Looking Ahead to Some Very Hard Problems

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Hard Problems

Hard Problems

Input size in #bits

P vs NP

Heuristics
and
approximation

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Input Size in Bits

Input Size in Bits

Input Size in Bits

Input Size in Bits

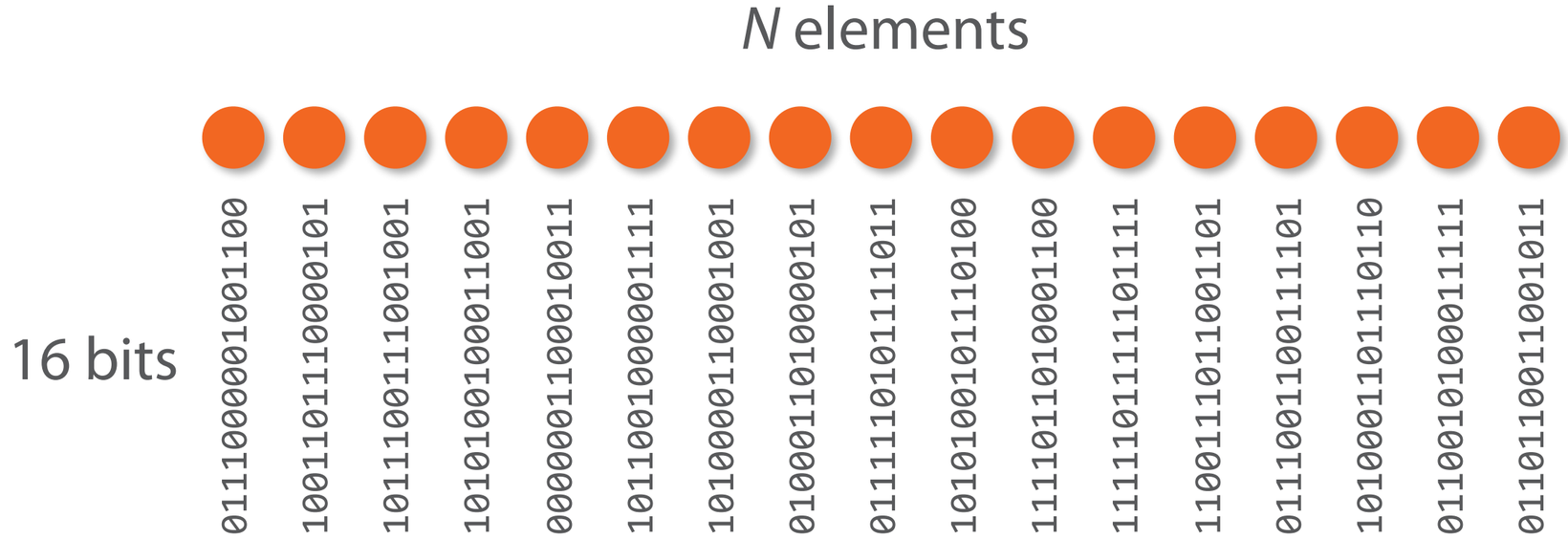
N elements

Input Size in Bits

$\Theta(N)$

Input Size in Bits

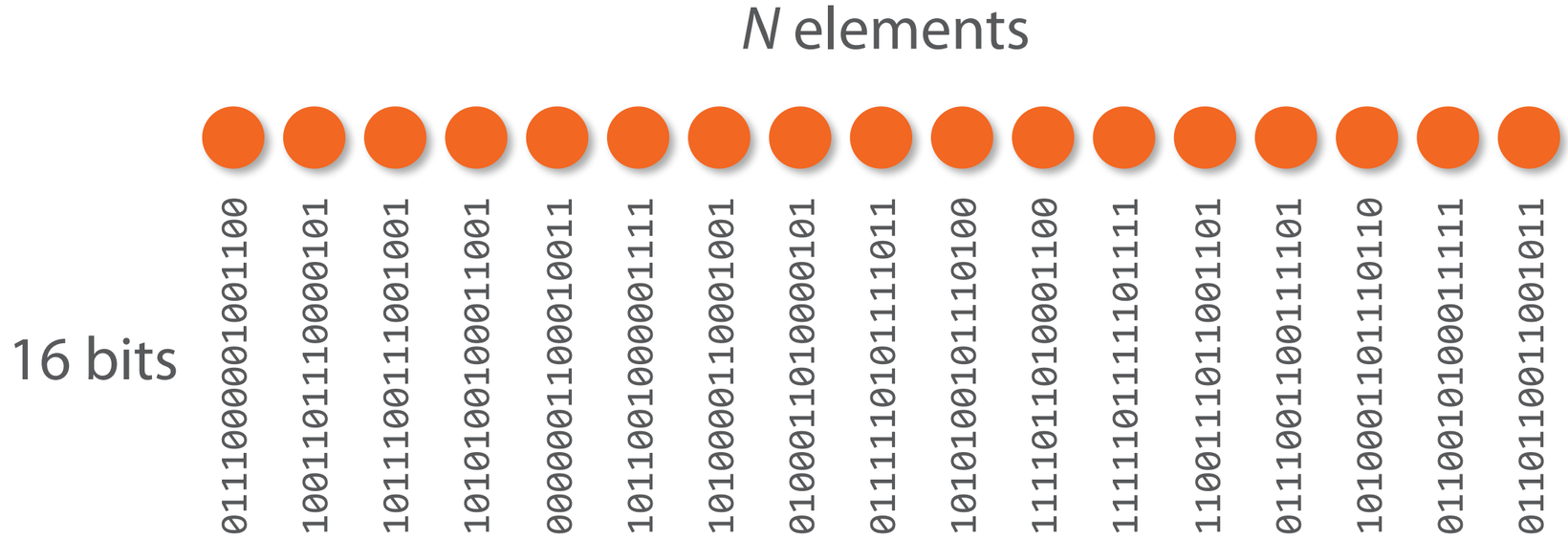
$\Theta(N)$



Input Size in Bits

$\Theta(N)$

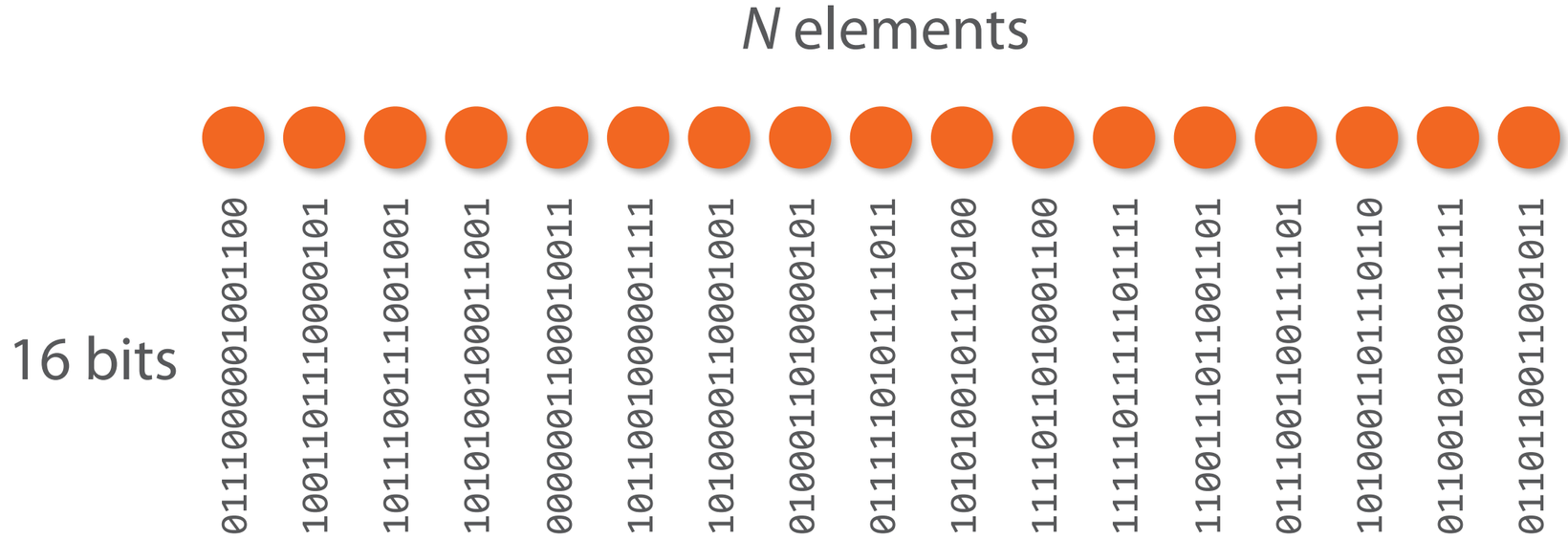
$\Theta(16 \cdot N)$



Input Size in Bits

$\Theta(N)$

$\Theta(16 \cdot N) = \Theta(N)$



Input Size in Bits

N elements

Input Size in Bits

Sorting…

N elements

Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

N elements

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Sorting…

$O(N \cdot \log_2 N)$

Input size

N elements

Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

Input size

#Halvings

N elements

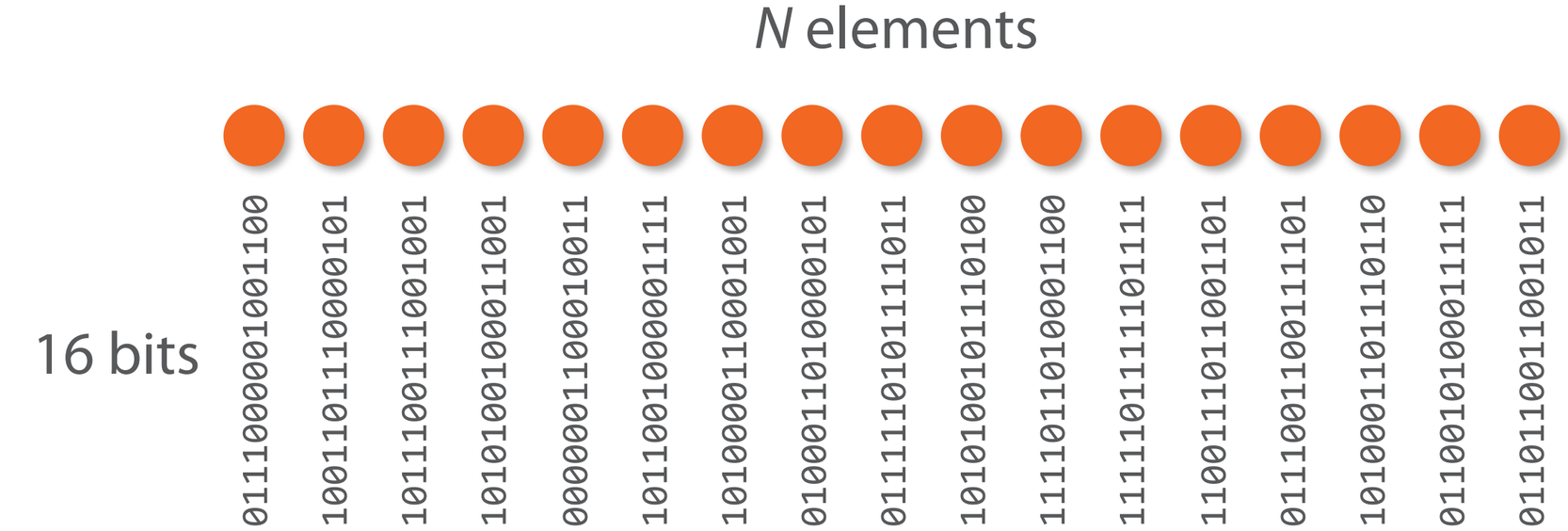
Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

Input size

#Halvings



Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

Input size

#Halvings

$O(16N$

16 bits

N elements

Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

Input size

#Halvings

16 bits

N elements

Input Size in Bits

Sorting…

$O(N \cdot \log_2 N)$

Input size

#Halvings

N elements

16 bits

$O(16N \cdot \log_2 N)$

$= O(N \cdot \log_2 N)$

Input Size in Bits

Knapsack capacity: 4

Weight: 2
Value: 3

Weight: 2
Value: 1

Weight:1
Value: 3

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4


Weight: 2
Value: 3


Weight: 2
Value: 1


Weight:1
Value: 3

For *N* items and capacity of *C*:

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4


Weight: 2
Value: 3


Weight: 2
Value: 1


Weight:1
Value: 3

For N items and capacity of C:

Dynamic programming: $\Theta(N \cdot C)$

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4


Weight: 2
Value: 3


Weight: 2
Value: 1


Weight:1
Value: 3

For N items and capacity of C:

Dynamic programming: $\Theta(N \cdot C)$

Items: $32 \cdot N$ bits

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4



Weight: 2
Value: 3

Weight: 2
Value: 1

Weight:1
Value: 3

For N items and capacity of C:

Dynamic programming: $\Theta(N \cdot C)$

Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
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	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4


Weight: 2
Value: 3


Weight: 2
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Weight:1
Value: 3

For N items and capacity of C:

Dynamic programming: $\Theta(N \cdot C)$

Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

Capacity: c bits gives a max capacity of 2^c

		Capacity				
		0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
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	i_3	0	3✓	3✓	6✓	6✓

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$\Theta(32 \cdot N$

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Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

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$\Theta(32 \cdot N \cdot 2^c)$

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		0	1	2	3	4
No items	i_0	0	0	0	0	0
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Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

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$\Theta(32 \cdot N \cdot 2^c) = \Theta(N \cdot 2^c)$

		Capacity				
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Dynamic programming: $\Theta(N \cdot C)$

Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

Capacity: c bits gives a max capacity of 2^c

$\Theta(32 \cdot N \cdot 2^c) = \Theta(N \cdot 2^c)$ Ouch!

		Capacity 0	1	2	3	4
No items	i_0	0	0	0	0	0
	i_1	0	0✗	3✓	3✓	3✓
	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Input Size in Bits

Knapsack capacity: 4


Weight: 2
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For N items and capacity of C:

Dynamic programming: $\Theta(N \cdot C)$

Items: $32 \cdot N$ bits or $2 \cdot 32 \cdot N$ bits

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$\Theta(32 \cdot N \cdot 2^c) = \Theta(N \cdot 2^c)$ Ouch!

		Capacity 0	1	2	3	4
No items	i_0	0	0	0	0	0
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	i_2	0	0✗	3✗	3✗	4✓
	i_3	0	3✓	3✓	6✓	6✓

Pseudo polynomial

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Input size in #b

P vs NP

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P vs NP

P vs NP

Polynomial

$5N^5 + 3N^4 + N$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

$\log_2 N$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

$\log_2 N$ is $O(N)$

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

$\log_2 N$ is $O(N)$

2^N

Not a polynomial

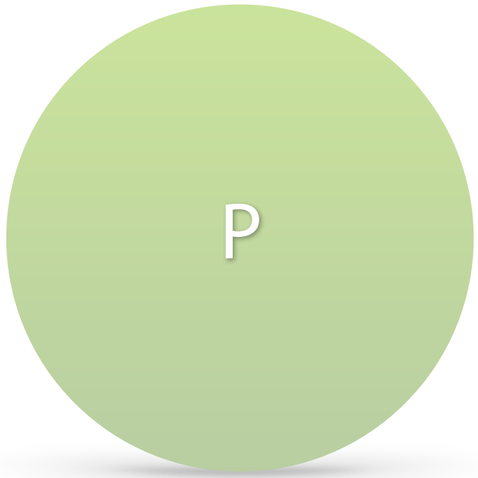
P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

$\log_2 N$ is $O(N)$



2^N

Not a polynomial

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

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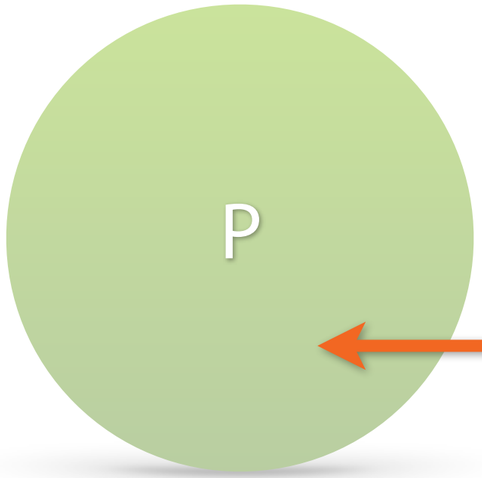

Solvable in polynomial time

2^N

Not a polynomial

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

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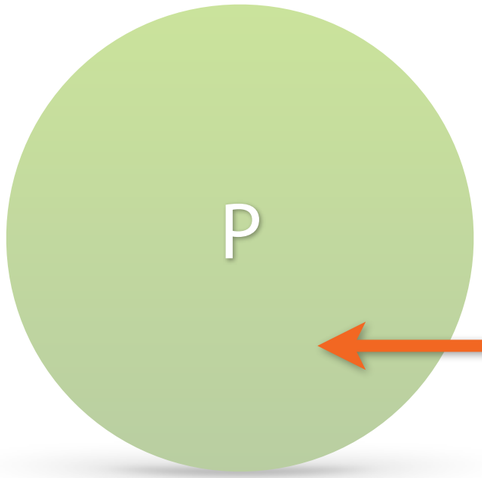

Solvable in polynomial time

Sorting

P vs NP

Polynomial

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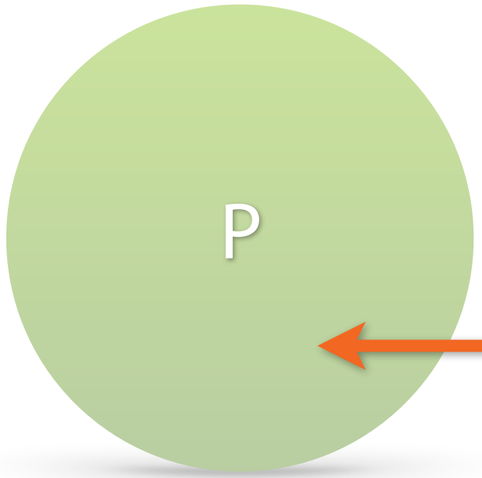

Solvable in polynomial time

Sorting

Traversing lists

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

$5N^3 + 3N^2 + N + 7$ is $O(N^3)$

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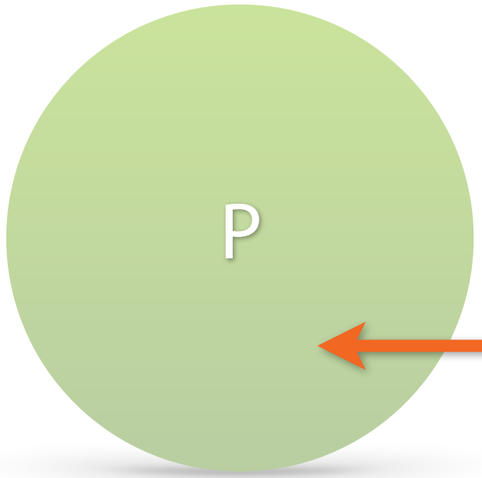

Solvable in polynomial time

Sorting

Traversing lists

Look-ups in hash tables

P vs NP

Polynomial

$5N^5 + 3N^4 + N$ is $O(N^5)$

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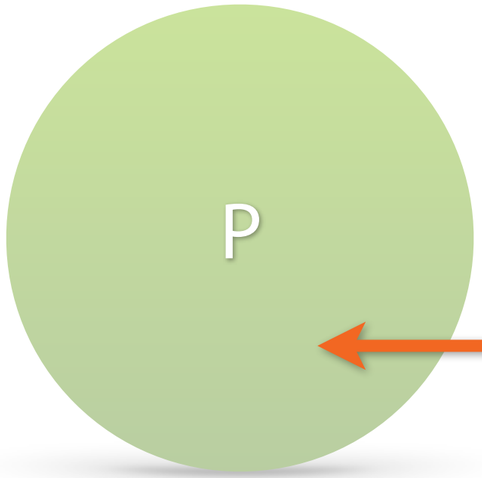


Solvable in polynomial time

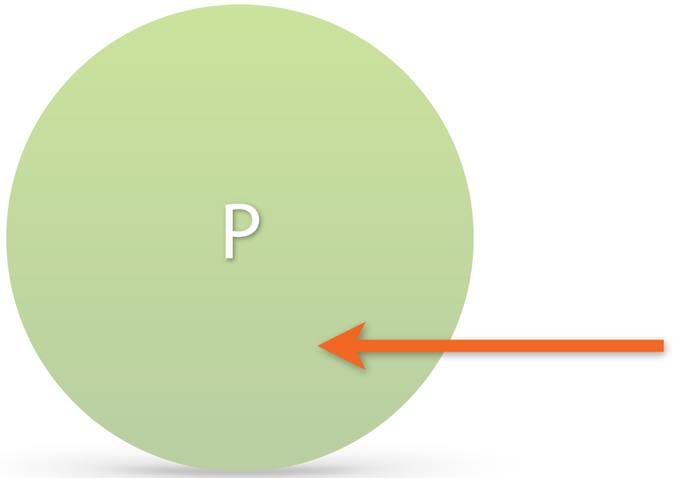
Sorting

Finding shortest route

Traversing lists

Look-ups in hash tables

P vs NP



Solvable in polynomial time

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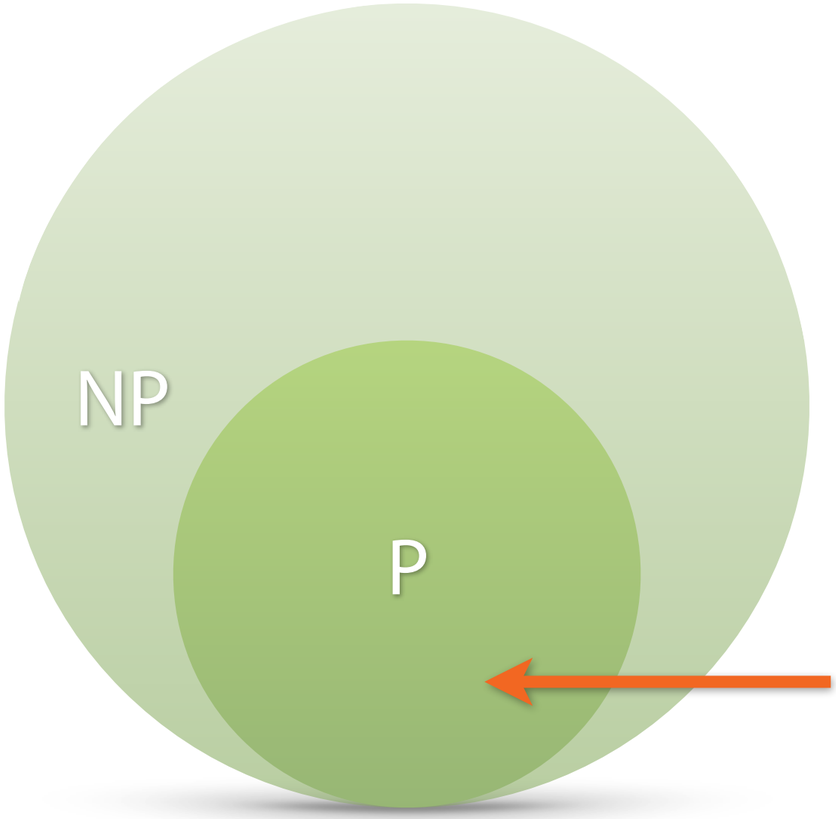
P vs NP



Solvable in polynomial time

Sorting

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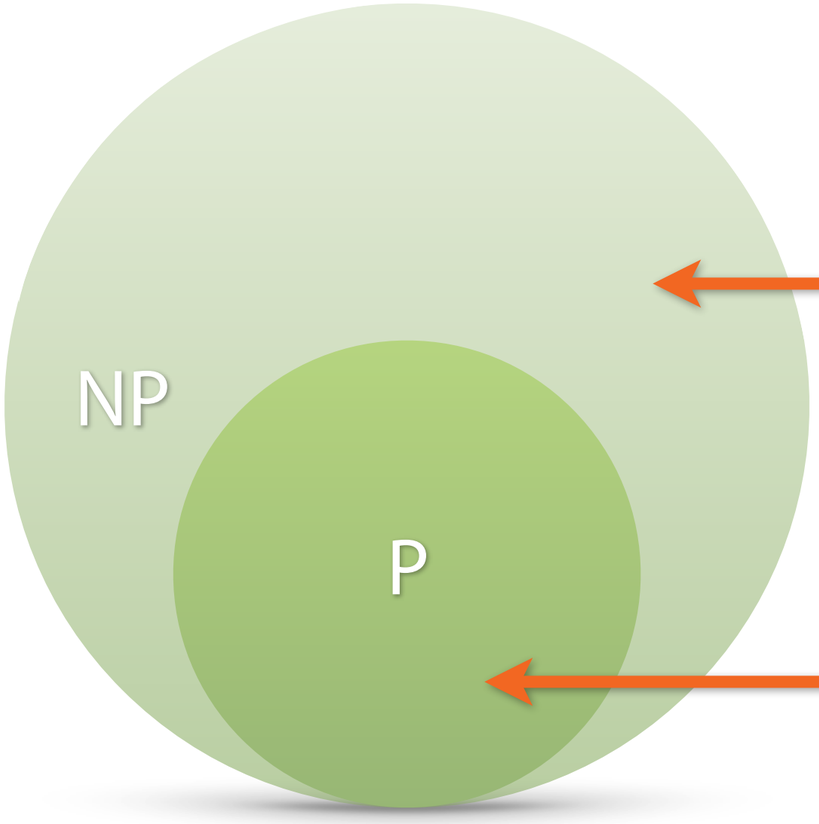
P vs NP



Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Sorting

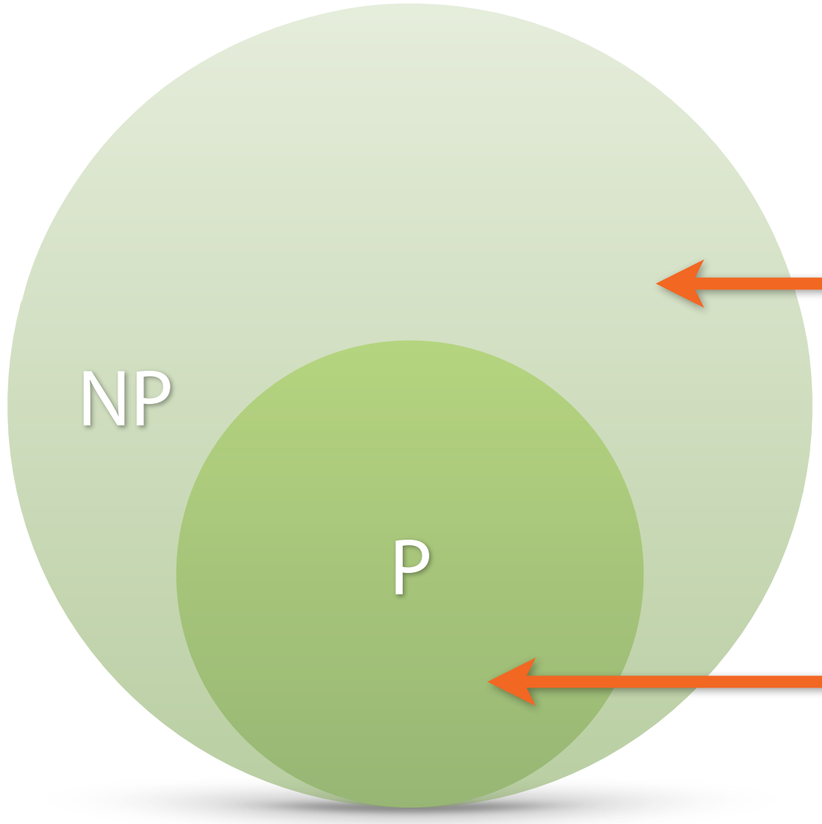
Finding shortest route

Traversing lists

Look-ups in hash tables

P vs NP

? Yes
No



Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Sorting

Finding shortest route

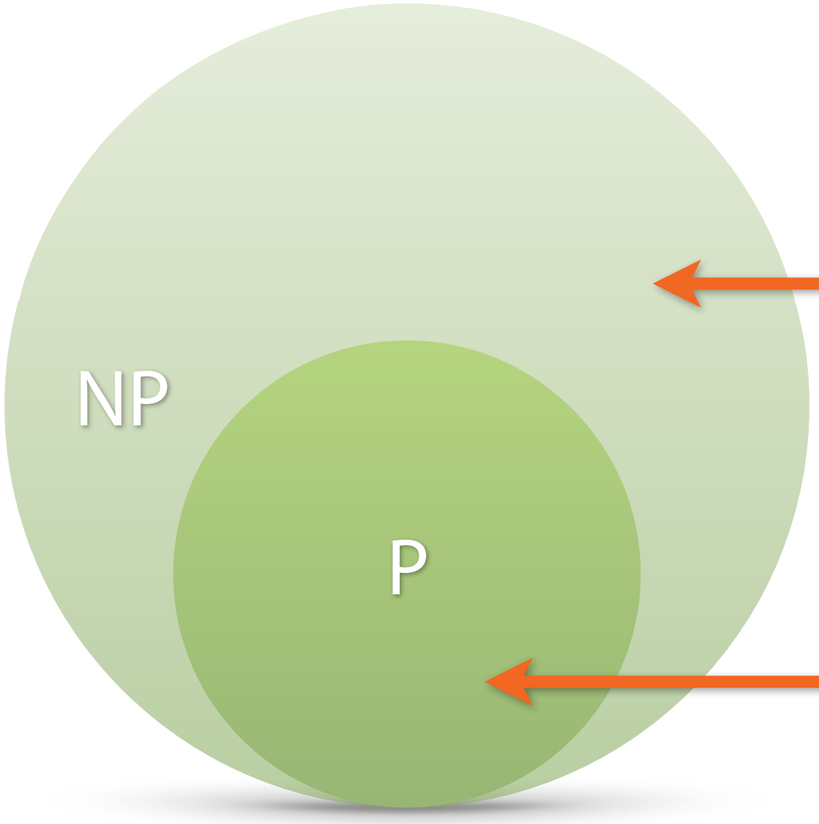
Traversing lists

Look-ups in hash tables

P vs NP

Does a route shorter than *L* exist?

? Yes
No



Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

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P vs NP

Does a route shorter than *L* exist?

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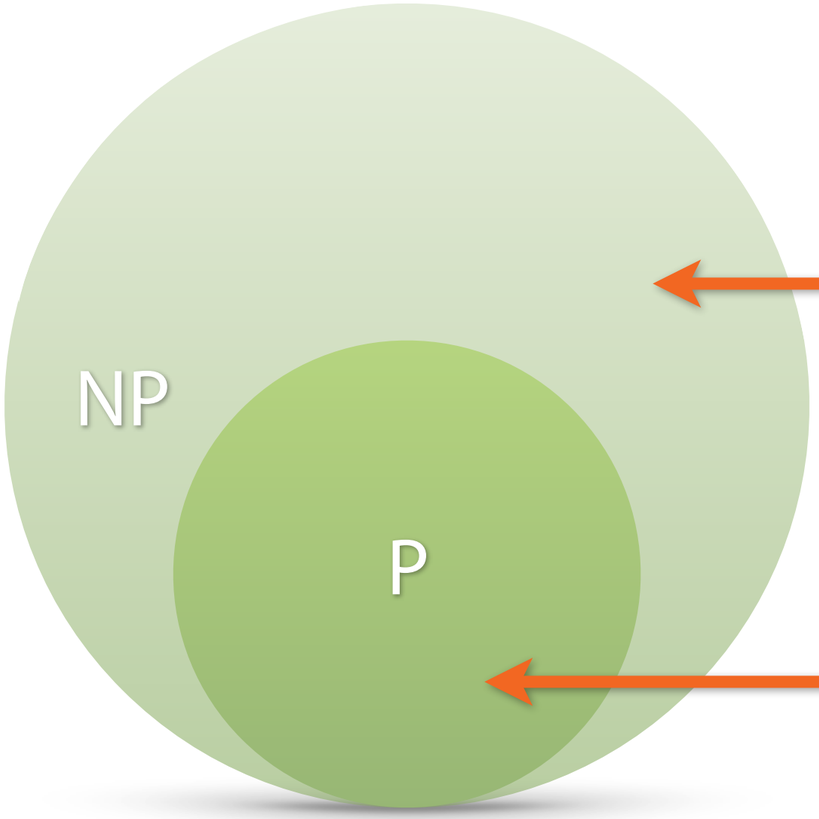
No



Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Sorting

Finding shortest route

Traversing lists

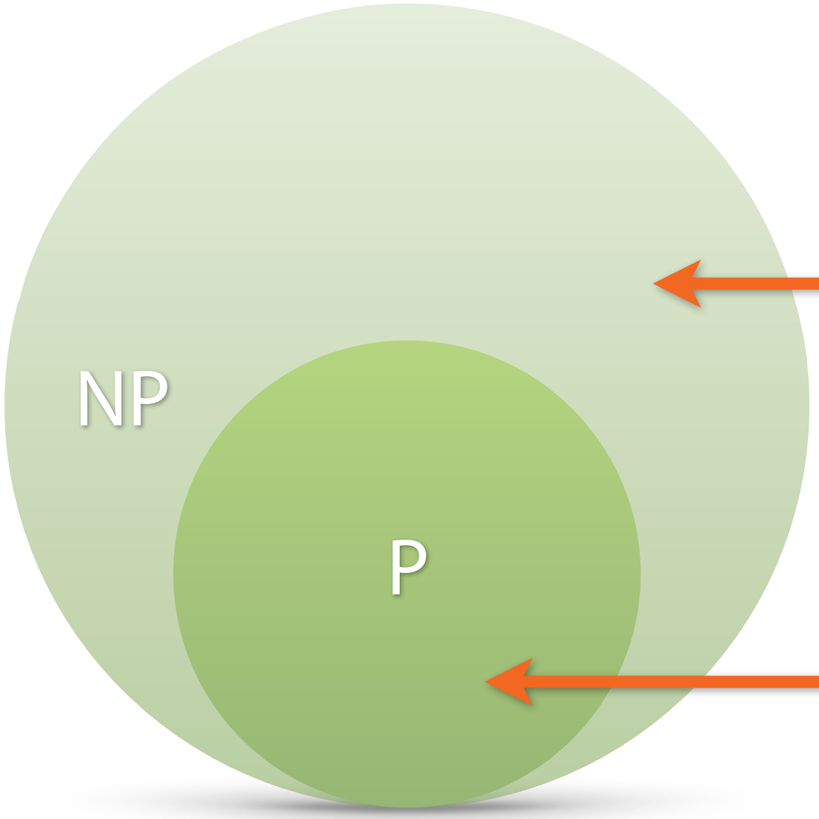
Look-ups in hash tables

P vs NP

Does a route shorter than *L* exist?

? Yes / No

Certificate: a concrete route



Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

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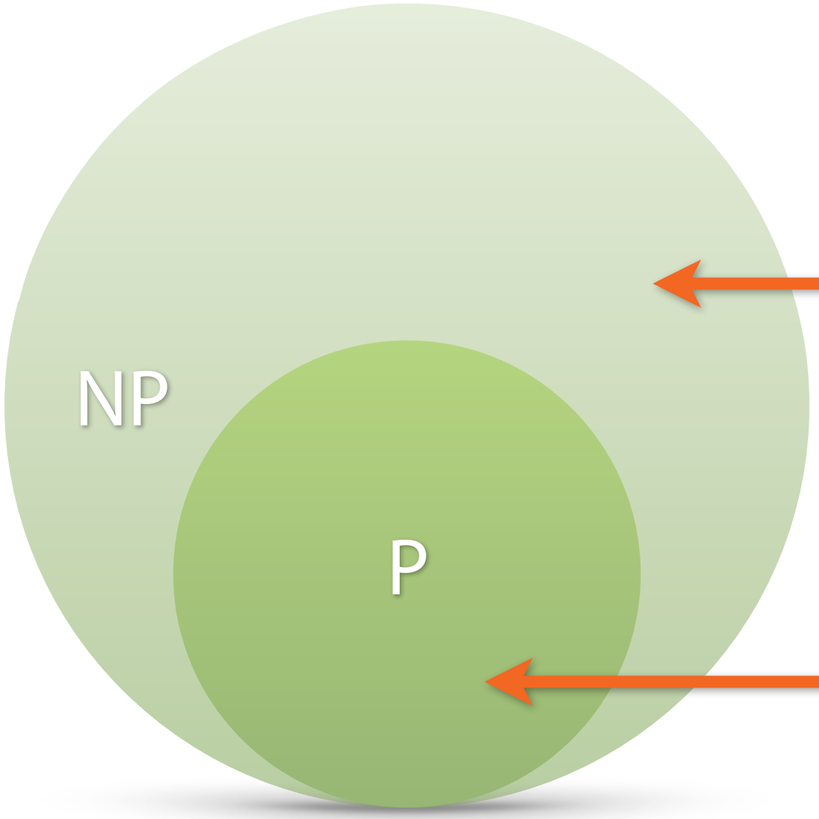
Look-ups in hash tables

P vs NP

Does a route shorter than *L* exist?

? Yes → Indeed
No ↗

Certificate: a concrete route



Decision problems ***verifiable*** in polynomial time

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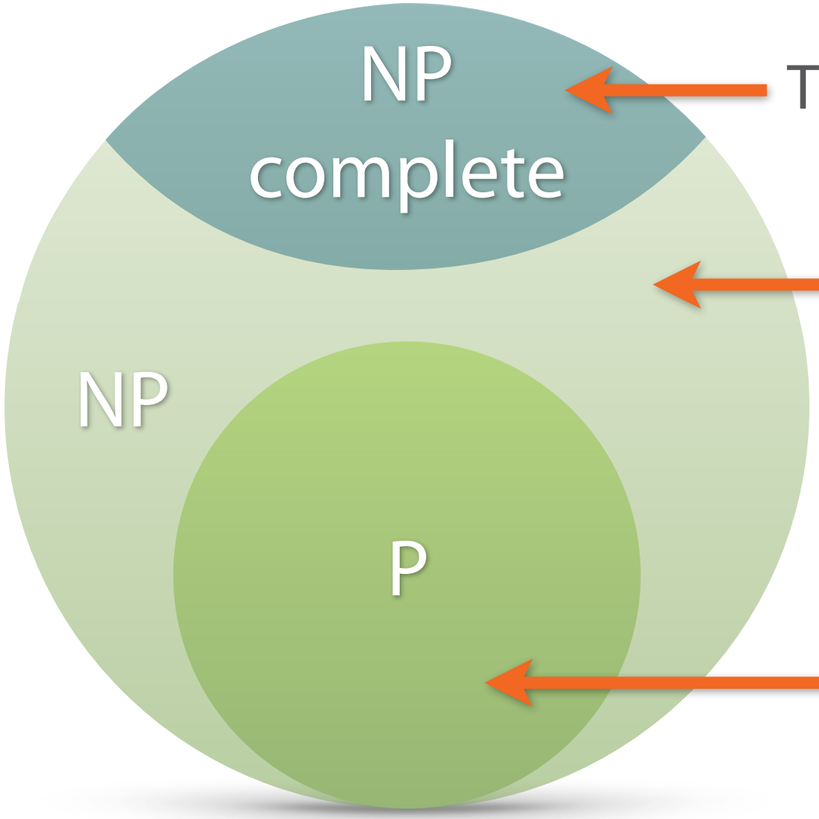

The hardest in NP

Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

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P vs NP

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No ↗

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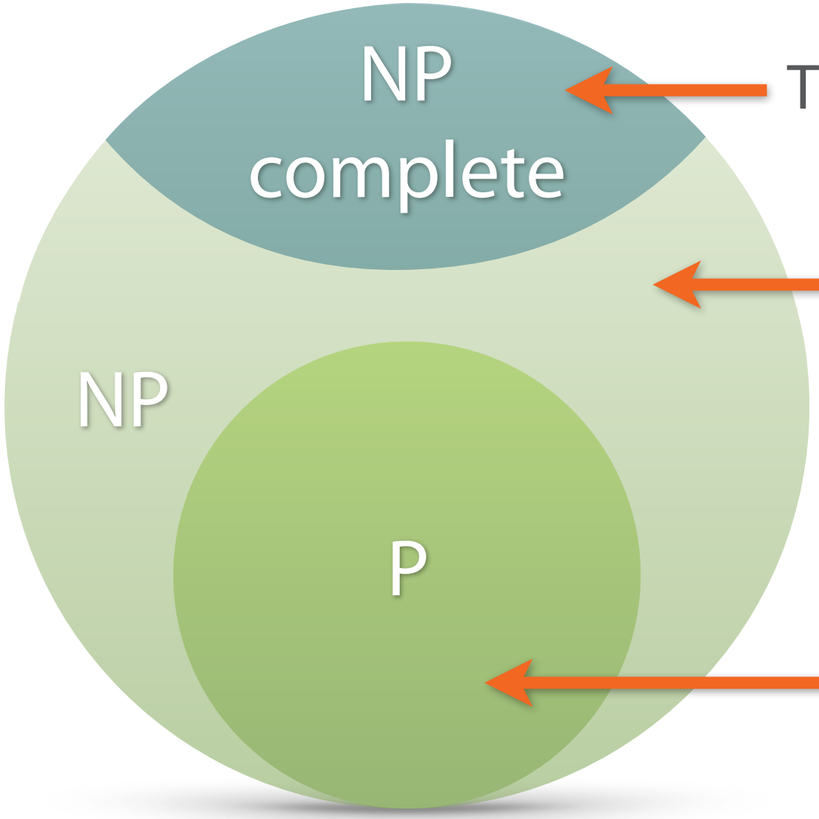


The hardest in NP

Subset sum

Decision problems ***verifiable*** in polynomial time

Sorting

Finding shortest route

Solvable in polynomial time

Traversing lists

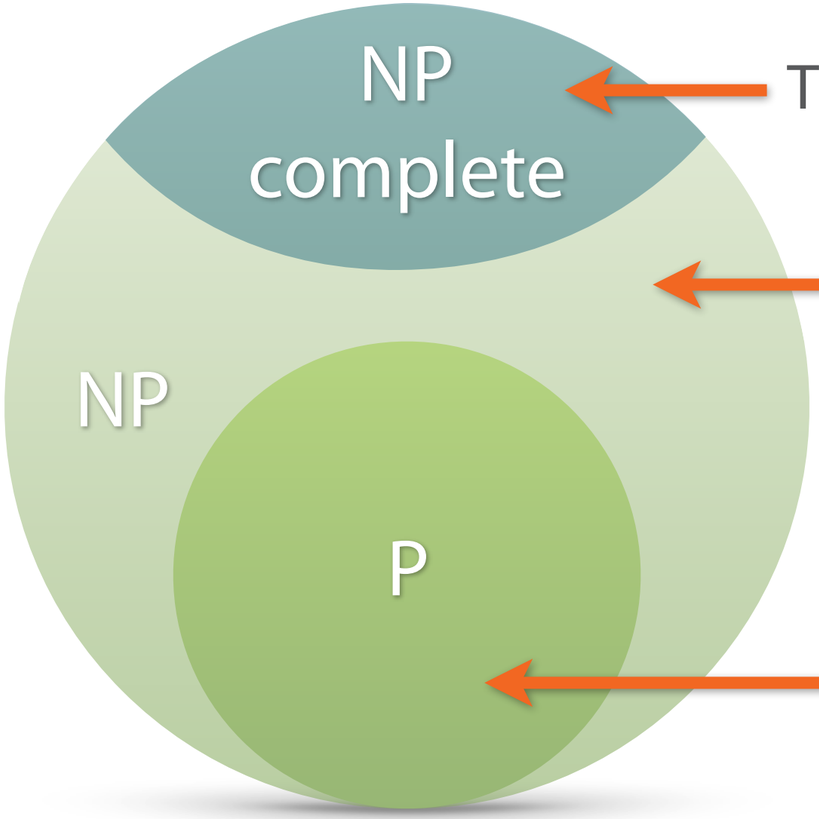
Look-ups in hash tables

P vs NP

Does a route shorter than *L* exist?

? Yes → Indeed
No ↗

Certificate: a concrete route



The hardest in NP

Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Subset sum

Knapsack (can a value ≥*V* be achieved?)

Sorting

Finding shortest route

Traversing lists

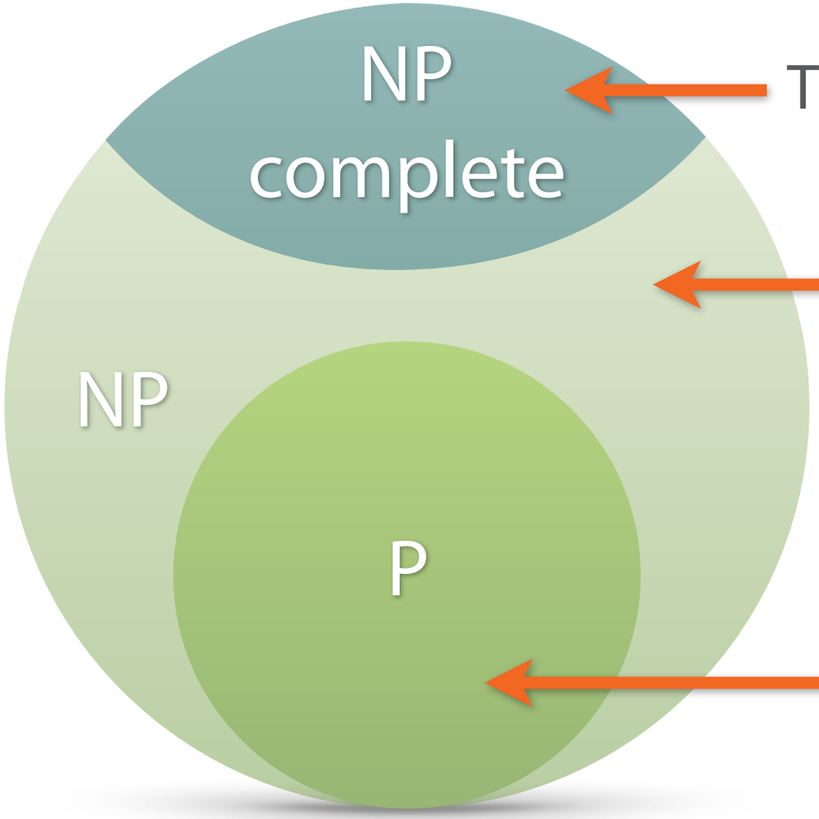
Look-ups in hash tables

P vs NP

Does a route shorter than *L* exist?

? Yes → Indeed
No ↗

Certificate: a concrete route



The hardest in NP

Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Subset sum

Orthogonal packing problem

Knapsack (can a value ≥*V* be achieved?)

Sorting

Finding shortest route

Traversing lists

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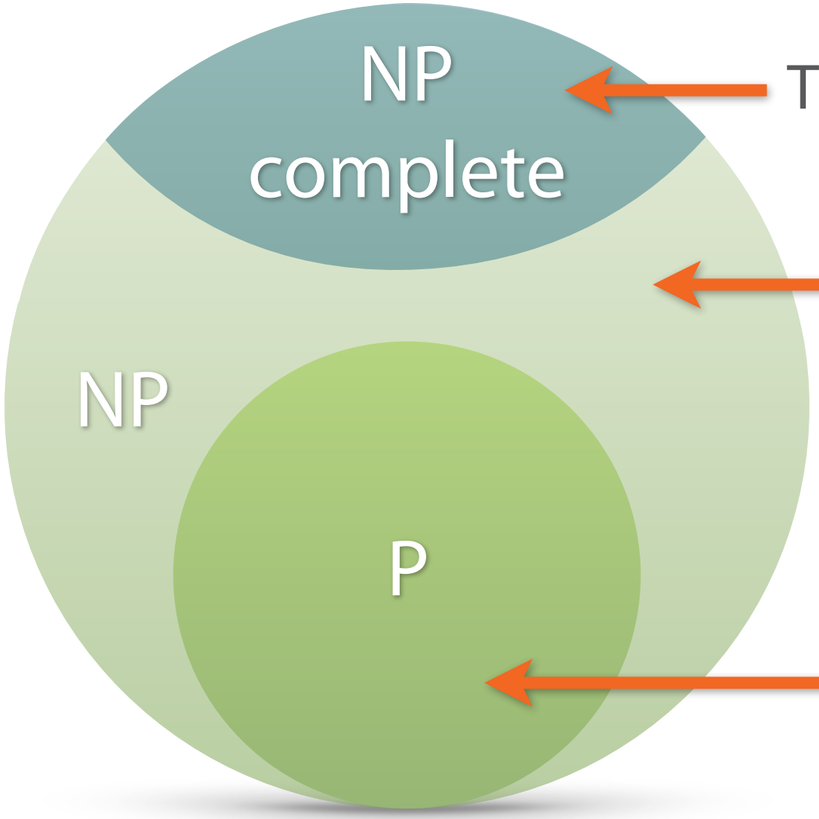
P vs NP

Does a route shorter than *L* exist?

? Yes → Indeed
No ↗

Certificate: a concrete route



The hardest in NP

Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Subset sum

Orthogonal packing problem

Knapsack (can a value ≥*V* be achieved?)

Sorting

Finding shortest route

Traversing lists

Look-ups in hash tables


Does a route
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test route
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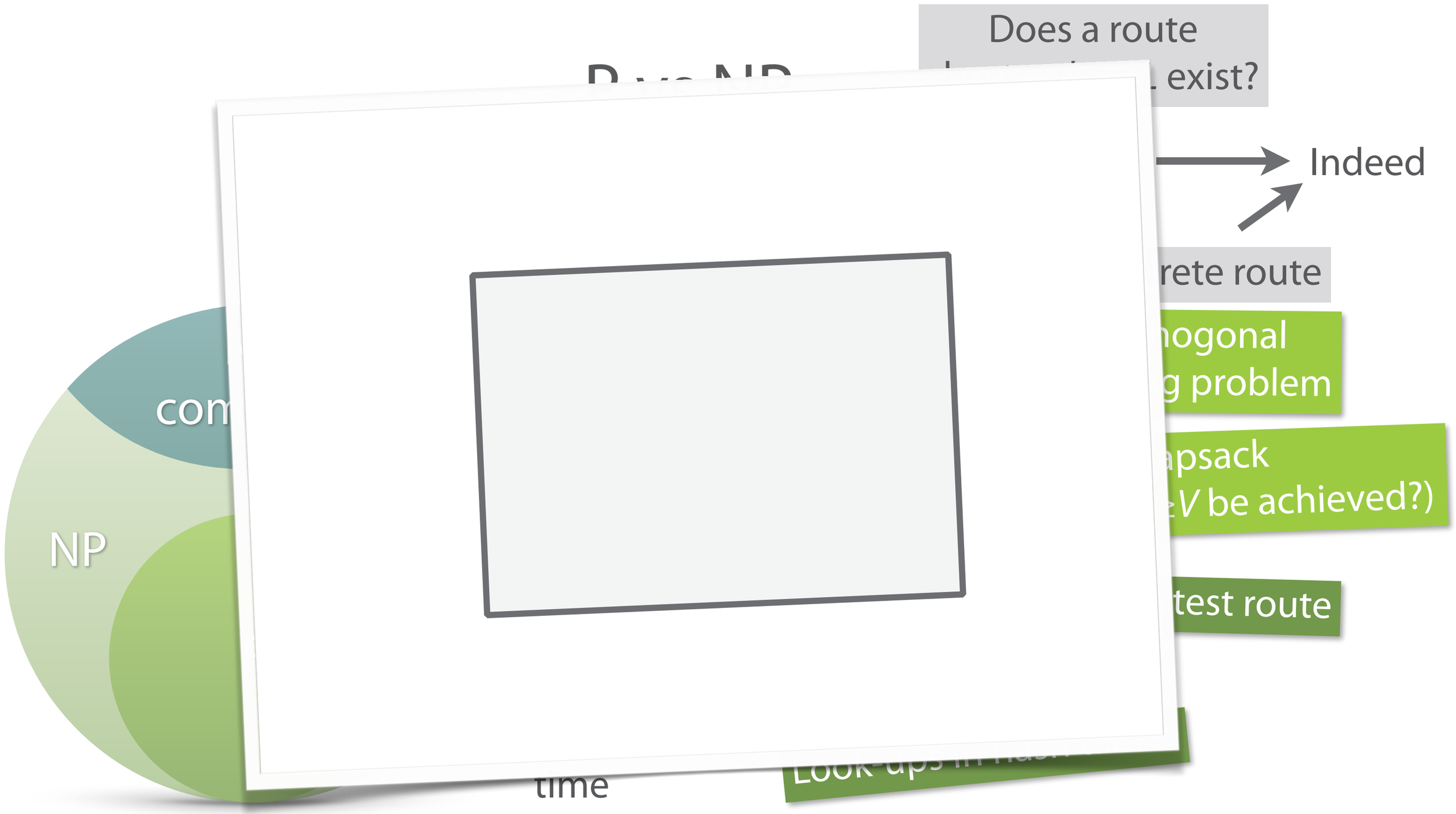
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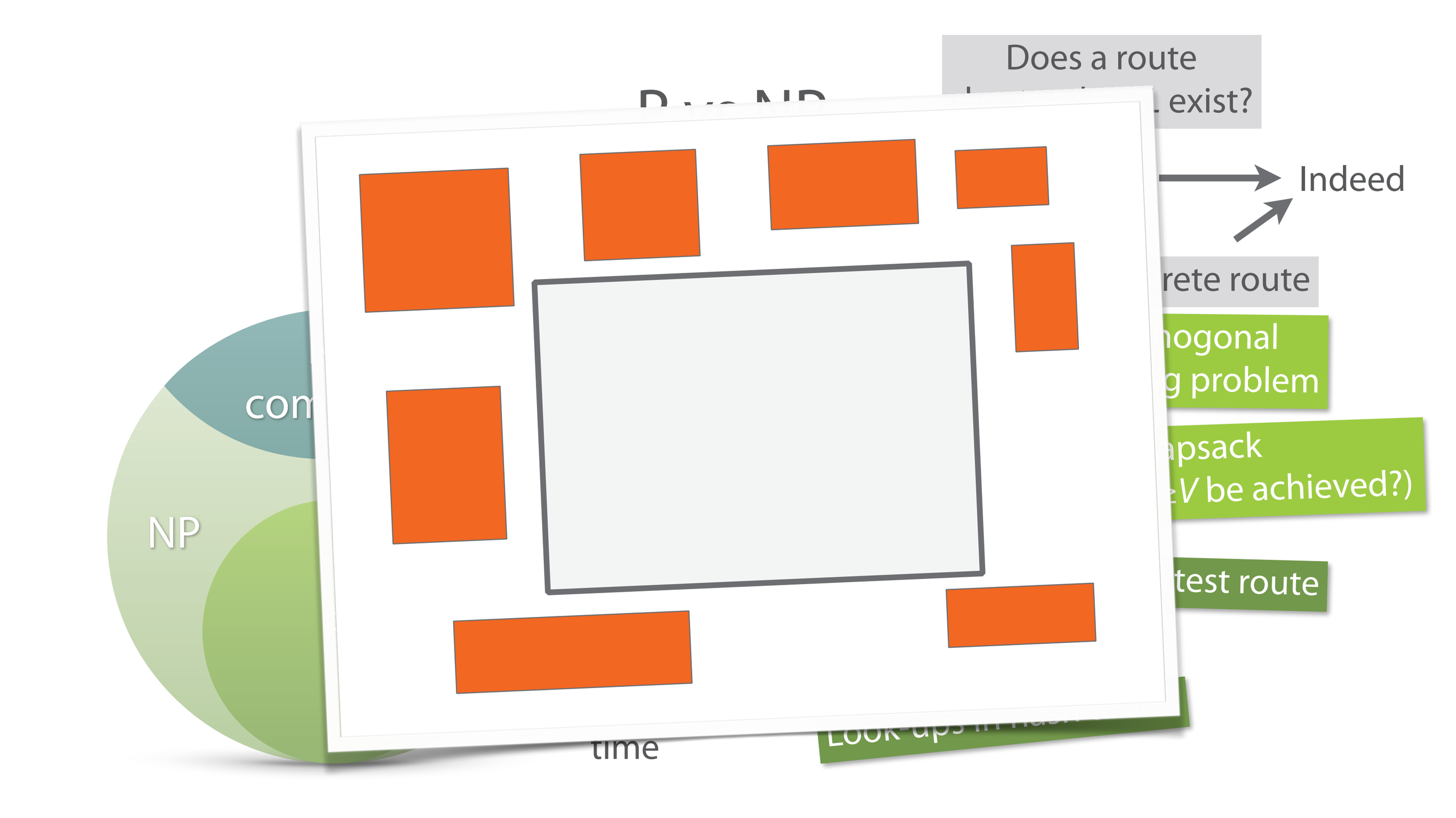
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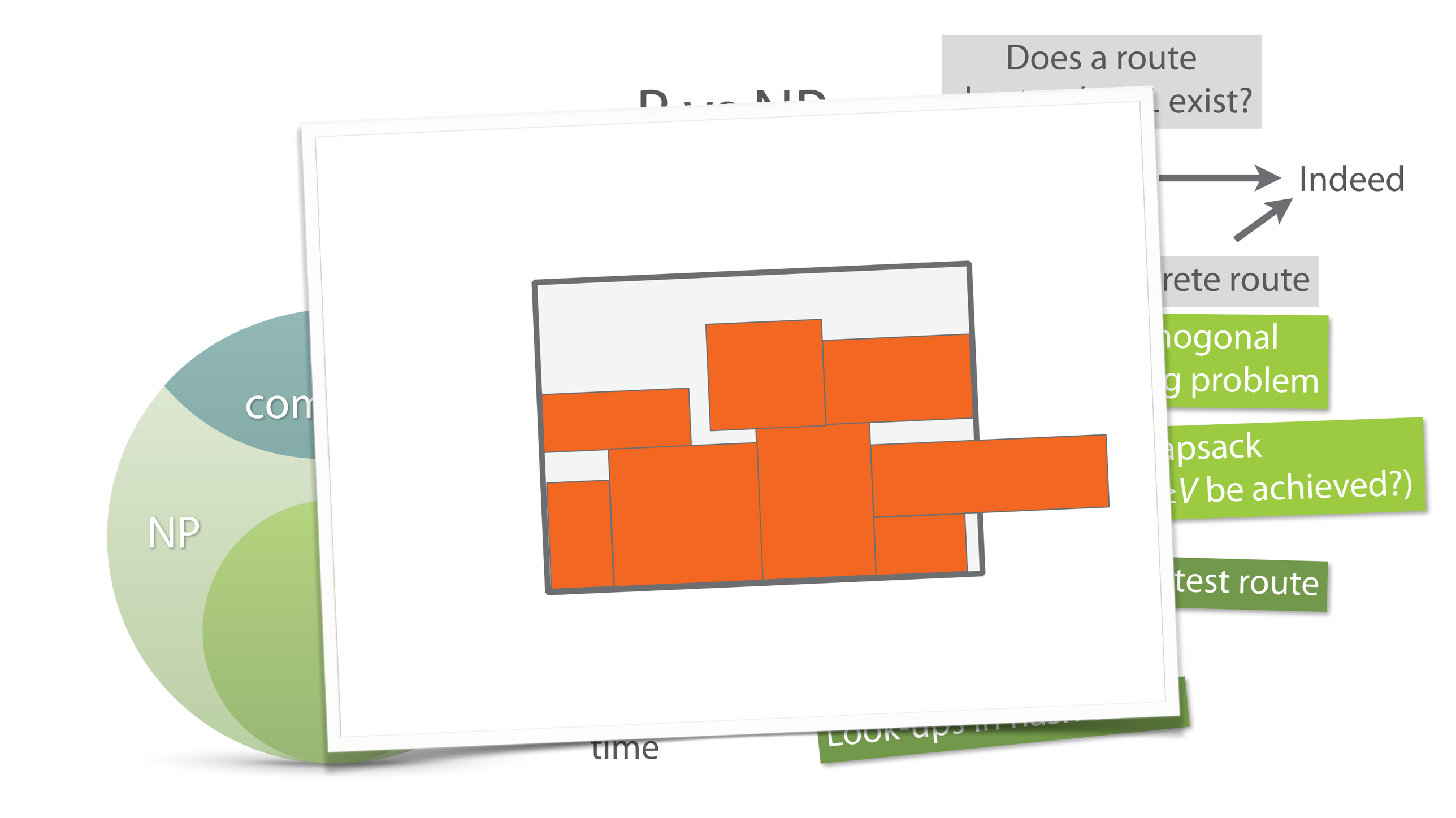
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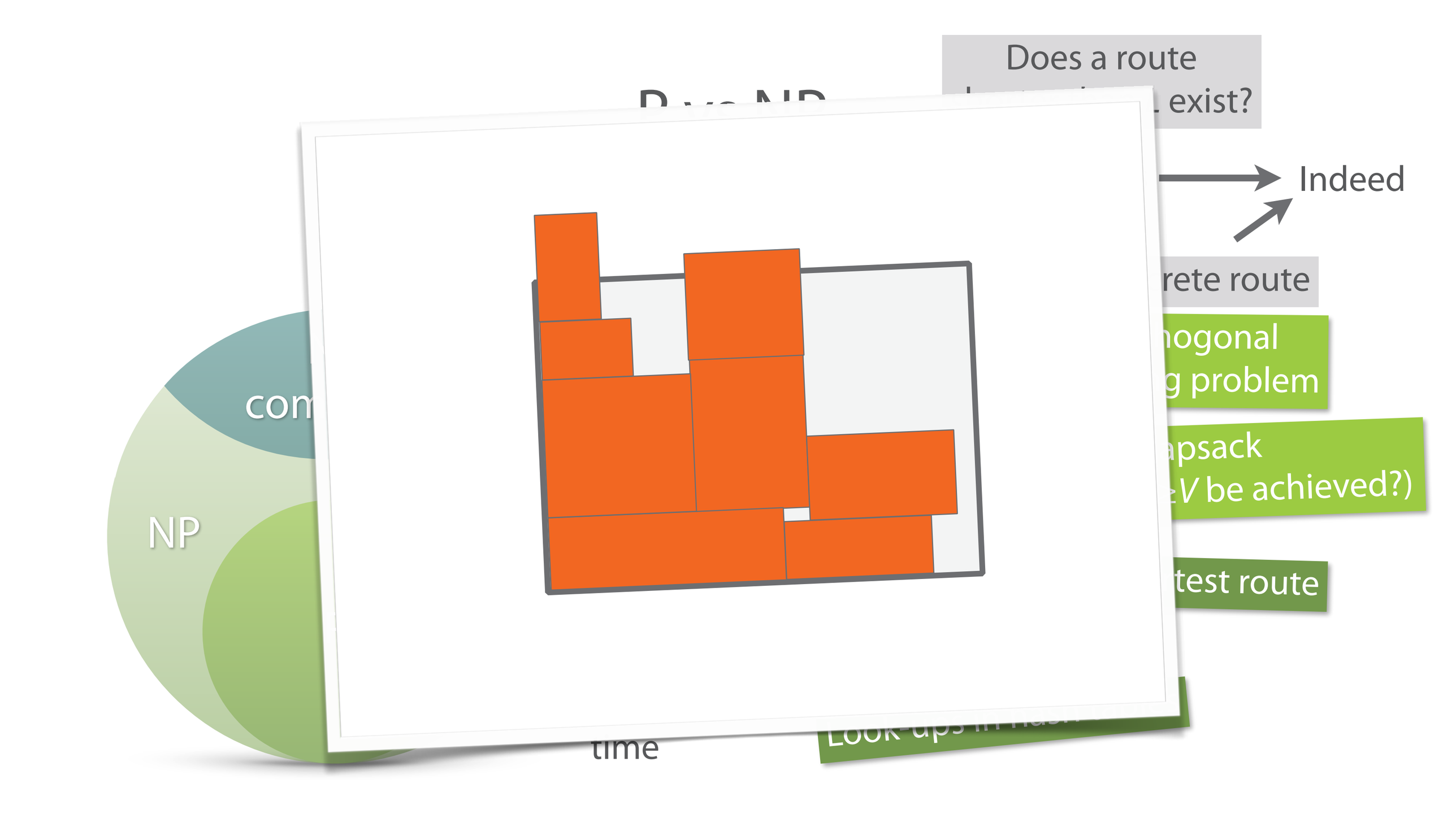
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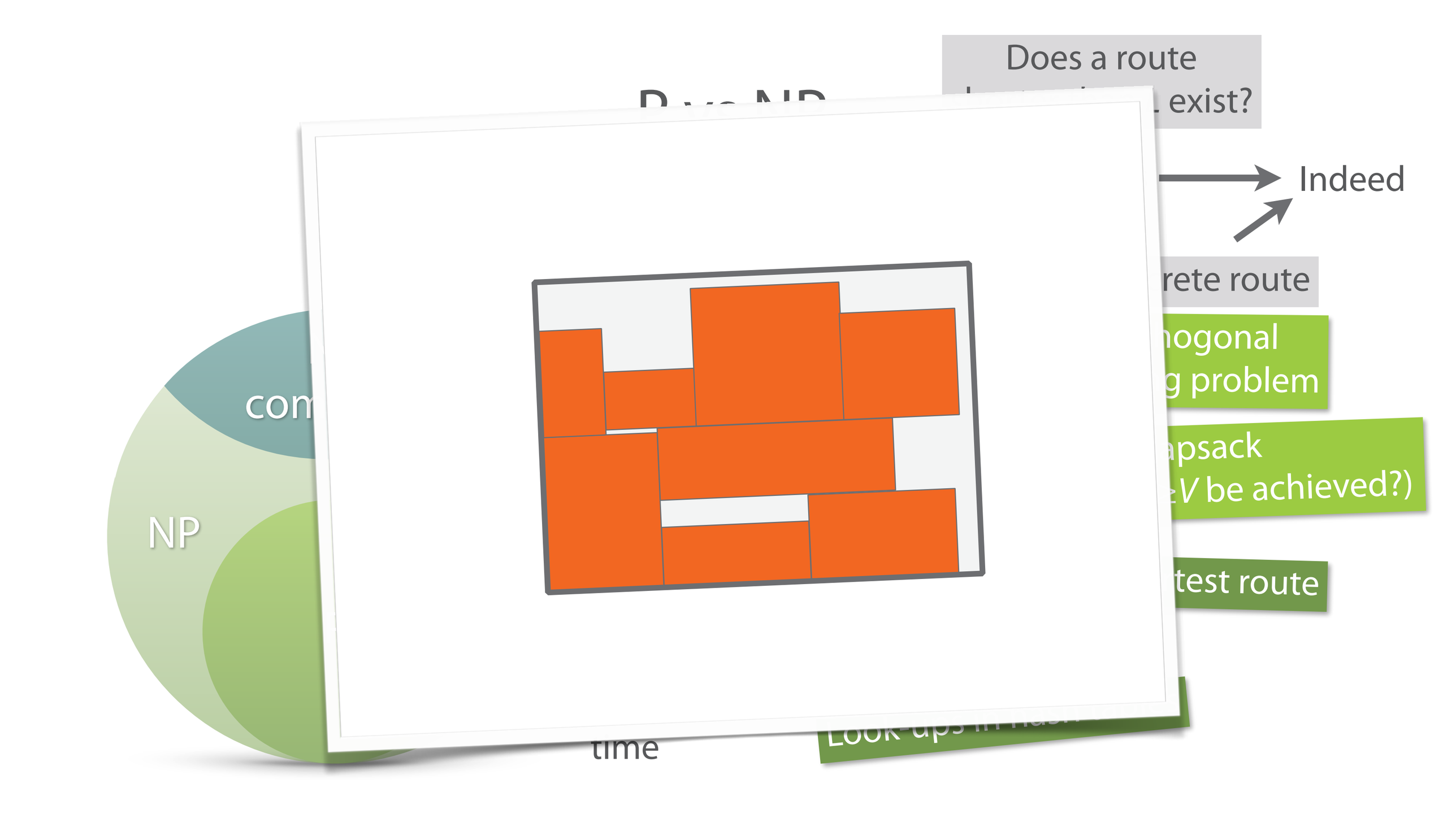
Does a route
exist?
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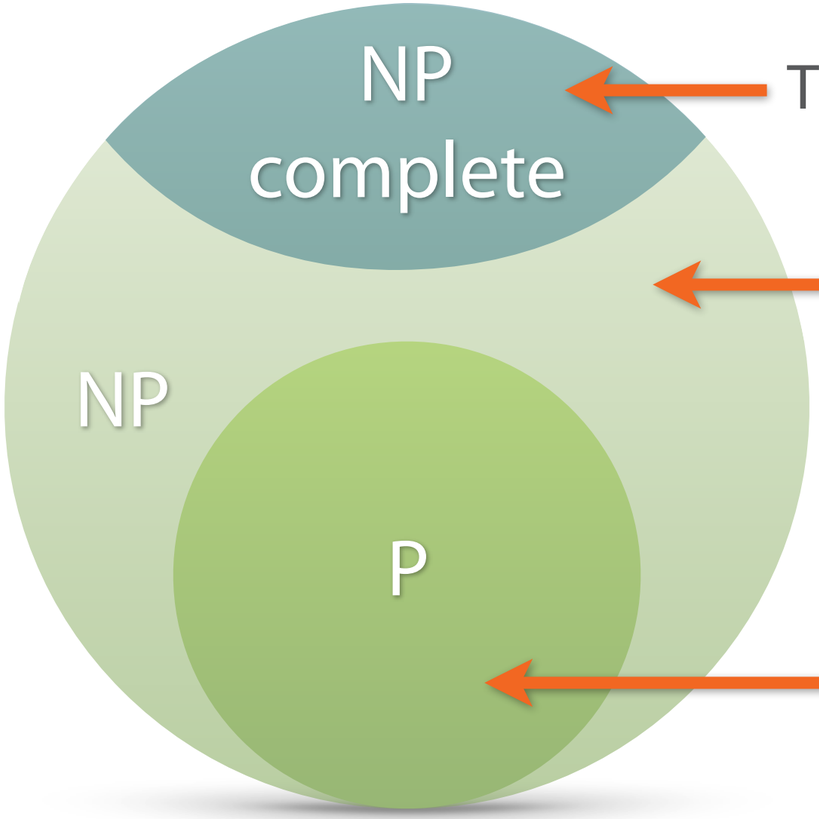
≥V be achieved?)
test route
com
NP
time

P vs NP

Does a route shorter than *L* exist?

? Yes → Indeed
No ↗

Certificate: a concrete route



The hardest in NP

Decision problems ***verifiable*** in polynomial time

Solvable in polynomial time

Subset sum

Orthogonal packing problem

Knapsack (can a value ≥*V* be achieved?)

Sorting

Finding shortest route

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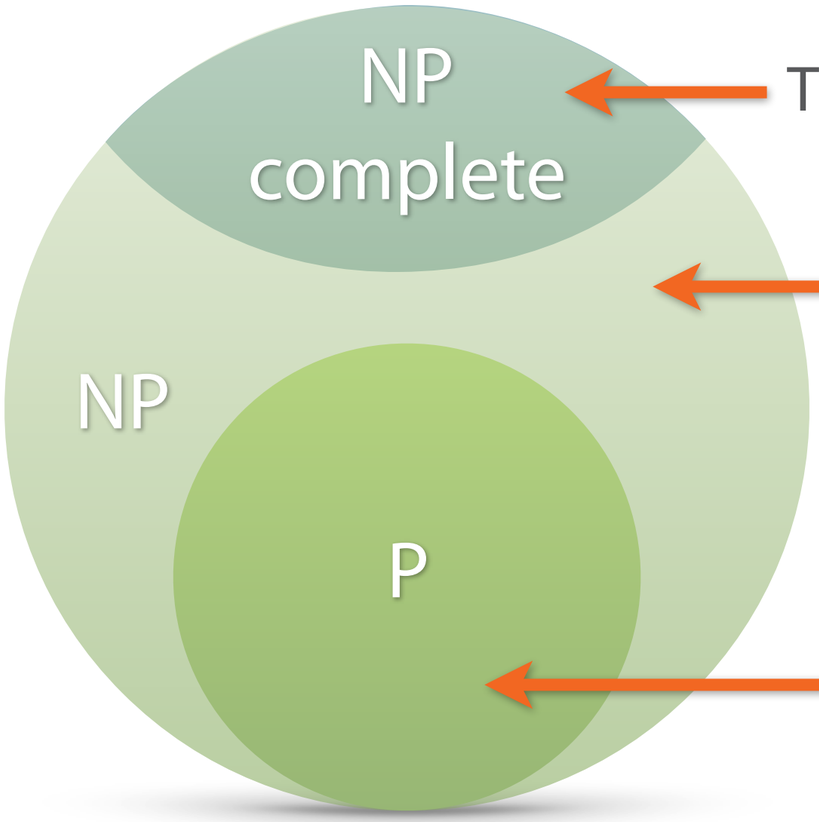
Look-ups in hash tables

P vs NP



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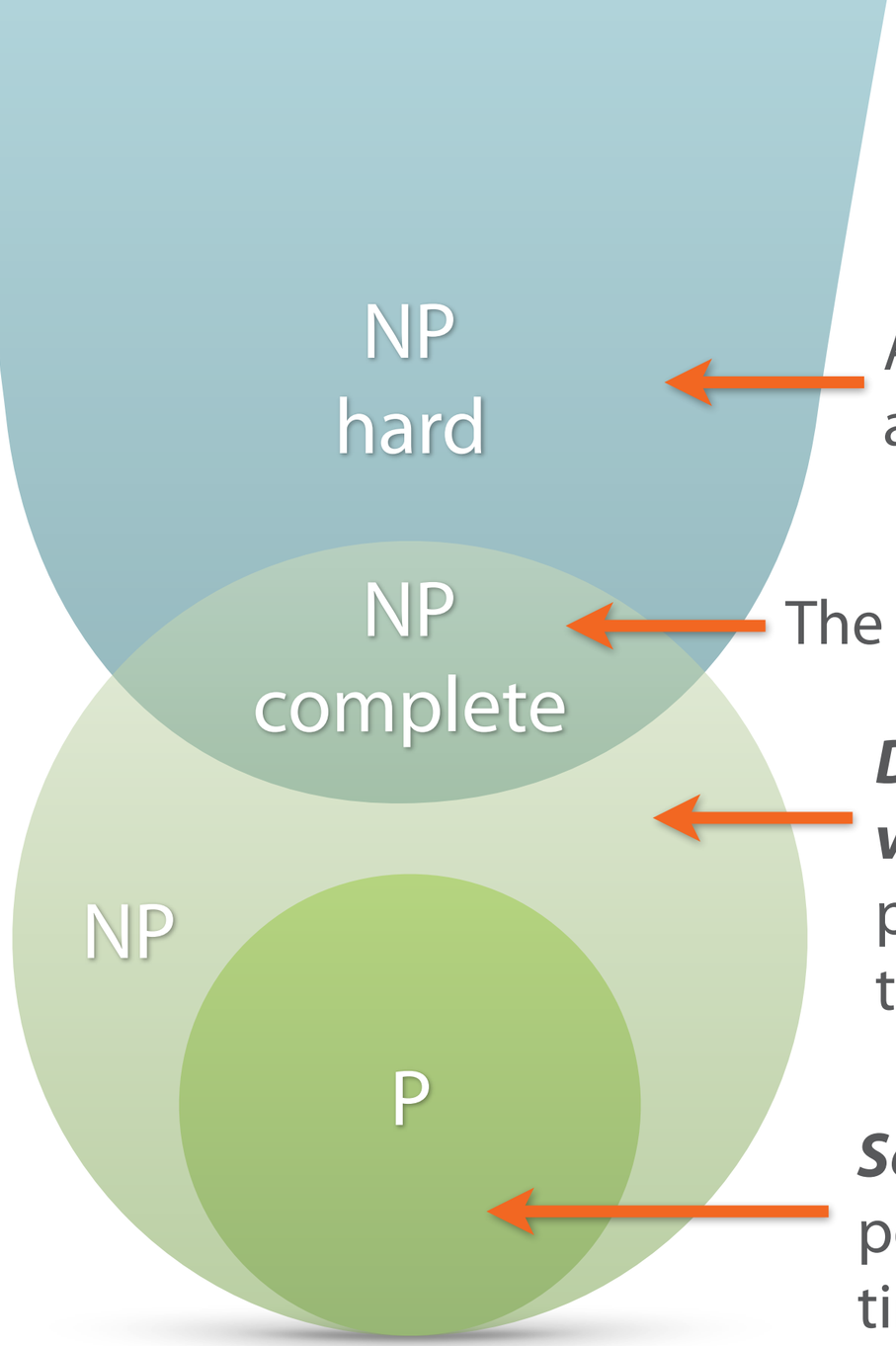

At least as hard as NP complete

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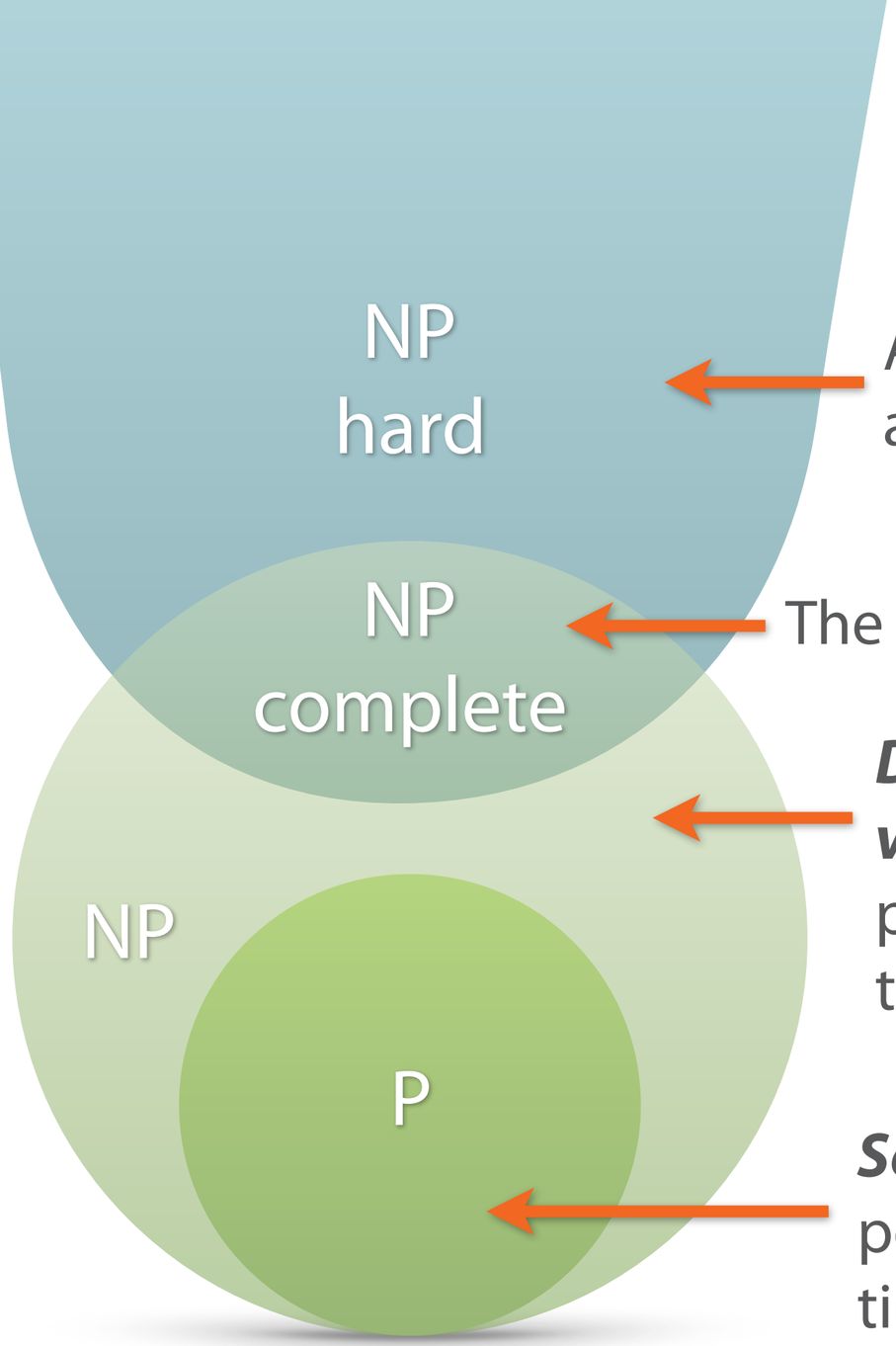

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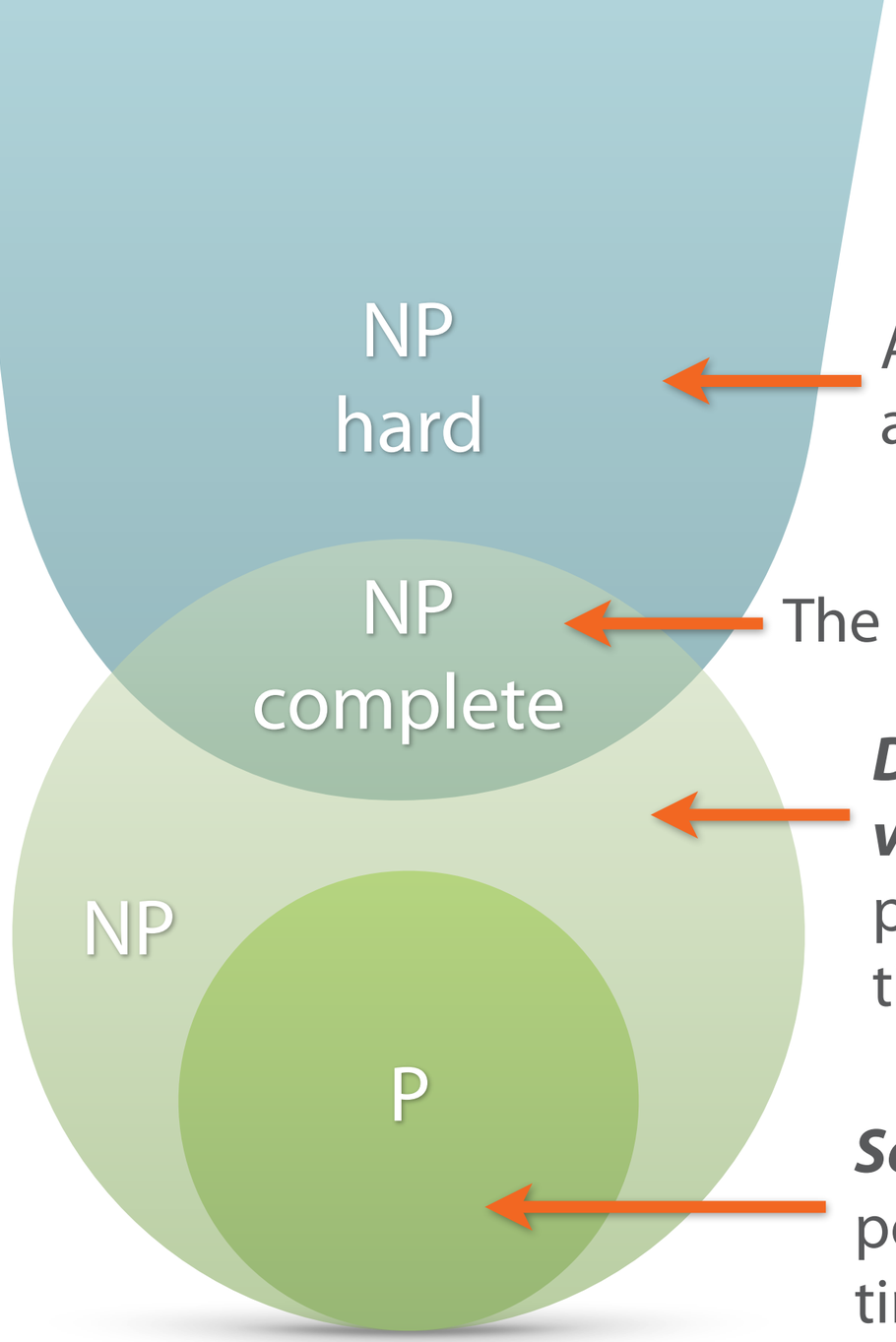

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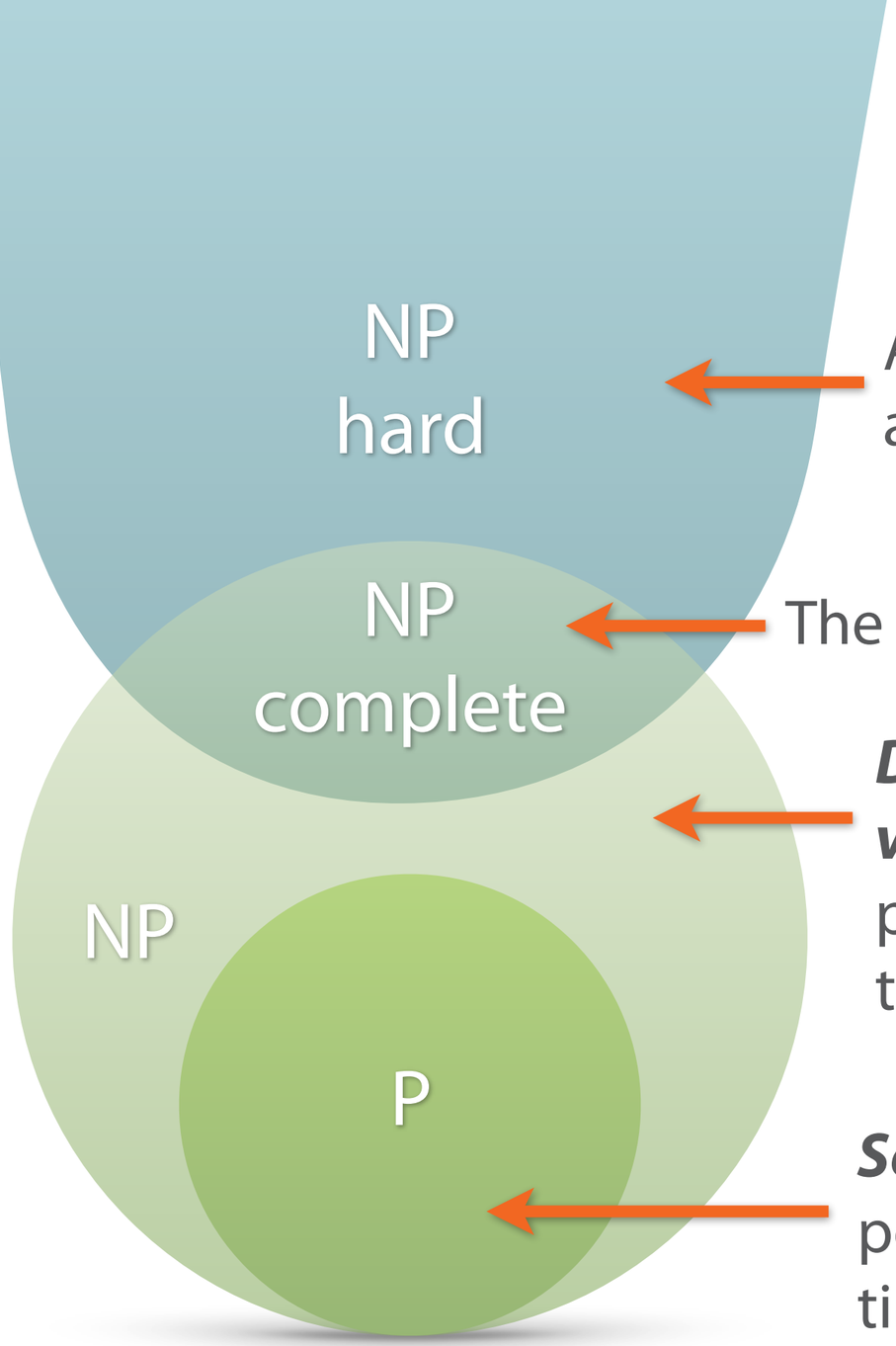

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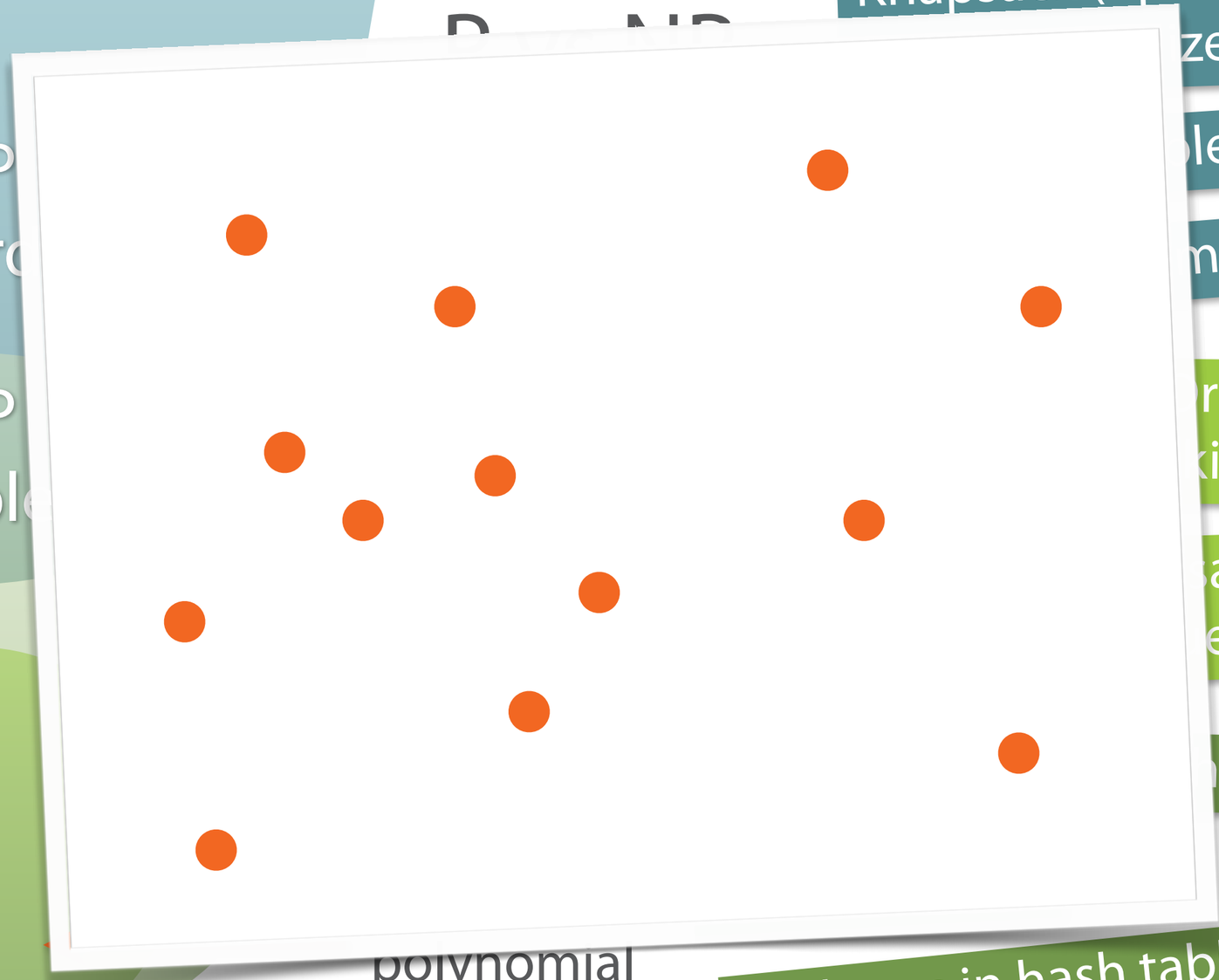

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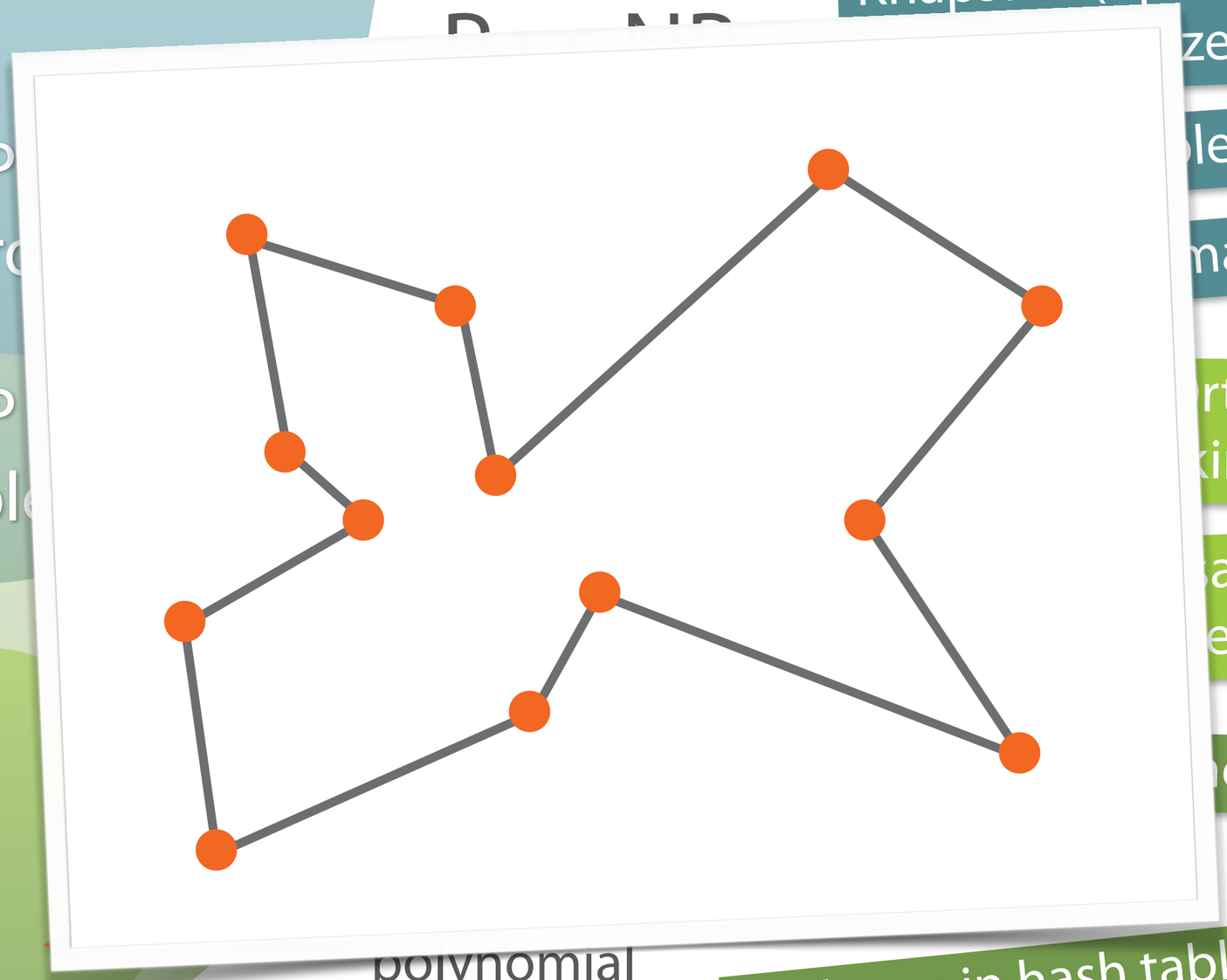

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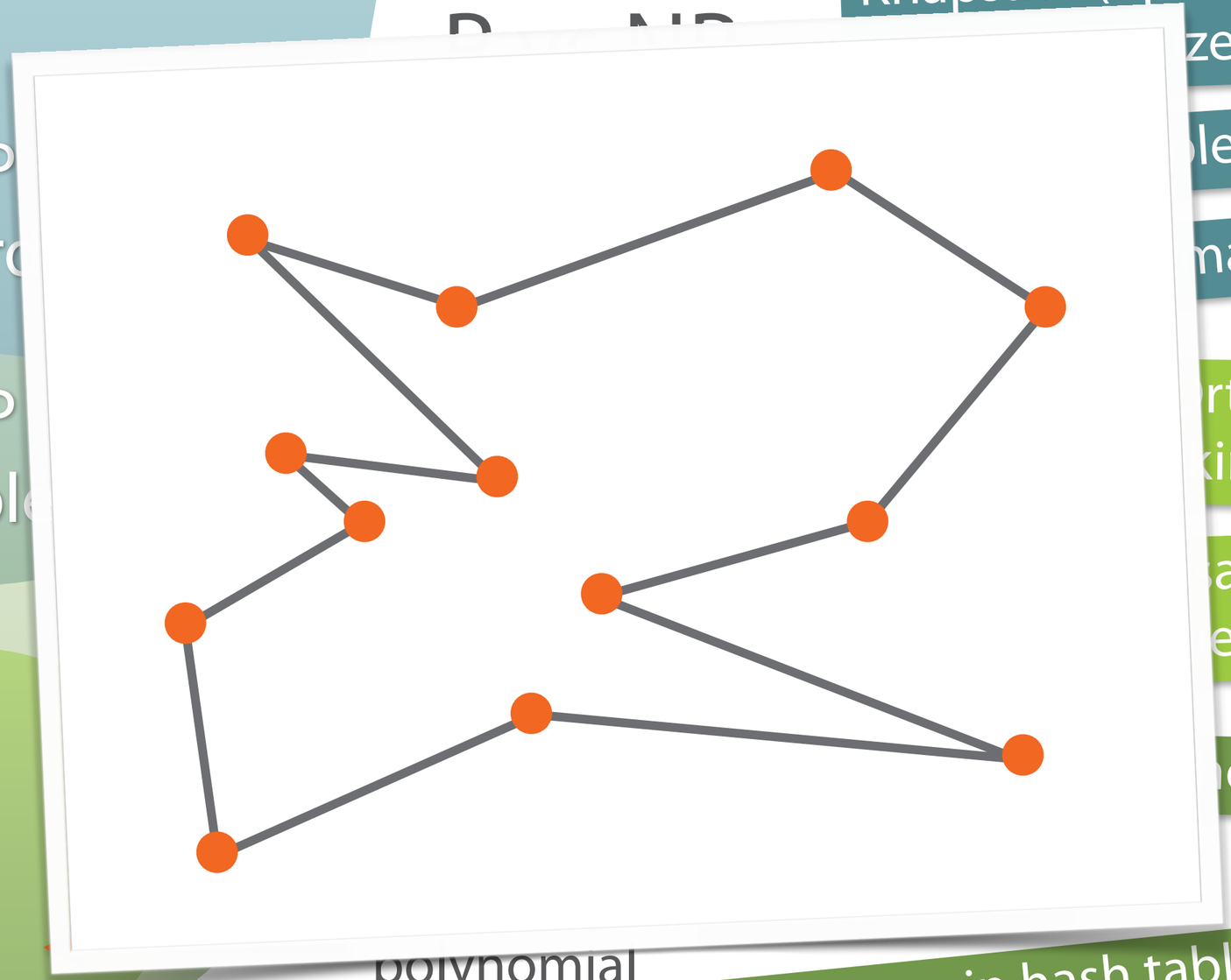

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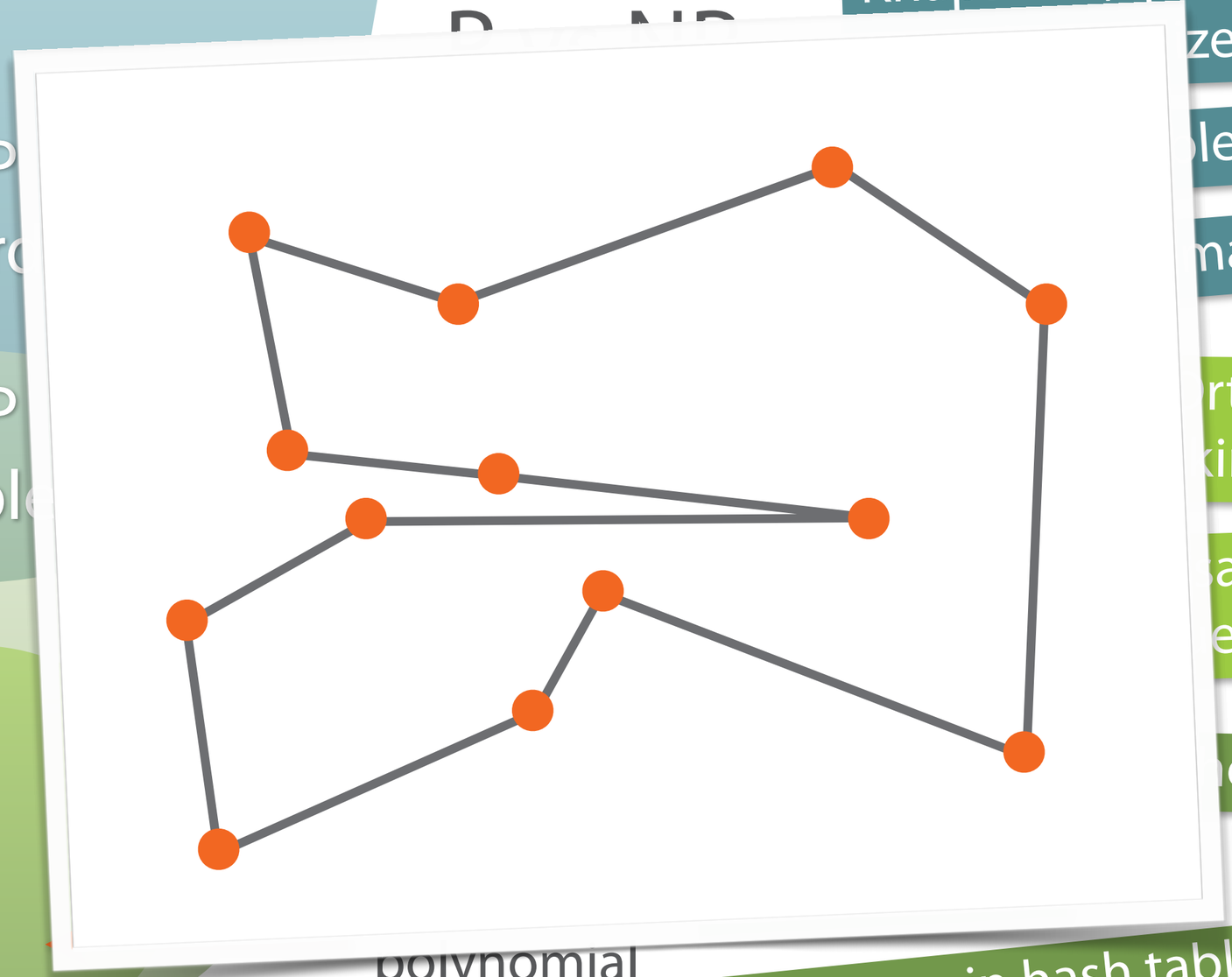

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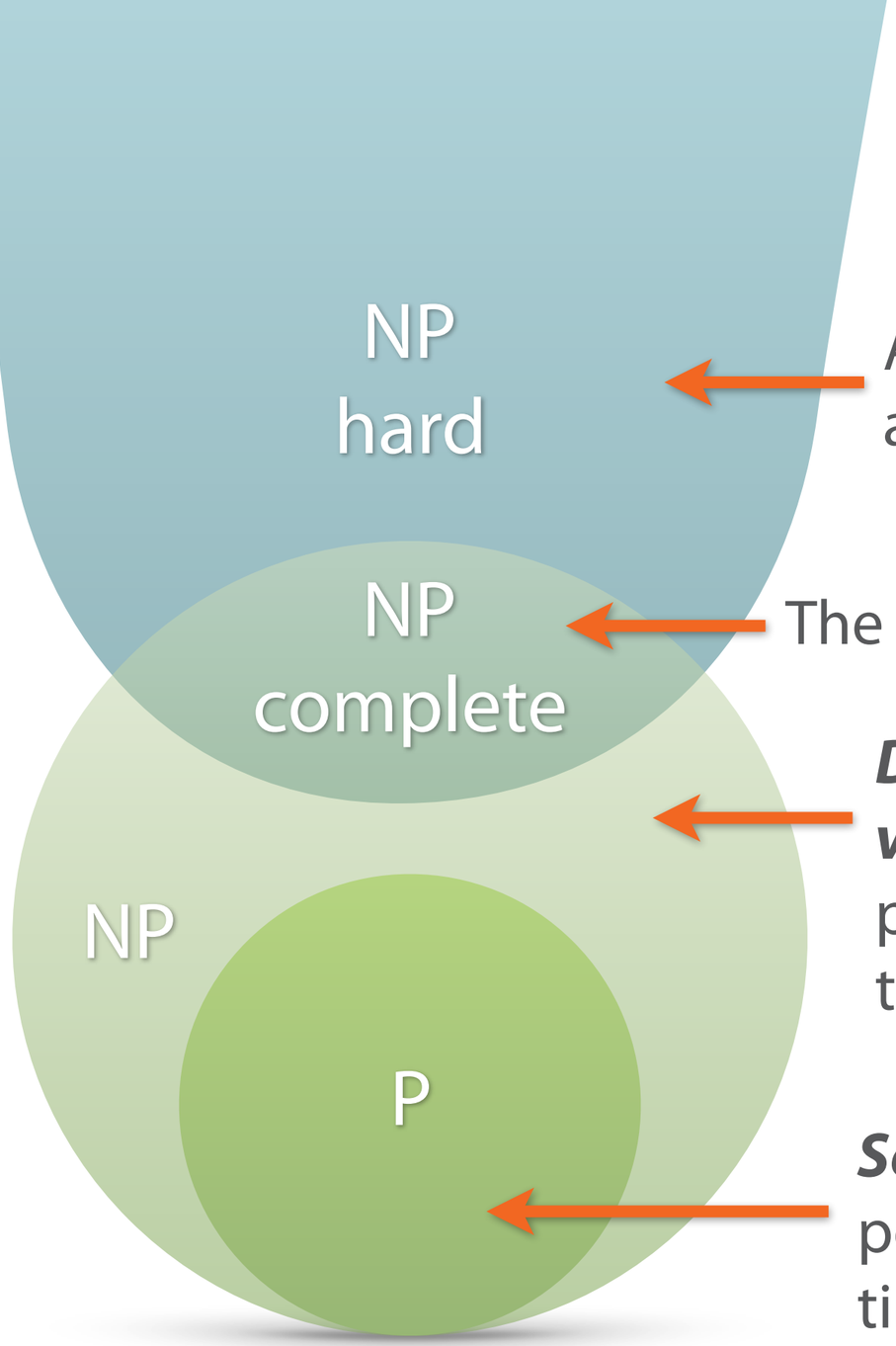
P vs NP



At least as hard as NP complete

Knapsack (optimization) (maximize *V*)

Halting problem

Traveling salesman

The hardest in NP

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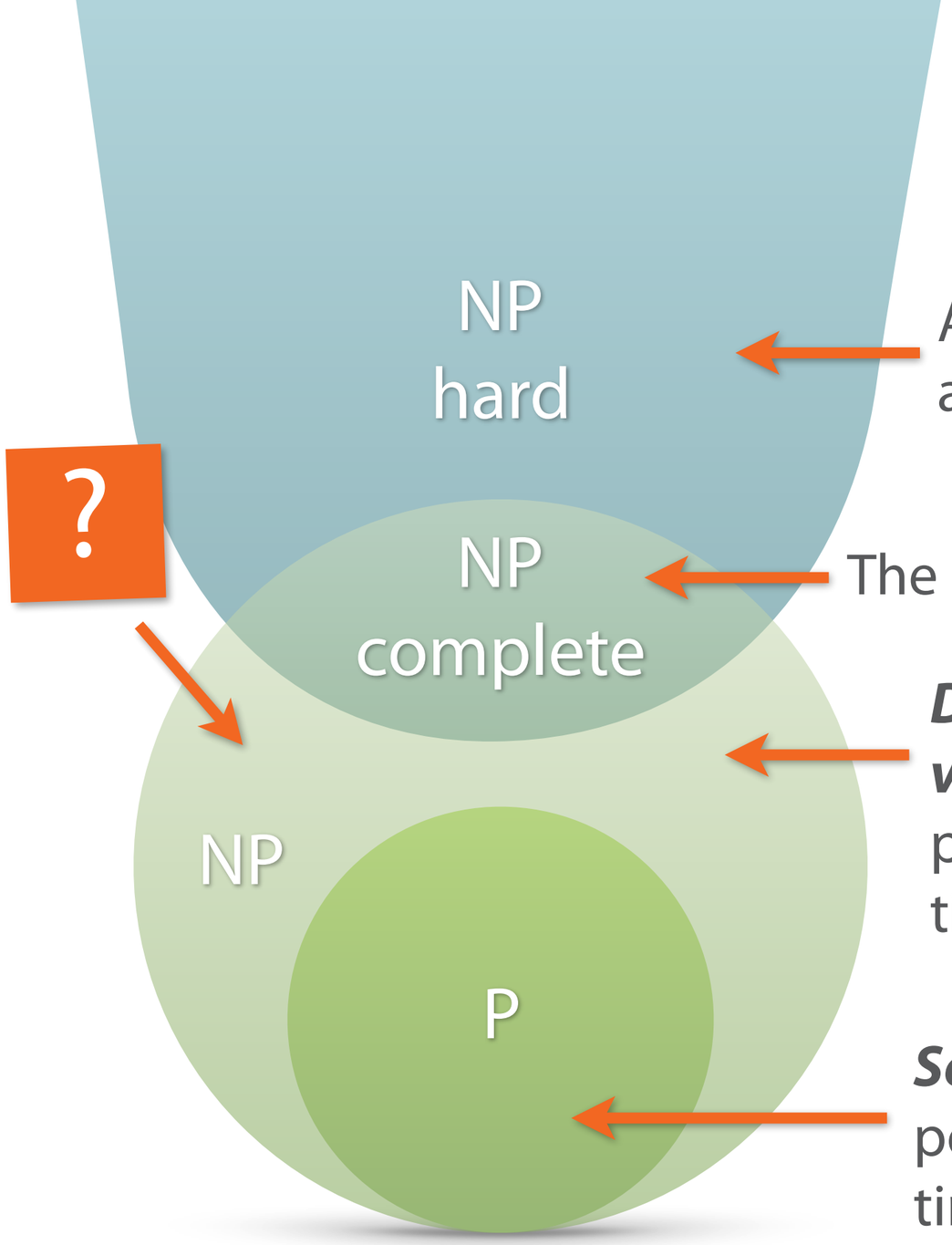

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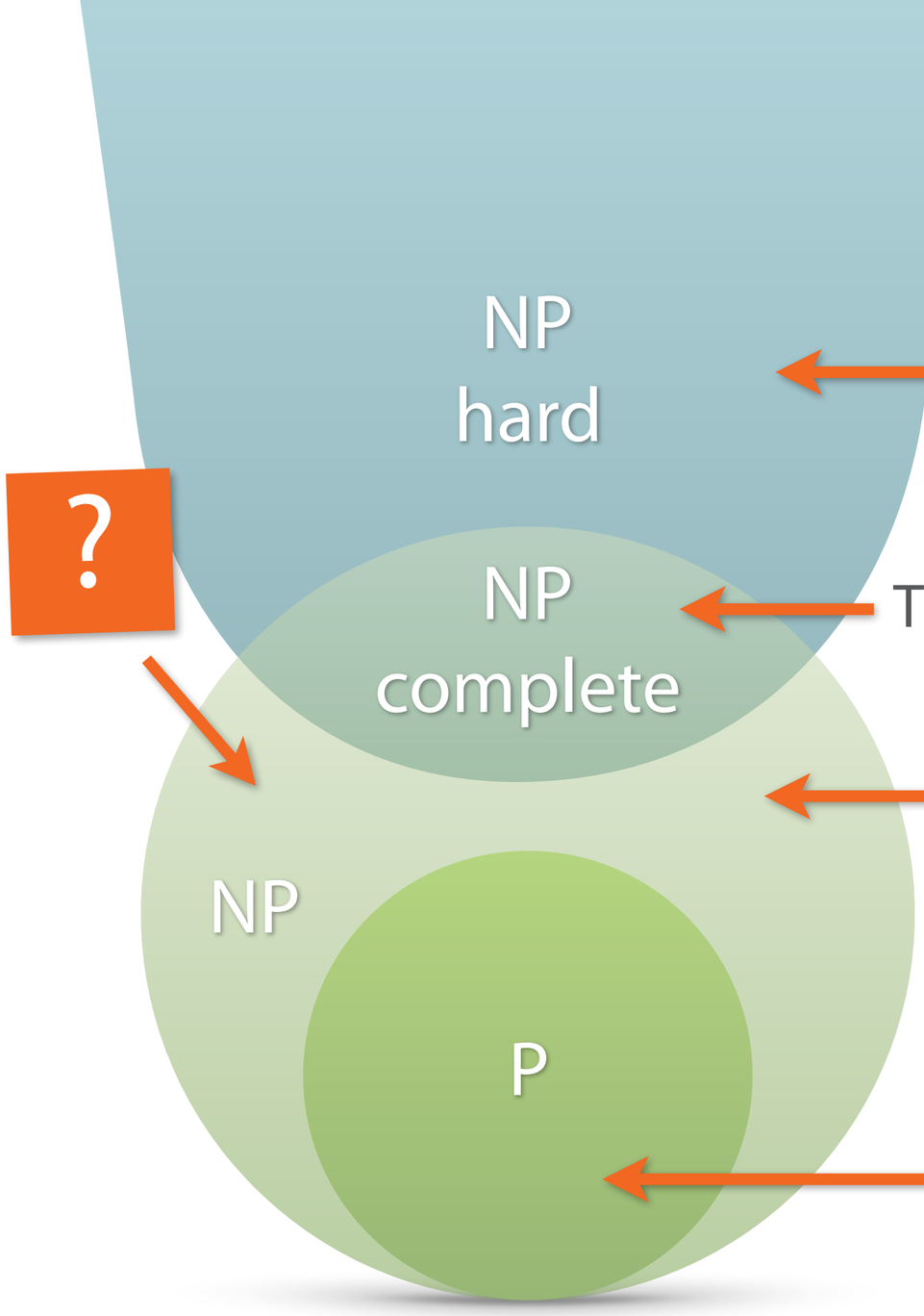

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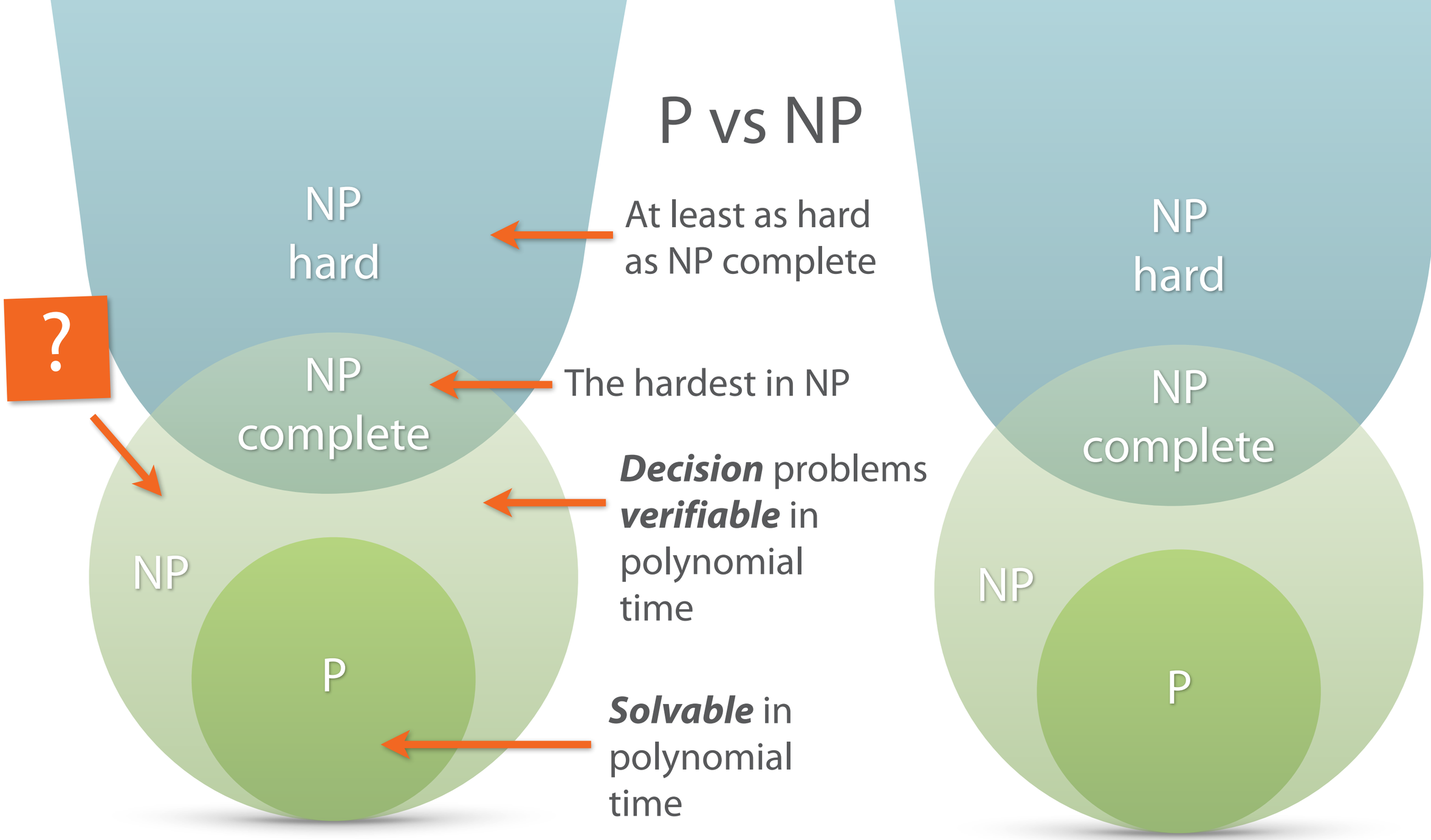
NP
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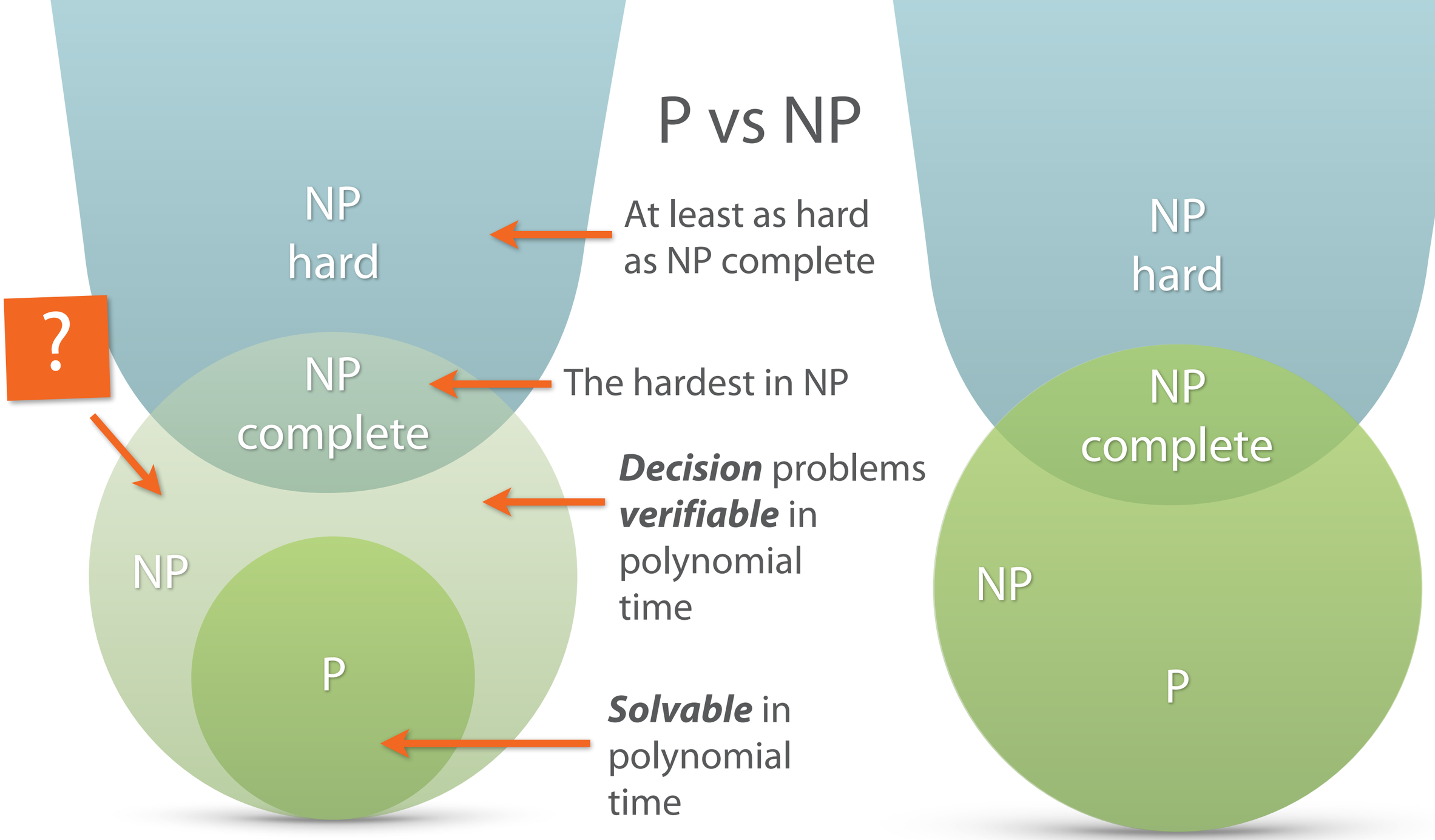
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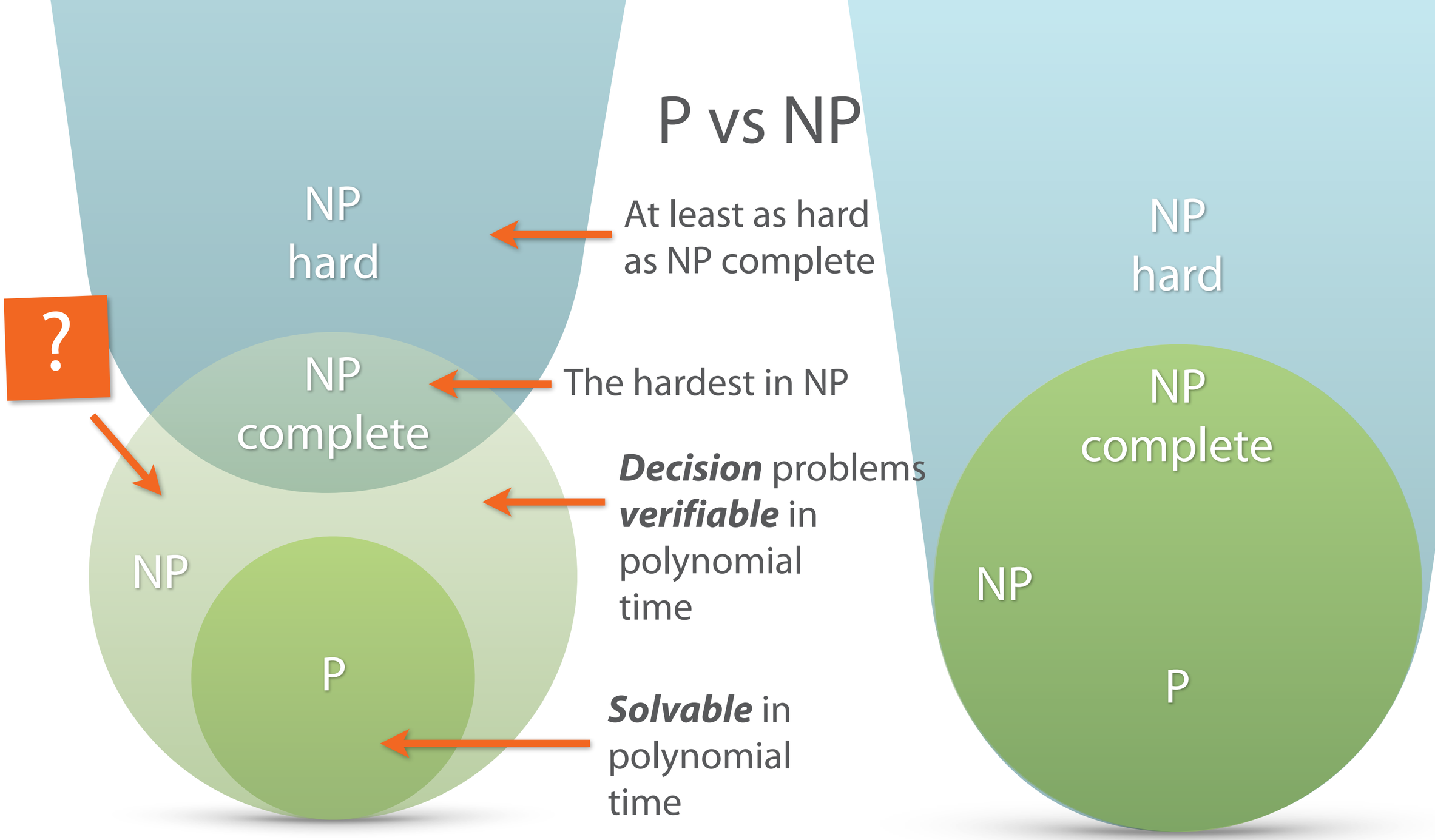
P vs NP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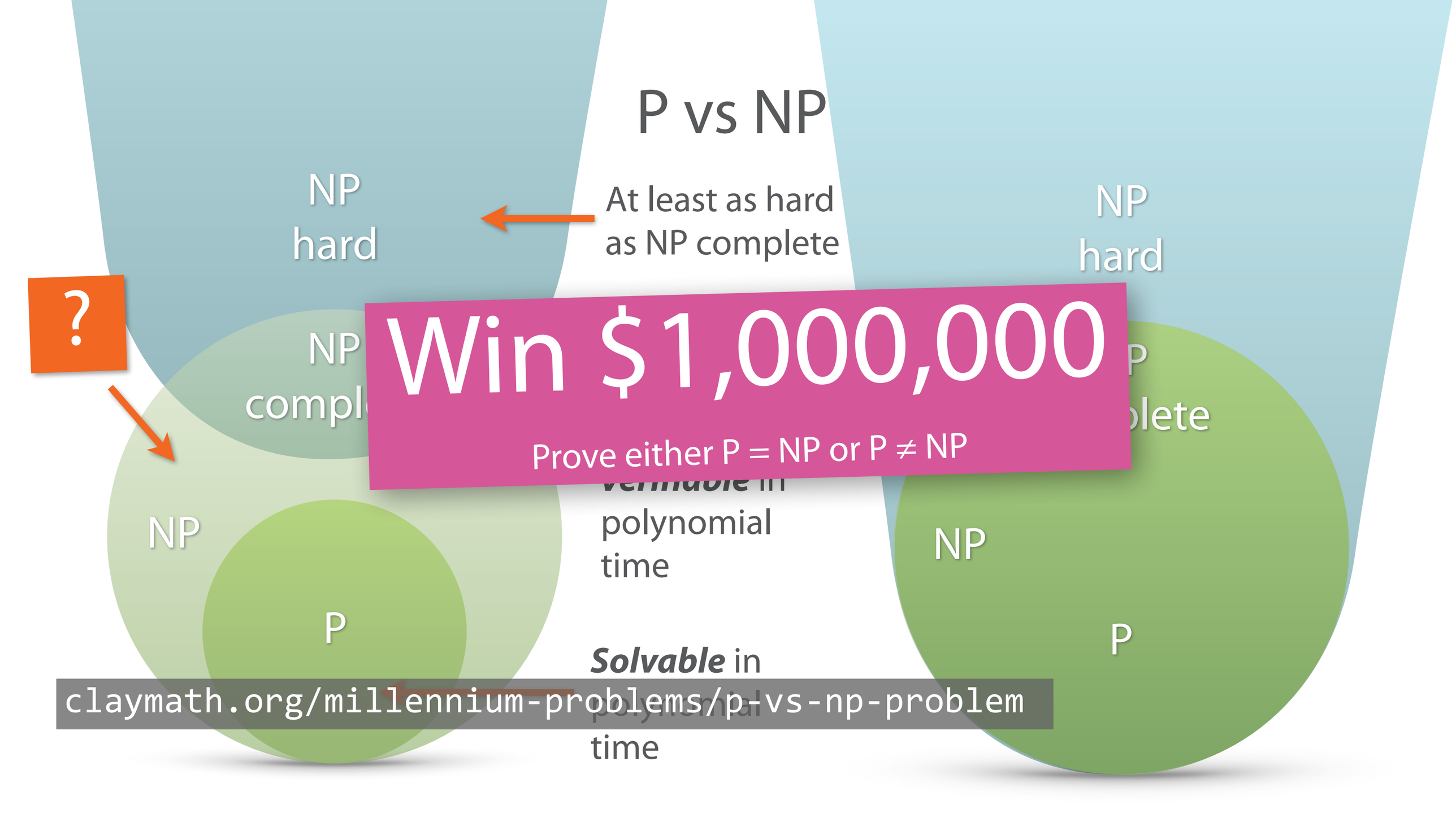
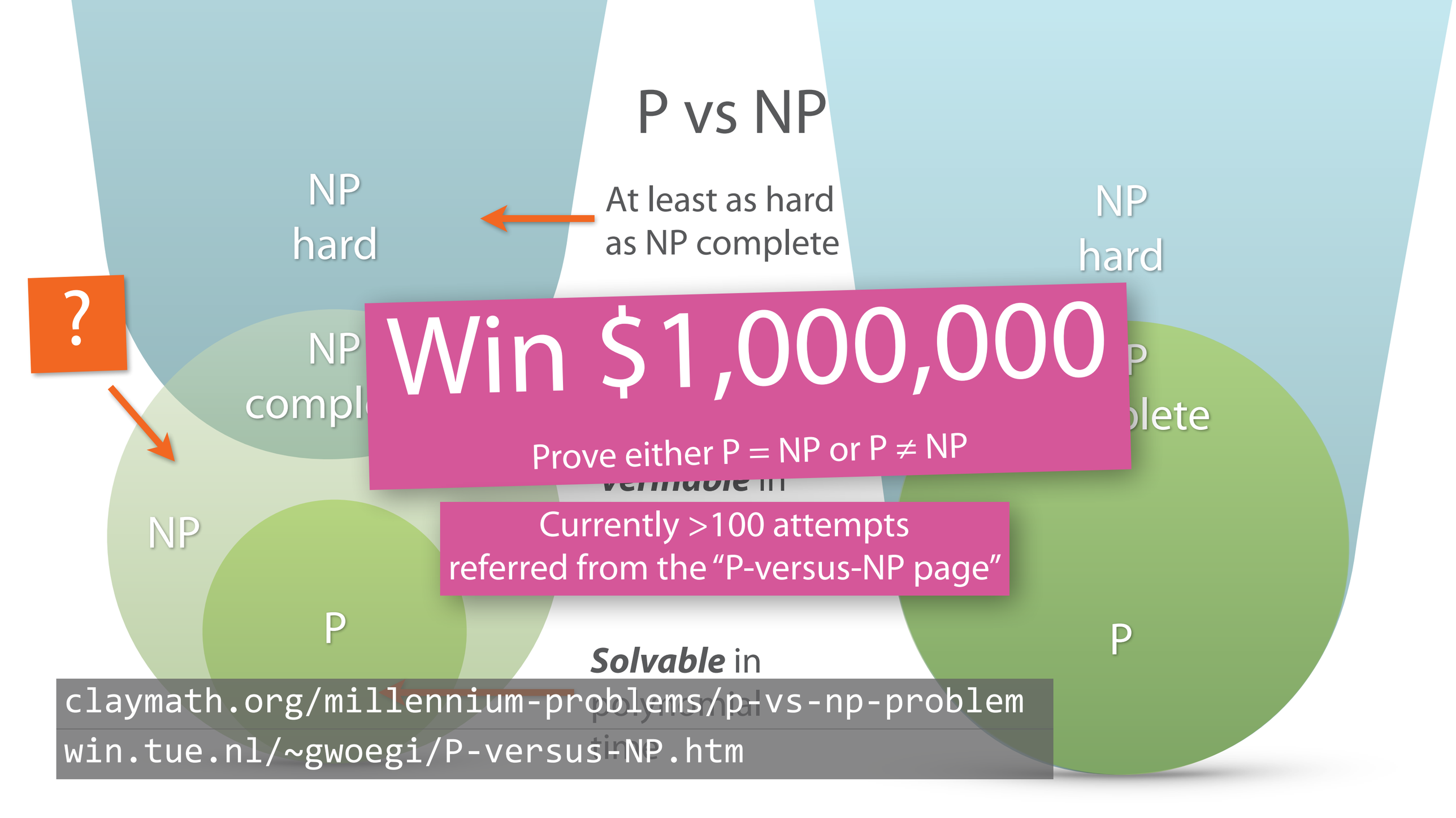

P vs NP
NP hard
At least as hard as NP complete
NP hard
?
Win $1,000,000
Prove either P = NP or P ≠ NP
NP
P
polynomial time
NP
P
claymath.org/millennium-problems/p-vs-np-problem


P vs NP
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Win $1,000,000
Prove either P = NP or P ≠ NP
Currently >100 attempts referred from the “P-versus-NP page”
NP
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Solvable in
claymath.org/millennium-problems/p-vs-np-problem
win.tue.nl/~gwoegi/P-versus-NP.htm

Hard Problems

Input size in #bits

P vs NP

Heuristics
and
approximation

Hard Problems

Input size in #bits

P vs NP

Heuristics
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Heuristics

Heuristics

Heuristics

Heuristics

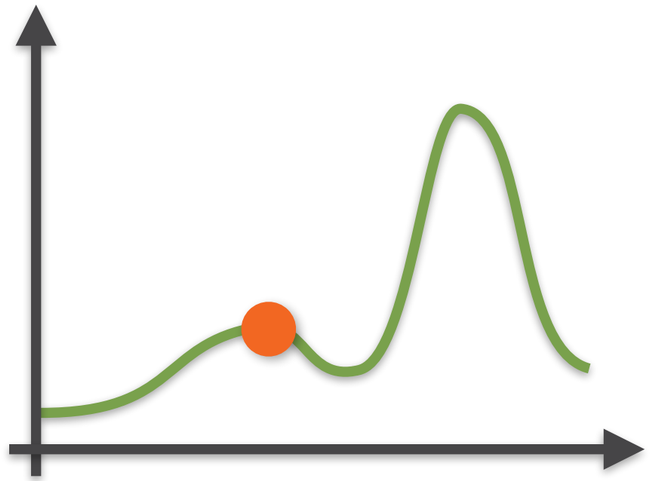
Heuristics

WANTED

Speed!

Trade with accuracy

Win simplicity?



Heuristics

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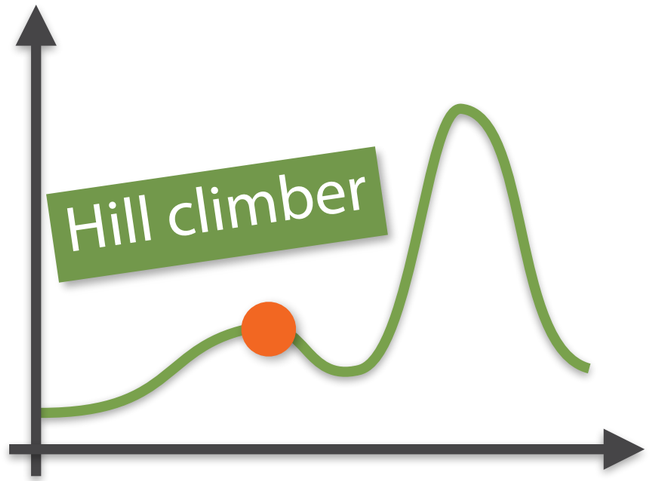

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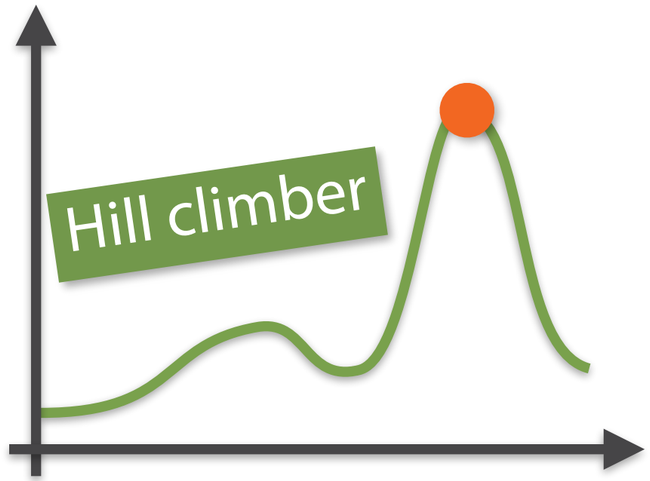

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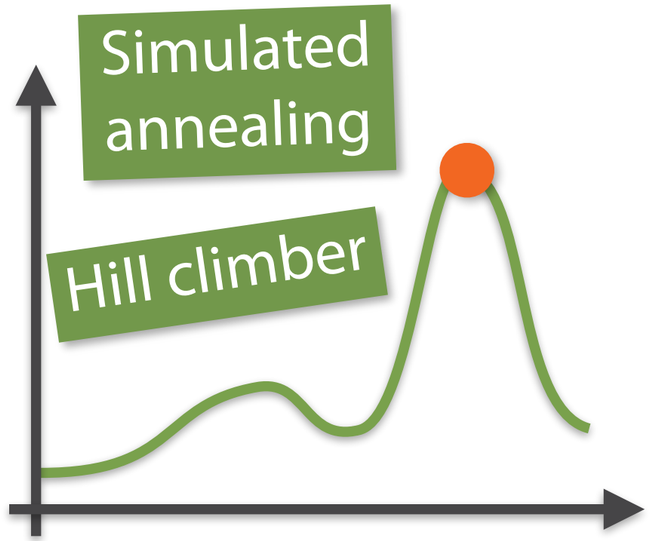

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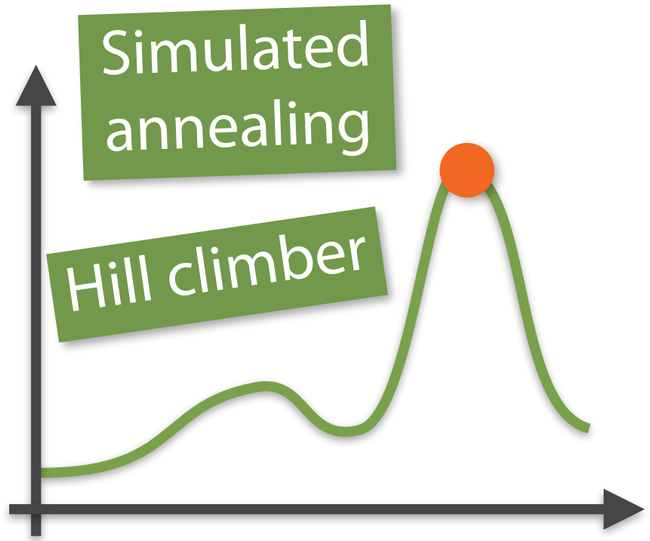

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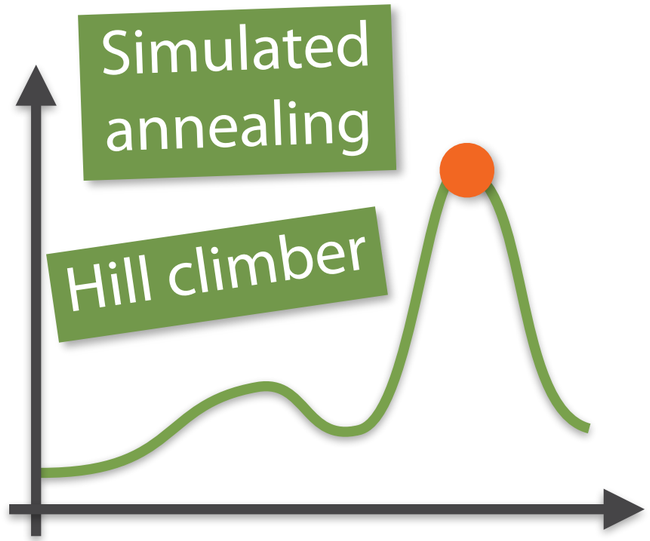

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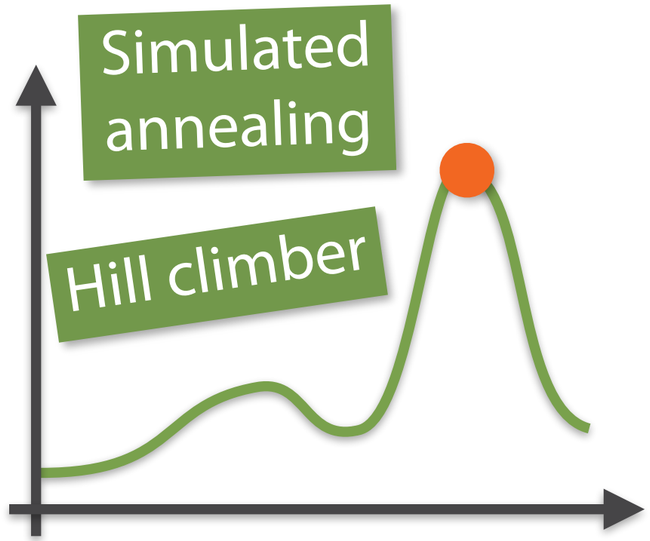

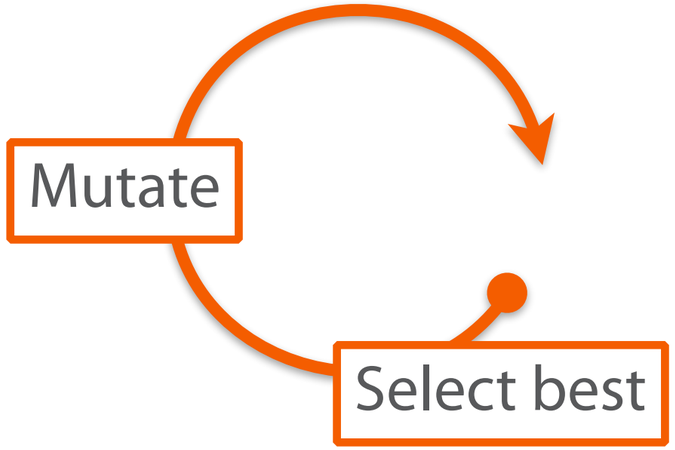

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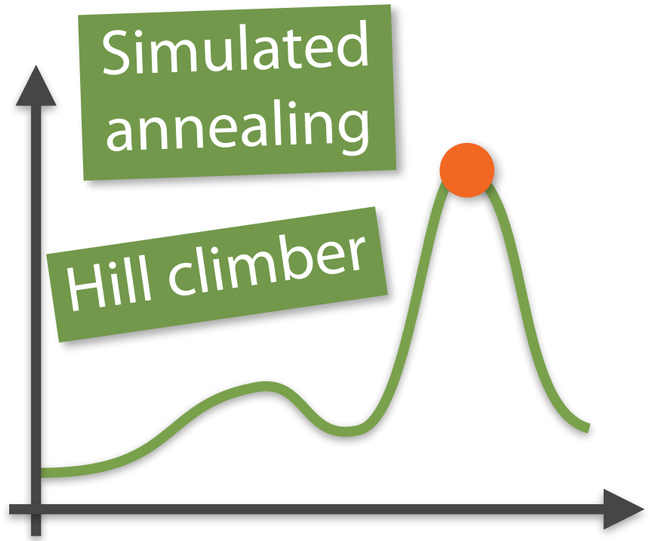
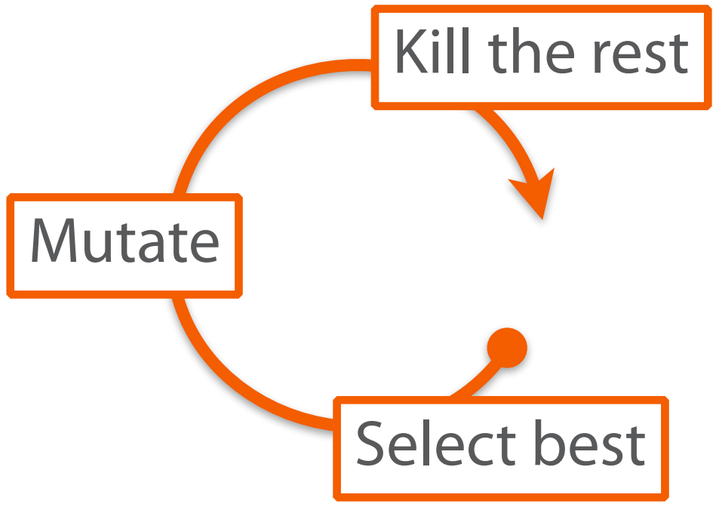
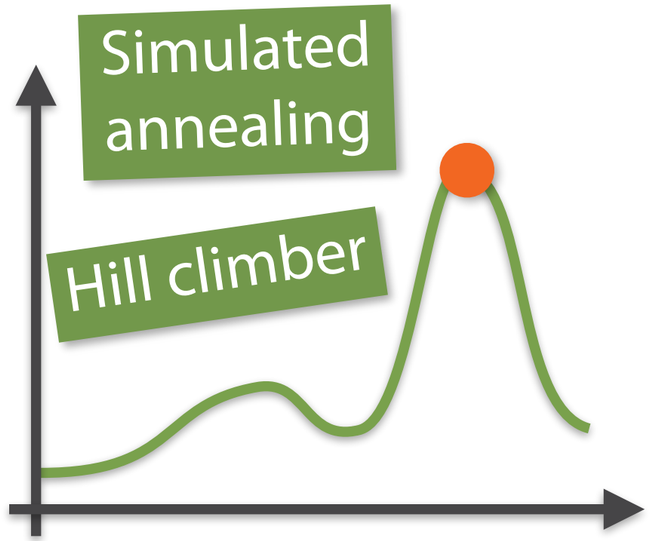




Heuristics



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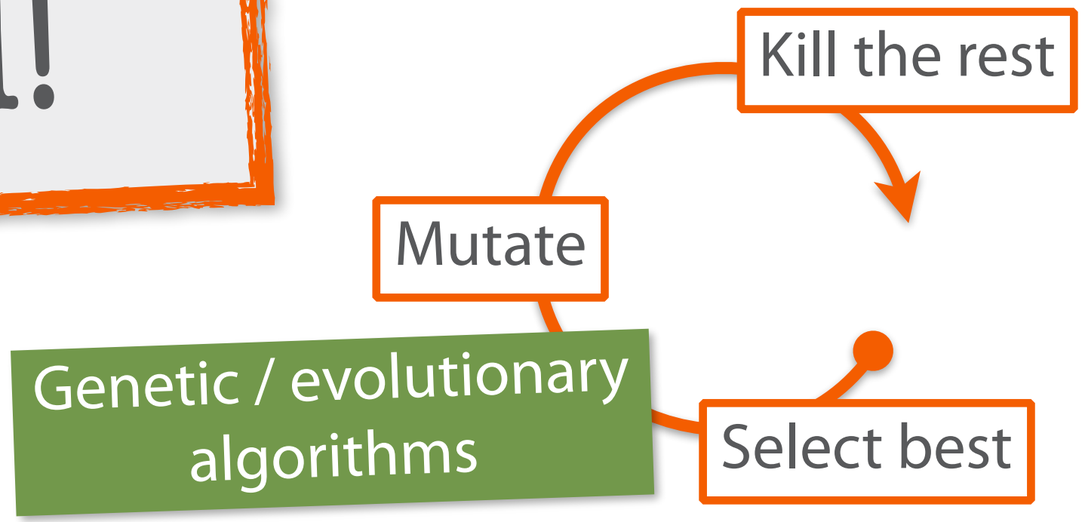

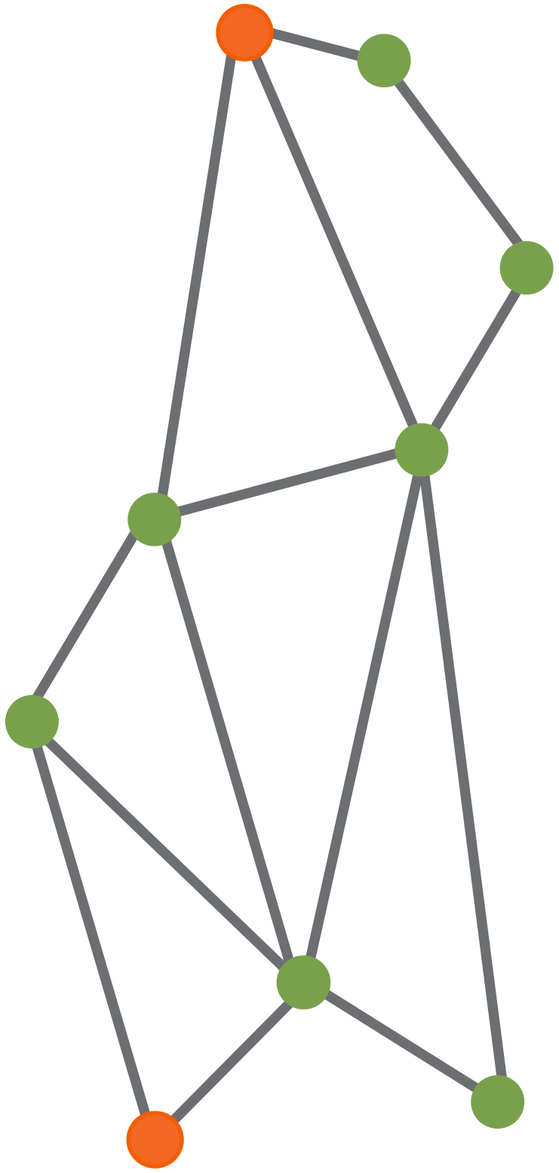

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Trade with accuracy

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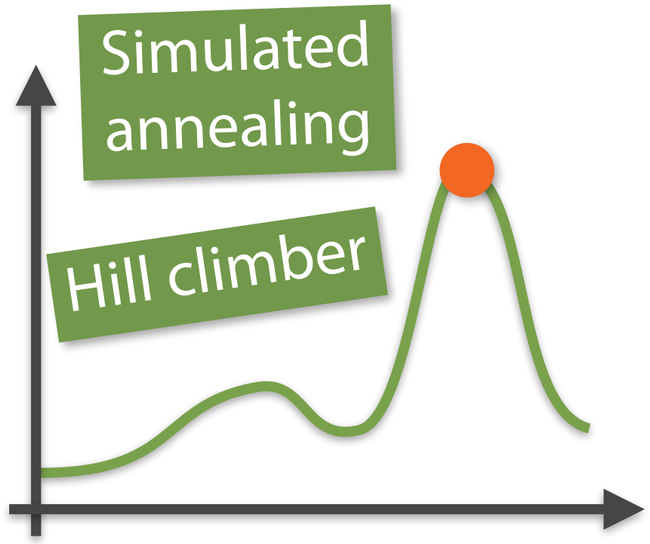
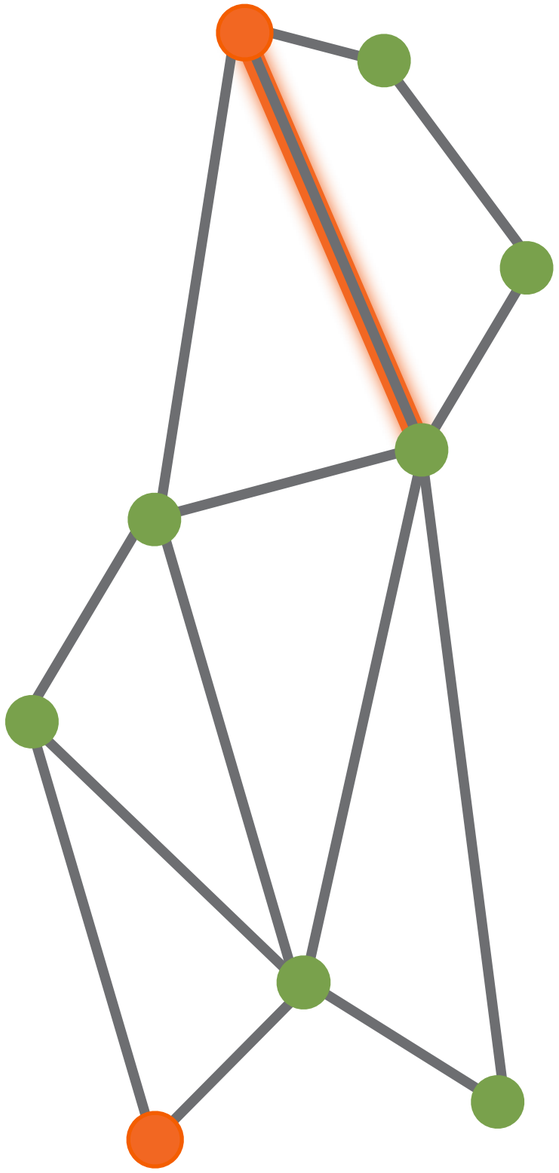


Kill the rest

Mutate

Genetic / evolutionary algorithms

Select best



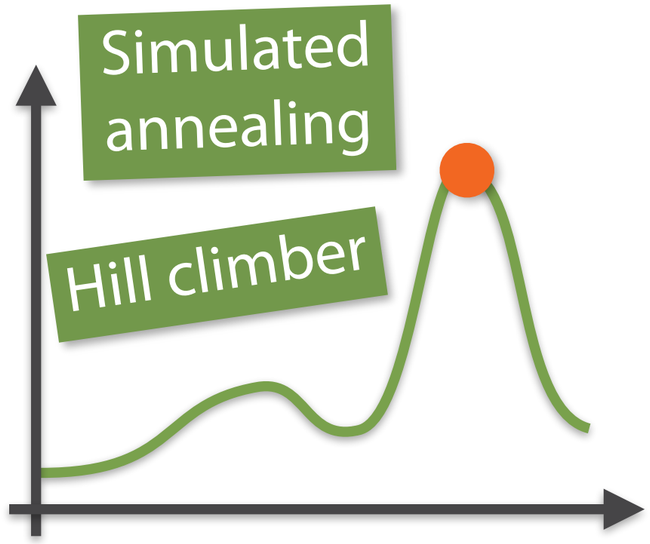
Heuristics

WANTED

Speed!

Trade with accuracy

Win simplicity?



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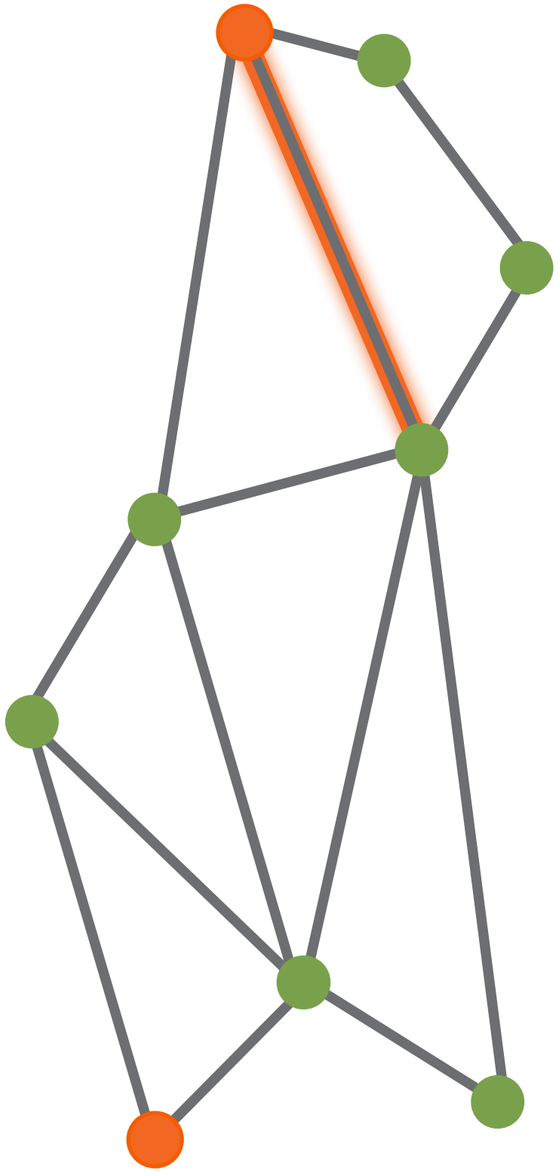


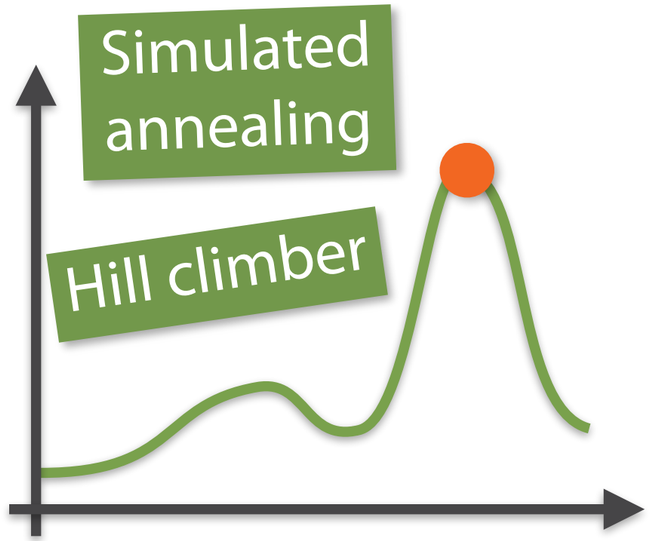
Distance ≥

WANTED

Speed!

Trade with accuracy

Win simplicity?



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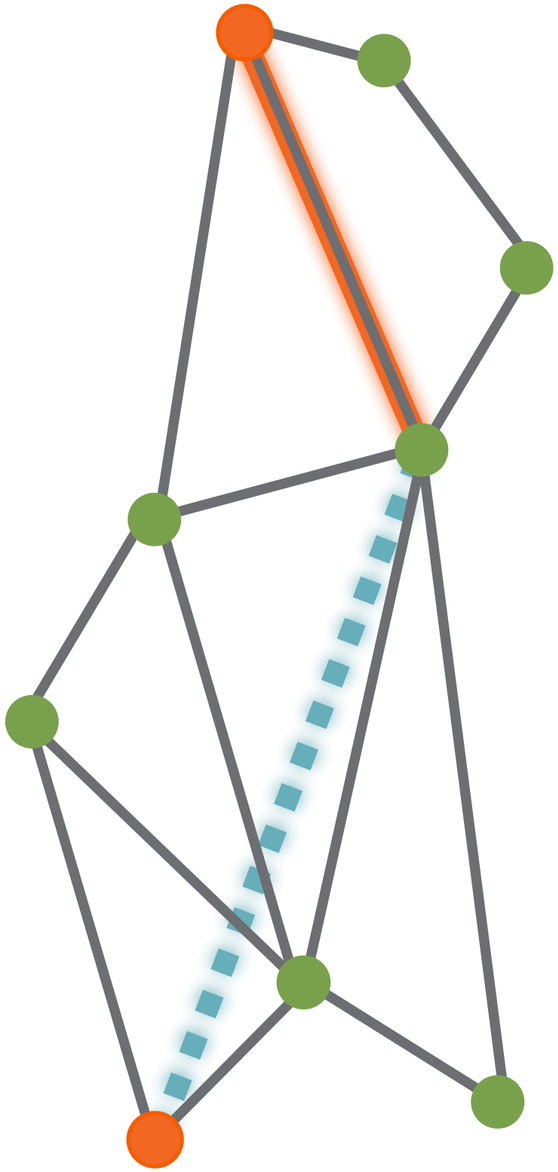
Heuristics



Distance ≥ +

WANTED

Speed!

Trade with accuracy

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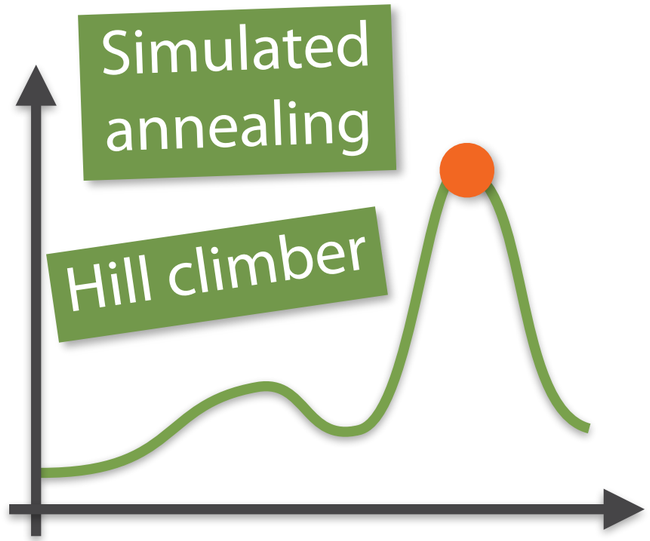

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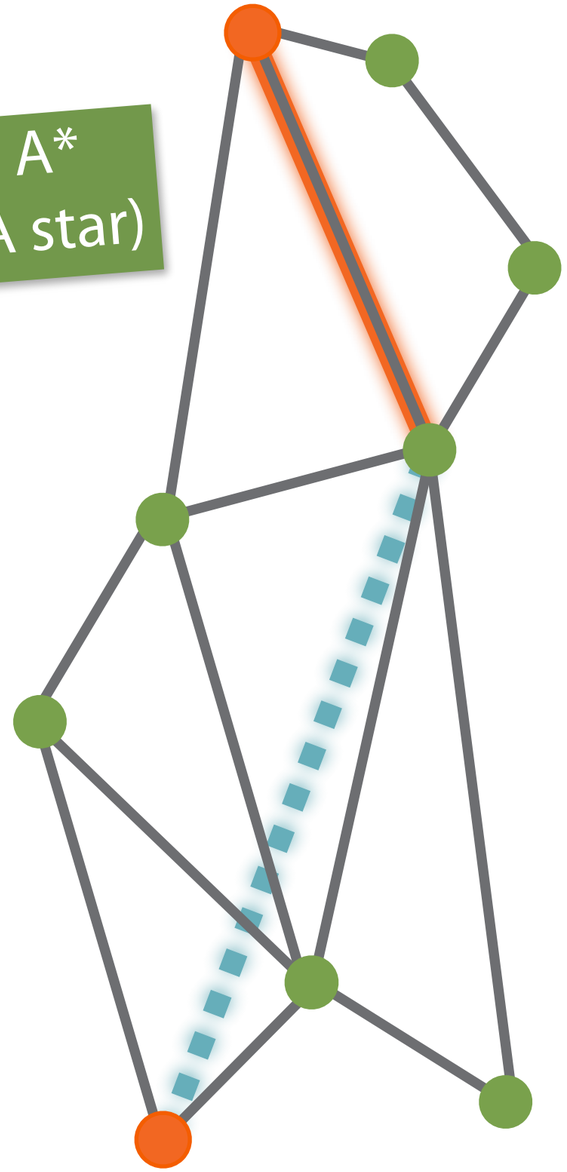
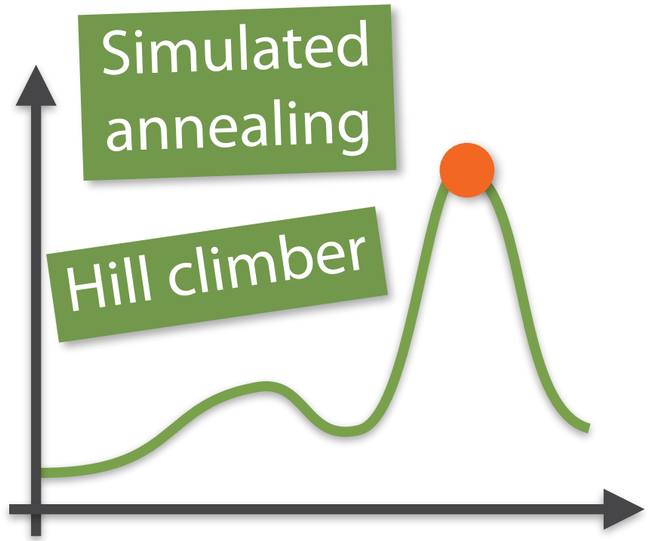


Distance ≥ +



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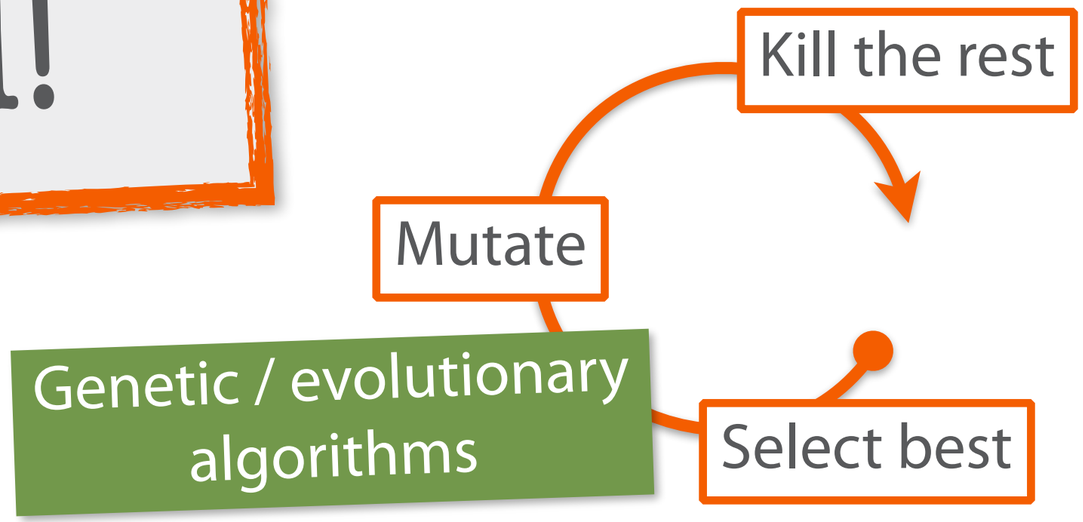

Approximation Algorithms

Approximation Algorithms

- for optimization problems

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Runs in polynomial time

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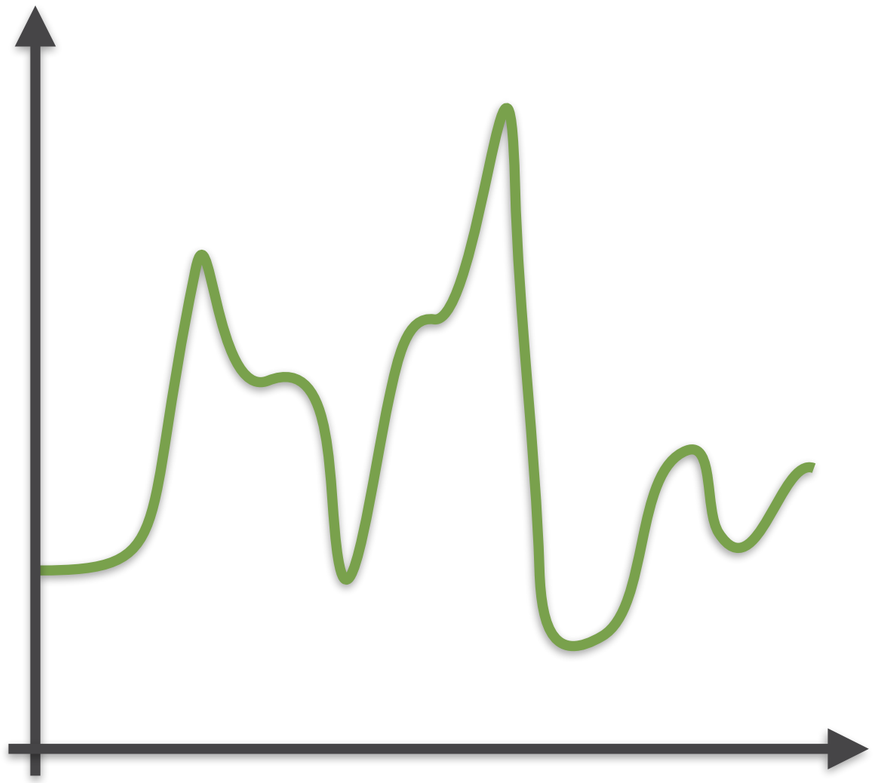
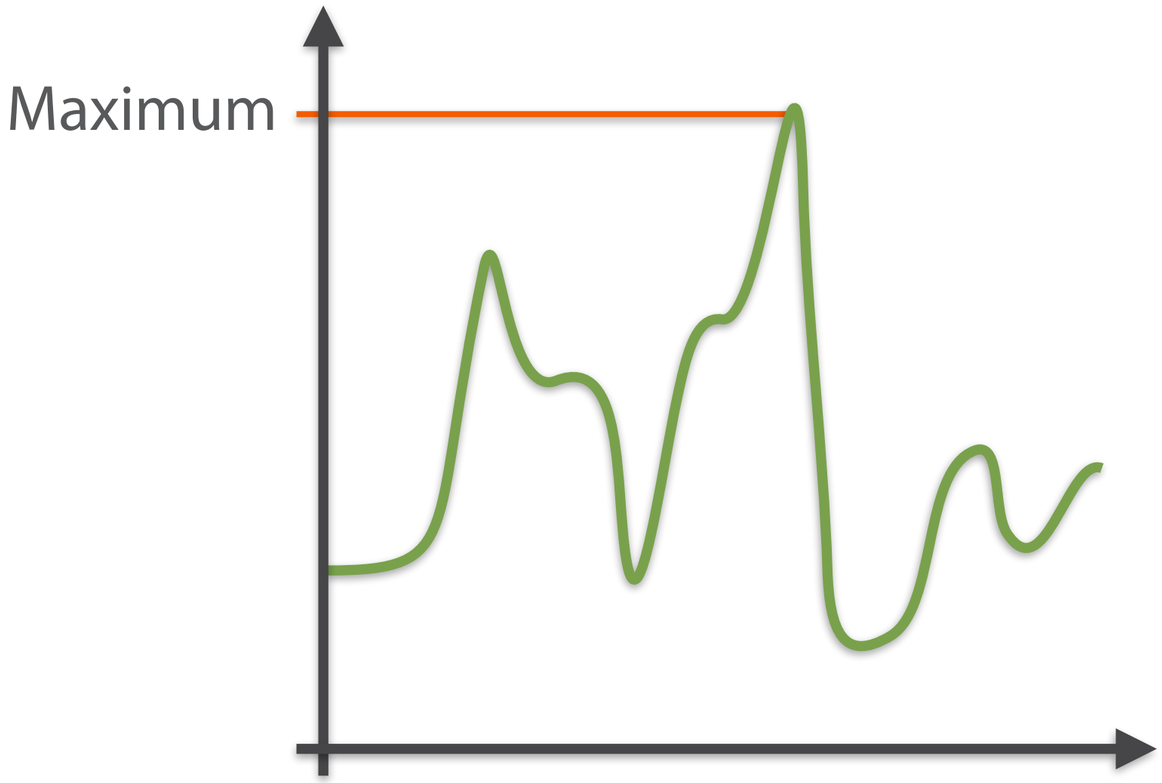


Approximation Algorithms

- for optimization problems

Runs in polynomial time

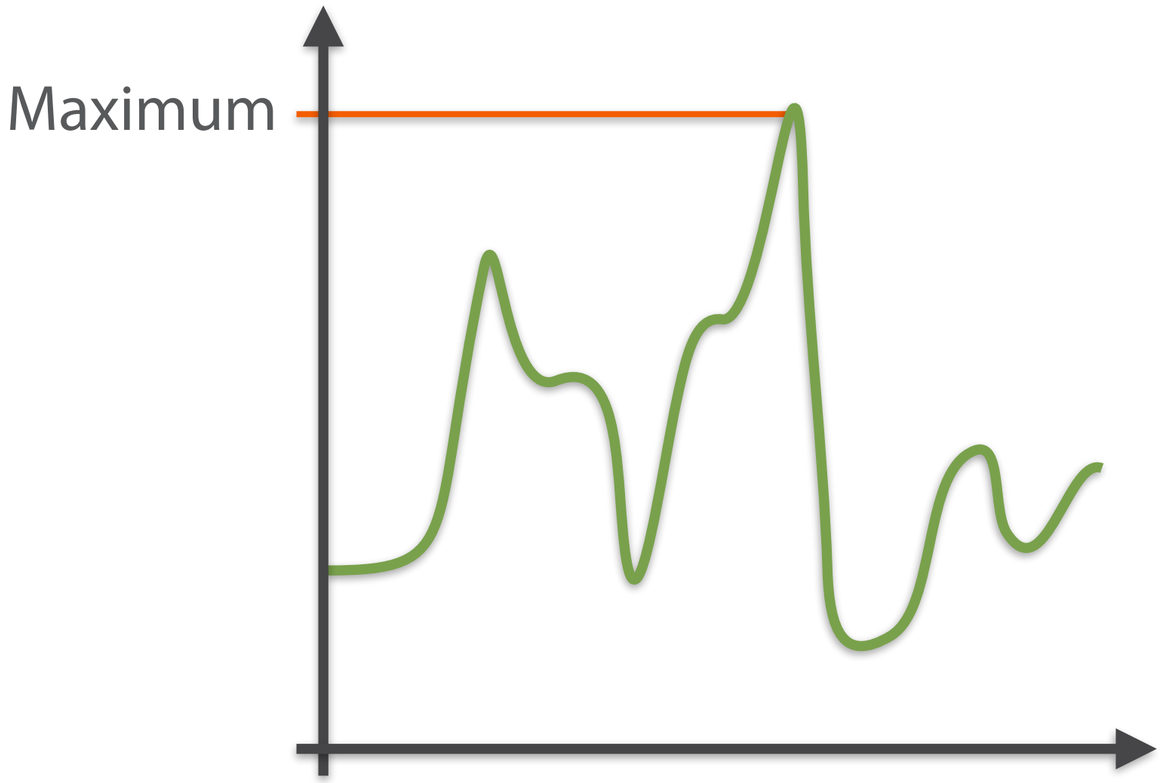


Approximation Algorithms

- for optimization problems

Runs in polynomial time

Allowed error: *p*

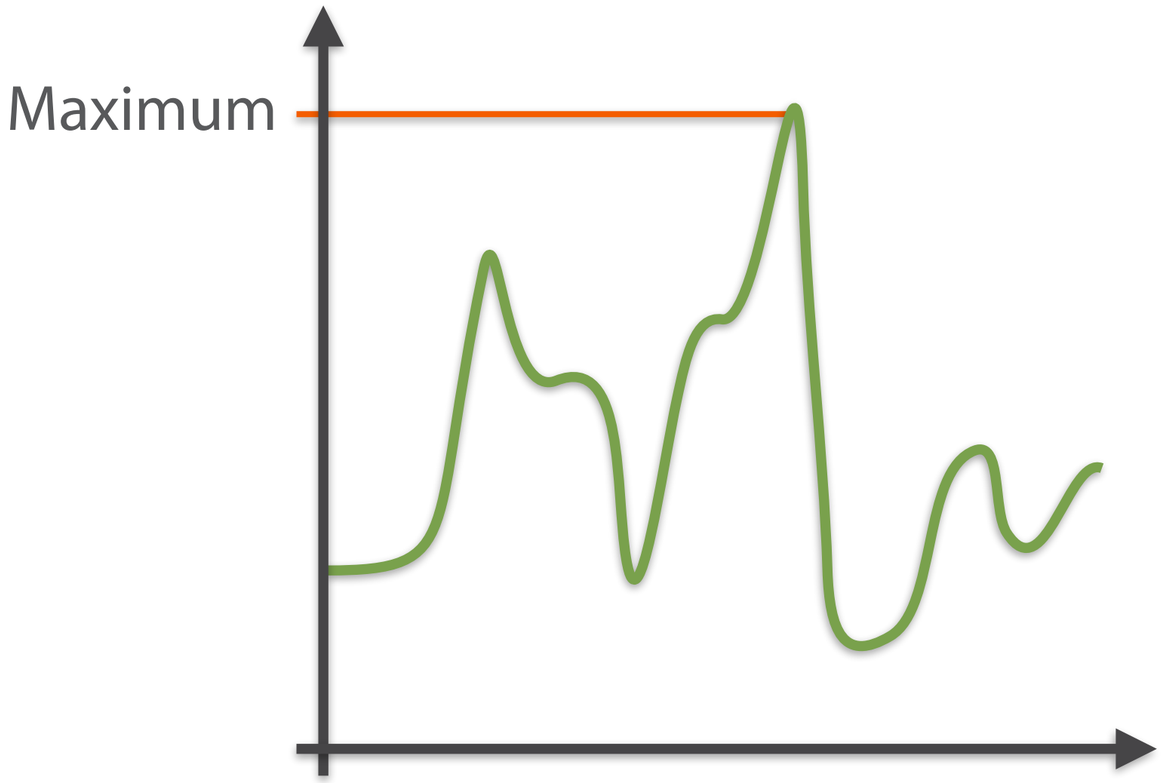


Approximation Algorithms

- for optimization problems

Runs in polynomial time

Allowed error: p $\quad 0 < p < 1$



Approximation Algorithms

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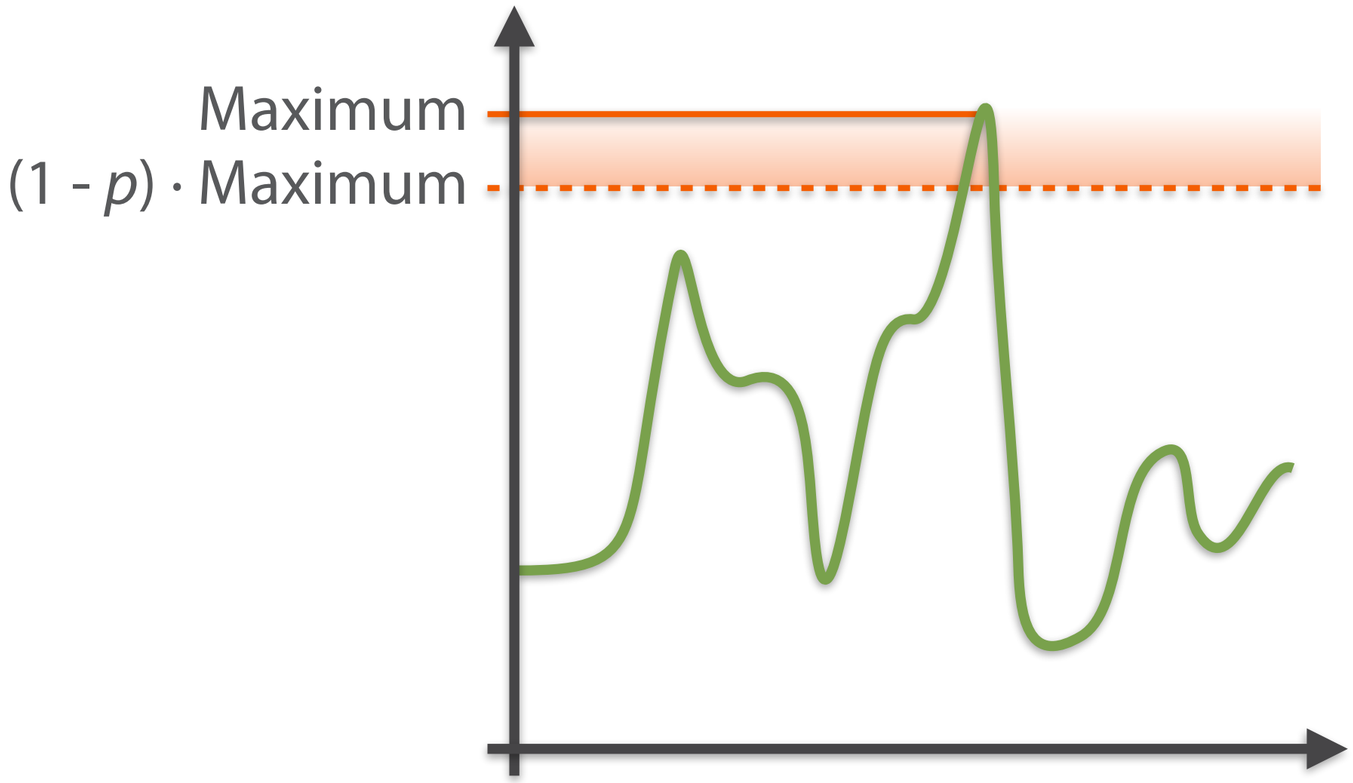


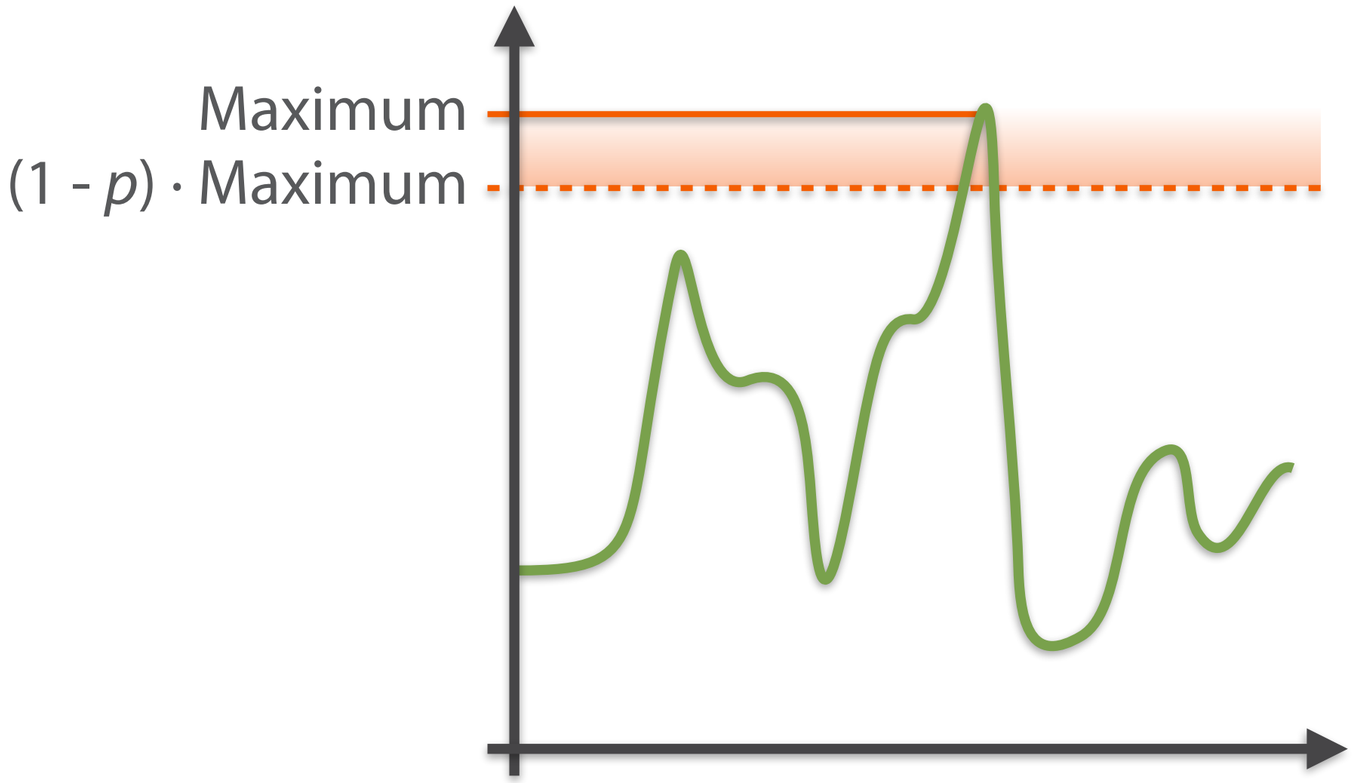
Approximation Algorithms

- for optimization problems

Runs in polynomial time

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



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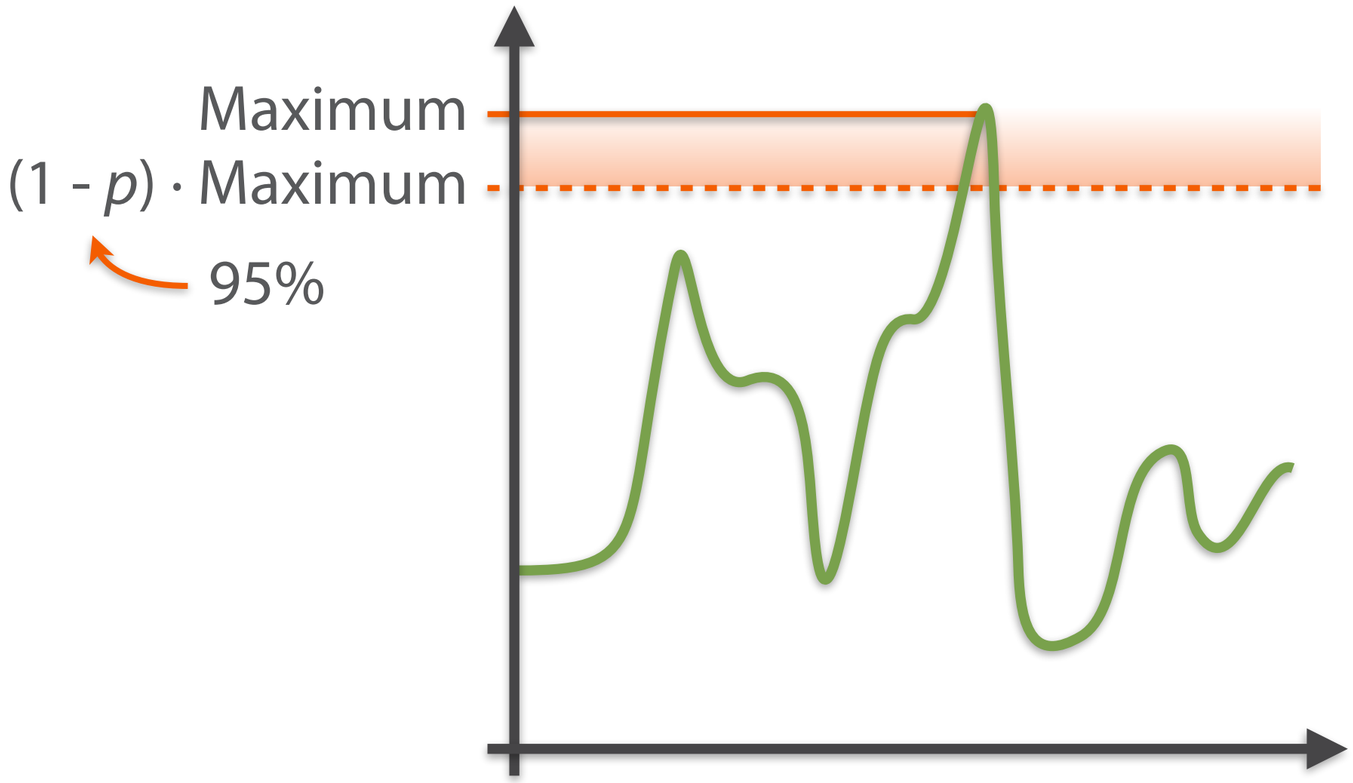

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Runs in polynomial time

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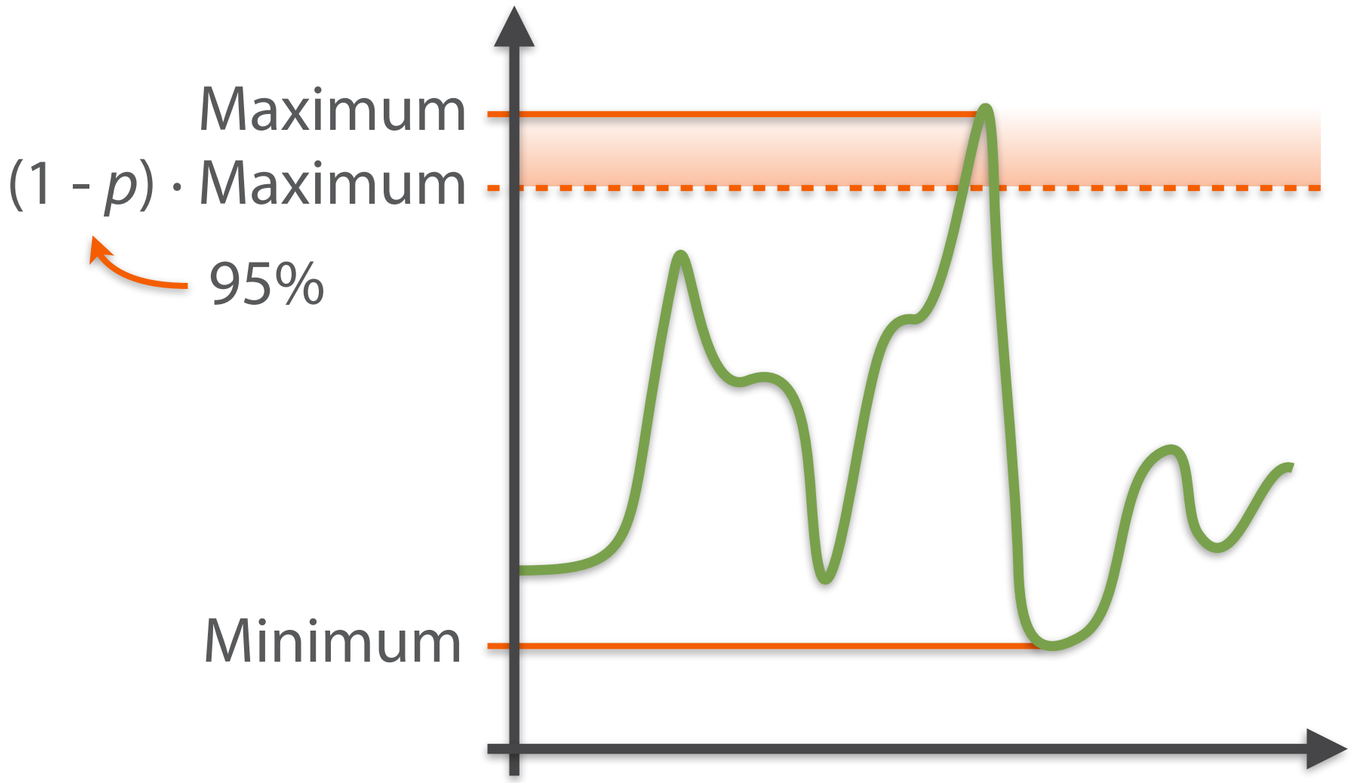

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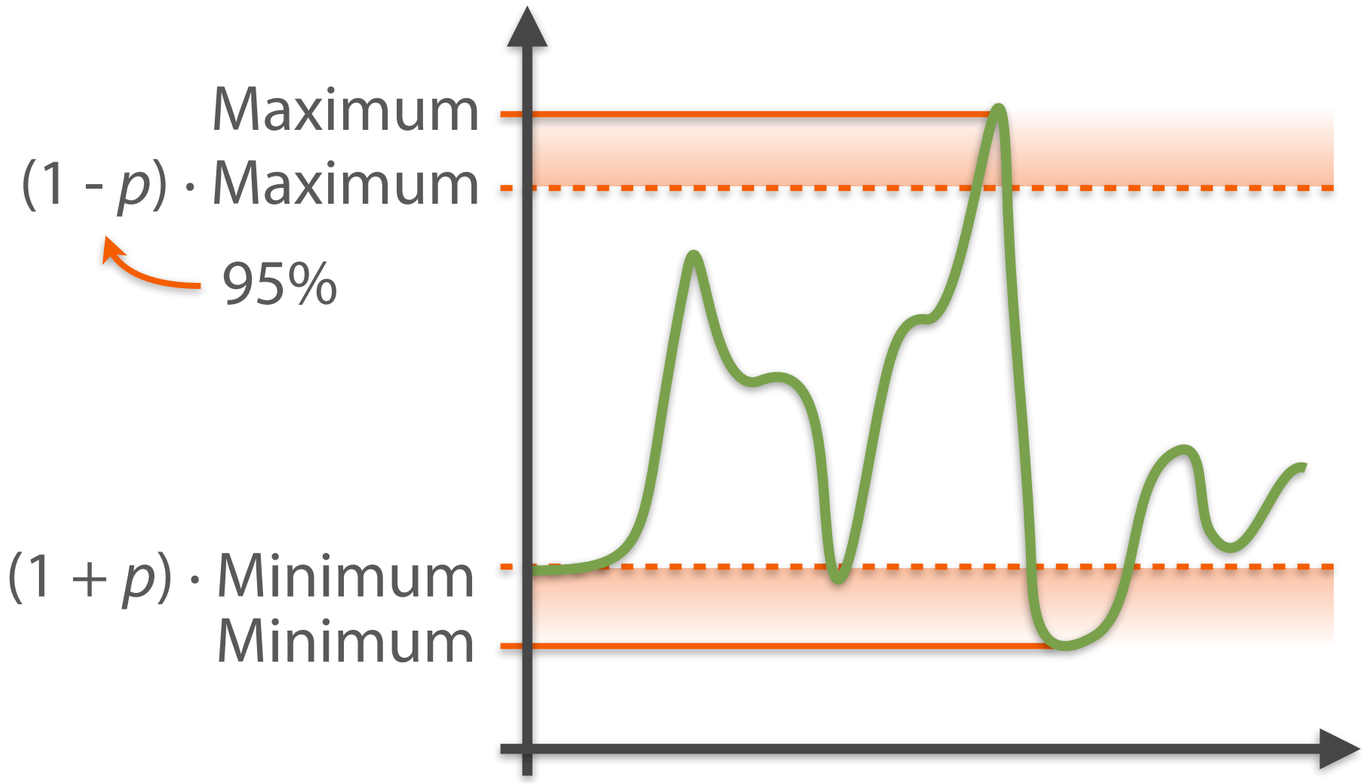

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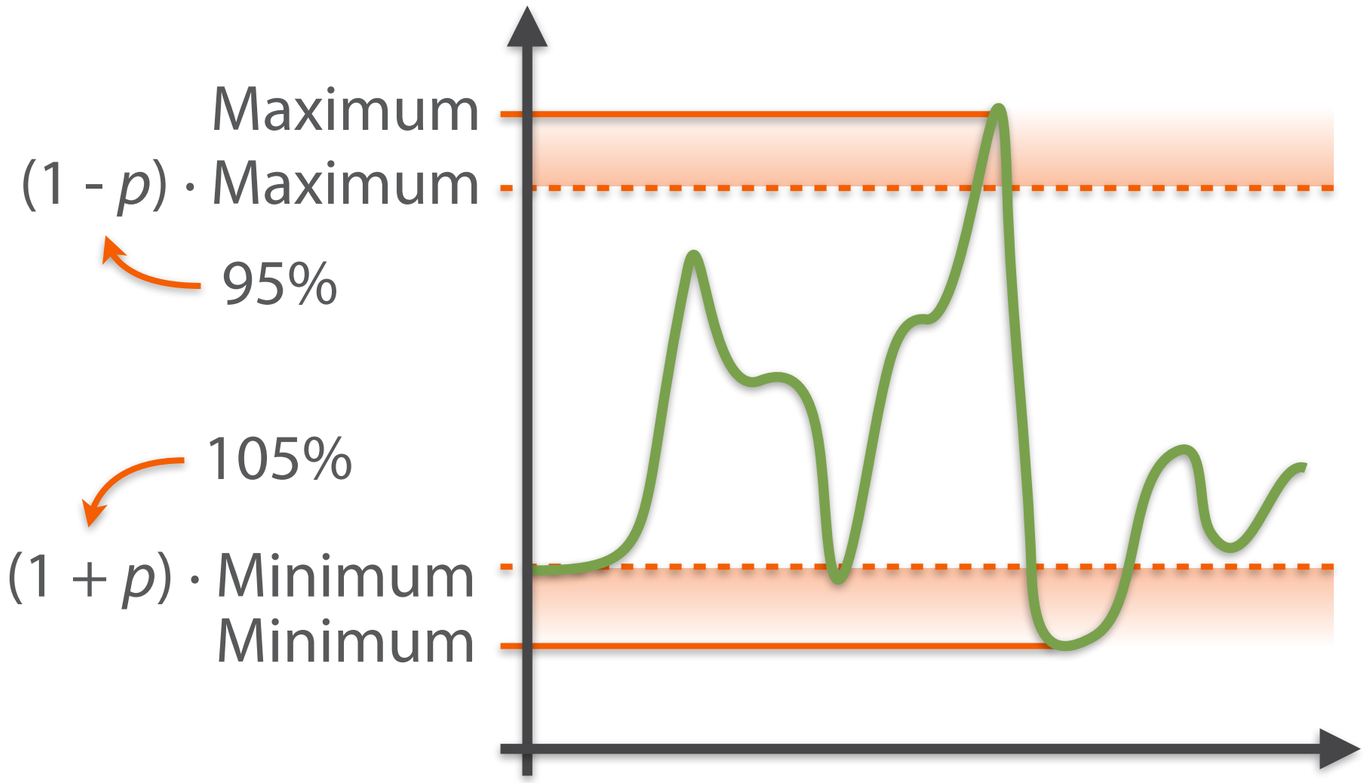

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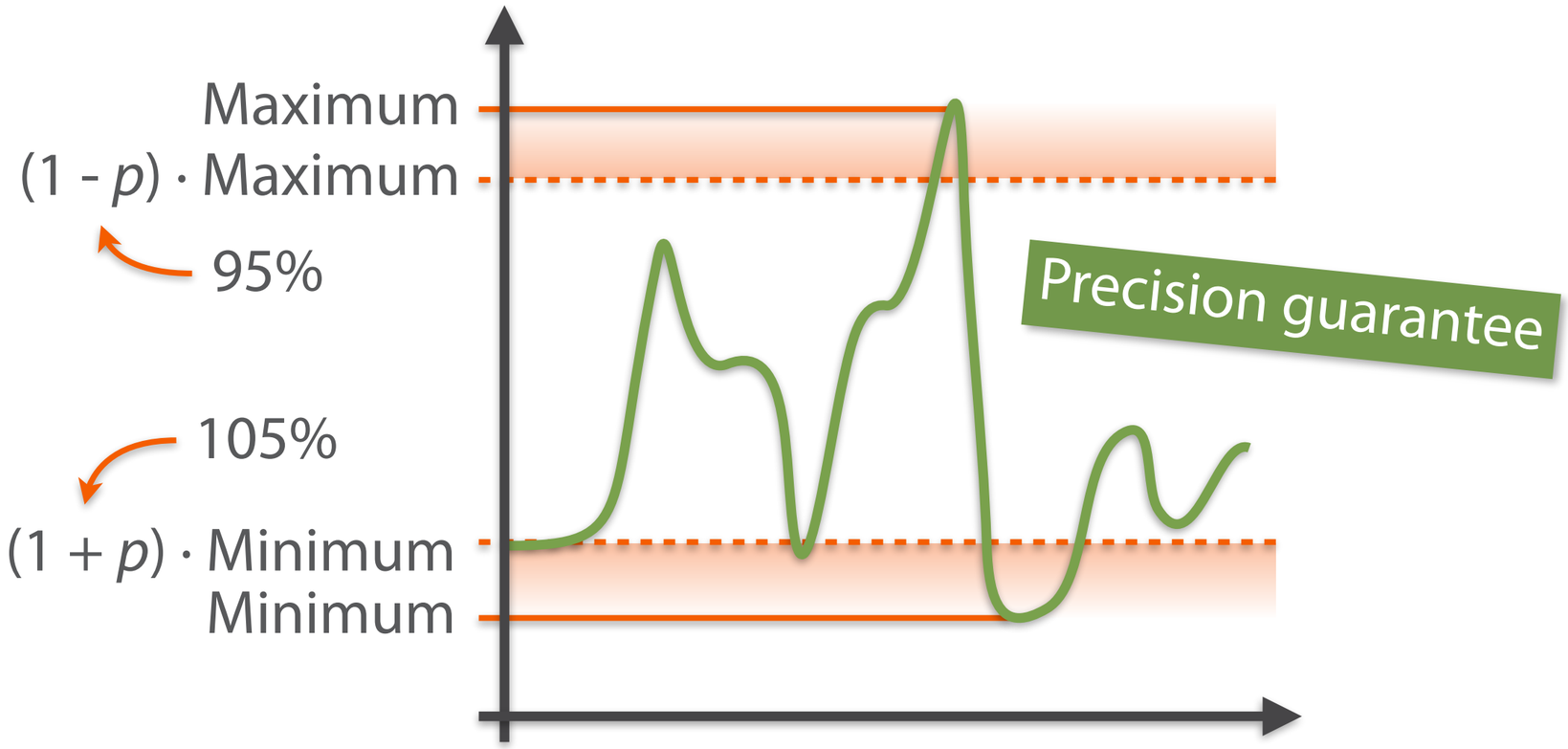


Approximation Algorithms

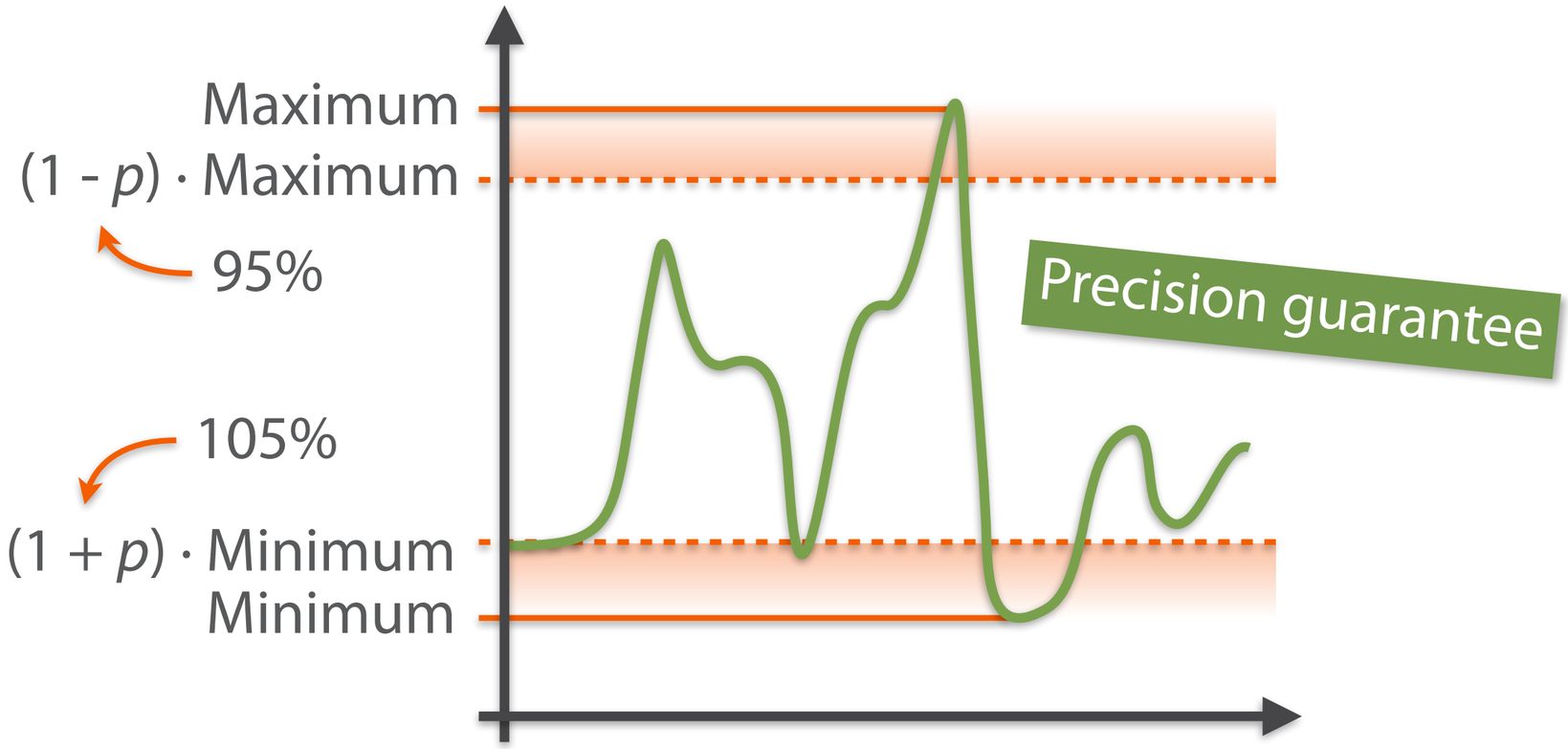
- for optimization problems

Runs in polynomial time

Polynomial-time approximation schemes (PTAS)

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



Approximation Algorithms

- for optimization problems

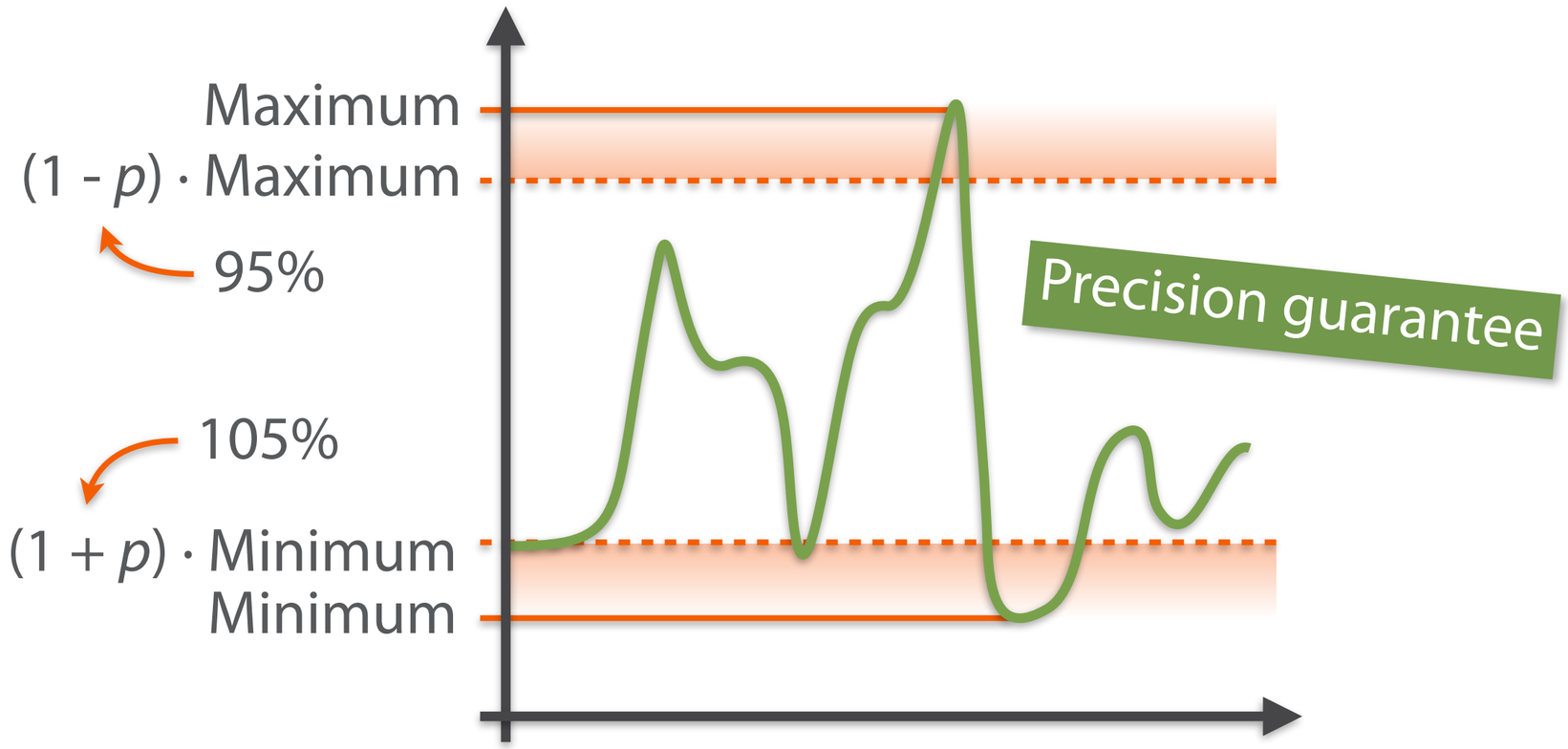
Runs in polynomial time

Polynomial-time approximation schemes (PTAS)

Complexity depends on error

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



Approximation Algorithms

- for optimization problems

Runs in polynomial time

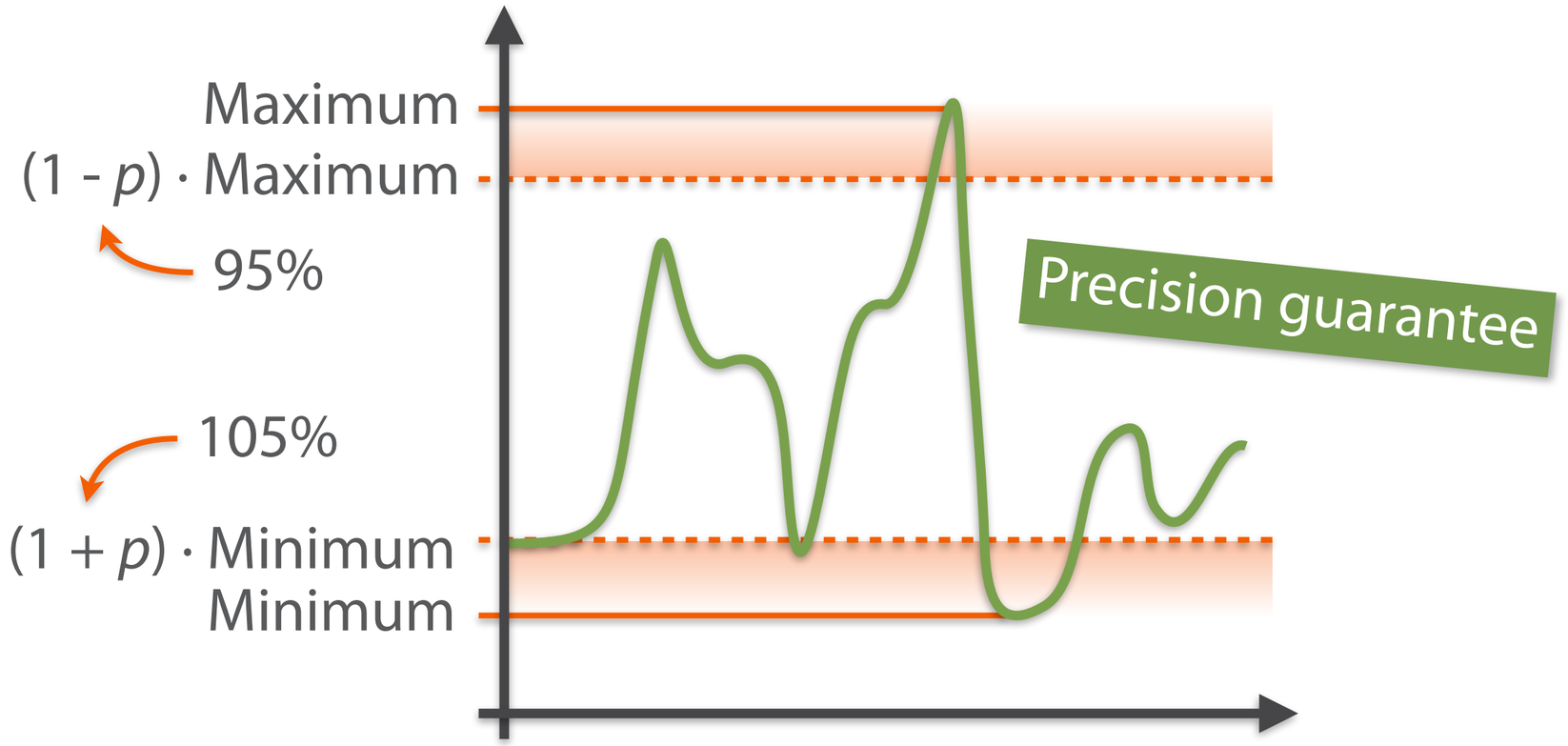
Polynomial-time approximation schemes (PTAS)

Complexity depends on error

$O(N^{1/p})$

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



Approximation Algorithms

- for optimization problems

Runs in polynomial time

Polynomial-time approximation schemes (PTAS)

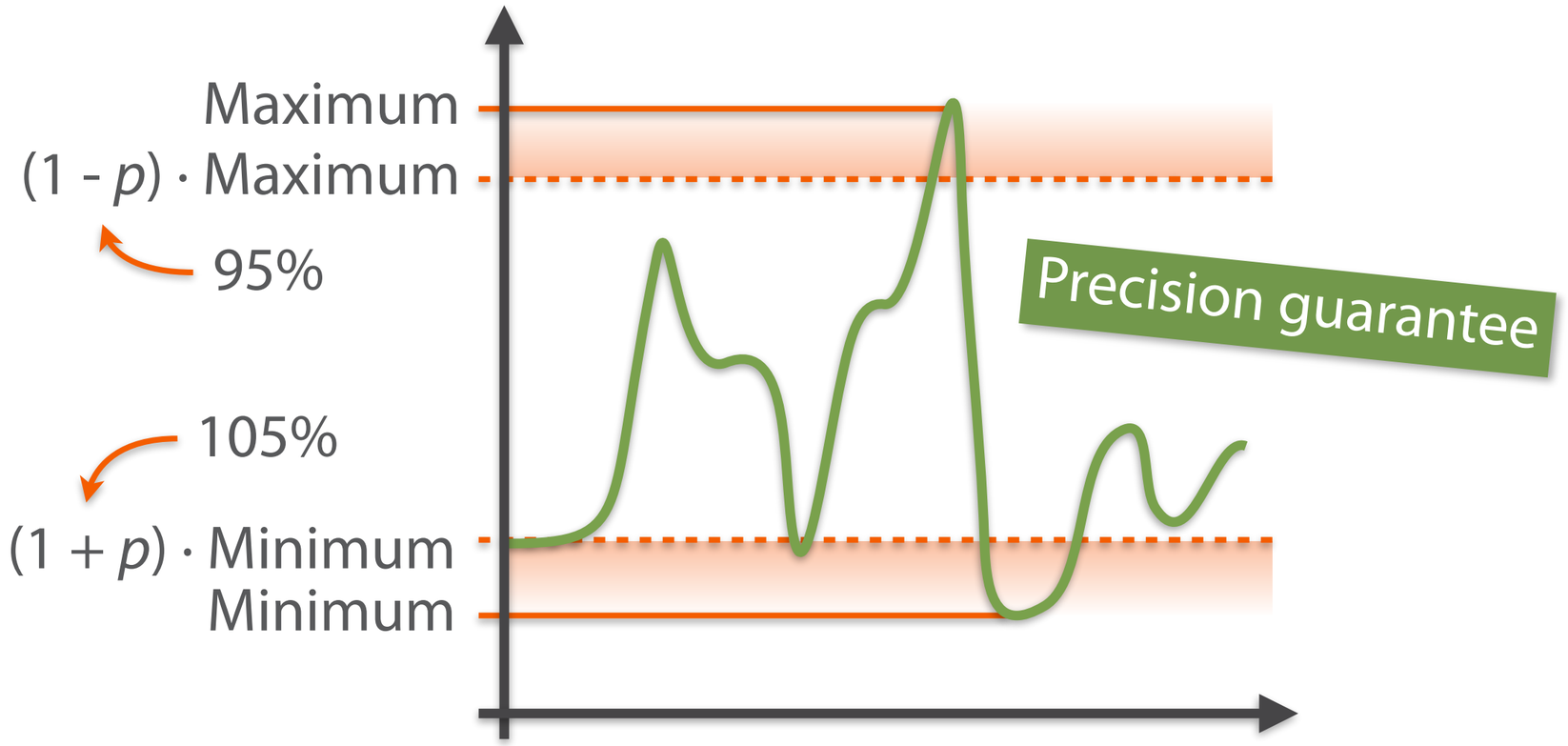
Complexity depends on error

$O(N^{1/p})$

$p = 0.1$: $O(N^{10})$

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



Approximation Algorithms

- for optimization problems

Runs in polynomial time

Polynomial-time approximation schemes (PTAS)

Complexity depends on error

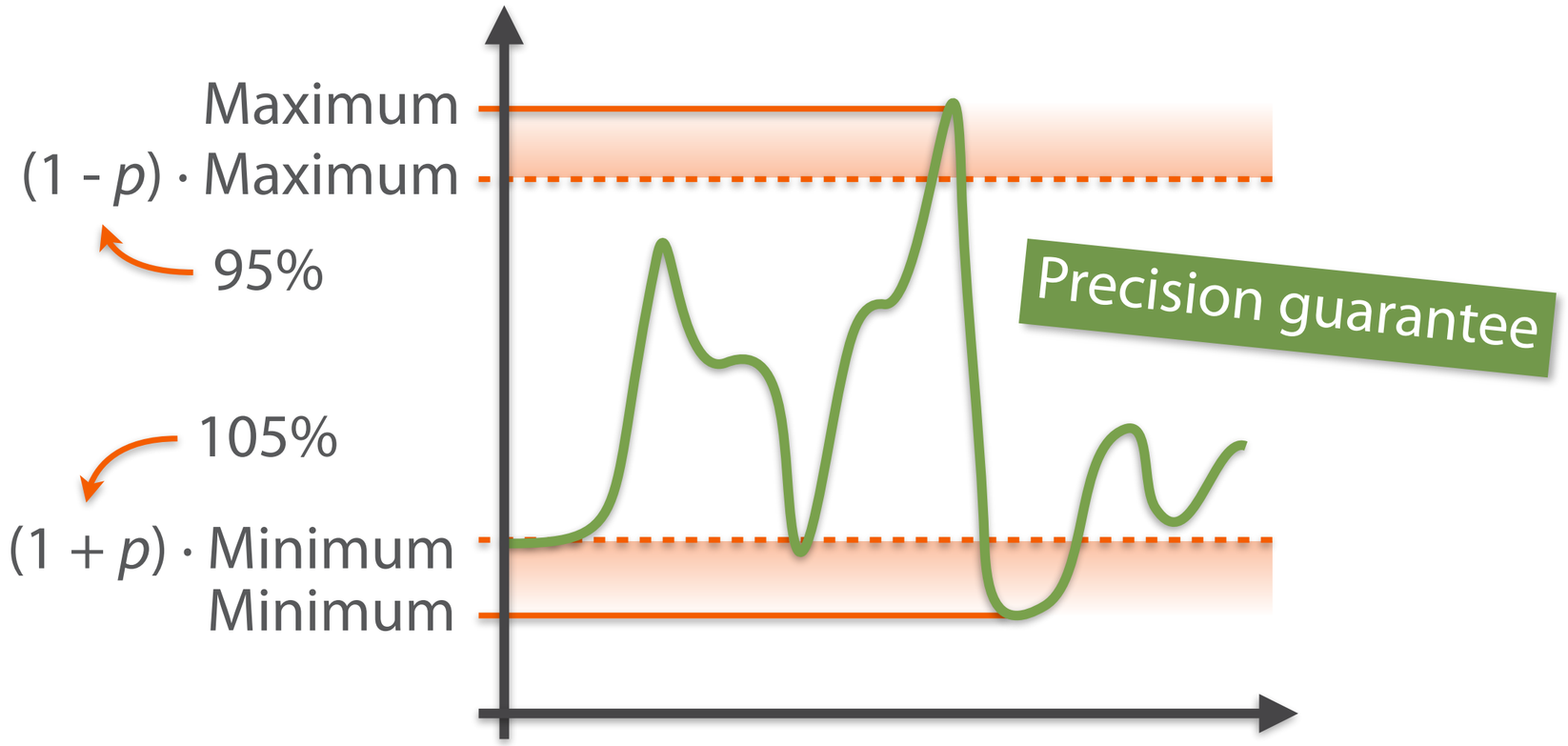
$O(N^{1/p})$

$p = 0.1$: $O(N^{10})$

$p = 0.05$: $O(N^{20})$

if $p = 0.05$

Allowed error: p $\quad 0 < p < 1$



Lessons Learned

Input size in #bits

P vs NP

Heuristics
and
approximation

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Complexity may cheat

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Input size in #bits

P: Solvable in polynomial time

P vs NP

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Input size in #bits

P: Solvable in polynomial time

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P vs NP

NP-hard: Generally awfully hard

Heuristics
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NP-hard: Generally awfully hard

P = NP unsolved!

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Exchange precision for speed

Heuristics and approximation

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P = NP unsolved!

Exchange precision for speed

Heuristics: Generally no worst-case

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