Algorithms for Polynomials

Research Coursework:
Introduction to Programming and Data Structures

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Polynomial Operations

Topic to be covered

- Representation
- Computing a polynomial
- Addition
- Subtraction
- Multiplication
- Division

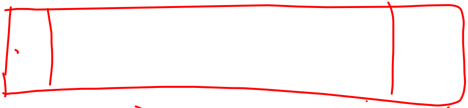
We will discuss polynomial of the form $P(x) = \sum_{i=0}^{n} a_i x^i$, i.e., polynomials with one varible.

$$p(x) = \sum_{i=0}^{n} a_i x^i$$

n- degree

$a_n \neq 0$

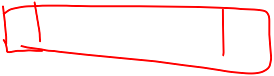
A a_0 ——— - - a_n



b_0 ——→ b_m

$(n+1)$

B



$m < n$

$C[i] = A[i] + B[i]$ $(m+1)$

max degree(C) =

0	1	2	3	4
10	5	1	0	0

A 0 1

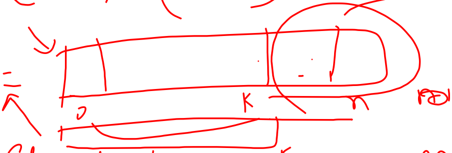
$$4x^2 + 3x + 2$$

$$-4x^2 - 3x + 5$$

C $0 \quad 0 \quad 7$

$C \rightarrow (n+1) \rightarrow$ at_most

$C =$



degree $(C) \geq K$

Check_leading_coefficient_

POLY = { float * A;
int size; →
int degree; }

$-5x^3 \; \cancel{+} - 4x^2 \; + \cancel{7}$ 3 4

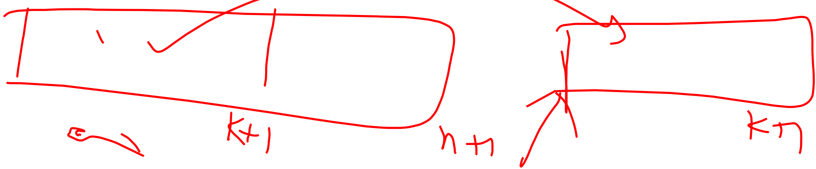
$5x^3 + 4x^2 + 2x + 3$ 3 4

$\longrightarrow$ $i = 3$

$13 + 2x + 4x^2 + 0x^3$ 3

2

$2 + 4x^2$



$3 \quad 2 \quad 1 \quad 0$

$A[n] \quad 5x^3 + 3x^2 + 2x + 2$

$B[n] \quad 4x^2 - 2x + 1$

$$C[i] = \sum_{j+k=i} a_j b_k$$

$C[n] \rightarrow$

$C[0] = A[0] \cdot B[0]$

$C[1] = A[0] \cdot B[1] + B[0] \cdot A[1]$

$C[2] = A[0] \cdot B[2] + A[1] B[1] + A[2] B[0]$

(l i)

for (j = 0 ; j < degree of A ; j++) {

~~for (k = 0 ; k < degree B ; k++) {~~

k = i - j if k >= 0 {

~~if (j + k == i)~~ { sum += A[j] · B[k]

} else break

~~{ }~~

}

$0 + \log(1) + \log n$

Given x, $P(x)$ $0+1+2+ \ldots + n-1$ $\frac{(n-1)(n-2)}{2}$

$P(x) = 5x^3 + 4x^2 + 3x + 2$ POLY P

P_1 P_0

$P(15)$

sum = 0

for (i = p.deg ; i > 0 ; i--) {

sum = sum + P[i]

sum = sum * x

}

sum = sum + P[0]

P[3] = 5

sum = 5x → i = 0

$(5x+4)x$

Division

$$A(x) = B(x) \cdot q(x) + r(x)$$

$A(x) \to m$, $B(x) \to n$, $q(x) \to Q$, $r(x) \to R$

$m \geq n$

Q degree $= n - m$

$0 \leq R$ degree $< n$

$A(x) = 5x^7 + 2x^2 + x + 5 \to 7$

$B(x) = 2x^4 + 3 \to 4$

$5x^3 + 0x^2 - 5x + 3$ $\quad Q[x] \to 3 \quad Q[3]$

$2x^4 + x^2 + 1 \;\Big)\; 5x^7 + 3x^4 + 2x + 1$ $\quad A[x] - R[x]$

$5x^7 + 5x^5 + 5x^3 \longrightarrow R[x]$

$0 \cdot \cancel{x^6} - 5x^5 + 3x^4 - 5x^3 + 2x + 1$

$-5x^5 \qquad - 5x^3 - 5x$

$3x^4 + 7x + 1$

$3x^4 + 3x^2 + 3$

$-3x^2 + 7x - 2$

for(i = 0 ; i < degree Q ; i - -) {

$\to B[m-i] / B[n-i]$

$\to Q[\text{deg} - i] = $ num

$A = A[x] - R[x]$

$\to R[x] = Q[x] \cdot B[x]$

}

Representation of Polynomials

$$P(x) = \sum_{i=0}^{n} a_i x^i$$

Different ways

How to store a polynomial?

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Different ways

How to store a polynomial?

1. Array: Useful when most of the coefficients are present
2. Linked List: Useful when very few coefficients are present
3. Any disadvantage?
4. Which is better

How to compute a polynomial

$$P(x) = \sum_{i=0}^{n} a_i x^i$$

How many multiplication and additions are required? Can We reduce multiplication further.

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Adding two polynomials

What will be the algorithm?

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Problem of over computation. Solution?

Adding two polynomials

What will be the algorithm?

What happen to the degree of new polynomial?

Problem of over computation. Solution?

Keep the degree stored.
Structure is required.

Division of a polynomial with another

Consider two polynomials:

$$f(x) = \sum_{i=0}^{n} a_i x^i,\ g(x) = \sum_{i=0}^{m} b_i x^i$$

Multiplication of two polynomials

Consider two polynomials:

$$f(x) = \sum_{i=0}^{n} a_i x^i,\ g(x) = \sum_{i=0}^{m} b_i x^i$$