Semester Project

Press design

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Statement on Academic Integrity

By submitting this report, I attest that the work contained within is the work of my partner and myself, alone. I attest that I shared the workload as evenly as possible between my partner and myself. I am proud of my work. I did not commit plagiarism by submitting the work of another person as my own.

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1 Introduction

The following document details the design of a 6 bar mechanism that will serve as a punch press, the mechanism's general configuration can be seen on figure 1.

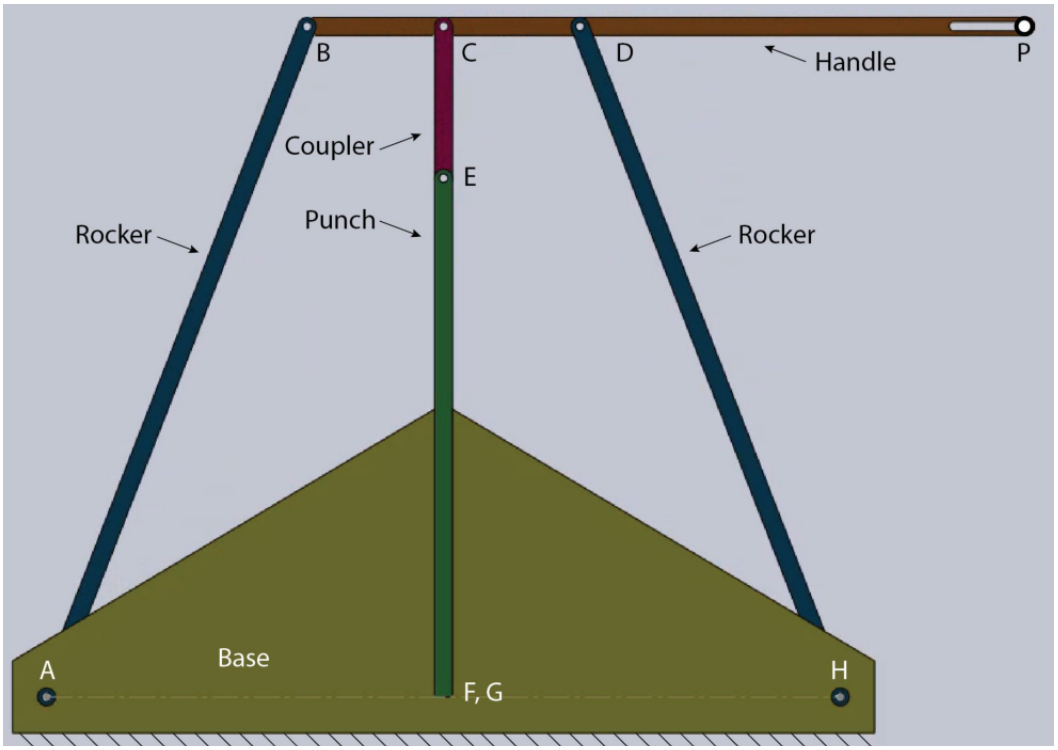


Figure 1: General configuration

The press purpose is to assemble two parts (placed at point G) together by means of press, interference or friction fit; this is done by having the user apply a force at point P that is then transmitted from the handle lever to the punch.

The main design objective is to not just be able to complete this motion, but to make it as easy as possible for the user to effectuate it, this was helped along by research provided by NASA and the US Department of Defense which (among other things) details the force that people can be expected to be able to exert in different positions.

However, a motion being easy to execute does not make it completely safe, as such investigating the possible injuries that prolonged use of the press may incur is of paramount importance in order to maintain the workers' physical health. The characteristics of these dangers were found with information obtained through the data available on the National Institute for Occupational Safety and Health's (NIOSH) home-

page which details the risk of work related upper limb musculoskeletal disorders, the most relevant kind of injury for a hand activated press.
Following that the document will quickly delve into the different factors that were modified to create different possible configurations of the press such as the punch's velocity and the dimensions of each link before finally moving into the details of the final configuration.

2 Ergonomic concerns

Stack, T., Ostrom, L. and Wilhelmsen, C. (2016) mentioned in their work Occupational Ergonomics that researchers have identified specific physical workplace risk factors involved in the development of Work-related Musculoskeletal Disorders (WMSDs). Exposure to these risk factors can result in:

- Decreased blood flow to muscles, nerves and joints;
- Nerve compression;
- Tendon or tendon sheath damage;
- Muscle, tendon, or ligament sprain or strain;
- Joint damage.

Also, they also say that prolonged exposure to the physical workplace risk factors can lead to permanent damage and a debilitating condition. When present for sufficient duration, frequency, or magnitude, physical workplace risk factors may contribute to the development of WMSDs. In addition, personal risk factors, such as physical conditioning, existing health problems, gender, age, work technique, hobbies, and organizational factors contribute to, but do not cause the development of WMSDs.
High force controls shall not be used except when the operator's nominal working position provides proper body support or limb support or both, e.g., seat backrest, foot support. Sustained (i.e., durations longer than 3 seconds) high force requirements shall be avoided.
That is why we need to take care about how the operator is going to be positioned when actuating the mechanism because he takes long journeys of work.
To this end, after having reviewed in detail various sources and information, the forces shown correspond to the fifth percentile of male data, we have found the following:

- When actuating high forces we can make use of the hands or feet;

- Using the hands we find that the pulling motion provides greater forces than vertical motions such as up or bottom.

- The pulling motion is going to have greater force when the elbow has a bigger degree of flexion.

We are looking for movements that provide high forces in neutral positions of the body because with that high forces we can consider the fifth percentile as a maximum.

3 Mechanism parameters

The mechanism has several base requirements that must be met, these are:

1. The punch must move from 4cm to 2cm.
2. The movement must occur within the span of one second.
3. Point P, where we expect the user to apply the force, must have an ergonomically reasonable path.
4. Points A, G & H must be colinear and within 120cm of each other.

The mechanism also has the following basic proportions, assuming X is in meters.

$\overline{AB}$	1x
$\overline{BC}$	0.19 x
$\overline{CD}$	0.19 x
$\overline{CE}$	0.211 x
$\overline{EF}$	0.721 x
$\overline{AG}$	0.5525 x
$\overline{GH}$	0.5525 x

Table 1: Base proportions

However, these proportions serve merely as a starting point and meeting them is not a strict requirement. Taking all of this into account, we have the following parameters that we may modify in order to meet the requirements.

1. Punch position, velocity and acceleration over 1 second.
    - Determines the motion of point P over time, modifying the ergonomics.

2. Link lengths.

    - Left and right rockers.
      Determine the press' minimum height and the handle's resting position.
    - Handle.
      Biggest determining factor for the press' horizontal length, greater length gives the user greater mechanical advantage, making the press' use easier.
    - Base length.
      Determines the minimum space required for the press to be installed, also affects the rockers' potential angles.
    - Coupler.
      Anchors the punch to the handle, determines the punch's movement range.
    - Punch.
      Size determines the movement range alongside the coupler.

It is worth noting that while proper materials and link dimensions were chosen for the final design, preliminary designs were simply tested as being square iron bars with a 2cm x 2 cm square cross section, thus their mass was simply calculated as:

$$m = 0.0004 \text{ m}^2 \cdot 7900 \text{ kg/m}^3 \cdot l \text{ m}$$

3.1 Punch functions

We may construct a function to define the movement of the punch over the second of operation, five different functions were chosen for testing purposes, details on their derivation may be found in the appendix.

1. Constant movement.

$$y = \dot{y} \cdot t \qquad \dot{y} = -2 \text{ cm/s} \qquad \ddot{y} = 0$$

   Despite the promised null acceleration, the punch and handle must still gain velocity from a starting resting position, this apparent contradiction results in "infinite" acceleration at the beginning and end of the movement that would result in a similarly infinite jerk that the user would feel as a shock when actuating the press so this profile is generally inferior to the rest.

2. Cosine defined movement.

$$y = a\cos bt + c \qquad \dot{y} = -ab\sin bt + c \qquad \ddot{y} = -ab^2\cos bt + c$$
$$a = \text{Free choice} \qquad b = \cos^{-1}\left(\frac{2}{a}\right) - c \qquad c = \cos^{-1}\left(\frac{4}{a}\right)$$

3. Sine defined movement.

$$y = a\sin bt + c \qquad \dot{y} = ab\cos bt + c \qquad \ddot{y} = -ab^2\sin bt + c$$
$$a = \text{Free choice} \qquad b = \sin^{-1}\left(\frac{2}{a}\right) - c \qquad c = \sin^{-1}\left(\frac{4}{a}\right)$$

4. Exponential defined movement.

$$y = -de^{at+b} + c \qquad \dot{y} = -ade^{at+b} \qquad \ddot{y} = -a^2de^{at+b}$$
$$d, b = \text{Free choice} \qquad c = 4 + de^b \qquad a = \ln\left(\frac{c-2}{d}\right) - b$$

5. Quadratic defined movement.

$$y = at^2 + bt + c \qquad \dot{y} = 2at + b \qquad \ddot{y} = 2a$$
$$b = \text{Free choice} \qquad c = 4 \qquad a = -b - 2$$

Manipulation of the free parameters within desmos led to the following graphs being chosen:

Cosine defined movement	a = 5
Sine defined movement	a = 4
Exponential defined movement	b = 0, d = 1
Quadratic defined movement	b = -4

Table 2: Movement function free coefficients

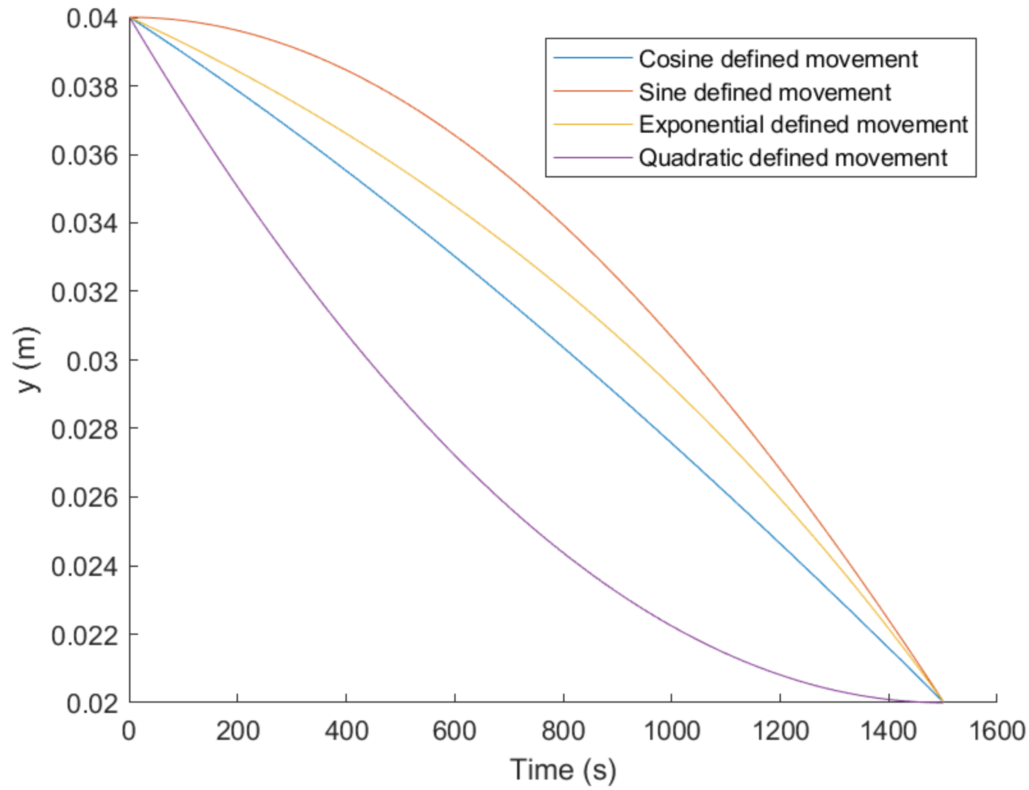


Figure 2: Movement Functions

These graphs were chosen visually through experimentation for their possible effects on the velocity, acceleration and jerk of point P.

3.2 Link lengths

Having chosen a set of functions to define the movement of the punch, it is now possible to choose the link lengths and find a configuration that can meet all of the requirements. Three different preliminary configurations were tested.

It is worth noting that while the function that defines y affects the jerk, it does not affect the force required to operate the press, so a function will not be chosen until an apt mechanism configuration has been found.

3.2.1 Configuration 1 (Base mechanism)

The first attempt simply left the mechanism in its initial configuration and assessed whether the results were appropriate. This had the following results.

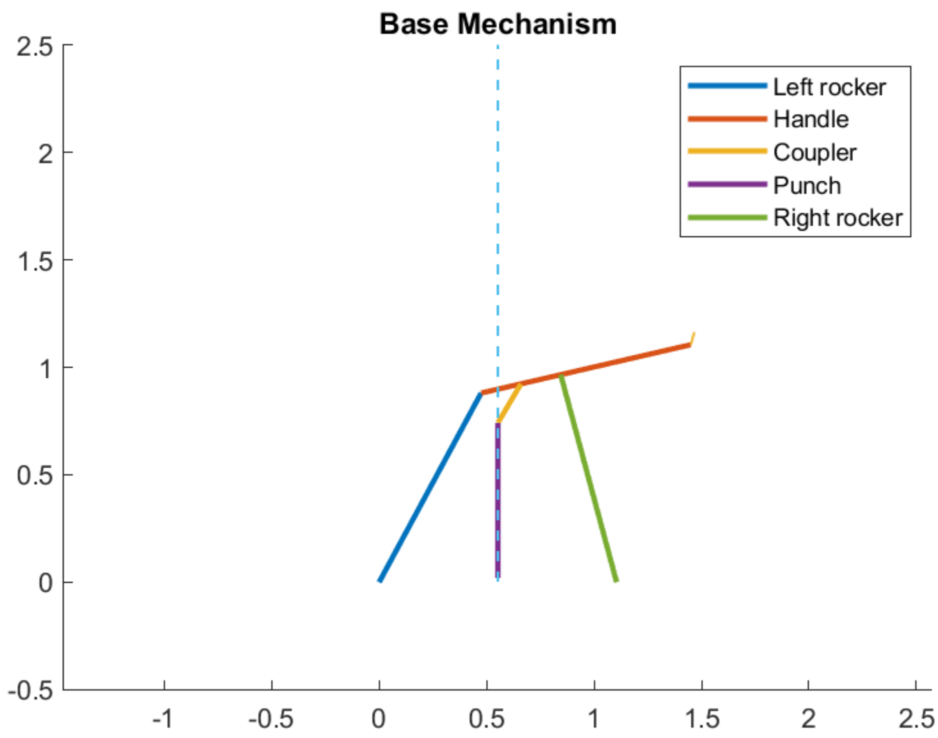


Figure 3: Base mechanism

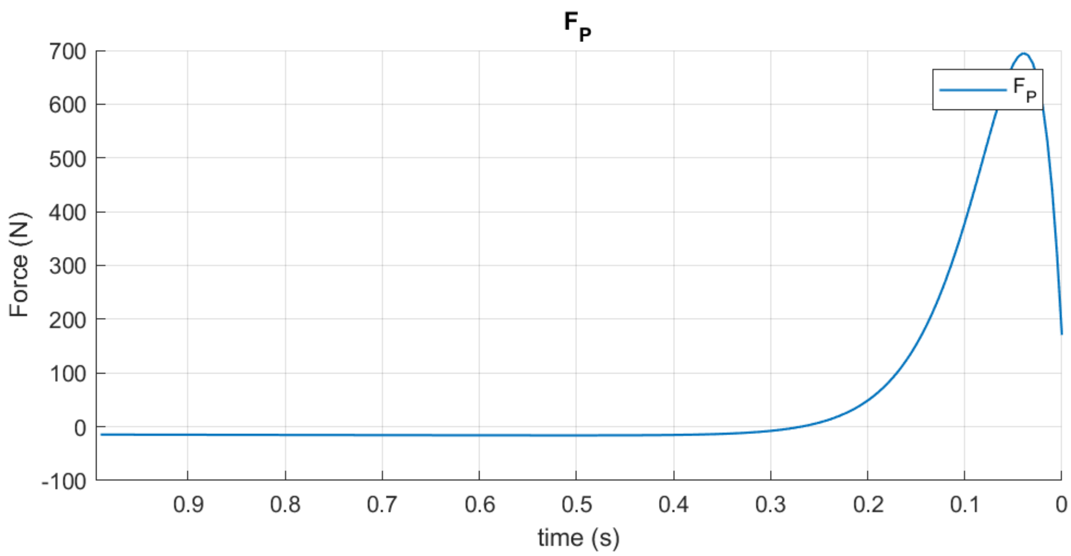


Figure 4: Base mechanism Fp

This press meets all of the directly stated requirements, having a nearly vertical movement path at point P and apt dimensions, however, it requires the user to apply a force of 700 Newtons, an act that may not be expected from the 5th percentile of people.

3.2.2 Configuration 2

This configuration tried to reduce the size of the body to reduce its mass, hoping to force a reduction of the forces affecting it.

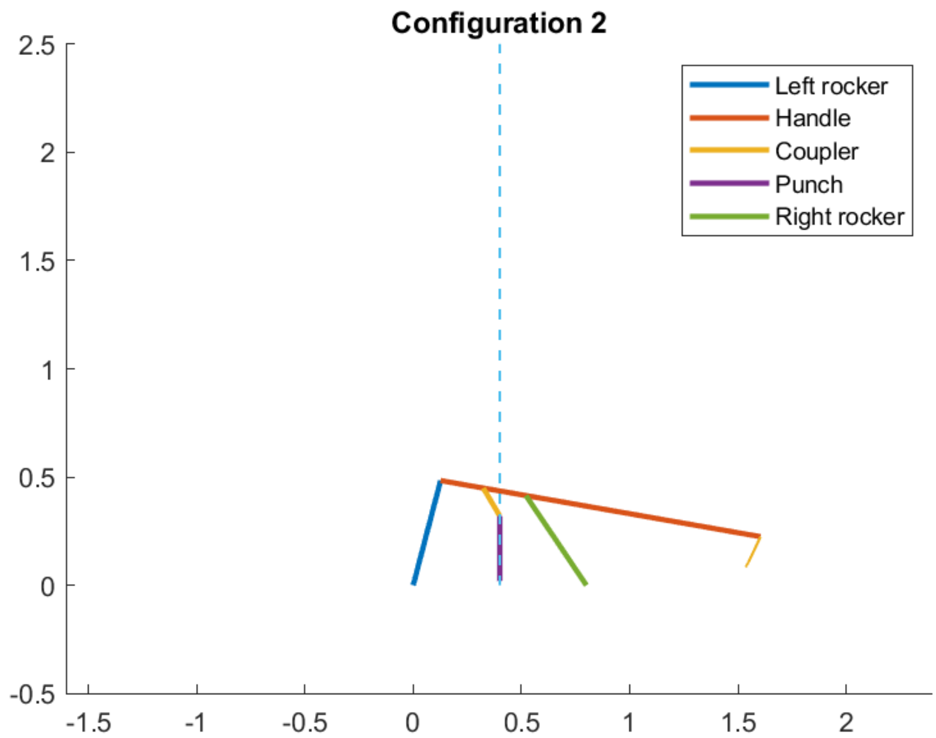


Figure 5: Configuration 2

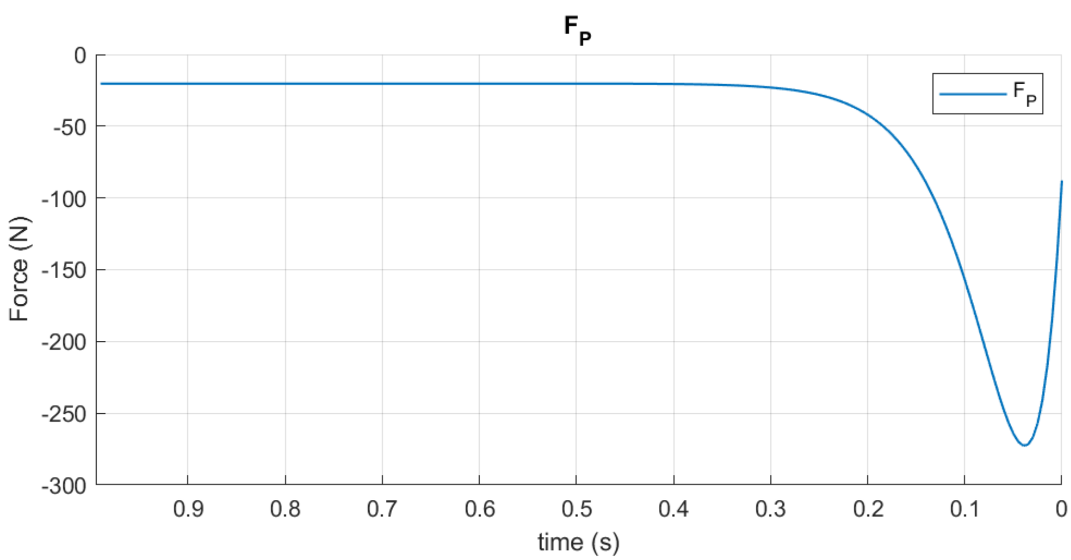


Figure 6: Configuration 2 Fp

While the force reduction was successful it is still not within the expectations for the 5th percentile of people, moreover the handle must now be pulled up instead of pushed down in order to complete the movement (an animation of this may be found within the appendix) which may result in awkward operation. As such it was decided to try a third configuration.

3.2.3 Configuration 3

This third configuration maintained most of the proportions from the initial mechanism, simply changing the lengths of the handle and right rocker, this had the following result.

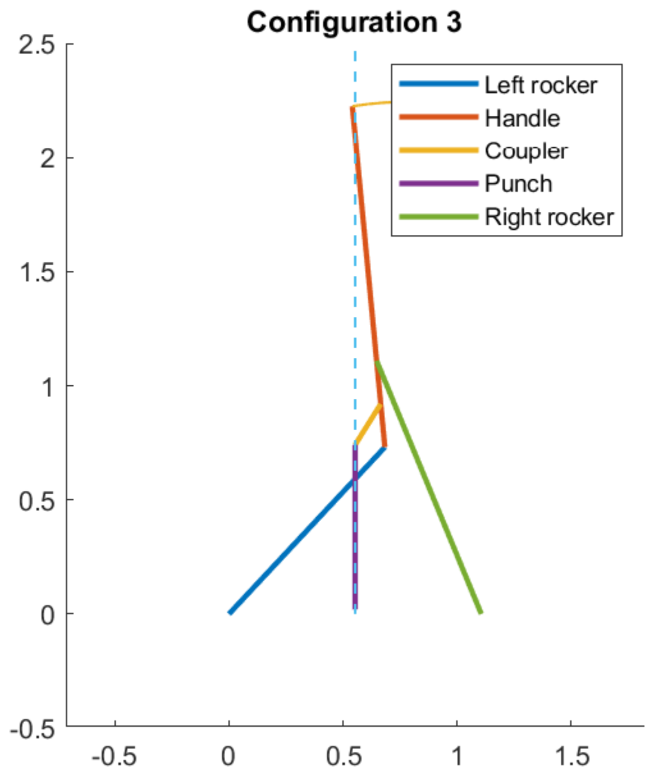


Figure 7: Configuration 3

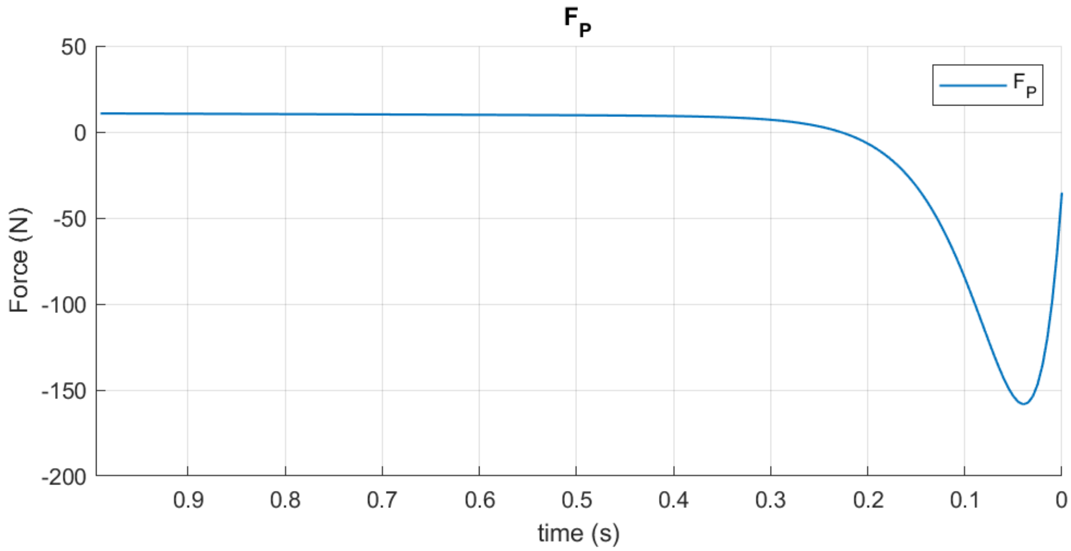


Figure 8: Configuration 3 Fp

This motion was mostly coincidental and the mechanism is quite clearly impossible to operate comfortably for most people due to its height (although this problem may be alleviated by designing a proper console), however, this configuration requires the user to input a force that is well within the 5th percentile due to the motion being almost completely horizontal. As such, the final mechanism will be a modification of this configuration.

4 Final design parameters

Following the 3 initial designs, the final design looked to maximize the length of point P's path as this reduced the force required.

This resulted in a mechanism with the following dimensions

Left rocker	0.75 m
Right rocker	0.75 m
Left rocker - Coupler	0.25 m
Right rocker - Coupler	0.15 m
Handle	1 m
Coupler	0.35 m
Punch	0.25 m
Left half of base	0.65 m
Right half of base	0.55 m

Table 3: Base proportions

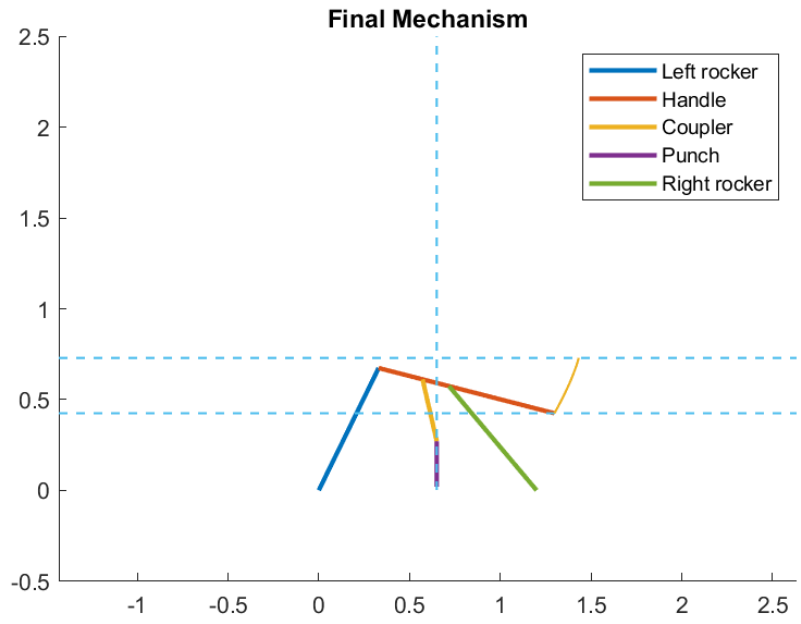


Figure 9: Final mechanism

4.1 Mobility design

The final mechanism may be represented via the following kinematic diagram.

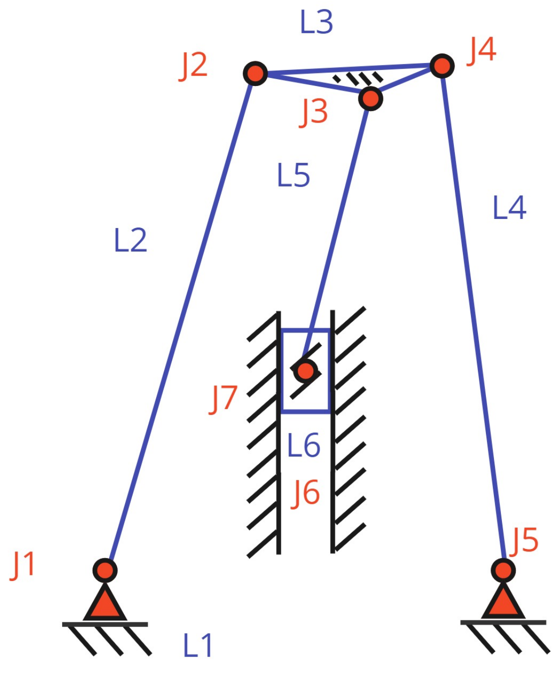


Figure 10: Final mechanism kinematic diagram

The link classes are the following.

Base (L1)	Ternary
Left rocker (L2)	Binary
Right rocker (L4)	Binary
Handle (L3)	Ternary
Coupler (L5)	Binary
Punch (L6)	Binary

Table 4: Final mechanism link classes

We require the mechanism to have a mobility of 1 in order for it to be possible to actuate by a single movement of the operator, we can determine whether our mechanism accomplishes this or not by using the Gruebler condition.

$$m = 3L - 2J - 3G$$

$$m = 3 \cdot 6 - 2 \cdot 7 - 3 \cdot 1 = 18 - 14 - 3$$

$$m = 1$$

While the press is a six bar mechanism, we can identify one four bar within it acting as a component.

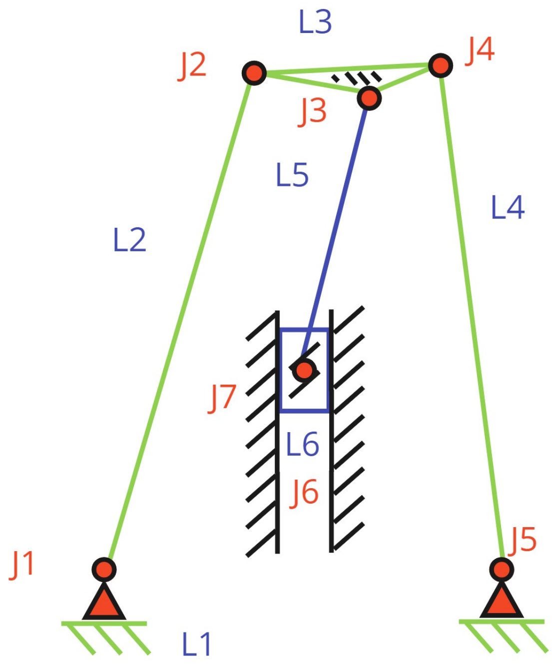


Figure 11: Final mechanism component four bar

We can classify this four bar mechanism as being Grashof or not by making use of the Grashof clasification.

$$S + L < P + Q$$

$$0.4 + 1.2 < 0.75 + 0.75$$

$$1.6 \not< 1.5$$

The four bar mechanism within the press is not Grashof.

Four Bar ABDH	RRR1

Table 5: Final mechanism link classes

4.2 Kinematic design

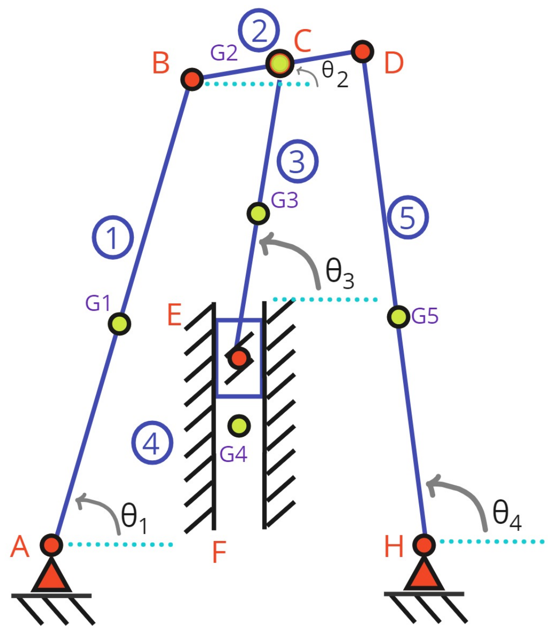


Figure 12: Final mechanism angle labels

The mechanism's respective angles, angular velocities and angular accelerations, as labeled on figure 12 over the second make use of the following vector loop equations.

Position vector loop equations

$$a\cos\theta_1 + \frac{b}{2}\cos\theta_2 - l\cos\theta_3 - \frac{d}{2} = 0 \quad \longleftarrow \textbf{Loop 1 Real component}$$

$$a\sin\theta_1 + \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y = 0 \quad \longleftarrow \textbf{Loop 1 Imaginary component}$$

$$c\cos\theta_4 - \frac{b}{2}\cos\theta_2 - l\cos\theta_3 + \frac{d}{2} = 0 \quad \longleftarrow \textbf{Loop 2 Real component}$$

$$c\sin\theta_4 - \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y = 0 \quad \longleftarrow \textbf{Loop 2 Imaginary component}$$

Velocity vector loop equations

$$-a\omega_1\sin\theta_1 - \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3 = 0 \quad \longleftarrow \textbf{Loop 1 Real component}$$

$$a\omega_1\cos\theta_1 + \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y} = 0 \quad \longleftarrow \textbf{Loop 1 Imaginary component}$$

$$-c\omega_4\sin\theta_4 + \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3 = 0 \quad \longleftarrow \textbf{Loop 2 Real component}$$

$$c\omega_4\cos\theta_4 - \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y} = 0 \quad \longleftarrow \textbf{Loop 2 Imaginary component}$$

Acceleration vector loop equations

$$-a\alpha_1\sin\theta_1 - a\omega_1^2\cos\theta_1 - \frac{b}{2}\alpha_2\sin\theta_2 - \frac{b}{2}\omega_2^2\cos\theta_2 + l\alpha_3\sin\theta_3 + l\omega_3^2\cos\theta_3 = 0 \quad \longleftarrow \textbf{Loop 1 Real component}$$

$$a\alpha_1 \cos\theta_1 - a\omega_1^2 \sin\theta_1 + \frac{b}{2}\alpha_2 \cos\theta_2 - \frac{b}{2}\omega_2^2 \sin\theta_2 - l\alpha_3 \cos\theta_3 + l\omega_3^2 \sin\theta_3 - \ddot{y} = 0 \longleftarrow \textbf{Loop 1 Imaginary component}$$

$$-c\alpha_4 \sin\theta_4 - c\omega_4^2 \cos\theta_4 + \frac{b}{2}\alpha_2 \sin\theta_2 + \frac{b}{2}\omega_2^2 \cos\theta_2 + l\alpha_3 \sin\theta_3 + l\omega_3^2 \cos\theta_3 = 0 \quad \longleftarrow \textbf{Loop 2 Real component}$$

$$c\alpha_4 \cos\theta_4 - c\omega_4^2 \sin\theta_4 - \frac{b}{2}\alpha_2 \cos\theta_2 + \frac{b}{2}\omega_2^2 \sin\theta_2 - l\alpha_3 \cos\theta_3 + l\omega_3^2 \sin\theta_3 - \ddot{y} = 0 \longleftarrow \textbf{Loop 2 Imaginary component}$$

Tangential Acceleration Equations

$$a_{t1} = -a\alpha_1 \sin\theta_1 + a\alpha_1 \cos\theta_1$$

$$a_{t2} = -\frac{b}{2}\alpha_2 \sin\theta_2 + \frac{b}{2}\alpha_2 \cos\theta_2$$

$$a_{t3} = l\alpha_3 \sin\theta_3 - l\alpha_3 \cos\theta_3$$

$$a_{t4} = -c\alpha_4 \sin\theta_4 + c\alpha_4 \cos\theta_4$$

Centripetal Acceleration Equations

$$a_{c1} = -a\omega_1^2 \cos\theta_1 - a\omega_1^2 \sin\theta_1$$

$$a_{c2} = -\frac{b}{2}\omega_2^2 \cos\theta_2 - \frac{b}{2}\omega_2^2 \sin\theta_2$$

$$a_{c3} = l\omega_3^2 \cos\theta_3 + l\omega_3^2 \sin\theta_3$$

$$a_{c4} = -c\omega_4^2 \cos\theta_4 - c\omega_4^2 \sin\theta_4$$

The derivation of these vector loop equations may be found within the appendix.

Their results are the following.

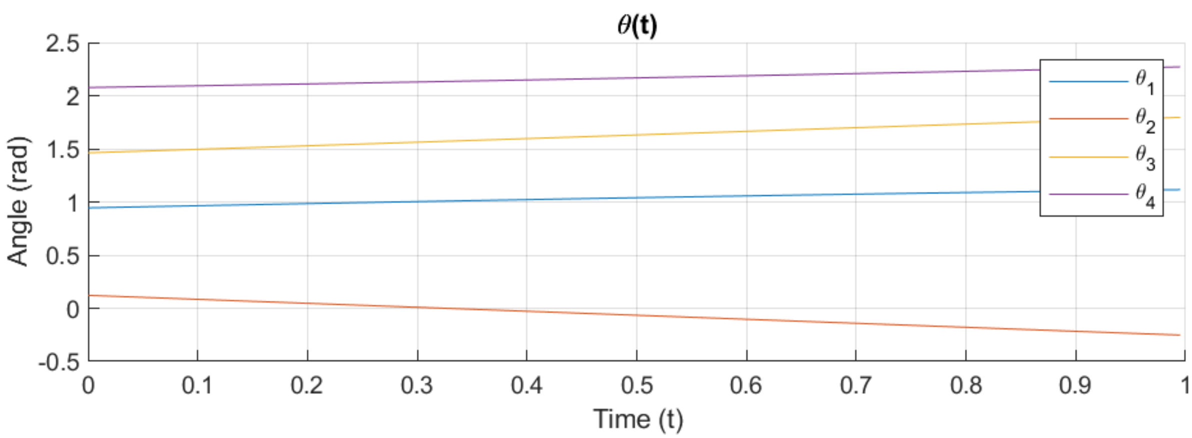


Figure 13: Final mechanism angles over time

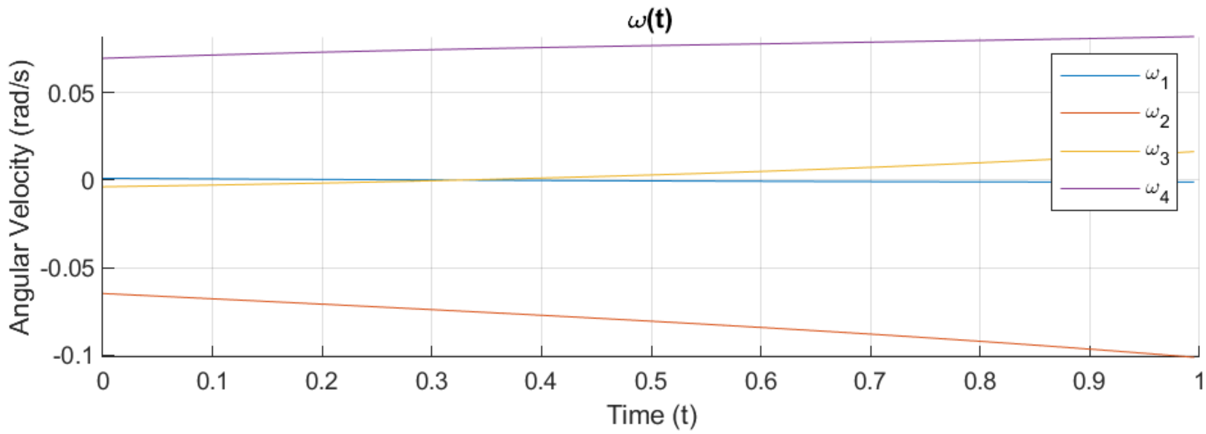


Figure 14: Final mechanism angular velocities over time

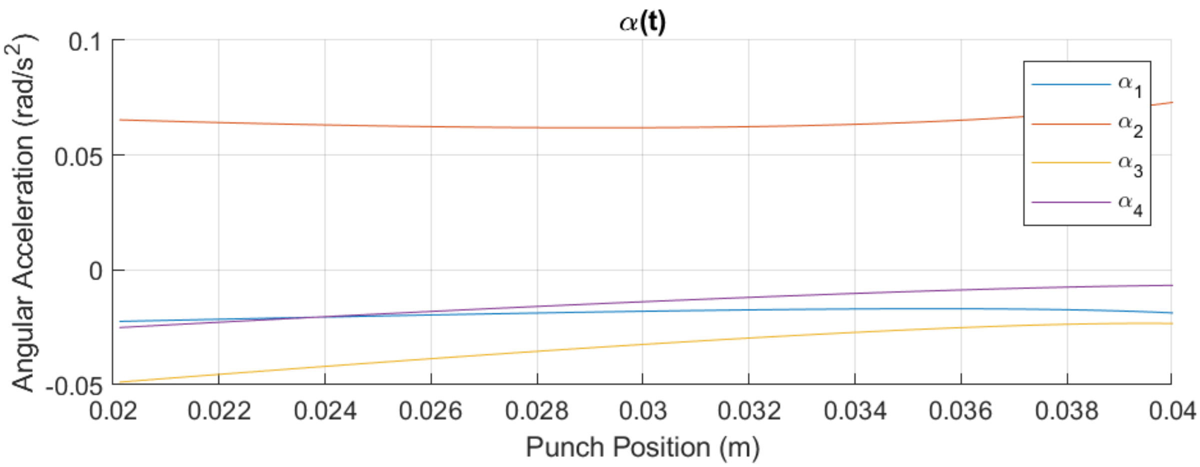


Figure 15: Final mechanism angular accelerations over time

The position, velocity and accelerations of each of the links' centers of mass make use of the following equations, their full derivation may be found within the appendix.

Center of Mass Positions

$$\vec{R}_{G_1/A} = \frac{\overline{AB}}{2} e^{j\theta_1}$$

$$\vec{R}_{G_2/B} = \overline{BP} e^{j\theta_2}$$

$$\vec{R}_{G_3/E} = \frac{\overline{CE}}{2} e^{j\theta_3}$$

$$\vec{R}_{G_4/F} = \frac{\overline{EF}}{2}$$

$$\vec{R}_{G_5/H} = \frac{\overline{DH}}{2} e^{j\theta_4}$$

Center of Mass Velocities

$$\dot{\vec{R}}_{G_1/A} = j\frac{\overline{AB}}{2}\omega_1 e^{j\theta_1}$$

$$\dot{\vec{R}}_{G_2/B} = j\overline{BP}\omega_2 e^{j\theta_2}$$

$$\dot{\vec{R}}_{G_3/E} = j\frac{\overline{CE}}{2}\omega_3 e^{j\theta_3}$$

$$\dot{\vec{R}}_{G_4/F} = 0$$

$$\dot{\vec{R}}_{G_5/H} = j\frac{\overline{DH}}{2}\omega_4 e^{j\theta_4}$$

Center of Mass Accelerations

$$\ddot{\vec{R}}_{G_1/A} = j\frac{\overline{AB}}{2}\alpha_1 e^{j\theta_1} - \frac{\overline{AB}}{2}\omega_1^2 ej\theta_1$$

$$\ddot{\vec{R}}_{G_2/B} = j\overline{BP}\alpha_2 e^{j\theta_2} - \overline{BP}\omega_2^2 ej\theta_2$$

$$\ddot{\vec{R}}_{G_3/E} = j\frac{\overline{CE}}{2}\alpha_3 e^{j\theta_3} - \frac{\overline{CE}}{2}\omega_3^2 ej\theta_3$$

$$\ddot{\vec{R}}_{G_4/F} = 0$$

$$\ddot{\vec{R}}_{G_5/H} = j\frac{\overline{DH}}{2}\alpha_4 e^{j\theta_4} - \frac{\overline{DH}}{2}\omega_4^2 ej\theta_4$$

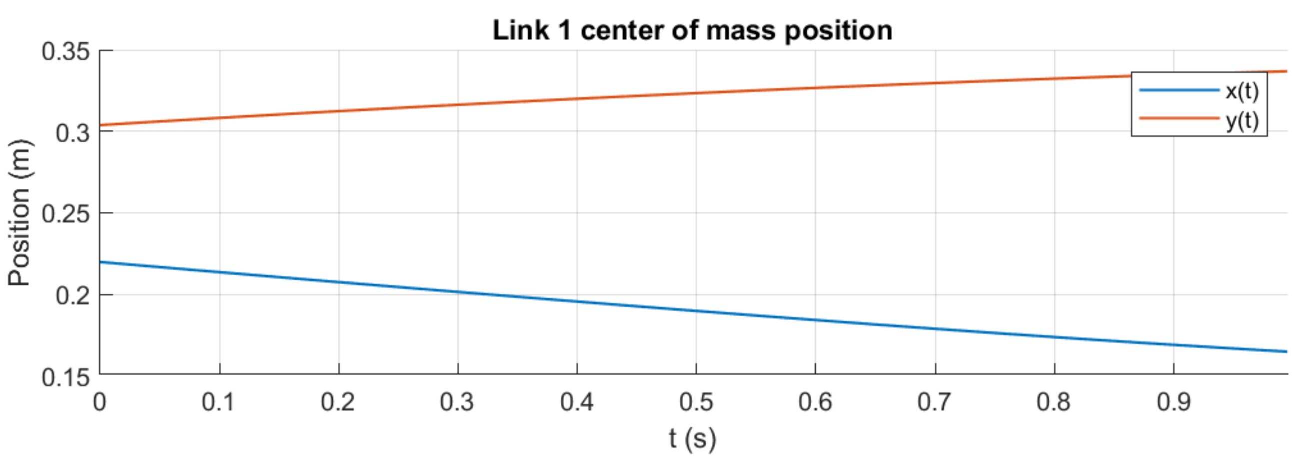


Figure 16: Link 1 center of mass position

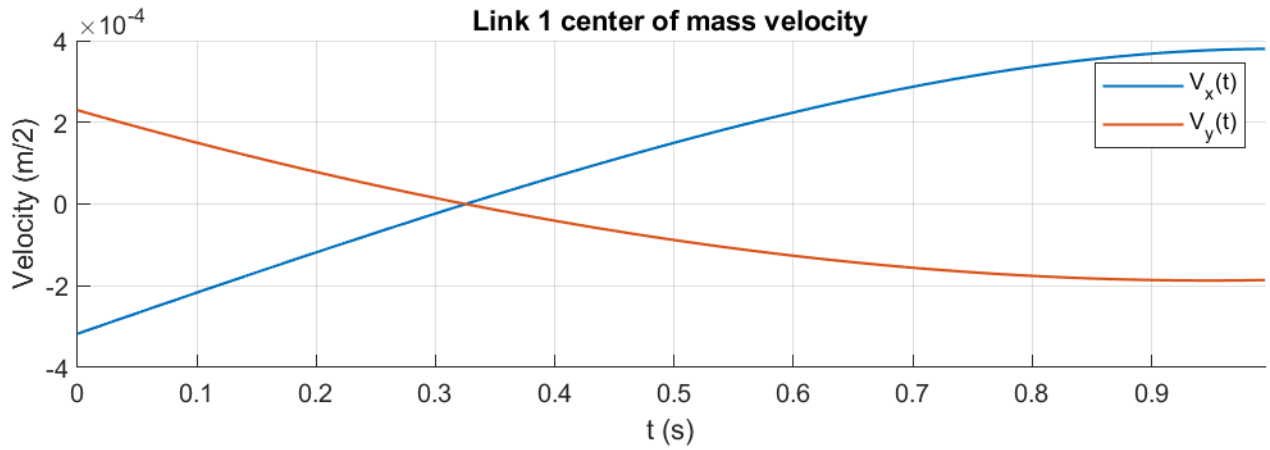


Figure 17: Link 1 center of mass velocity

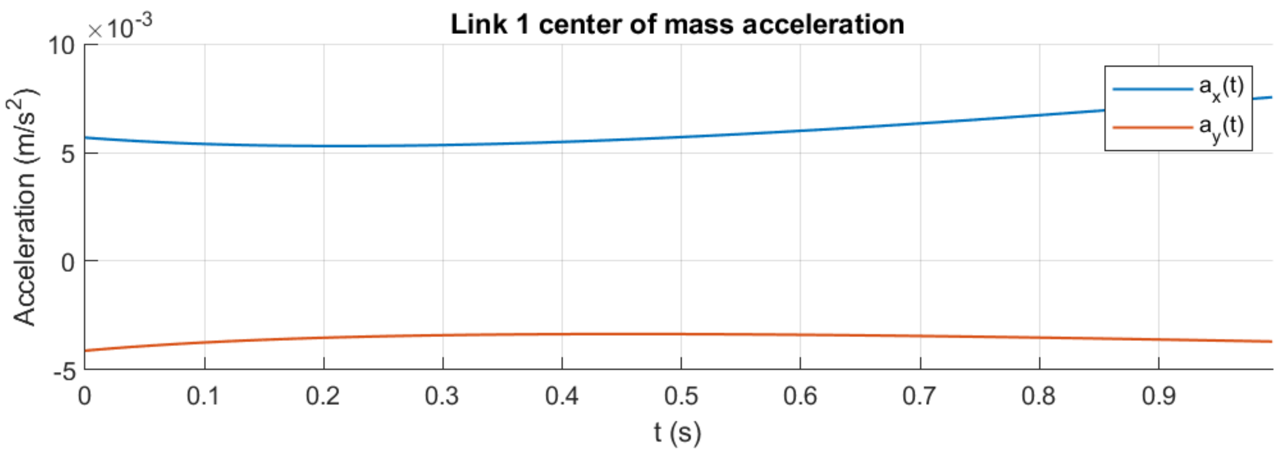


Figure 18: Link 1 center of mass acceleration

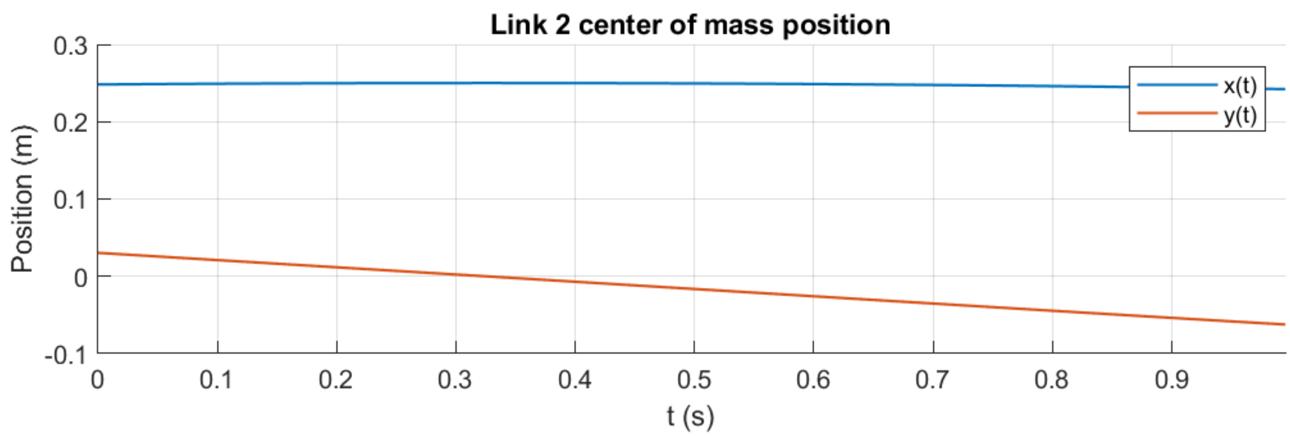


Figure 19: Link 2 center of mass position

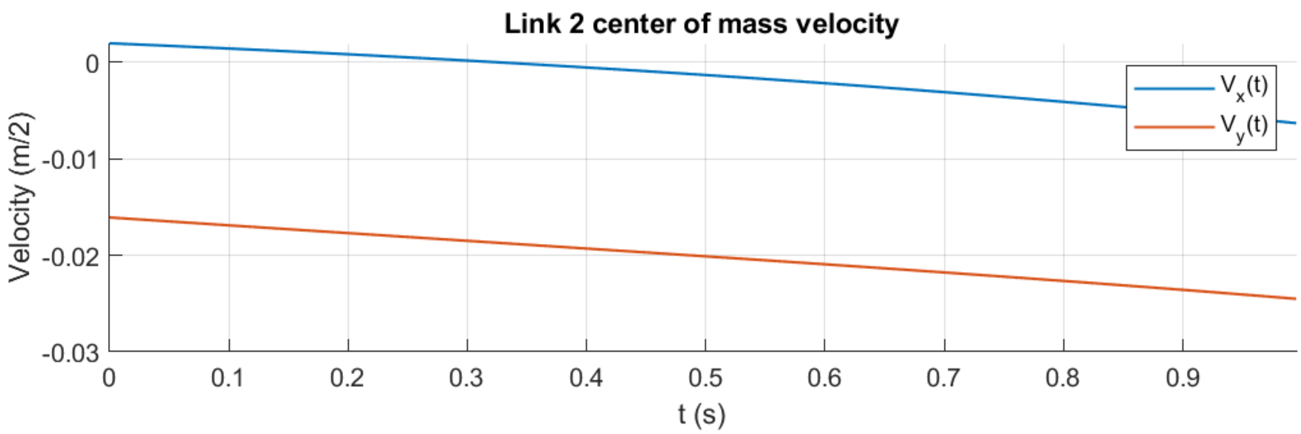


Figure 20: Link 2 center of mass velocity

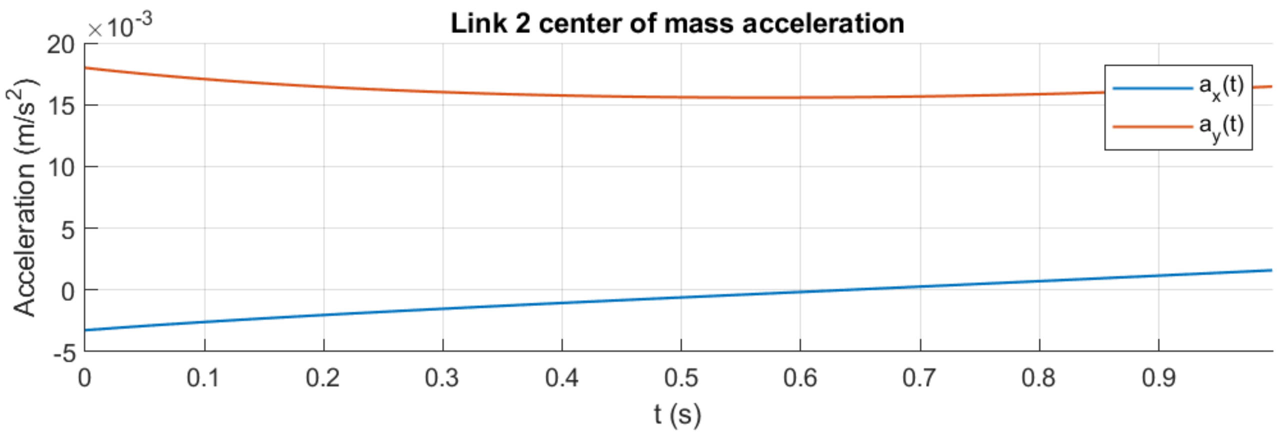


Figure 21: Link 2 center of mass acceleration

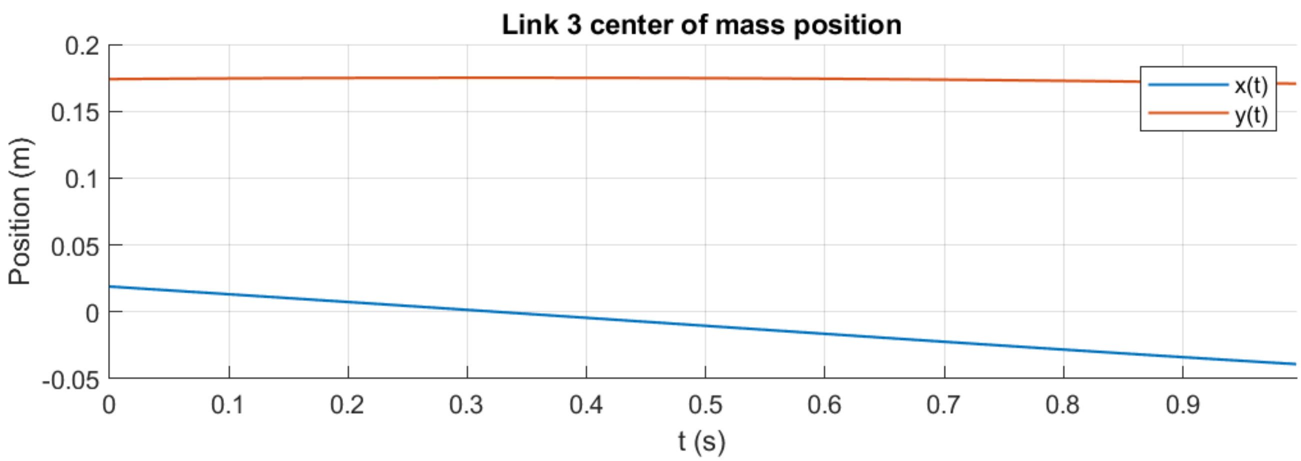


Figure 22: Link 3 center of mass position

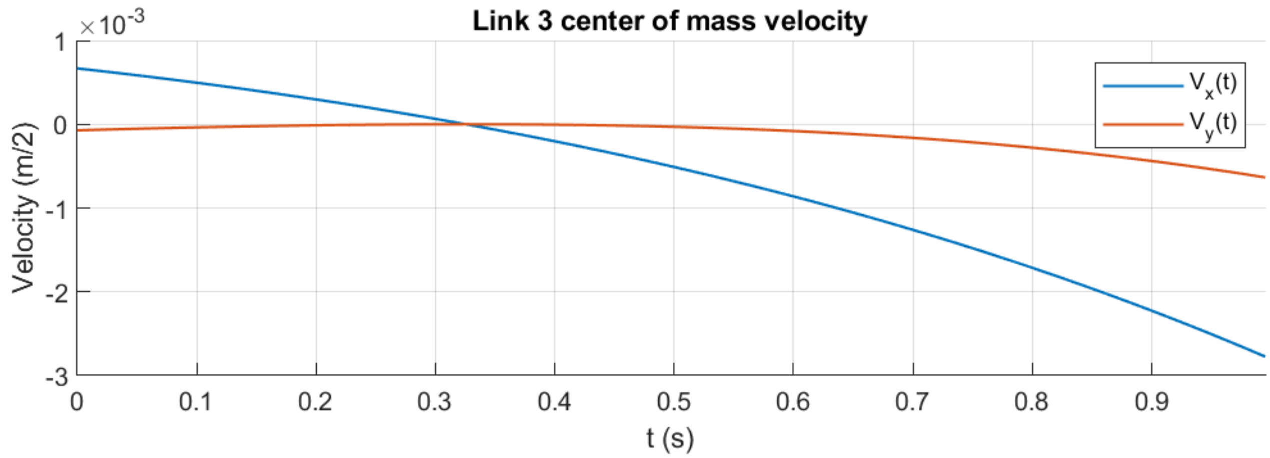


Figure 23: Link 3 center of mass velocity

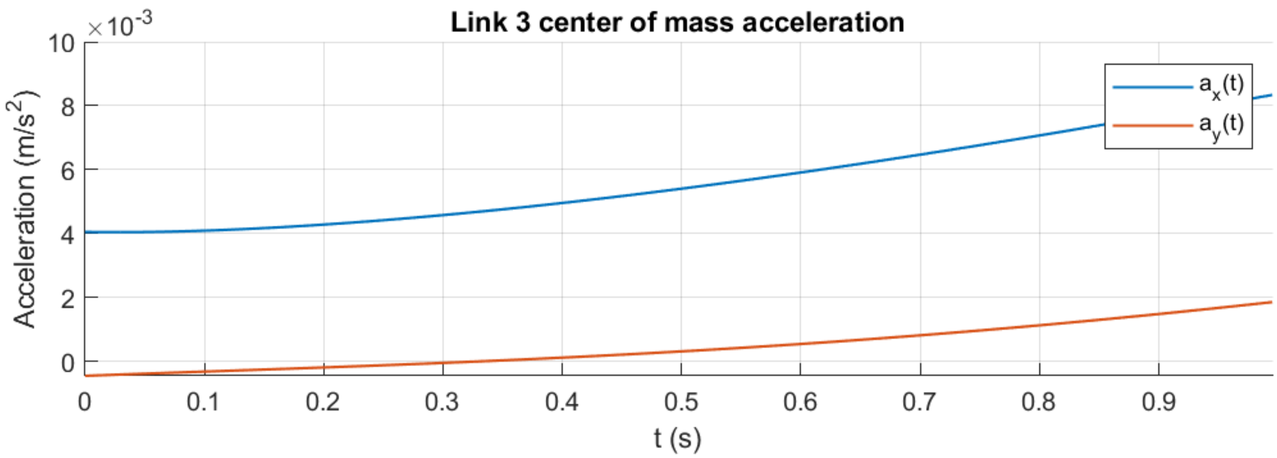


Figure 24: Link 3 center of mass acceleration

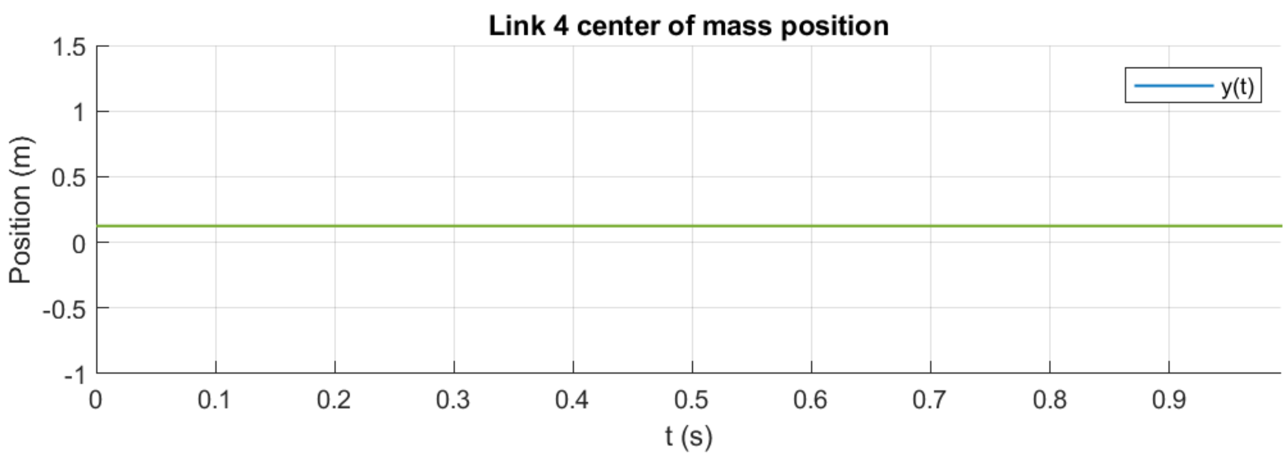


Figure 25: Link 4 center of mass position

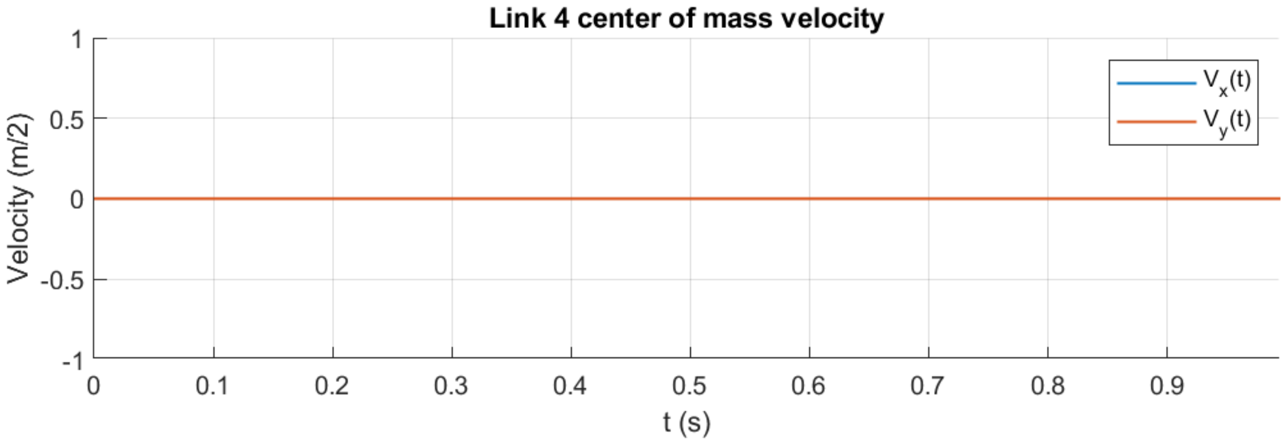


Figure 26: Link 4 center of mass velocity

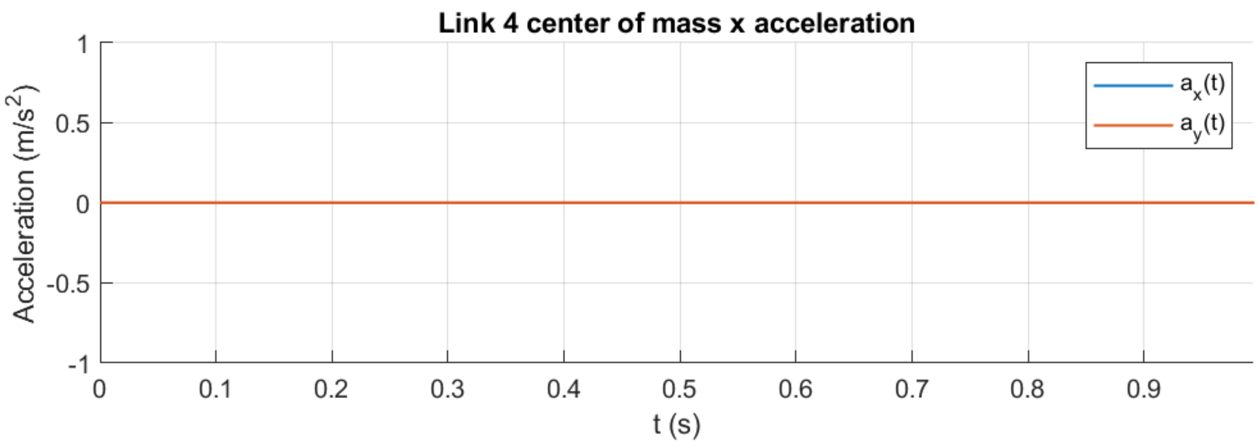


Figure 27: Link 4 center of mass acceleration

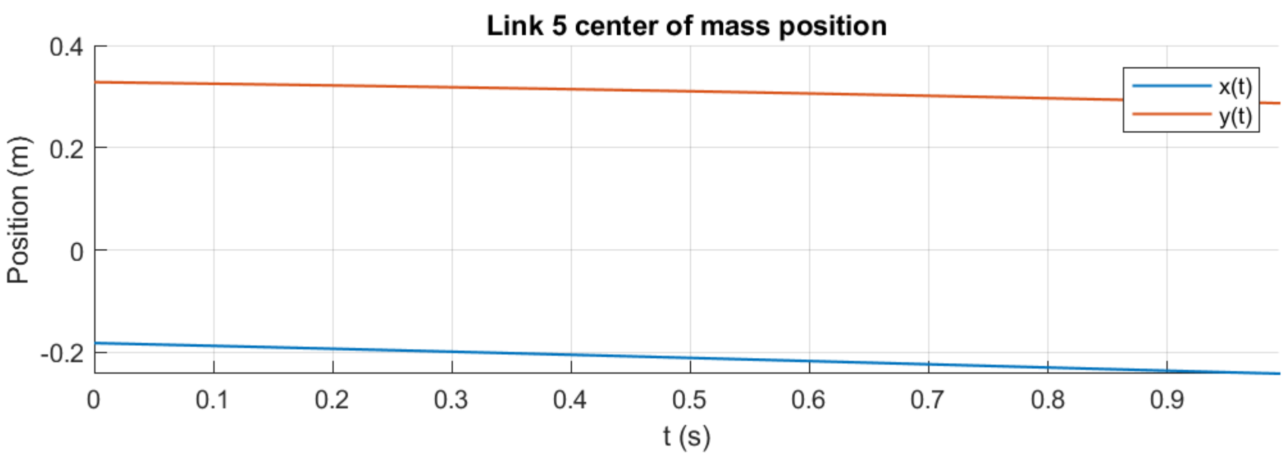


Figure 28: Link 5 center of mass position

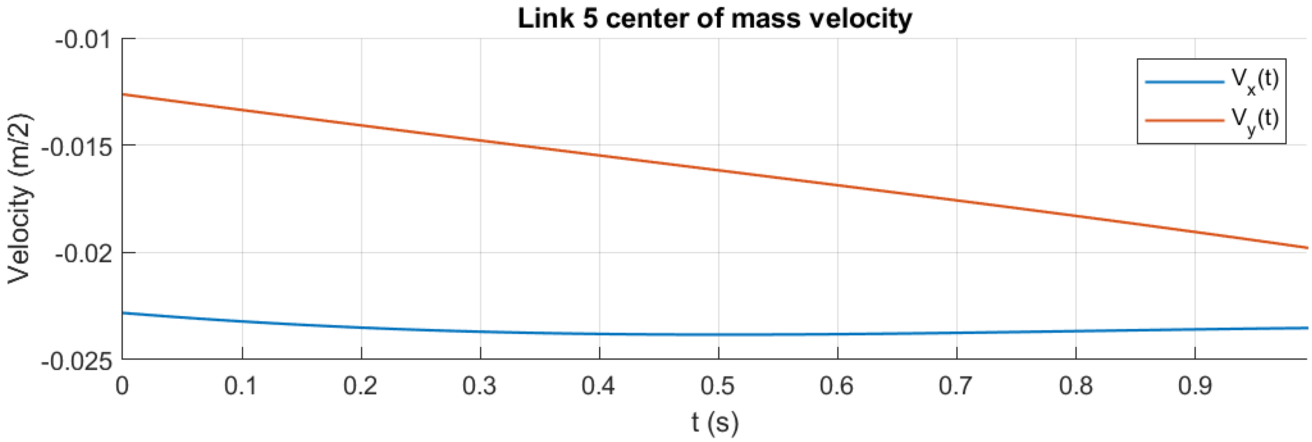


Figure 29: Link 5 center of mass velocity

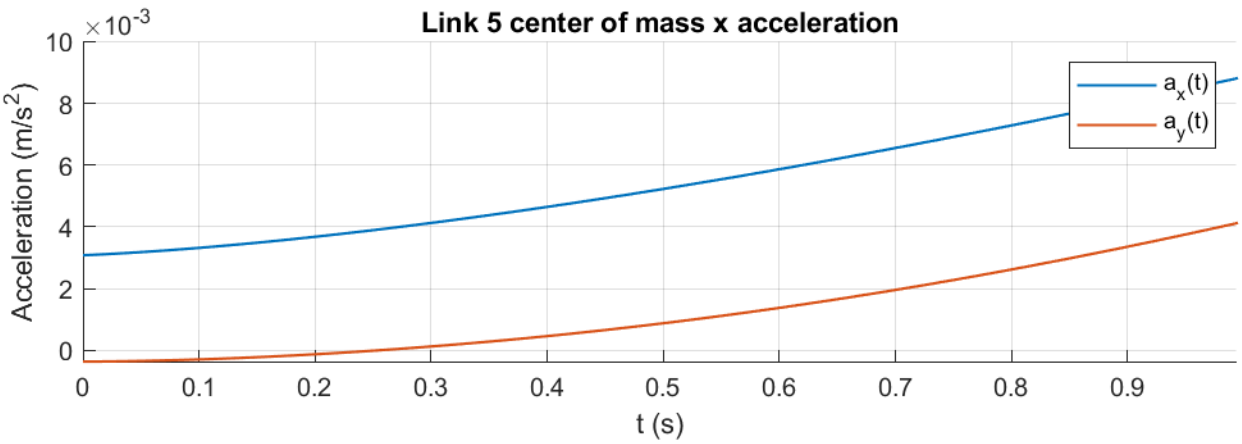


Figure 30: Link 5 center of mass acceleration

4.3 Dynamic design

The press was built from iron with a density of 7900 kg/m^3, with a cross section area of 4 cm^2, resulting in the following link masses.

Left rocker	2.37 kg
Right rocker	2.37 kg
Handle	3.16 kg
Coupler	1.1060 kg
Punch	0.79 kg

Table 6: Final mechanism link masses

The forces present at each point of the mechanism are the following, the free body diagrams and force balances may be found within the appendix.

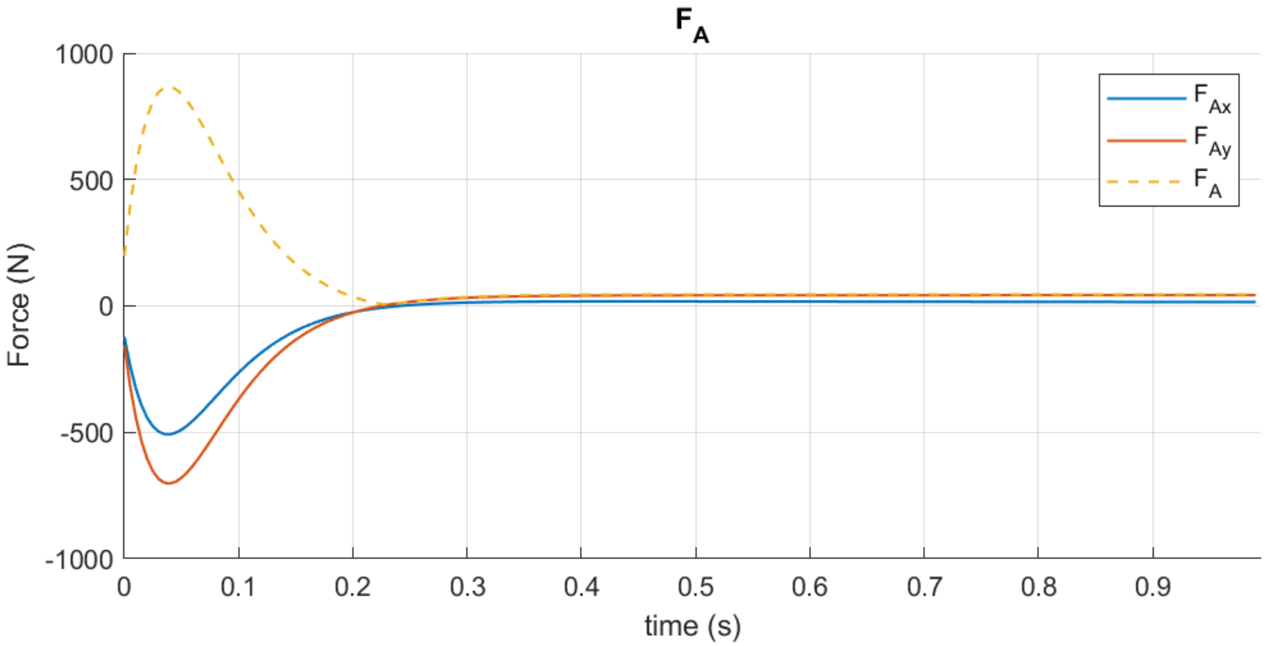


Figure 31: Force A

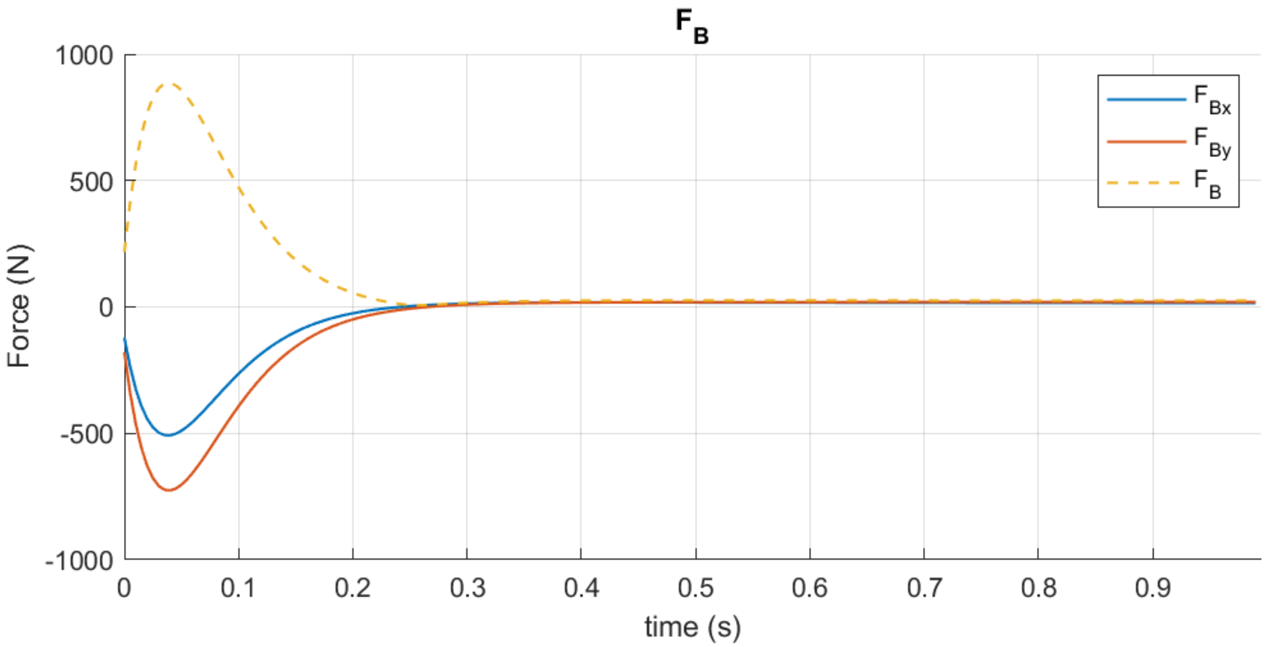


Figure 32: Force B

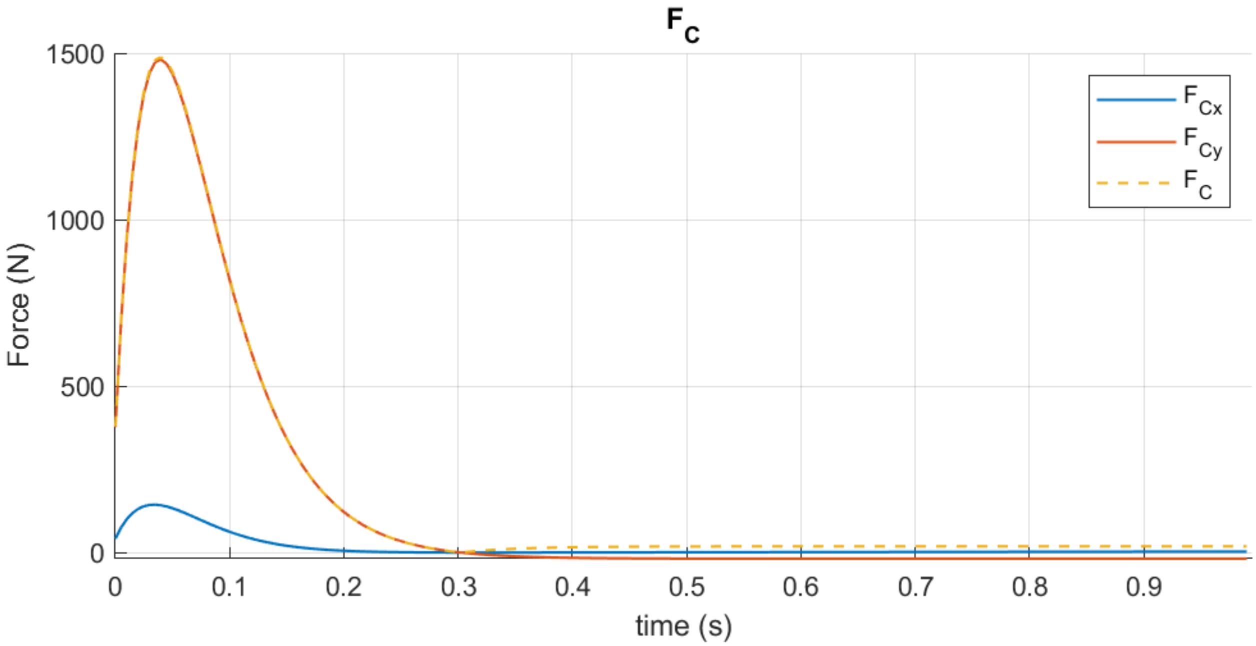


Figure 33: Force C

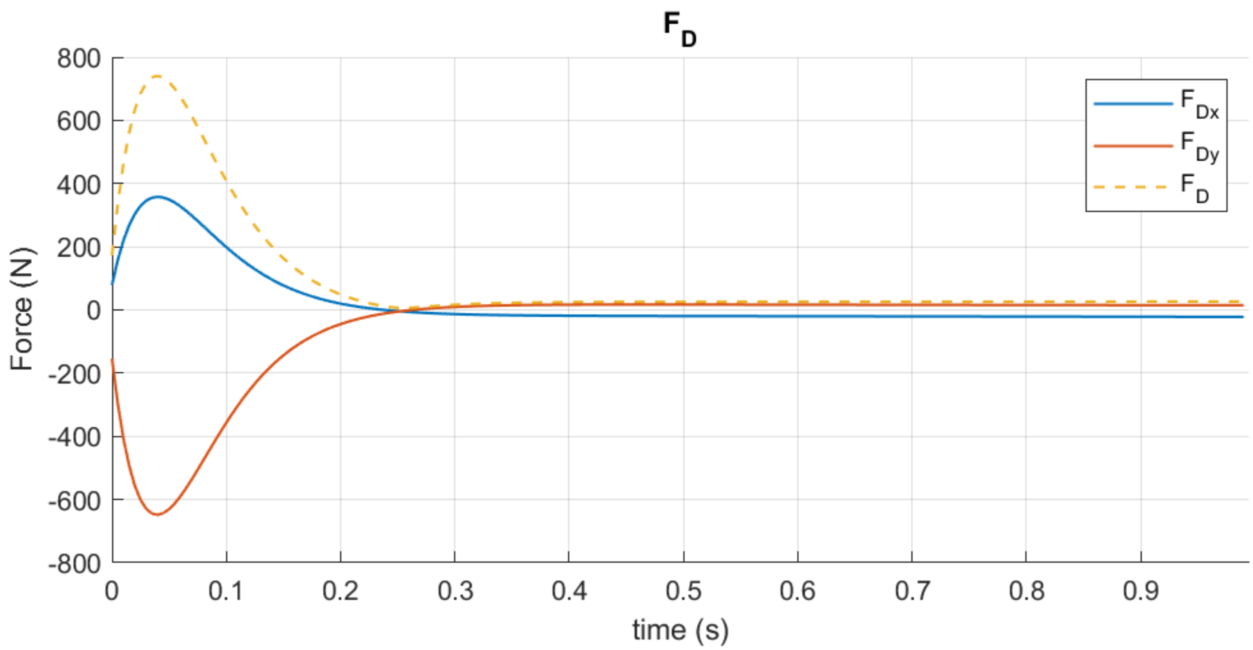


Figure 34: Force D

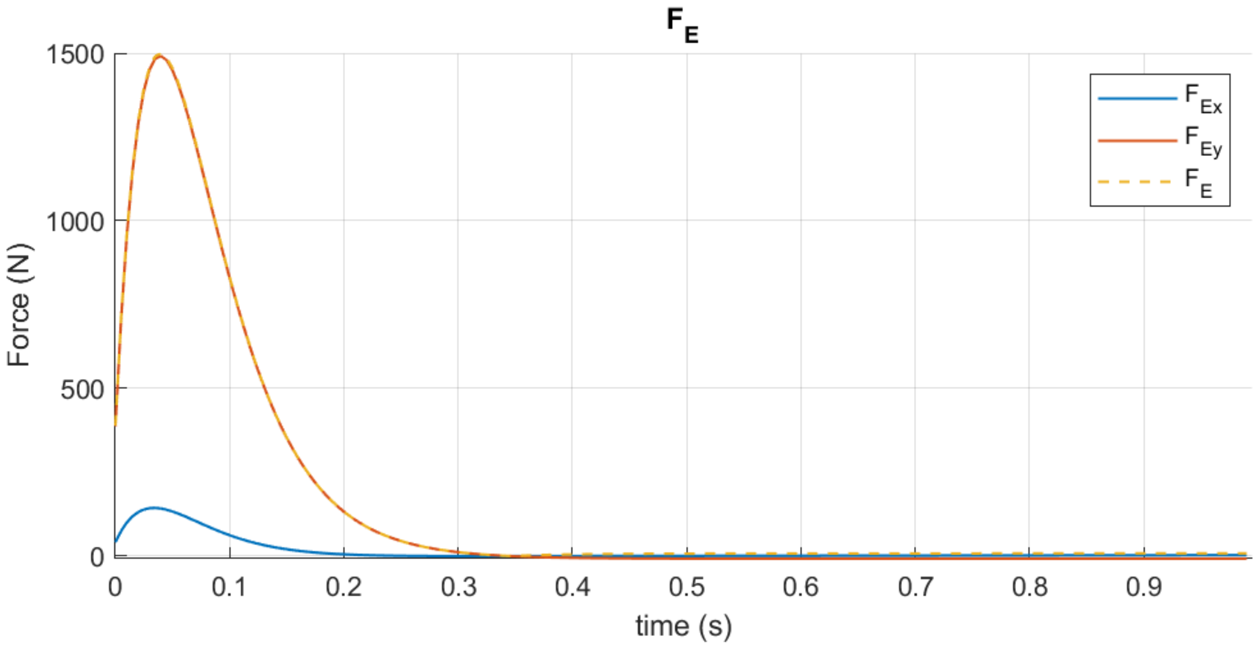


Figure 35: Force E

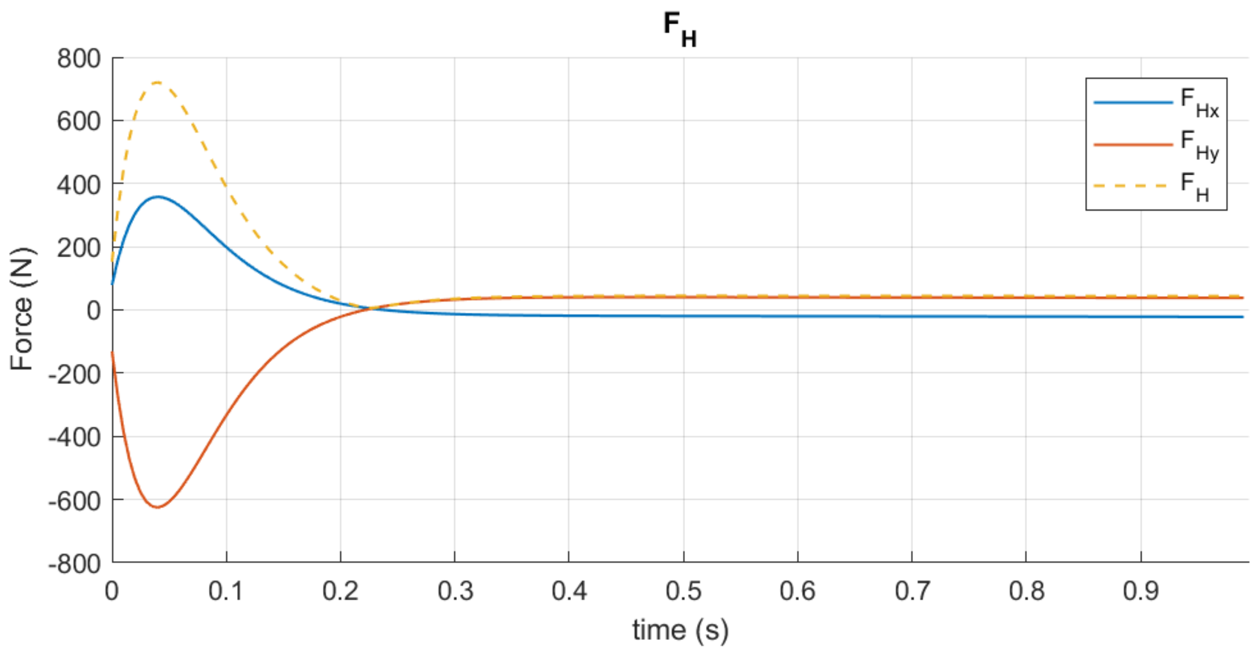


Figure 36: Force H

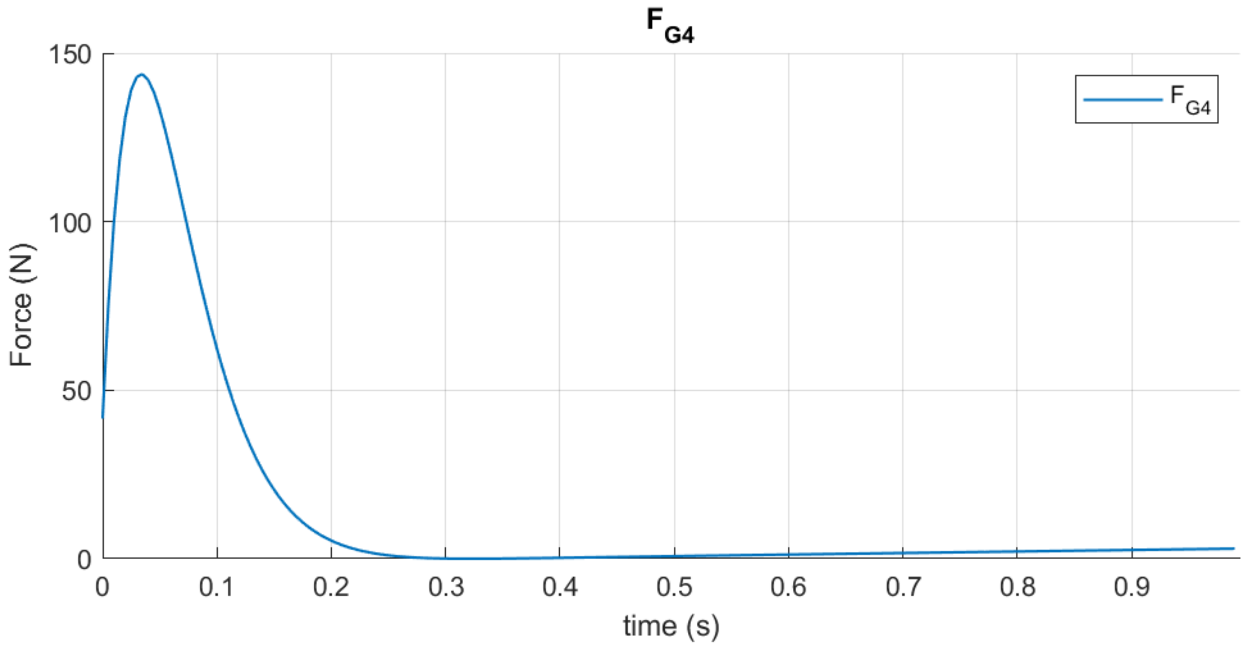


Figure 37: Force G4

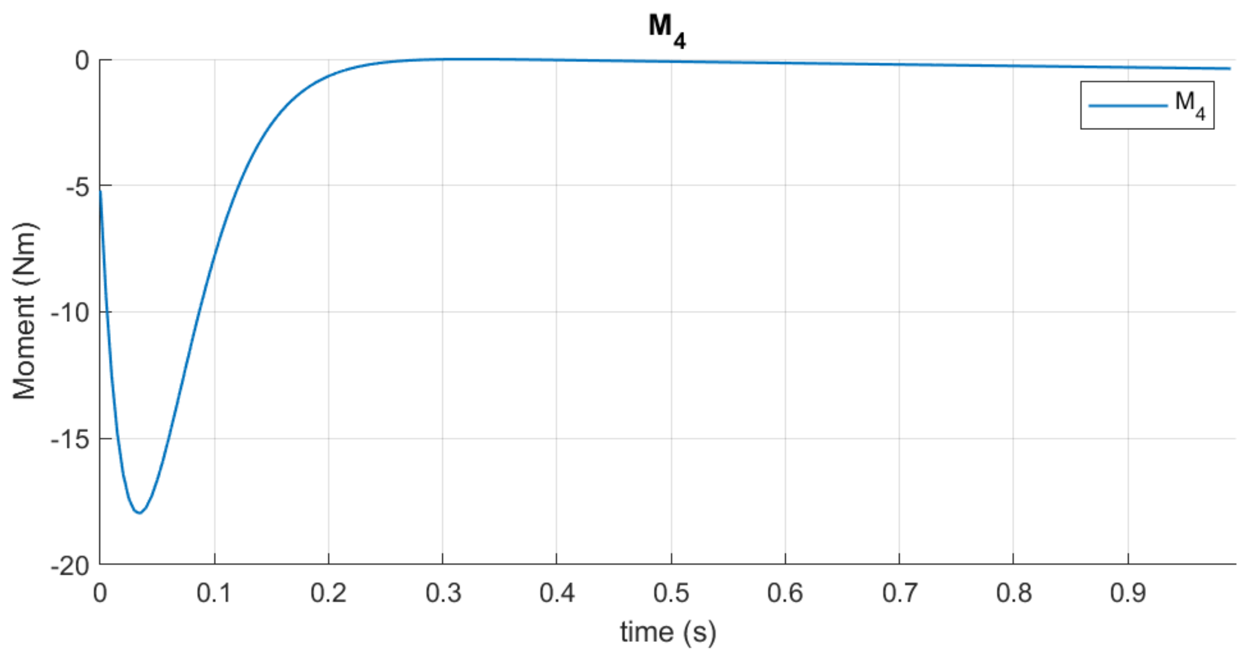


Figure 38: Reaction moment at G4

The force required to actuate the press is the following.

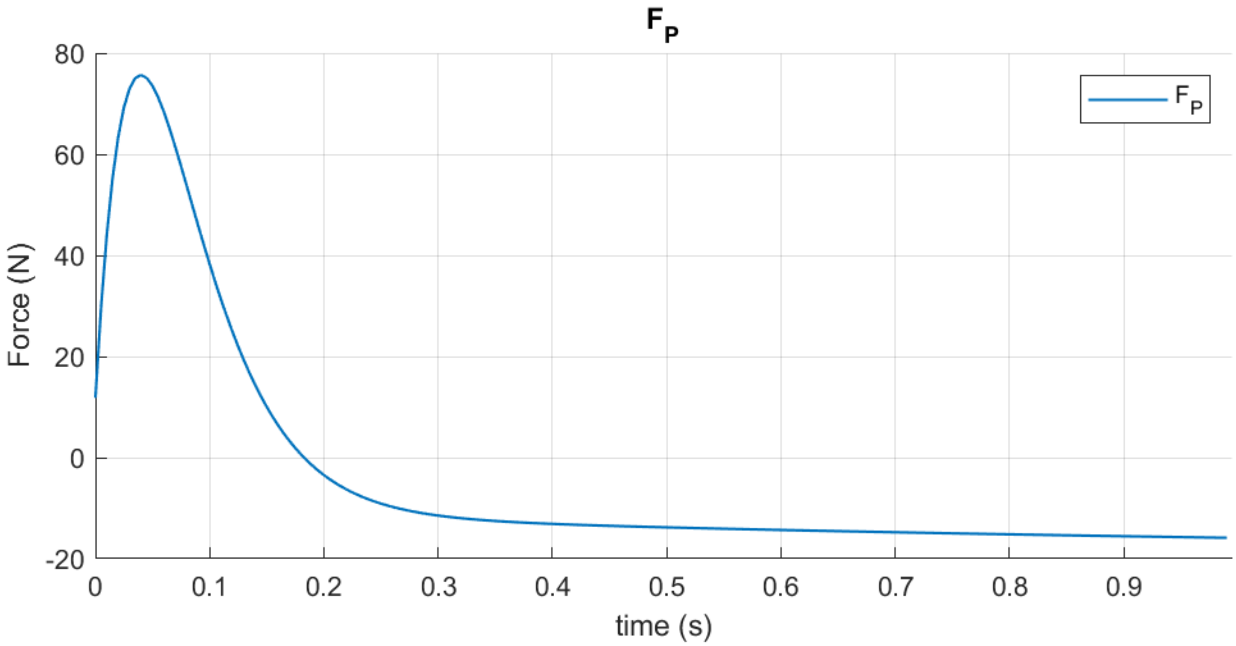


Figure 39: Actuation force

The power that the user must supply in order to actuate the press is the following. Its maximum value is 4.96 Watts.

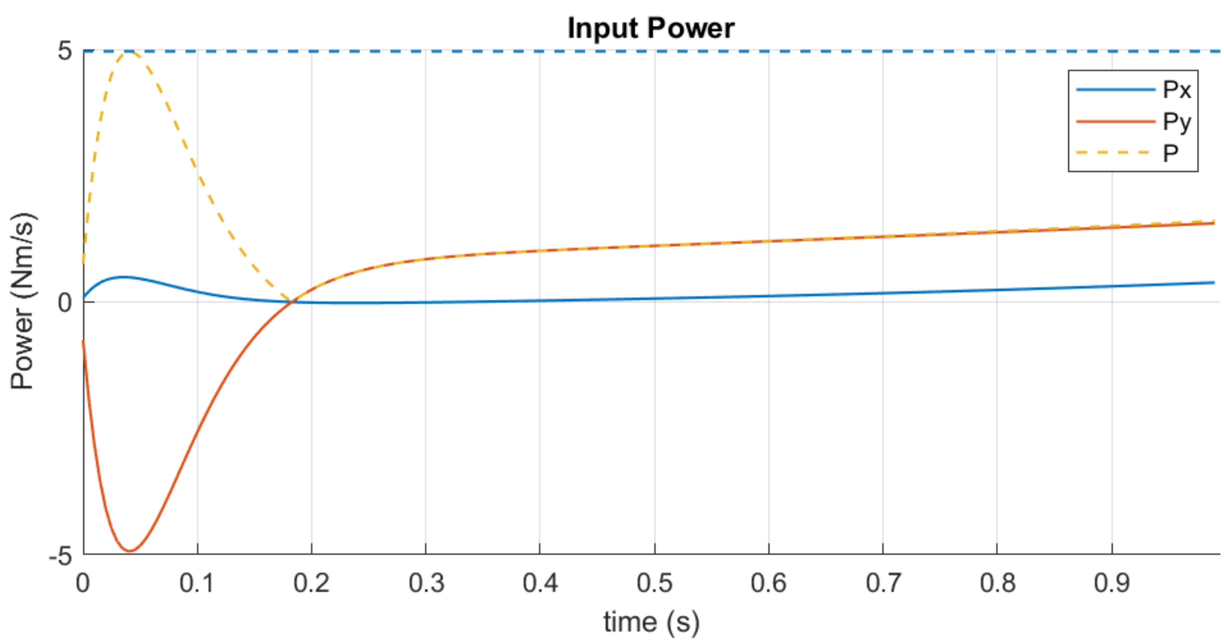


Figure 40: Actuation power

According to the MIL-STD-1472F, a male 5th percentile operator may be expected to produce 93 N of force when moving their arm down and 116 N when moving it up when it has between 90 and 120 degrees of elbow flexion (which is a reasonable range to expect a press to be used at) with females generally being able to produce 2/3rds of this force (according to point 5.4.4.2). Using the velocity of point P on the handle we can find the range of power that a 5th percentile operator may produce using this press.

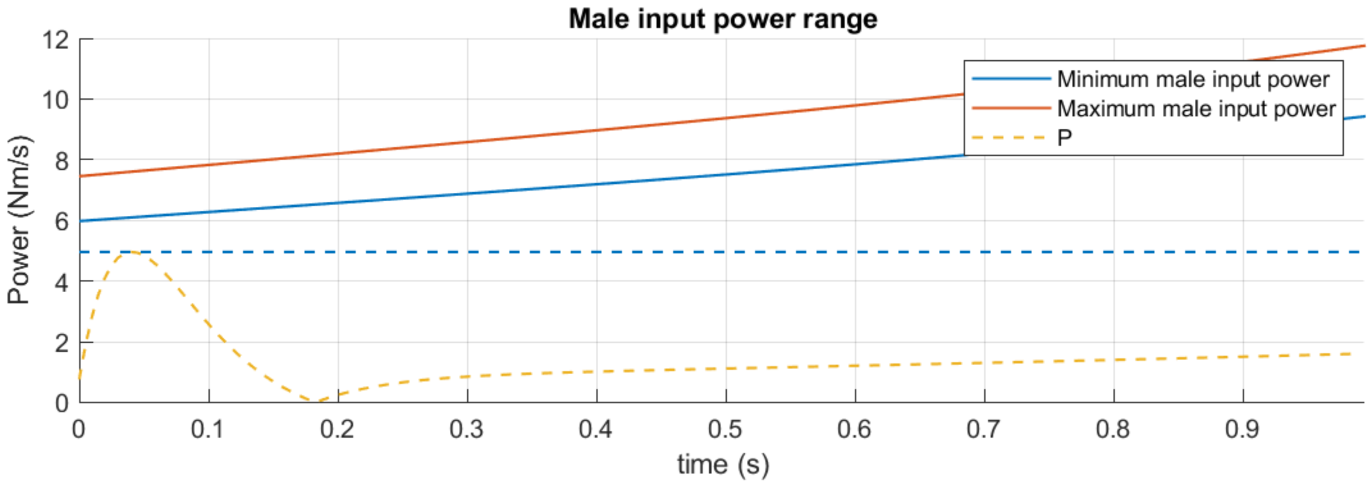


Figure 41: Male operator input power

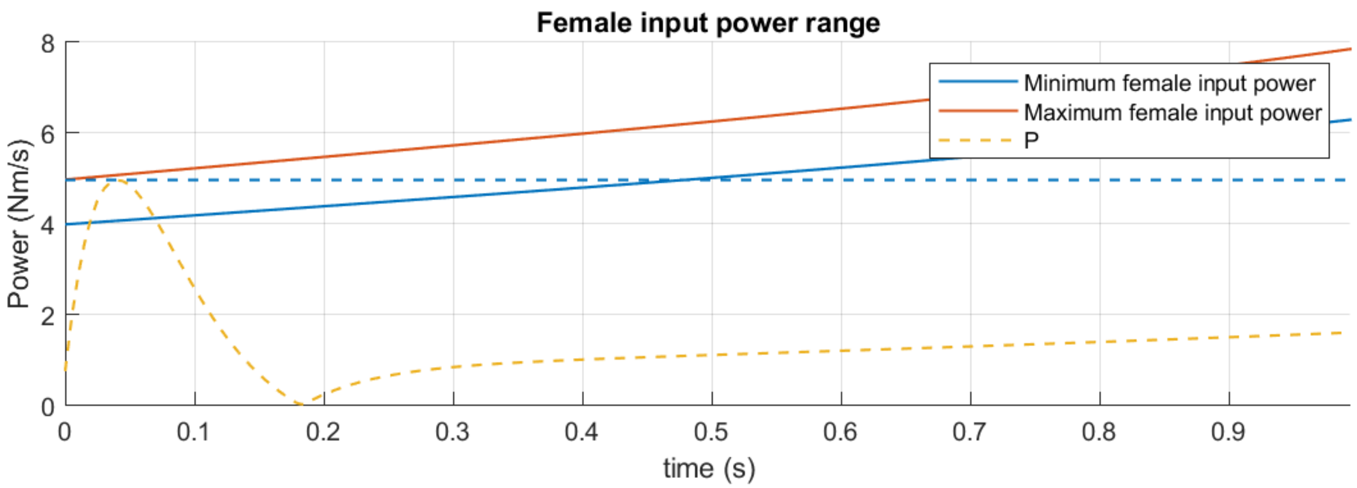


Figure 42: Female operator input power

It is worth nothing that the minimum is the power produced with the left arm while the maximum is that produced with the right arm.

However, before deciding whether this is appropriate we must know how fast the operators would need to move their arms to accomplish this, the following figure shows that this velocity would at most be slightly above 10 cm/s, which is perfectly reasonable.

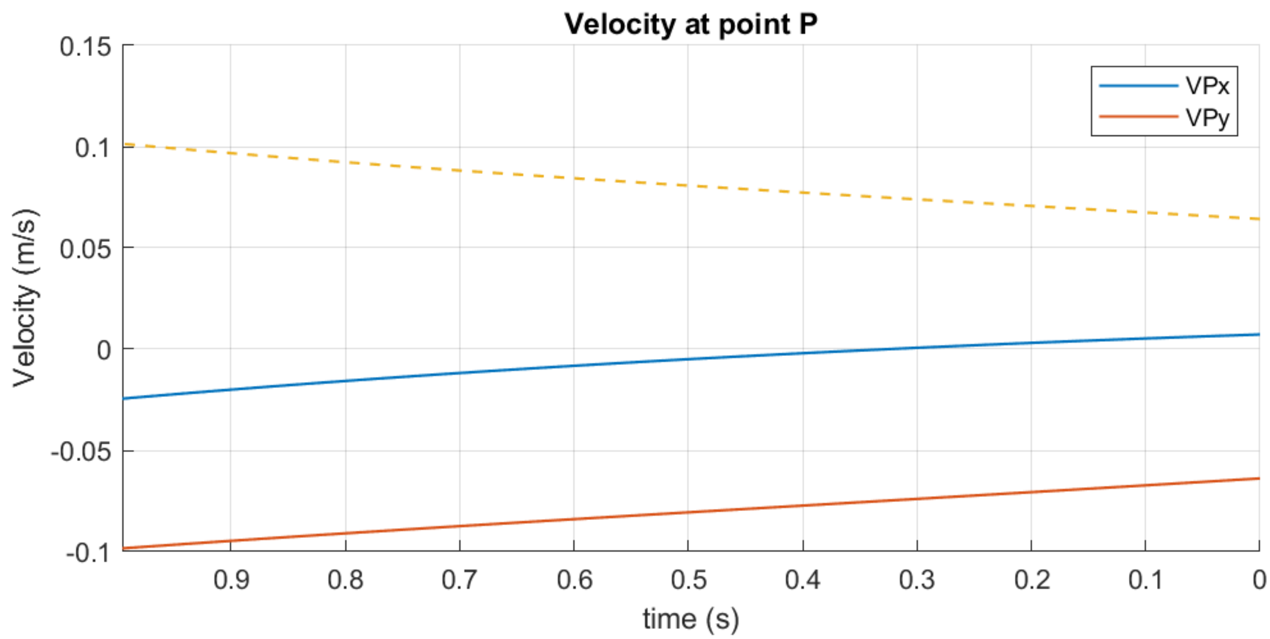


Figure 43: Input velocity

This shows that a male operator would be able to use the press with either arm, however, a female operator would only be able to use her right arm (or both) to provide the required power.

4.4 Console design

Based on the infomation provided by Theresa Stack, Lee T. Ostrom, And Cheryl A. Wilhelmsen in ”Occupational Ergonomics, A Practical Approach”, we can discard ideas of working stations because they recommend sitting or standing position depending on some factors of the work the person is going to do in a journey.

We should not consider a standing position for long periods of time to perform a job. Long periods of standing work can cause back pain, leg swelling, problems with blood circulation, sore feet, and tired muscles.

Also all the day sitting is not recommendable at all because it is not good for the body, worker should be able to change its posture with an ergonomic chair.

With the calculation done, we notice that the worker must reach a force near to 80 N or 17.98 lb but just for a short period of time, less than 0.1 seconds, most of the time of the movement , then, looking at the Table 11.1 Criteria for Designing a Workstation of the Occupational Ergonomics book, we see that for heavy load and/or with high forces – greater than 10 lb sitting is not recommendable.

TABLE 11.1 Criteria for Designing a Workstation

Parameters	Standing Workstation	Sit–Stand/Leaning Workstation	Sit to Stand Workstation	Sitting Workstation	Special Considerations
Heavy load and/or with high forces – greater than 10 lb	Work surface height 6–16 in. below elbow height	Not recommended	Not recommended	Not recommended	Provide a place for short breaks. A lean station or a chair
Intermittent work with moderately high forces – 2–10 lb	Work surface height 4–6 in. below elbow height	Work surface height 4–6 in. below elbow height	Not recommended	Not recommended	Provide a place for short breaks. A lean station or a chair
Extended reach envelope	If variable tasks are required	If variable tasks are required	If moderately fin work is required and/or forces are less than 10 lb while sitting	If fin or precise work is required and/or forces are less than 2 lb	Ensure worker can reach items required without attaining awkward postures
Variable work surface heights	If forces greater than 10 lb are required. Work surface height 6–16 in. below elbow height	If moderately high forces are required – 2–10 lb	If moderately fin work is required and/or forces are less than 10 lb while sitting	If work can be performed at approximately elbow height and forces are less than 2 lb	Ensure worker does not attain sustained awkward postures

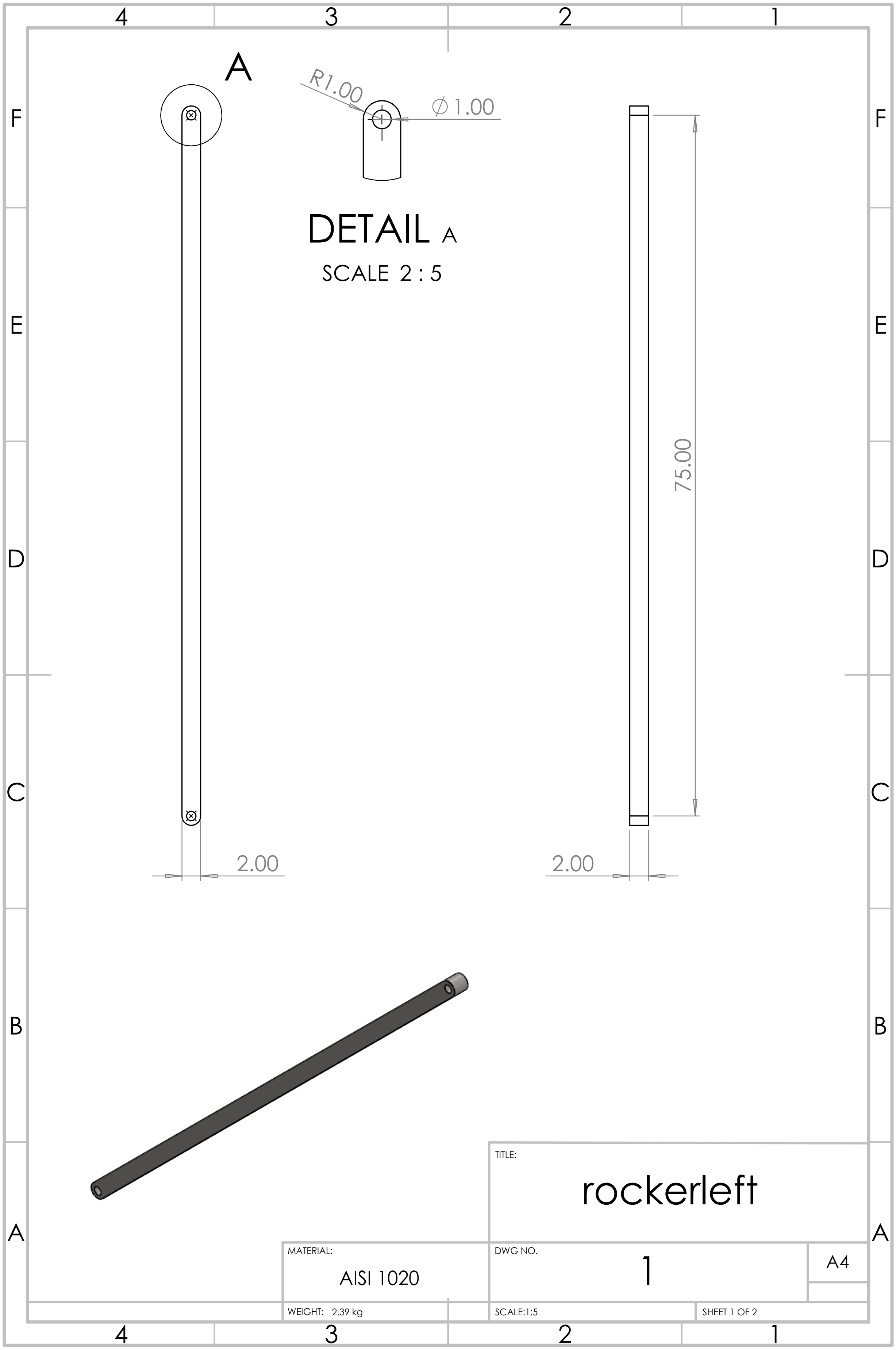
Figure 44: Table 11.1 from "Occupational Ergonomics"

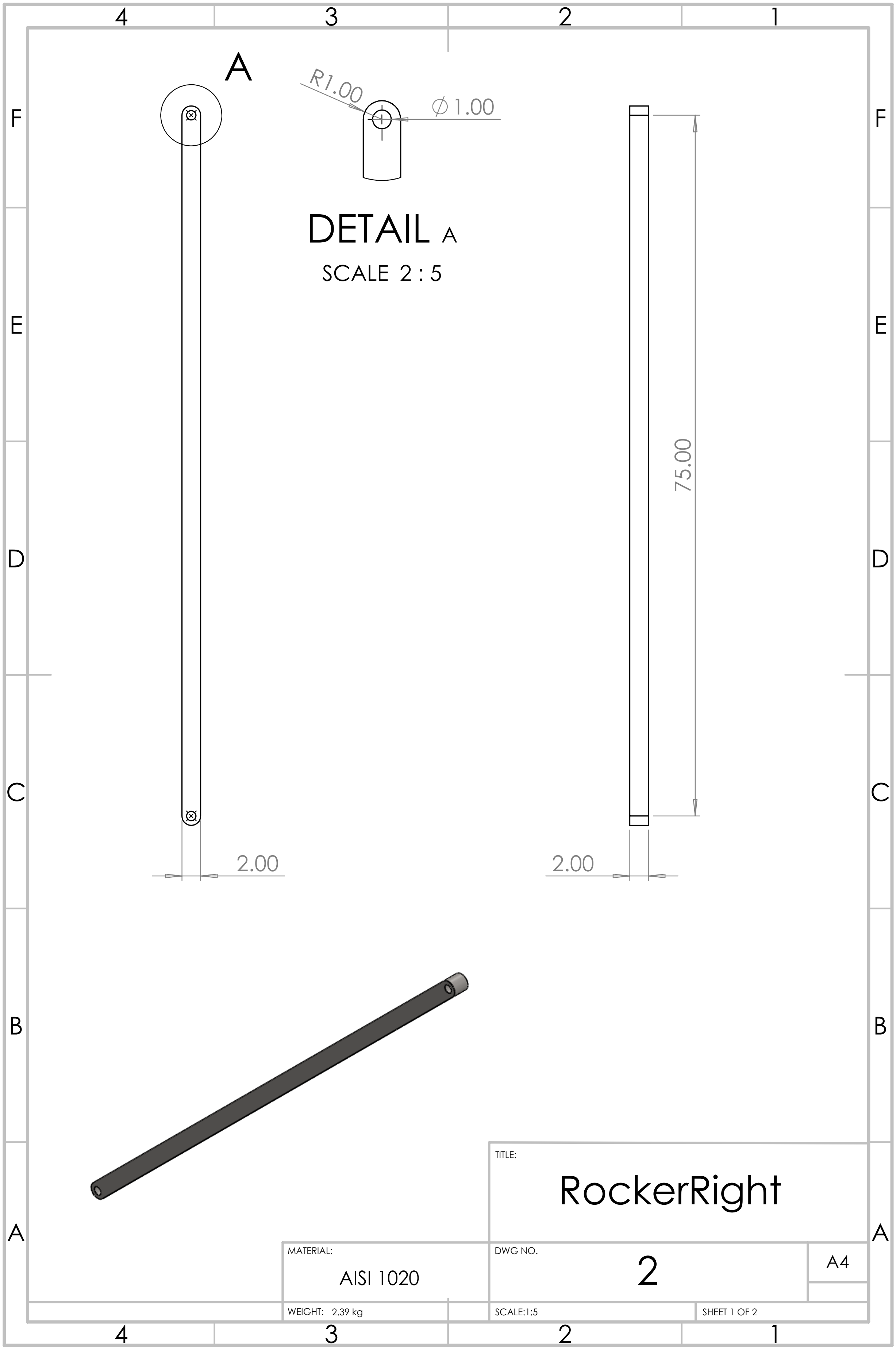
A sit-stand workstation would be great for this range of forces and also because of the repeatability, but because of the movement, leaning the arms is not comfortable and useful.

Then, standing position is going to be the viable option. For this, let's consider the requirements of neutral positions when standing is done.

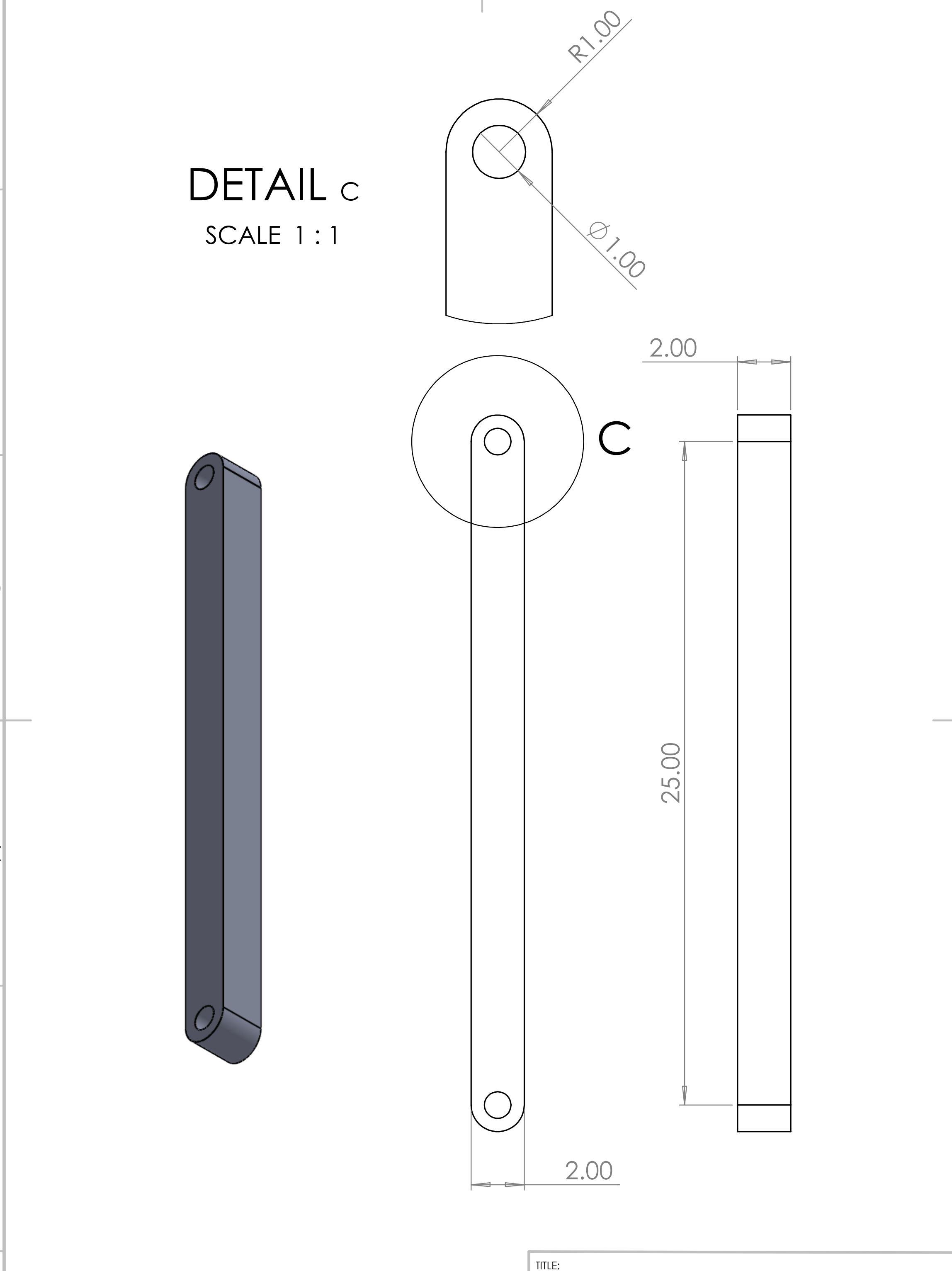
From "Occupational Ergonomics" in figure 2.2 Standing neutral postures, a standing neutral position should have:

- Wrist is in line with the forearm. It is either bent up (extension) nor bent down (flexion).
- Forearm rests with the thumb up. It is not rotated.
- Elbow with the angle between the forearm and upper arm is close to a right angle (90°). Some extension (up to 110°) may be desirable.
- Upper arm: hangs straight down. No abduction, adduction, flexion or extension.
- Shoulder: are in resting position, neither hunched up or nor pulled down, and not pulled forward or back.
- Neck: the head is balanced on the spinal column. It is not titled forward, back or either side. Its is not rotated to the left or right.
- Back: the spine naturally assumes a S-shaped curve. The upper spine (thoracic region) is bent gently out; the lower spine (lumbar region) is gently in. The spine is not rotated, twisted or bent to the left or right.


A
R1.00
⌀1.00
DETAIL A
SCALE 2 : 5
75.00
2.00
2.00
TITLE:
rockerleft
MATERIAL:
AISI 1020
DWG NO.
1
A4
WEIGHT: 2.39 kg
SCALE:1:5
SHEET 1 OF 2



A
R1.00
⌀1.00
DETAIL A
SCALE 2 : 5
75.00
2.00
2.00
TITLE:
RockerRight
MATERIAL:
AISI 1020
DWG NO.
2
A4
WEIGHT: 2.39 kg
SCALE:1:5
SHEET 1 OF 2



TITLE: Punch

MATERIAL: AISI 1020

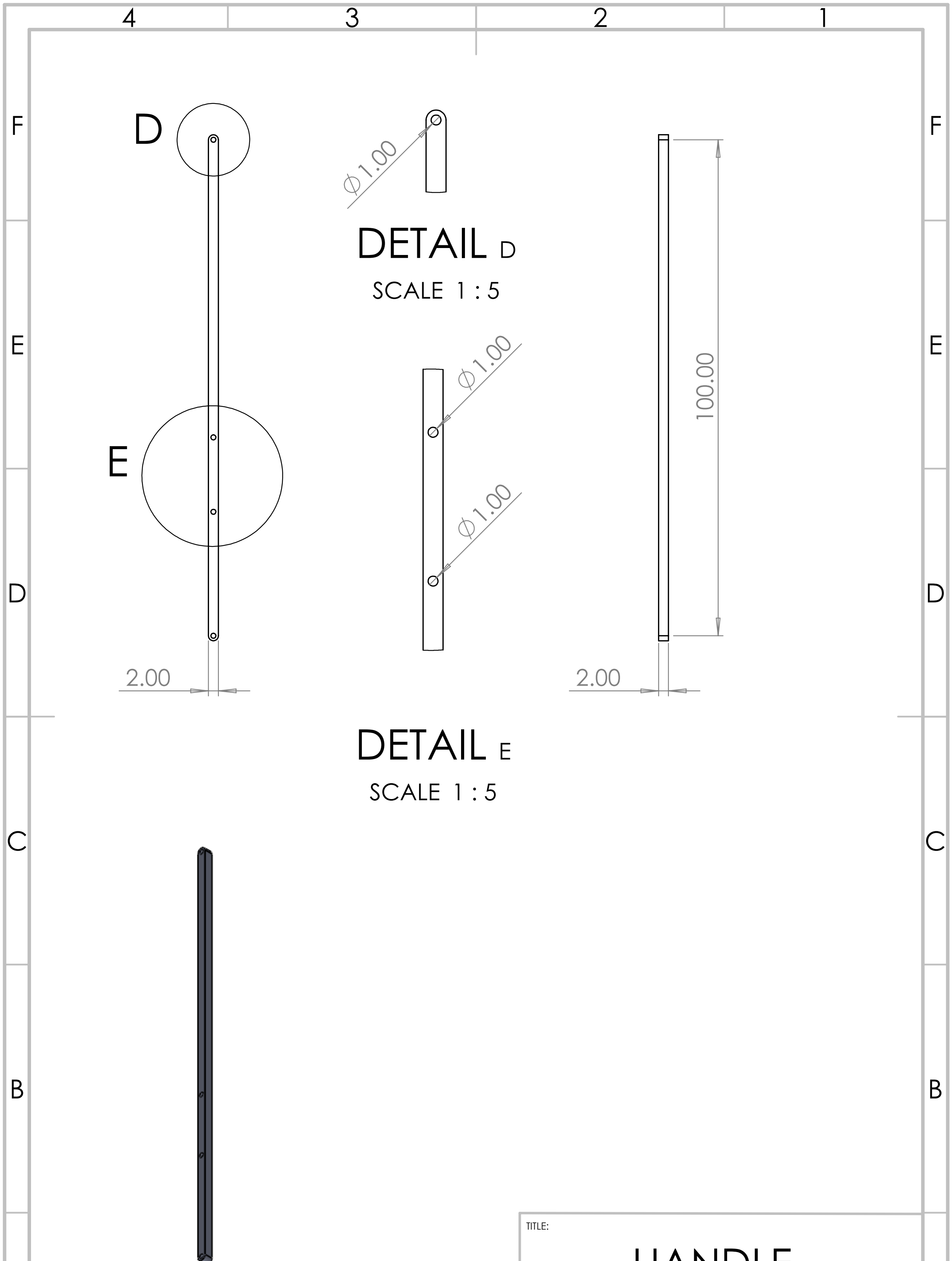
DWG NO. 3

A4

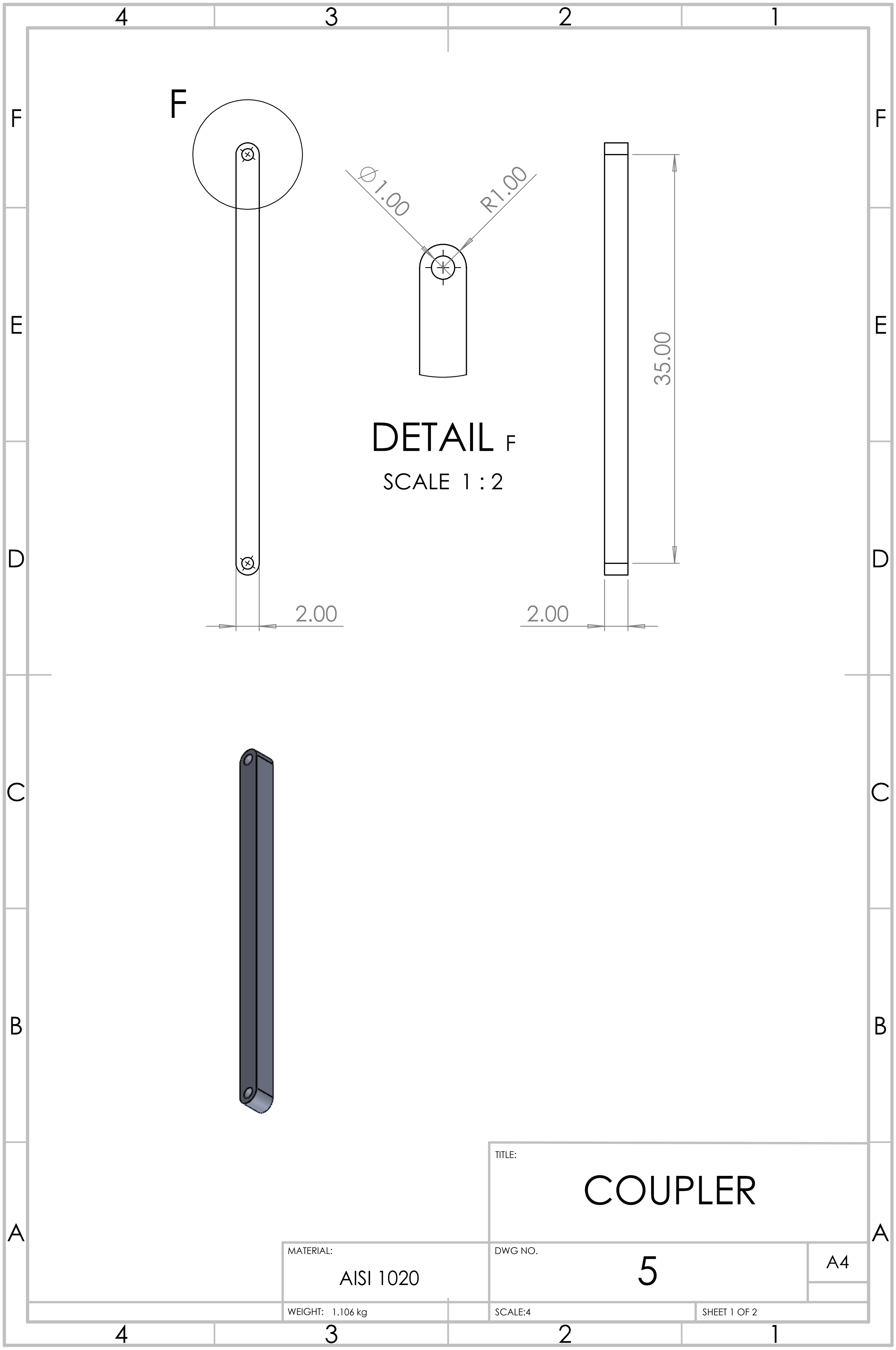
WEIGHT: 2.39 kg

SCALE:1:2

SHEET 1 OF 2



	TITLE: HANDLE	
MATERIAL: AISI 1020	DWG NO. 4	A4
WEIGHT: 3.16 kg	SCALE:1:10	SHEET 1 OF 2


F
⌀1.00
R1.00
DETAIL F
SCALE 1 : 2
35.00
2.00
2.00
TITLE:
COUPLER
MATERIAL:
AISI 1020
DWG NO.
5
A4
WEIGHT: 1.106 kg
SCALE:4
SHEET 1 OF 2

5 Conclusion

The press design went through several changes over the course of this project and finding one that properly satisfied our needs was quite the challenge, as not only did we need to complete the required motions, but the press had to be usable by just about anybody and its use had to be comfortable to them. Thus the main challenge became finding a configuration that had an ergonomic enough trajectory for point P while also keeping the force required to actuate it down, this required a great deal of iteration and scrapped designs that aren't shown in this document (besides some of the most interesting or useful ones) and over the course of this we found that the forces decreased if:

- The handle was larger.
- The overall dimensions of the mechanism became smaller.
- The path of point P became longer.

While the first is easy enough to intuit, the other two came as surprises as we initially expected the forces to grow as the mechanism's size was reduced and had simply not thought about the effect the path of point P could have. This last point came to us thanks in part to the configuration seen in section 3.2.3, this caused us to focus our efforts on maximizing this path while maintaining adequate proportions.

The final design itself managed to meet our requirements well enough to be satisfactory, however it is not without faults, the base is on the very limit of being acceptable at 120 cm and the path of point P, while acceptable, moves towards the press instead of the user which is not ergonomically excellent. Moreover, while a 5th percentile male operator should theoretically be capable of actuating the press with either arm, a female operator would be required to use her right arm lest she not have enough strength to supply the required power.

Were we to design a new press or similar mechanism, our plan of action would be to see how each parameter, whether we can directly control it or not, affects the requirements in order to understand what it is that we must modify in order to obtain the desired results.

Overall, the main takeaway of this project is the way ergonomics shape the design of any mechanism that a human will operate, after all, designing a press that simply completes the required motion would be fairly simple, but designing one that takes the human factor into account in order to make the operator's job as easy as possible creates a completely new, and incredibly important challenge.

6 Appendix

MATLAB code

Velocity function derivations

6.1 Kinematics

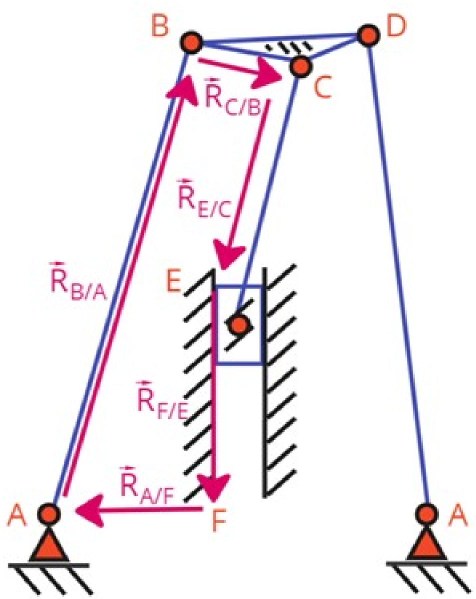


Figure 45: Mechanism 3 Vector Loop 1

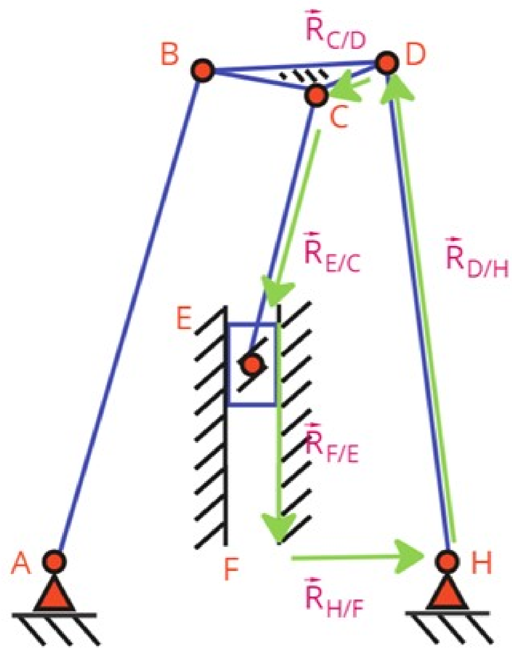


Figure 46: Mechanism 3 Vector Loop 2

As seen in figures 45 and 46 this mechanism needs two vector loops in order to be properly defined. This may be written in the following way.

Loop 1

$$\vec{R}_{B/A} + \vec{R}_{C/B} + \vec{R}_{E/C} + \vec{R}_{A/F} = 0$$

$$\Downarrow$$

$$ae^{j\theta_1} + \frac{b}{2}e^{j\theta_2} - le^{j\theta_3} - r\hat{j} - \frac{d}{2}\hat{\imath} - y\hat{j} = 0$$

$$\Downarrow$$

$$a\left(\cos\theta_1 + j\sin\theta_1\right) + \frac{b}{2}\left(\cos\theta_2 + j\sin\theta_2\right) - l\left(\cos\theta_3 + j\sin\theta_3\right) - r\hat{j} - \frac{d}{2}\hat{\imath} - y\hat{j} = 0$$

$$\Downarrow$$

$$a\cos\theta_1 + \frac{b}{2}\cos\theta_2 - l\cos\theta_3 - \frac{d}{2} = 0 \quad \longleftarrow \textbf{Real component}$$

$$a\sin\theta_1 + \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y = 0 \quad \longleftarrow \textbf{ Imaginary component}$$

Loop 2

$$\vec{R}_{D/H} + \vec{R}_{C/D} + \vec{R}_{E/C} + \vec{R}_{F/E} + \vec{R}_{H/F} = 0$$

$$\Downarrow$$

$$ce^{j\theta_4} - \frac{b}{2}e^{j\theta_2} - le^{j\theta_3} - r\hat{j} + \frac{d}{2}\hat{i} - y\hat{j} = 0$$

$$\Downarrow$$

$$c\left(\cos\theta_4 + j\sin\theta_4\right) - \frac{b}{2}\left(\cos\theta_2 + j\sin\theta_2\right) - l\left(\cos\theta_3 + j\sin\theta_3\right) - r\hat{j} + \frac{d}{2}\hat{i} - y\hat{j} = 0$$

$$\Downarrow$$

$$c\cos\theta_4 - \frac{b}{2}\cos\theta_2 - l\cos\theta_3 + \frac{d}{2} = 0 \quad \longleftarrow \textbf{ Real component}$$

$$c\sin\theta_4 - \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y = 0 \quad \longleftarrow \textbf{ Imaginary component}$$

We may find an expression for the acceleration by deriving this twice.

Start with loop 1's velocity

$$\frac{d}{dt}\left[a\cos\theta_1 + \frac{b}{2}\cos\theta_2 - l\cos\theta_3 - \frac{d}{2}\right] = 0$$

$$-a\omega_1\sin\theta_1 - \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3 = 0 \quad \longleftarrow \textbf{ Real component}$$

$$\frac{d}{dt}\left[a\sin\theta_1 + \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y\right] = 0$$

$$a\omega_1\cos\theta_1 + \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y} = 0 \quad \longleftarrow \textbf{ Imaginary component}$$

Now find loop 1's acceleration

$$\frac{d}{dt}\left[-a\omega_1\sin\theta_1 - \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3\right] = 0$$

$$-a\alpha_1\sin\theta_1 - a\omega_1^2\cos\theta_1 - \frac{b}{2}\alpha_2\sin\theta_2 - \frac{b}{2}\omega_2^2\cos\theta_2 + l\alpha_3\sin\theta_3 + l\omega_3^2\cos\theta_3 = 0 \quad \longleftarrow \textbf{ Real component}$$

$$\frac{d}{dt}\left[a\omega_1\cos\theta_1 + \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y}\right] = 0$$

$$a\alpha_1\cos\theta_1 - a\omega_1^2\sin\theta_1 + \frac{b}{2}\alpha_2\cos\theta_2 - \frac{b}{2}\omega_2^2\sin\theta_2 - l\alpha_3\cos\theta_3 + l\omega_3^2\sin\theta_3 - \ddot{y} = 0 \longleftarrow \textbf{ Imaginary component}$$

Continue with loop 2's velocity

$$\frac{d}{dt}\left[c\cos\theta_4 - \frac{b}{2}\cos\theta_2 - l\cos\theta_3 + \frac{d}{2}\right] = 0$$

$$-c\omega_4\sin\theta_4 + \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3 = 0 \quad \longleftarrow \textbf{Real component}$$

$$\frac{d}{dt}\left[c\sin\theta_4 - \frac{b}{2}\sin\theta_2 - l\sin\theta_3 - r - y\right] = 0$$

$$c\omega_4\cos\theta_4 - \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y} = 0 \quad \longleftarrow \textbf{Imaginary component}$$

Now find loop 2's acceleration

$$\frac{d}{dt}\left[-c\omega_4\sin\theta_4 + \frac{b}{2}\omega_2\sin\theta_2 + l\omega_3\sin\theta_3\right] = 0$$

$$-c\alpha_4\sin\theta_4 - c\omega_4^2\cos\theta_4 + \frac{b}{2}\alpha_2\sin\theta_2 + \frac{b}{2}\omega_2^2\cos\theta_2 + l\alpha_3\sin\theta_3 + l\omega_3^2\cos\theta_3 = 0 \quad \longleftarrow \textbf{Real component}$$

$$\frac{d}{dt}\left[c\omega_4\cos\theta_4 - \frac{b}{2}\omega_2\cos\theta_2 - l\omega_3\cos\theta_3 - \dot{y}\right] = 0$$

$$c\alpha_4\cos\theta_4 - c\omega_4^2\sin\theta_4 - \frac{b}{2}\alpha_2\cos\theta_2 + \frac{b}{2}\omega_2^2\sin\theta_2 - l\alpha_3\cos\theta_3 + l\omega_3^2\sin\theta_3 - \ddot{y} = 0 \longleftarrow \textbf{Imaginary component}$$

The resulting equations may now be used in the Newton-Raphson solver to find the mechanism's kinematics.

6.2 Dynamics

6.2.1 Link 1 Free Body Diagram

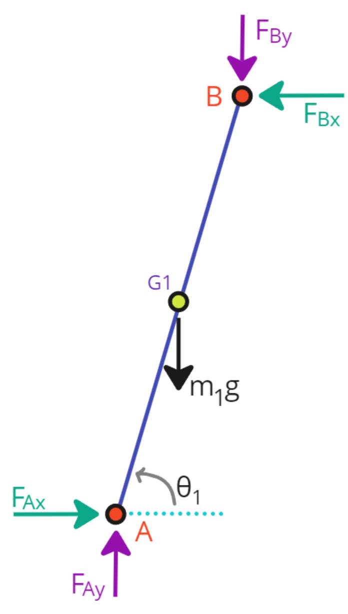


Figure 47: Mechanism 3 link 1 free body diagram

Making use of the above free body diagram we can set up the following equations of motion:

Force sums

$$\sum F_x = F_{Ax} - F_{Bx} = m_1 a_{G_1x}$$

$$\sum F_y = F_{Ay} - F_{By} - m_1 g = m_1 a_{G_1y}$$

Moment sum

$$\sum M_{G_1} = \vec{R}_{A/G_1} \times (F_{Ax} + jF_{Ay}) + \vec{R}_{B/G_1} \times (-F_{Bx} - jF_{By}) = I_{G_1}\alpha_1$$

$$\Downarrow$$

$$\vec{R}_{A/G_1} \times (F_{Ax} + jF_{Ay}) = Im\left[\bar{R}_{A/G_1} \cdot (F_{Ax} + jF_{Ay})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\frac{\overline{AB}}{2}\cos\theta_1 + j\frac{\overline{AB}}{2}\sin\theta_1\right) \cdot (F_{Ax} + jF_{Ay})\right]$$

$$\Downarrow$$

$$\vec{R}_{A/G_1} \times (F_{Ax} + jF_{Ay}) = -\frac{\overline{AB}}{2}F_{Ay}\cos\theta_1 + \frac{\overline{AB}}{2}F_{Ax}\sin\theta_1$$

$$\downarrow$$

$$\vec{R}_{B/G_1} \times (-F_{Bx} - jF_{By}) = Im\left[\left(\frac{\overline{AB}}{2}\cos\theta_1 - j\frac{\overline{AB}}{2}\sin\theta_1\right) \cdot (-F_{Bx} - jF_{By})\right]$$

$$\Downarrow$$

$$\vec{R}_{B/G_1} \times (-F_{Bx} - jF_{By}) = -\frac{\overline{AB}}{2}F_{By}\cos\theta_1 + \frac{\overline{AB}}{2}F_{Bx}\sin\theta_1$$

$$\downarrow$$

$$\sum M_{G_1} = -\frac{\overline{AB}}{2}F_{Ay}\cos\theta_1 + \frac{\overline{AB}}{2}F_{Ax}\sin\theta_1 - \frac{\overline{AB}}{2}F_{By}\cos\theta_1 + \frac{\overline{AB}}{2}F_{Bx}\sin\theta_1 = I_{G_1}\alpha_1$$

6.2.2 Link 2 Free Body Diagram

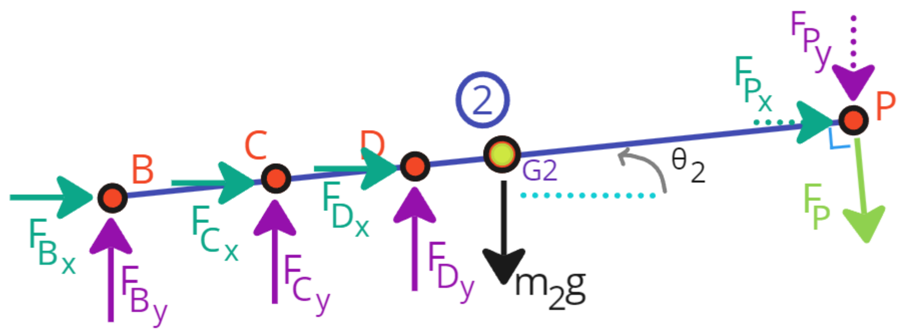


Figure 48: Mechanism 3 link 2 free body diagram

Making use of the above free body diagram we can set up the following equations of motion:

Force sums

$$\sum F_x = F_{Bx} + F_{Cx} - F_{Dx} + F_P \sin\theta_2 = m_2 a_{G_2x}$$

$$\sum F_y = F_{By} + F_{Cy} + F_{Dy} + F_{Cy} - F_P \cos\theta_2 - m_2 g = m_2 a_{G_2y}$$

Moment sum

$$\sum M_{G_2} = \vec{R}_{B/G_2} \times (F_{Bx} + jF_{By}) + \vec{R}_{C/G_2} \times (F_{Cx} + jF_{Cy}) + \vec{R}_{D/G_2} \times (F_{Dx} + jF_{Dy}) - \frac{\overline{BP}}{2} F_P$$

$$\vec{R}_{B/G_2} \times (F_{Bx} + jF_{By}) = Im\left[\bar{R}_{B/G_2} \cdot (F_{Bx} + jF_{By})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\frac{\overline{BP}}{2}\cos\theta_2 + j\frac{\overline{BP}}{2}\sin\theta_2\right) \cdot (F_{Bx} + jF_{By})\right]$$

$$\Downarrow$$

$$\vec{R}_{B/G_2} \times (F_{Bx} + jF_{By}) = -\frac{\overline{BP}}{2} F_{By}\cos\theta_2 + \frac{\overline{BP}}{2} F_{Bx}\sin\theta_2$$

$$\downarrow$$

$$\vec{R}_{C/G_2} \times (F_{Cx} + jF_{Cy}) = Im\left[\bar{R}_{C/G_2} \cdot (F_{Cx} + jF_{Cy})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\overline{CG_2}\cos\theta_2 + j\overline{CG_2}\sin\theta_2\right) \cdot (F_{Cx} + jF_{Cy})\right]$$

$$\Downarrow$$

$$\vec{R}_{C/G_2} \times (F_{Cx} + jF_{Cy}) = -\overline{CG_2} F_{Cy}\cos\theta_2 + \overline{CG_2} F_{Cx}\sin\theta_2$$

$$\downarrow$$

$$\vec{R}_{D/G_2} \times (F_{Dx} + jF_{Dy}) = Im\left[\bar{R}_{D/G_2} \cdot (F_{Dx} + jF_{Dy})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\overline{DG_2}\cos\theta_2 + j\overline{DG_2}\sin\theta_2\right) \cdot (F_{Dx} + jF_{Dy})\right]$$

$$\Downarrow$$

$$\vec{R}_{D/G_2} \times (F_{Dx} + jF_{Dy}) = -\overline{DG_2}F_{Dy}\cos\theta_2 + \overline{DG_2}F_{Dx}\sin\theta_2$$

$$\downarrow$$

$$\sum M_{G_2} = -\frac{\overline{BP}}{2}F_{By}\cos\theta_2 + \frac{\overline{BP}}{2}F_{Bx}\sin\theta_2 - \overline{CG_2}F_{Cy}\cos\theta_2 + \overline{CG_2}F_{Cx}\sin\theta_2 - \overline{DG_2}F_{Dy}\cos\theta_2$$
$$+\overline{DG_2}F_{Dx}\sin\theta_2 - \frac{\overline{BP}}{2}F_P$$

6.2.3 Link 3 Free Body Diagram

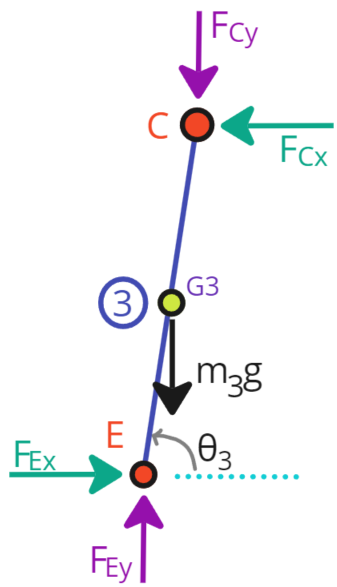


Figure 49: Mechanism 3 link 3 free body diagram

Making use of the above free body diagram we can set up the following equations of motion:

Force sums

$$\sum F_x = F_{Ex} - F_{Cx} = m_3 a_{G_3x}$$
$$\sum F_y = F_{Ey} - F_{Cy} - m_3 g = m_3 a_{G_3y}$$

Moment sum

$$\sum M_{G_3} = \vec{R}_{E/G_3} \times (F_{Ex} + jF_{Ey}) + \vec{R}_{C/G_3} \times (-F_{Cx} - jF_{Cy}) = I_{G_3}\alpha_3$$

$$\Downarrow$$

$$\vec{R}_{E/G_3} \times (F_{Ex} + jF_{Ey}) = Im\left[\bar{R}_{E/G_3} \cdot (F_{Ex} + jF_{Ey})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\frac{\overline{EC}}{2}\cos\theta_3 + j\frac{\overline{EC}}{2}\sin\theta_3\right) \cdot (F_{Ex} + jF_{Ey})\right]$$

$$\Downarrow$$

$$\vec{R}_{E/G_3} \times (F_{Ex} + jF_{Ey}) = -\frac{\overline{EC}}{2}F_{Ey}\cos\theta_3 + \frac{\overline{EC}}{2}F_{Ex}\sin\theta_3$$

$$\downarrow$$

$$\vec{R}_{C/G_3} \times (-F_{Cx} - jF_{Cy}) = Im\left[\left(\frac{\overline{EC}}{2}\cos\theta_3 - j\frac{\overline{EC}}{2}\sin\theta_3\right) \cdot (-F_{Cx} - jF_{Cy})\right]$$

$$\Downarrow$$

$$\vec{R}_{C/G_3} \times (-F_{Cx} - jF_{Cy}) = -\frac{\overline{EC}}{2}F_{Cy}\cos\theta_3 + \frac{\overline{EC}}{2}F_{Cx}\sin\theta_3$$

$$\downarrow$$

$$\sum M_{G_3} = -\frac{\overline{EC}}{2}F_{Ey}\cos\theta_3 + \frac{\overline{EC}}{2}F_{Ex}\sin\theta_3 - \frac{\overline{EC}}{2}F_{Cy}\cos\theta_3 + \frac{\overline{EC}}{2}F_{Cx}\sin\theta_3 = I_{G_3}\alpha_3$$

6.2.4 Link 4 Free Body Diagram

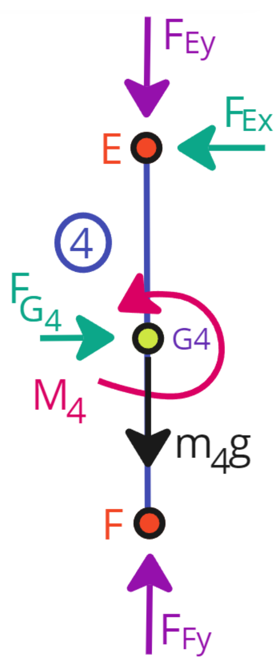


Figure 50: Mechanism 3 link 4 free body diagram

Making use of the above free body diagram we can set up the following equations of motion:

Force sums

$$\sum F_x = F_{G_4} - F_{Ex} = m_4 a_{G_4 x}$$

$$\sum F_y = F_{Fy} - F_{Ey} - m_4 g = m_4 a_{G_4 y}$$

Moment sum

$$\sum M_{G_4} = -M_4 + \frac{\overline{EF}}{2} F_{Ex} = 0$$

6.2.5 Link 5 Free Body Diagram

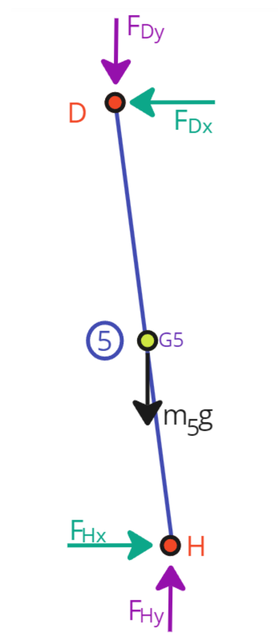


Figure 51: Mechanism 3 link 5 free body diagram

Making use of the above free body diagram we can set up the following equations of motion:

Force sums

$$\sum F_x = F_{Hx} + F_{Dx} = m_5 a_{G_5 x}$$

$$\sum F_y = F_{Hy} - F_{Dy} - m_5 g = m_5 a_{G_5 y}$$

Moment sum

$$\sum M_{G_5} = \vec{R}_{H/G_5} \times (F_{Hx} + jF_{Hy}) + \vec{R}_{D/G_5} \times (F_{Dx} - jF_{Dy}) = I_{G_5} \alpha_4$$

$$\Downarrow$$

$$\vec{R}_{H/G_5} \times (F_{Hx} + jF_{Hy}) = Im\left[\bar{R}_{H/G_5} \cdot (F_{Hx} + jF_{Hy})\right]$$

$$\Downarrow$$

$$Im\left[\left(-\frac{\overline{HD}}{2}\cos\theta_4 + j\frac{\overline{HD}}{2}\sin\theta_4\right)\cdot(F_{Hx}+jF_{Hy})\right]$$

$$\Downarrow$$

$$\vec{R}_{H/G_5}\times(F_{Hx}+jF_{Hy}) = -\frac{\overline{HD}}{2}F_{Hy}\cos\theta_4 + \frac{\overline{HD}}{2}F_{Hx}\sin\theta_4$$

$$\downarrow$$

$$\vec{R}_{D/G_5}\times(F_{Dx}-jF_{Dy}) = Im\left[\left(\frac{\overline{HD}}{2}\cos\theta_4 + j\frac{\overline{HD}}{2}\sin\theta_4\right)\cdot(F_{Dx}-jF_{Dy})\right]$$

$$\Downarrow$$

$$\vec{R}_{D/G_5}\times(F_{Dx}-jF_{Dy}) = -\frac{\overline{HD}}{2}F_{Dy}\cos\theta_4 + \frac{\overline{HD}}{2}F_{Dx}\sin\theta_4$$

$$\downarrow$$

$$\sum M_{G_5} = -\frac{\overline{HD}}{2}F_{Hy}\cos\theta_4 + \frac{\overline{HD}}{2}F_{Hx}\sin\theta_4 - \frac{\overline{HD}}{2}F_{Dy}\cos\theta_4 + \frac{\overline{HD}}{2}F_{Dx}\sin\theta_4 = I_{G_5}\alpha_4$$

6.2.6 Matrix

The system of equations can be represented by the following matrix equation.

$$\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_1 & -k_2 & k_1 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & s2 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & -c2 & 0 & 0 \\
0 & 0 & k_3 & -k_4 & -k_5 & k_6 & -k7 & k8 & 0 & 0 & 0 & 0 & -BP/2 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & K9 & -K10 & 0 & 0 & K9 & -K10 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & LEG & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K11 & -K12 & 0 & 0 & K11 & -K12 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{Ax} \\ F_{Ay} \\ F_{Bx} \\ F_{By} \\ F_{C_x} \\ F_{C_y} \\ F_{D_x} \\ F_{D_y} \\ F_{E_x} \\ F_{E_y} \\ F_{H_x} \\ F_{H_y} \\ F_P \\ M_4 \\ F_{G_4}
\end{bmatrix}$$

$$
= \begin{bmatrix} m_1 a_{G_1x} \\ m_1 a_{G_1y} + m_1 \cdot g \\ I_{G_1}\alpha_1 \\ m_2 a_{G_2x} \\ m_2 a_{G_2y} + m_2 \cdot g \\ I_{G_2}\alpha_2 \\ m_3 a_{G_3x} \\ m_3 a_{G_3y} + m_3 \cdot g \\ I_{G_3}\alpha_3 \\ m_4 a_{G_4x} \\ m_4 a_{G_4y} + m_4 \cdot g - F_{Fy} \\ 0 \\ m_5 a_{G_5x} \\ m_5 a_{G_5y} + m_5 \cdot g \\ I_{G_5}\alpha_4 \end{bmatrix}
$$

Where

$$
\begin{array}{lllll}
k_1 = \frac{\overline{AB}}{2}\sin\theta_1 & k_2 = \frac{\overline{AB}}{2}\cos\theta_1 & k_3 = \frac{\overline{BP}}{2}\sin\theta_2 & k_4 = \frac{BP}{2}\cos\theta_2 & k_5 = \overline{CG_2}\sin\theta_2 \\
k_6 = \overline{CG_2}\cos\theta_2 & k_7 = \overline{DG_2}\sin\theta_2 & k_8 = \overline{DG_2}\cos\theta_2 & k_9 = \frac{\overline{EC}}{2}\sin\theta_3 & k_{10} = \frac{\overline{EC}}{2}\cos\theta_3 \\
k_{11} = \frac{\overline{HD}}{2}\sin\theta_4 & k_{12} = \frac{\overline{HD}}{2}\sin\theta_4 & s2 = \sin\theta_2 & c2 = \cos\theta_2 &
\end{array}
$$

This matrix equation may now be used in conjunction with a computer program to solve the system at every point in time.