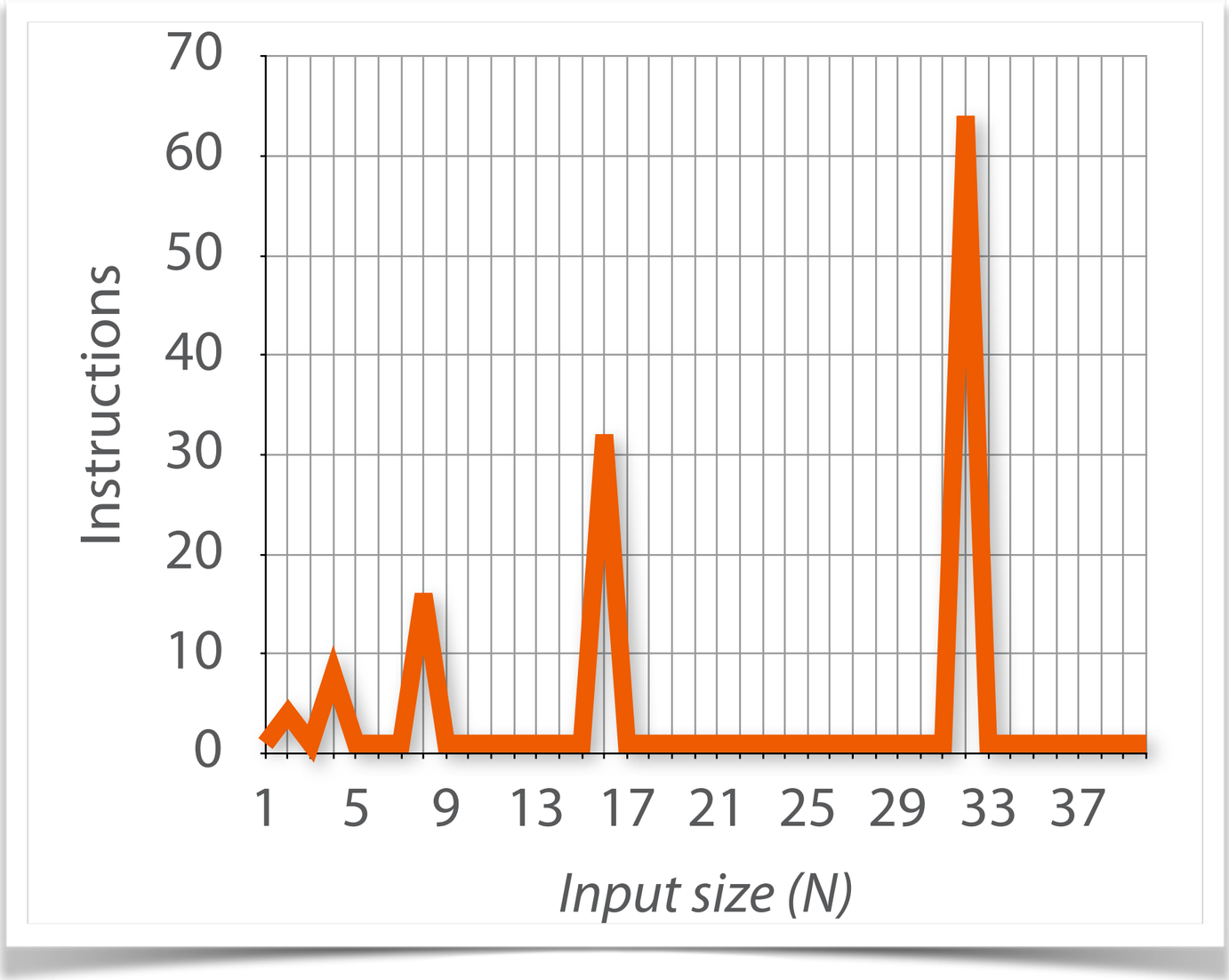
Complexity of multiple calls

Typically: $O(1)$ per call

Housekeeping: $O(N)$

M calls:

Amortized Complexity



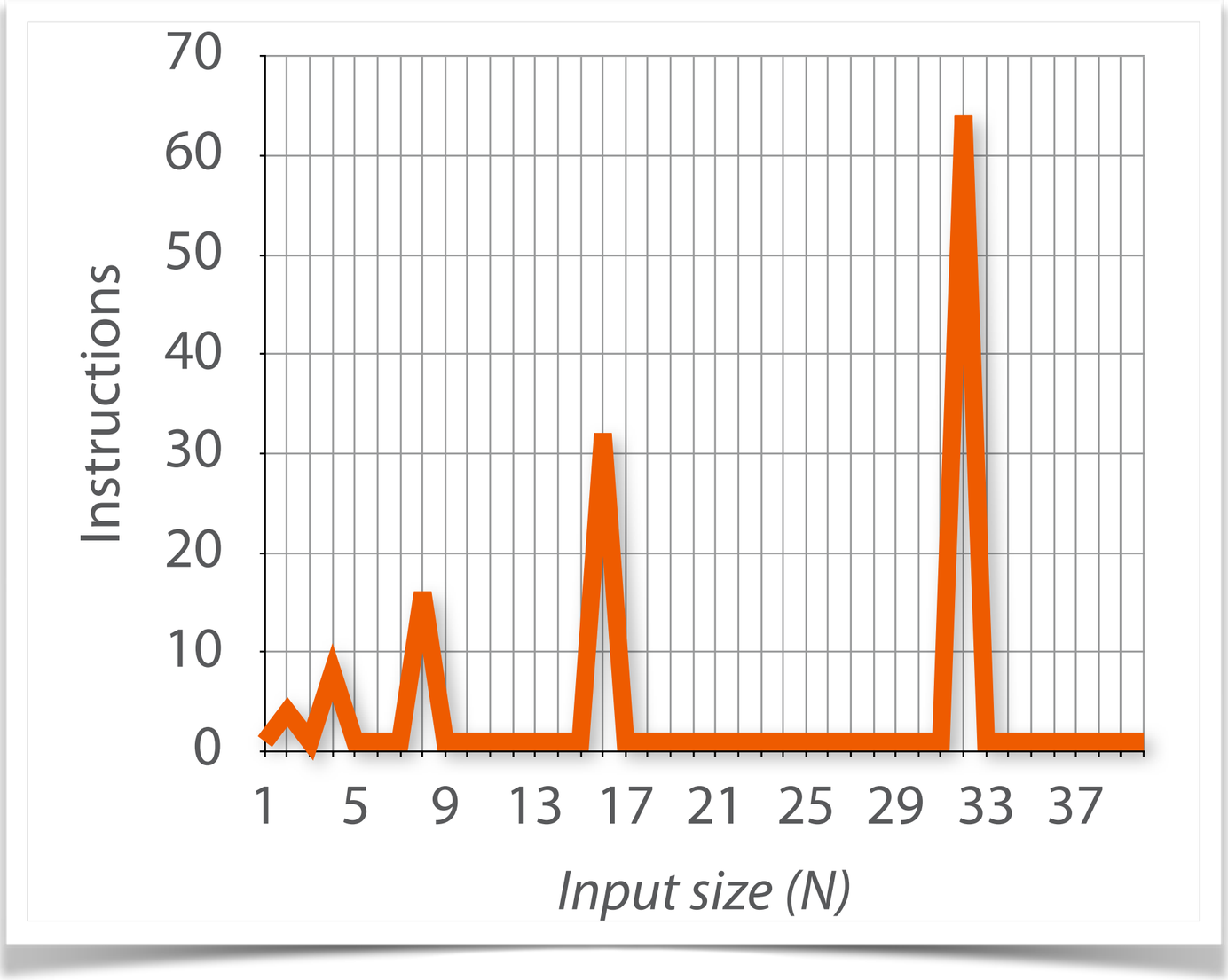
Complexity of multiple calls

Typically: $O(1)$ per call

Housekeeping: $O(N)$

M calls: $O(M)$ or $O(M \cdot N)$?

Amortized Complexity



Complexity of multiple calls

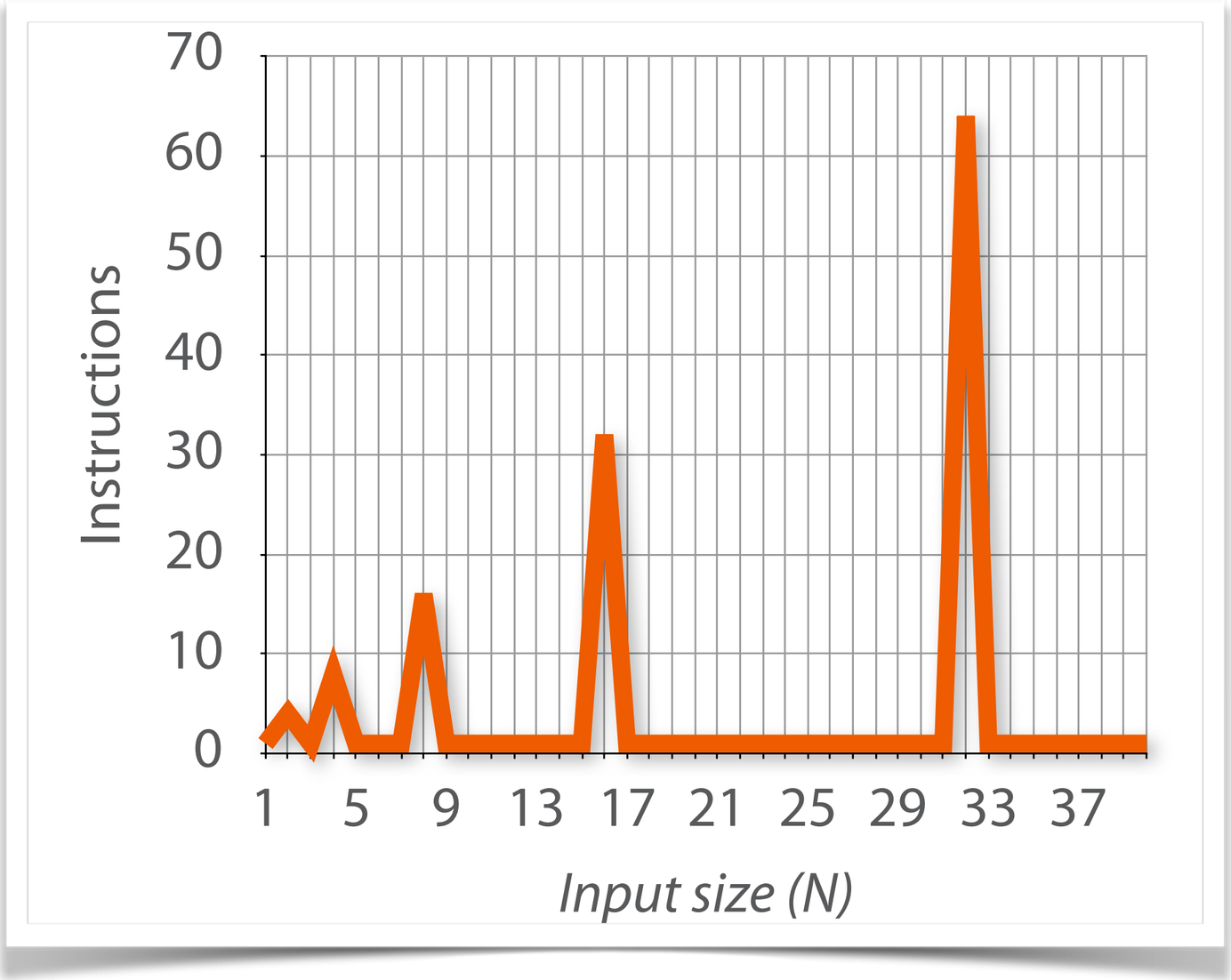
Typically: $O(1)$ per call

Housekeeping: $O(N)$

M calls: $O(M)$ or $O(M \cdot N)$?

Housekeeping frequency?

Amortized Complexity



Complexity of multiple calls

Typically: $O(1)$ per call

Housekeeping: $O(N)$

M calls: $O(M)$ or $O(M \cdot N)$?

Housekeeping frequency?

Housekeeping complexity?

Lessons Learned

Lessons Learned

Count instructions

Lessons Learned

Count instructions

Look at the *nature* of the curve

Lessons Learned

Count instructions

Look at the *nature* of the curve

Hard part:
Ignore constants

Lessons Learned

Count instructions

Look at the *nature* of the curve

Hard part: Ignore constants

$O(\ldots)$

$\Theta(\ldots)$

$\Omega(\ldots)$

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Look at the *nature* of the curve

Hard part:
Ignore constants

Recursive methods:
Fold out

Lessons Learned

Count instructions

Look at the *nature* of the curve

Hard part:
Ignore constants

Recursive methods:
Fold out

Amortized analysis:
Even out the housekeeping