Ballistic Deposition and Surface Growth

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1 Acknowledgement

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Once again, thank you, Dr. Sayeed, for your guidance and mentorship, and for helping us achieve our research objectives.

2 Introduction

The term "Ballistic" in the field of physics is used as an adjective to describe the motion and behavior of objects that are launched through the medium of air and achieve motion due to the force of gravity, The term "Deposition" refers to the process by which materials or particles are deposited or laid down on a surface or a substrate. This can occur through various mechanisms, such as sedimentation, condensation, and precipitation, etc. overall, "Ballistic Deposition" refers to the study of a system that undergoes random layer by layer particle depositions as seen in nature.

The Ballistic Deposition Model was originally designed to study the process of sedimentation analytically by taking into account the least number of environment variables, i.e. to make the simplest model to describe the process of sedimentation. This model was first proposed in 1959 by, Marjorie J. Vold in her paper "A numerical approach to the problem of sediment volume."[6]

The goal of this project was to create a simulation program to model the Ballistic Deposition process and do further analysis on the changes in surface growth by adding in further complexities and environment variables such as initial seed, sticking probabilities, poisoned surfaces, etc. The simulation program was written in FORTRAN and was Plotted using GNUPLOT.

The process of Ballistic Deposition in nature is a 3-Dimensional process but due to visualizing limitations we simulate a 2-Dimensional version where we can clearly visualize the data as well as study the properties in depth to further define those properties to the 3-Dimensional Model.

As mentioned, studying sedimentation was the main motive to develop the surface growth models, yet apart from sedimentation there exists several surface growth processes in nature that closely relate the the model's studied above. for example, Propagation of flame fronts i.e. A sheet of paper that is half burnt horizontally when observed shows the interface of the burnt and unburnt sections of the paper forms a rough pattern similar to our deposition models Fig. 1

Figure 1: Interface of burnt paper shows closely related surface roughnening patterns as our deposition models

Frost formation or snow particles Deposition i.e. a window prone to frost formation in cold climatic conditions forms a rough surface growth of ice. Ballistic Deposition closely mimics such surface growth as the basic rules following the deposition are closely related with added variables such as pressure dependence, temperature dependence, particle size variance, multi-directional deposition, etc.. Fig. 2

Figure 2: Frost formation on a window having rough surface growth [5]

Both these examples follow similar surface growth yet also vary a lot from the idealistic point of view of surface growth due to the fact that natural surface growth is bound to have several complications that we tend to avoid in our models.

Apart from these examples of natural growth, certain number of lab generated surface growth experiments are also conducted to back our surface growth simulations for ideal cases. two of these experimental studies are discussed below that include 'Structure and Mechanical Properties of High-Porosity Macroscopic Agglomerates Formed by Random Ballistic Deposition' [2] by Jurgen Blum and Rainer Schraple, and 'Deposition Kinetics of Particles at a Solid Surface Governed by the Ballistic Deposition Model' [3] by Ph. Carl, P. Schaaf, J.-C. Voegel, J.-F. Stoltz, Z. Adamczyk, and B. Senger.

3 Description of the Model

The process of deposition begins with taking a 2D empty box of desired length where, identical particles are thrown from the maximum height of the box at random. These particles keep moving further down until and unless a certain condition is met at which the particle becomes stationary. The conditions mentioned above are for us to define so as to mimic the process seen in nature.

The simplest of such processes is the Random Ballistic Deposition (RBD) where the condition for the particles to become stationary is if and only if there is a particle below it, or it has reached the minimum height of the box. Although such processes are not seen in nature, the surface growth for them can be simply determined.

Adding a few more conditions to the falling particles takes us further towards Ballistic Deposition (BD) and mimics the natural process better than that by RBD. the following conditions are added to the RBD to simulate BD: the particles become stationary If there is another particle beside it at that point, if there is a particle below it, or it has reached the minimum height of the box.

This framework works neatly for studying the natural deposition processes as the properties studied from BD Simulations agree with the experimental results that were studied and verified.

3.1 Simulation Details

We begin modeling our simulation by selecting the initial variables i.e. Length of the Box (L), Number of Monte Carlo Cycles (nmc).[1][2] We further create an array of 'L' elements to store the maximum height of the system at each point after every particle being deposited. i.e.

$$\mathbf{H}[x_1, x_2, x_3, ..., x_L]$$

where, x_i refers to the maximum height of the system at i^{th} position [3] Now, we initialize our lattice of length 'L' and maximum initial height at all points to be 0 (since no particles are deposited). We begin by selecting a random point using a random integer 'r' between 1 to L and drop the particle from the maximum height of our lattice from the point 'r' and update the new maximum height at r^{th} element in $\mathbf{H}$ following our deposition conditions based on the type of deposition being studied. NOTE: it is sufficient enough to have a single array of 'L' elements that stores the maximum heights at every point instead of a 2D array storing the information of all the points of our system since, it's computationally expensive to have a 2D array than a 1D array. The methodology mentioned above remains the same irrespective of the type of deposition being studied unless and until the initial conditions for the system are mentioned.

Since, we use uniform random numbers to generate our deposition model, we need to repeat our simulation N number of times and average it to get the actual results. [4]

[1] Monte Carlo Methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.

[2] NOTE: There is no measure for 'time' in our simulation program. Hence, we use 1 unit of time to be equal to 'L' number of particles deposited (This is done to make sure that our system is particle concentration invariant.)

[3] NOTE: Natural surface growth neither follows lattice deposition nor has identical particles being deposited but due to the fact that it is computationally expensive to simulate purely natural surface growth, we simulate Lattice deposition of identical particles.

[4] In our simulation model, we have repeated 1 set of simulation $N = 100$ number of times and averaged the result over to get better results. higher the value of N higher the results as well as the computation time.

3.1.1 Random Ballistic Deposition

For Random Ballistic Deposition Simulation (RBD), after setting up the values for 'L', 'nmc' and '**H**' respectively as mentioned above. Now select a random number 'r' between 1 and L and now add the following condition to update **H** with respect to 'r'

$$\mathbf{H}[x_r] = x_r + 1$$

where, x_r is the r^{th} element in **H** . We, repeat this process [Length * No. Of Monte Carlo Cycle] times and note down the coordinates after every particle is deposited. i.e

$$(x, y) = (r, \mathbf{H}[r])$$

NOTE: Since, no particle interacts with its adjacent neighbours it is sufficient to proceed without adding boundary conditions at 'r' = 1 or L.

On visual analysis of simulating Random Ballistic Deposition Fig. 3 it can be seen clearly that the surface roughness increases as a function of time.

NOTE: We are only categorising 'roughness' based on physical visualization and have not yet quantified the values for surface roughness.

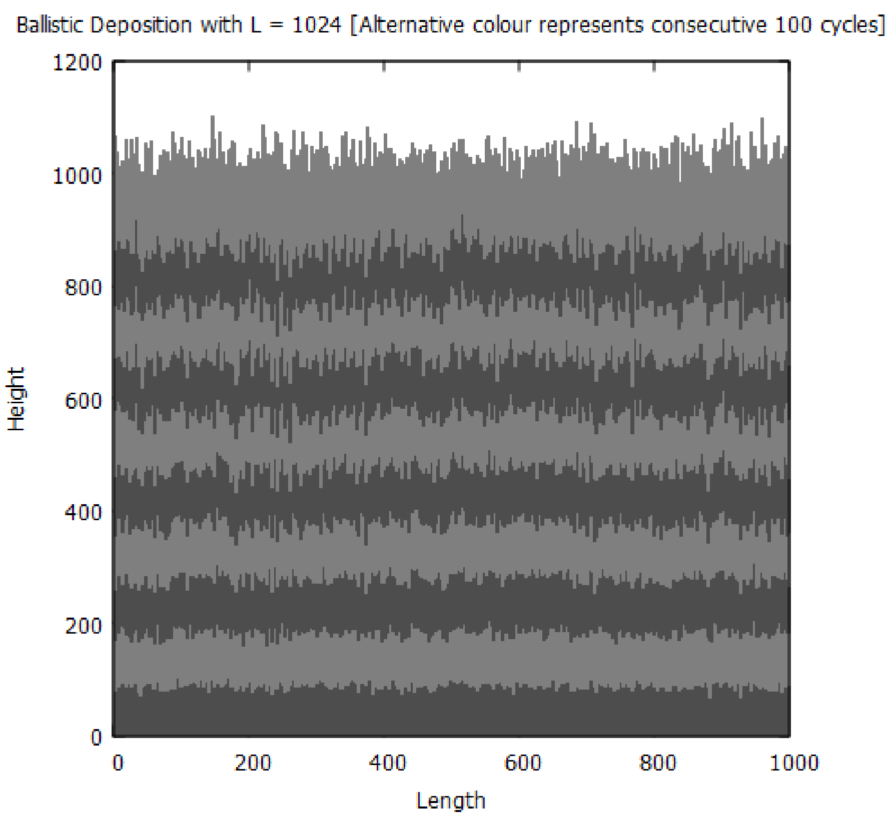

Figure 3: Random Ballistic Deposition Simulation in a box of length = 1024 units and No. of Monte Carlo Cycles = 1000

3.1.2 Ballistic Deposition

For Ballistic Deposition we follow similar procedure as mentioned above in RBD with additional conditions. The particle that is being deposited does interact with its adjacent neighbours causing the particle to become stationary with respect to its adjacent neighbour. This interaction is termed as 'sticking' where the particle 'sticks' to its neighbour before settling down as seen in RBD and can be achieved with the following Condition:

$$\mathbf{H}[x_r] = max(\mathbf{H}[x_{r-1}], \mathbf{H}[x_r] + 1, \mathbf{H}[x_{r+1}])$$

where, we also apply the boundary conditions i.e.

for r = 1, $\mathbf{H}[x_{r-1}] = \mathbf{H}[x_L]$ and for r = L; $\mathbf{H}[x_{r+1}] = \mathbf{H}[x_1]$.

These conditions are sufficient enough to proceed further and note down the coordinates after every particle is deposited i.e. $(x, y) = (r, \mathbf{H}[r])$.

As seen in RBD, we also repeat this process [Length * No. Of Monte Carlo Cycle] times. NOTE: In Natural Deposition processes, the interaction between particles can be temporary and not 'strict' i.e. the particles can further move down to attain equilibrium. This is one of the complexities we observe in nature that can be tricky to simulate. Our main aim is to make the simplest model and study it to further add complexities, Hence, BD is the simplest model to demonstrate Natural Deposition processes.

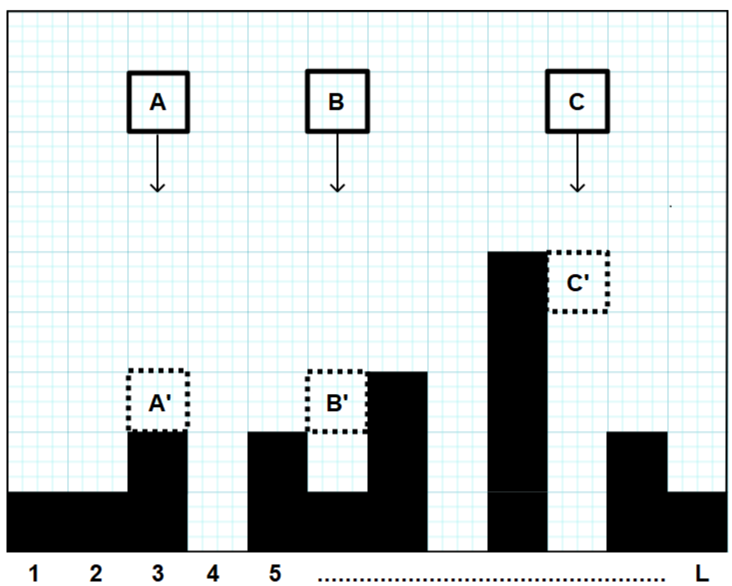

Figure 4: Ballistic Deposition Model: Starting initially with a flat substrate, particles are droped one at a time randomly from a height greater than the maximum height of the system and stick upon first contact. A', B', C' are sticking positions of particles A, B, C

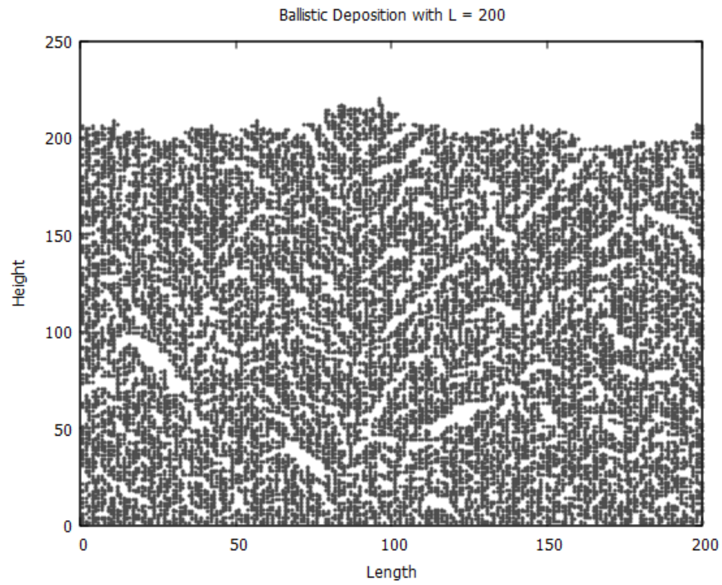

Figure 5: Simulation of the Ballistic Deposition model for L = 200 and No. of Cycles = 100

From Fig. 5 We can clearly see by the visualization that a lot of the portion in our lattice remained empty and particles can not reach those area's due to the strict 'sticking' of particles. This leads for our surface growth to have a 'Porous'[5] structure.

[5]Porous: : possessing or full of pores/permeable

3.2 Results and Discussion

Upon visual analysis, we choose uniform random surface growth while performing our simulation. This is done purely to mimic Natural surface growth as it is observed that the mean height of the surface increases linearly with respect to time. Though the mean height increases linearly, we visually observe that the surface roughness increases as a function of time. Another characteristic property of this type of surface growth is that, we observe that as the roughness increases as a function of time, so does the deviation of heights from the mean height at a given point in time. this deviation of heights can be quantified using standard deviation ($\sigma = \sqrt{\langle \mathbf{H}^2 \rangle - \langle \mathbf{H} \rangle^2}$). It is sufficiant enough to say that roughness of our surface is directly proportional to σ as both are seen to be an increasing function of time, and since we have not quantified the concept of roughness, we can clearly say that knowing σ gives us enough information about the surface roughness itself. Hence, we use Standard Deviation to quantify the surface roughness of BD. Lets use the symbol $W(t)$[6] and call it the 'Surface Width' instead of using σ to define Standard deviation ($W(t) = \sigma$)

In the algorithm above we defined 1 unit of time to be equal 1 Monte Carlo cycle i.e. to have 'L' number of particles being randomly deposited. Therefore, we now run our simulation again while calculating W(t) using the Std. deviation formula mentioned above every Monte Carlo Cycle and note down the data of time and W(t) and further plot our data i.e t v/s W(t) to analyze our surface growth.

3.2.1 Random Ballistic Deposition

For Random Ballistic Deposition it is easy to calculate the standard deviation W(t) as a function of time using statistical analysis and is given as $W(t) = t^{0.5}$ which matches precisely with our simulation as expected.

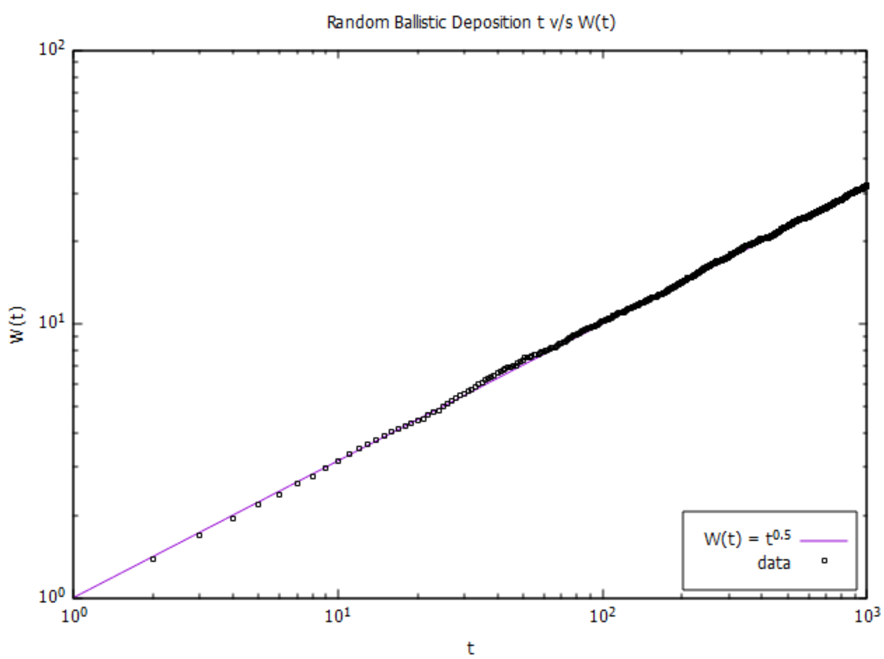

Figure 6: Surface Width as a function of time for Random Ballistic Deposition with the function $W(t) = t^{0.5}$ coinciding with our data signifying that our simulation model follows accurately to the theoretical calculation

[6]W(t) is the standard deviation of the array $\mathbf{H}$ at time 't'

NOTE: We get the same results irrespective of the Length chosen for our box. Hence, the function $W(t) = t^{0.5}$ is sufficient and enough to describe surface growth for RBD.

Particles are deposited on a surface at random in Random Ballistic Deposition, replicating the behaviour of particles in a gas or liquid. Statistical methods can be used to calculate the standard deviation of the surface height after a particular number of particles have been placed as follows:

Let $h(x)$ be the height of the surface at position x after n particles have been deposited. We can define the average height of the surface as:

$$H = \frac{1}{L}\int_0^L h(x)dx$$

where L is the length of the surface. The variance in height is:

$$\sigma^2 = \frac{1}{L}\int_0^L (h(x) - H)^2 dx$$

where σ is the standard deviation of the height distribution or the Surface Width. In Random Ballistic Deposition, each particle is independent of interactions with any other particles. Let $\Delta h(x)$ be the height of the surface at position x after one particle has been deposited. Therefore, after n particles are deposited,

$$h(x) = \mathbf{H} + \sum_{i=1}^{n} \Delta h_i(x)$$

where $\Delta h_i(x)$ is the height change due to the ith particle deposition. Hence, after n particles being deposited, Variance term becomes:

$$\sigma^2 = \frac{1}{L}\int_0^L \left[H + \sum_{i=1}^{n} \Delta h_i(x) - H\right]^2 dx$$

$$\sigma^2 = \frac{1}{L}\int_0^L \left[\sum_{i=1}^{n} \Delta h_i(x)\right]^2 dx$$

The height change due to the ith particle deposition, $\Delta h_i(x)$, is a random variable with mean 0 and variance $\sigma_0^2 = 1$, where σ_0 is the standard deviation of the height change due to a single particle deposition. Assuming that the height changes due to particle depositions are uncorrelated, we can write:

$$\mathrm{Var}\left[\sum_{i=1}^{n} \Delta h_i(x)\right] = \sum_{i=1}^{n} \mathrm{Var}\left[\Delta h_i(x)\right] = n\sigma_0^2 = n$$

Substituting this expression into the equation for σ^2, we obtain:

$$\sigma^2 = \frac{1}{L}\int_0^L n^2 dx = \frac{1}{L}[L^2] = L$$

Now, since in our simulation we use 1 time step = L particles randomly deposited we get,

$$\sigma = \sqrt{L} = \sqrt{t}$$

3.2.2 Ballistic Deposition

For Ballistic Deposition, we observe that the mean standard deviation W(t) does behave in the same manner as that seen in RBD, though it is very short lived (only in the range between t = 1 to 10), this region where BD initial behaves like RBD is known as Poisson's regime, after which the the function of time that defines W(t) changes to some t^β such that $W(t) \propto t^\beta$ which is also known as Growth Regime and later starts saturating at higher values of 't' to a certain point depending upon the Length of the Box that is known as Saturation Regime i.e. W(t) for BD is also a function of 'L' hence, we'll use W(L,t) to define the surface width for BD as a function of Length as well as time.

On further plotting the surface width for different values of 'L' as a function of time on log-scale Fig.5, we get 3 insights out of our plot.

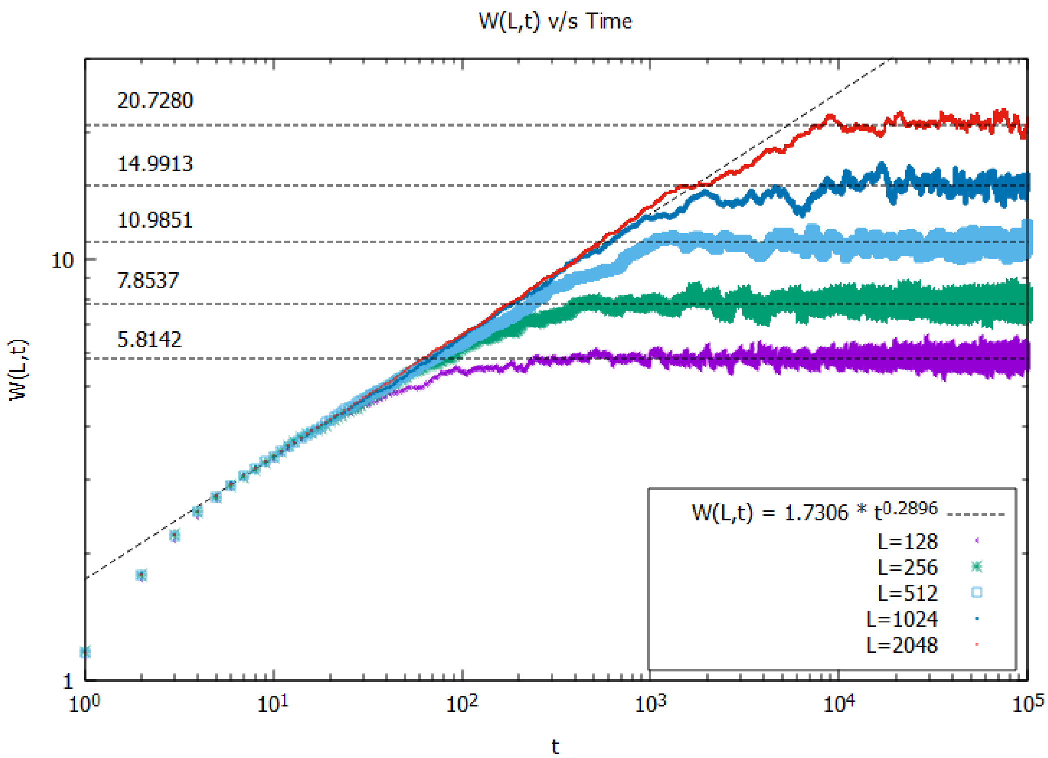

Figure 7: Surface Width as a function of time for and ensamble average of Ballistic Deposition for different values of L on a logarithmic scale

1. The Plot behaves irrespective of Length before saturating and is a function of time given as, $W(L,t) \sim t^\beta$ for, $t < t_{sat}$

2. the saturation width $W_{sat}(L,t)$ is some function of L^α i.e $W_{sat}(L,t) \sim L^\alpha$

3. the time at which the surface width starts saturating is also some function of L^z i.e. $t_{sat} \sim L^z$

NOTE: Analytical solution to the values of these exponents for Ballistic Deposition becomes far more complex than that of Random Ballistic Deposition. Hence, instead of doing the analytical proof, we proceed to study the relationships of these exponents with each other instead.

Performing a Least Square fit to Fig.7 in the growth region, we clearly see that the value of $\beta = 0.2896 \pm 0.1\%$ i.e. for $t < t_{sat}$, we have $W(L,t) \sim t^{0.2896}$. To get the values of the remaining exponents i.e. α and z, we do the following plots. For the value of α, we plot L v/s W_{sat} Fig. 8

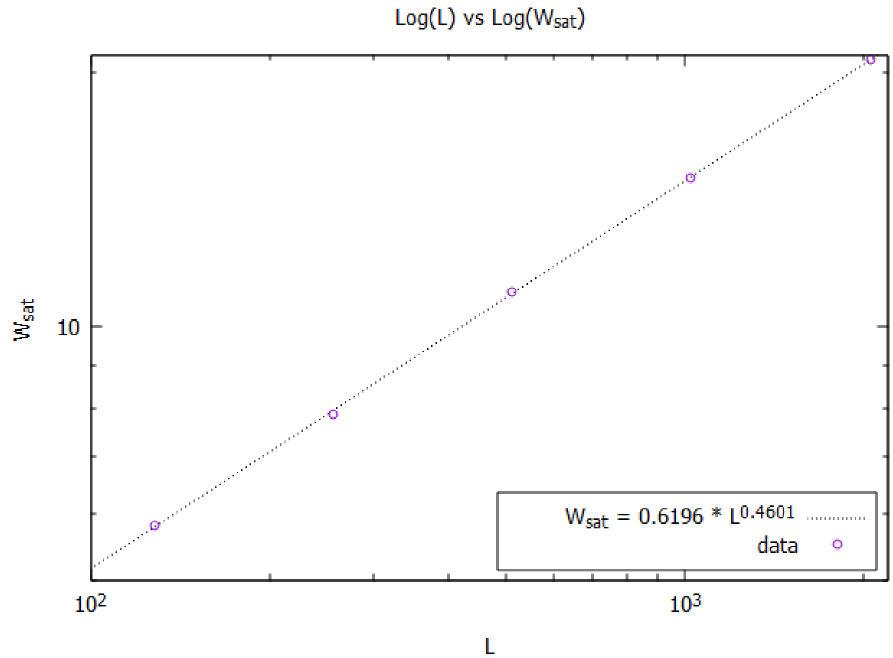

Figure 8: W_{sat} as a function of L

After performing Least Square Fit, The value for $\alpha = 0.4601 \pm 0.1\%$ i.e. $W_{sat} \sim L^{0.4601}$. Similarly, we plot L v/s t_{sat} Fig. 9.

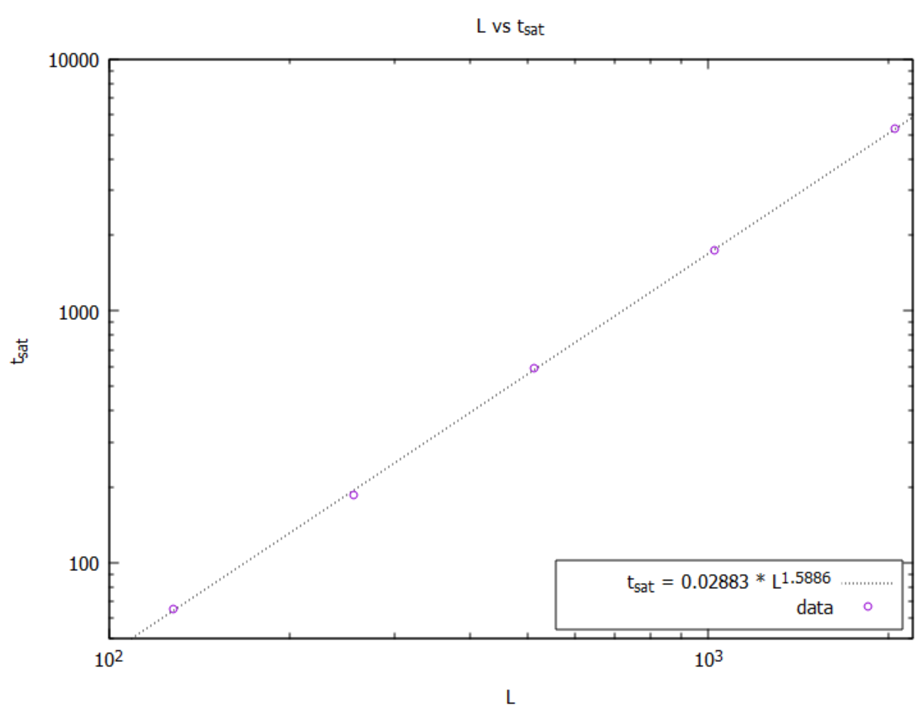

Figure 9: t_{sat} as a function of L

On analysis of the plot, the value for $z = 1.5886 \pm 0.1\%$, i.e. $t_{sat} \sim L^{1.5886}$.
The Values of the exponents are observed to be;

1. $\beta = 0.2896 \pm 0.1\%$,

2. $\alpha = 0.4601 \pm 0.1\%$, and

3. $z = 1.5886 \pm 0.1\%$.

We also note that $z = 1.5886 = \frac{0.4601}{0.2896} = \frac{\alpha}{\beta}$

These exponents are known as Scaling Exponents and aren't independent. Due to this property, we can further fit all the curves for different values of L in fig. 5 onto a single curve. This is done using the Family-Vicsek Scaling Relation [4] i.e.

$$W(L,t) \sim L^{\alpha} f(\frac{t}{L^{z}})$$

Therefore, when plotted $\frac{W(L,t)}{L^{\alpha}}$ v/s $\frac{t}{L^{z}}$ we get the following curve, Fig. 8 that fits excellently as expected as per the growth regime and so forth.

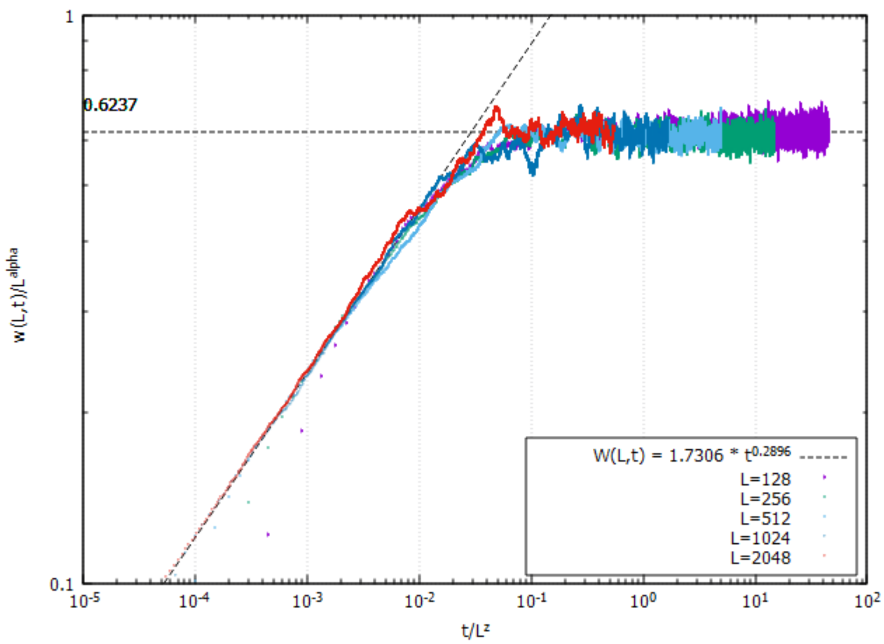

Figure 10: Family-Vicsek Scaling Relation $[\frac{W(L,t)}{L^{\alpha}}$ v/s $\frac{t}{L^{z}}]$ i.e. the general curve for Ballistic Deposition

It is important to to note that saturation time t_{sat} and the saturation width W_{sat} increases with the system size, suggesting that the saturation phenomenon has a 'Finite Size Effect' [1] i.e. a system with infinite size ($L = \infty$) DOES NOT Saturate.

BD growth process develop 'Correlations' unlike RBD that leads to saturation, implying that surface heights at each points depends on that of neighbouring sites. The microscopic origin of correlations is UNKNOWN but it can be further studied.

According to our rules of deposition, the particle that is being deposited at a cite depends upon neighbouring heights at that site. As more and more particles are deposited randomly, this dependence extends out to the entire system that leads to correlation. The characteristic distance of correlation is known as correlation length denoted by ξ_l.

At beginning, BD behaves like RBD having no correlations.as time increases, so does ξ_l.

For finite systems, ξ_l can have a maximum value thats equal to L. when this value is achieved, the entire system becomes correlated resulting in saturation of surface width. i.e.

$$\xi_l \sim L \qquad [t > t_{sat}]$$

$$\xi_l \sim t^{1/z} \qquad [t < t_{sat}]$$

Random Sticking Probability

The Ballistic Deposition Model that was studied before had strict particle interaction rules i.e. once the particle came close to another particle, it became stationary. We now introduce the concept of Sticking Probabilities i.e. we randomly choose, whether the particle after coming in contact with another particle becomes stationary or descends further down. It is important to note that in the case of Random Ballistic Deposition, this 'Sticking Probability' (lets call it P_s) $P_s \sim 0$ that signifies that our particle never interacts with another particle, and in the case of Ballistic Deposition, $P_s \sim 1$ that signifies that our particle always 'sticks' to the first particle that it comes in contact with i.e. RBD and BD are the extreme cases for all values of P_s. In nature, P_s can be variable depending upon the environmental and materialistic properties or can also have a constant value. We consider 'Random Sticking Probability' case in order to mimic nature as accurately as possible.

In random sticking probability case of BD, we choose 2 random numbers 's' and 'P_s' $\in [0, 1]$ and apply the following conditions:

1. If $s < P_s$ then the particle moves 1 unit down if there isn't another particle below it repeats the procedure for the new position unless it becomes stationary or becomes stationary if there is a particle below it.

2. If $s \geq P_s$ then the particle becomes stationary at that point itself.

On performing similar analysis as done in Ballistic Deposition, from the results obtained we see that the exponents α, β, and z remain UNCHANGED though there is a change in the value of Surface width at saturation, i.e. W_{sat} for Random Sticking Probability is higher than W_{sat} with $P_s = 1$ which is expected since we see no saturation at $P_s = 0$ i.e. surface width saturation for RBD is at ∞.

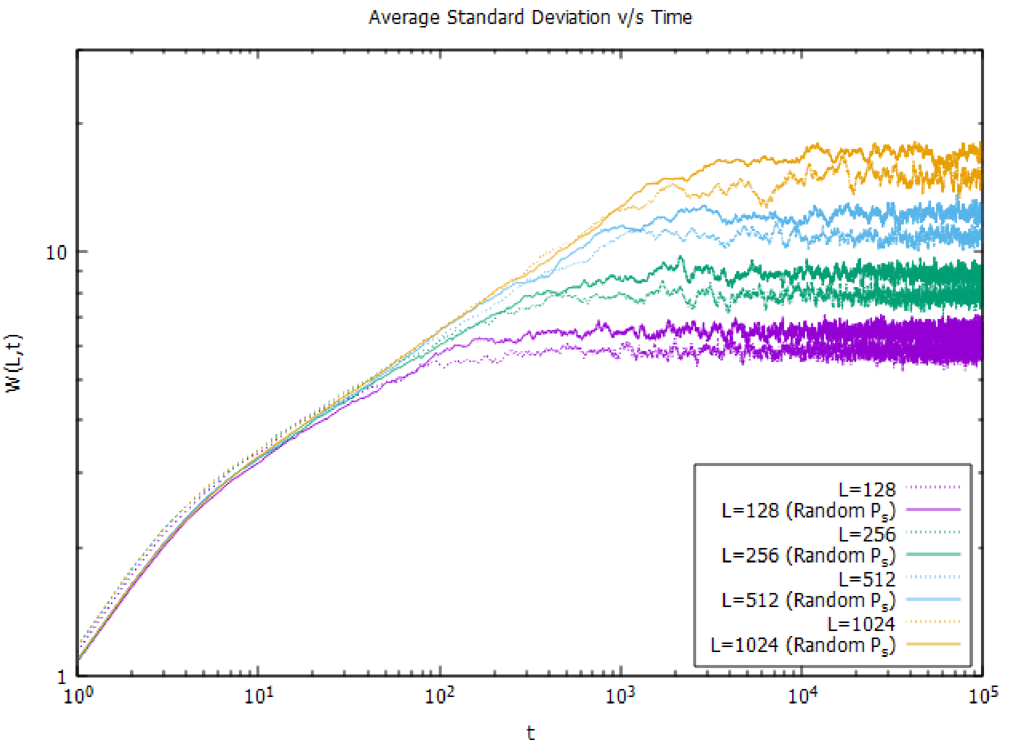

Figure 11: Surface Width as a function of time with comparison of surface growth between Ballistic Deposition having $P_s = 1$ and $P_s \in [0, 1]$

From Fig. 11 It is seen that the exponents remain unchanged, we can conclude the fact that they are particle interaction invariant for identical particles. the reason we only study identical particles is due to the fact that adding particles of various sizes can become too complex and computationally expensive for our computers to render at higher values of Monte Carlo Cycles.

Initial Seed

The Ballistic Deposition Model begins with the assumption of the initial surface width $W(L, 0) = 0$ i.e. a 'Plane Surface'. Such a surface is only possible idealistically though throughout nature, there exists NO plane surface. Hence to make our simulation mimic this property, We initiate our simulation with an 'Initial Seed' i.e. Initializing all elements excluding one of $\mathbf{H}$ to be 0. For simplicity we use the midpoint of our system i.e. $\mathbf{H}[int(\frac{L}{2})] \neq 0$ and let our system evolve.

The Following Fig.10 Shows an example of a simulated model of BD for size of the box L = 100 with an initial seed at the midpoint i.e. $\mathbf{H}[50] = 100$. On further analysis for different values of initial seeds and Lengths we still see saturation of the surface width.

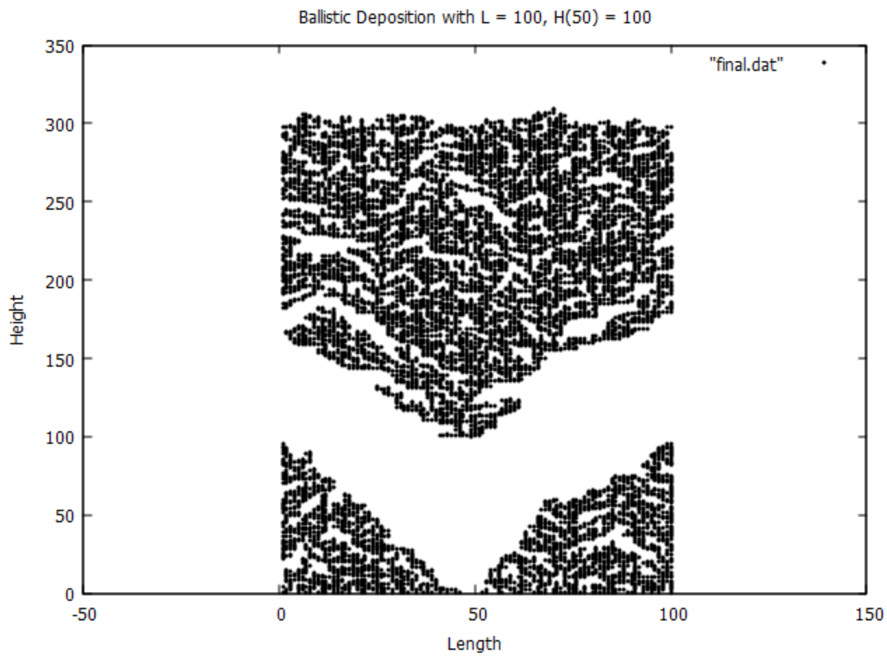

Figure 12: Surface Width as a function of time for BD (L = 100) with an initial seed (i.e. initializing **H**[50] = 100)

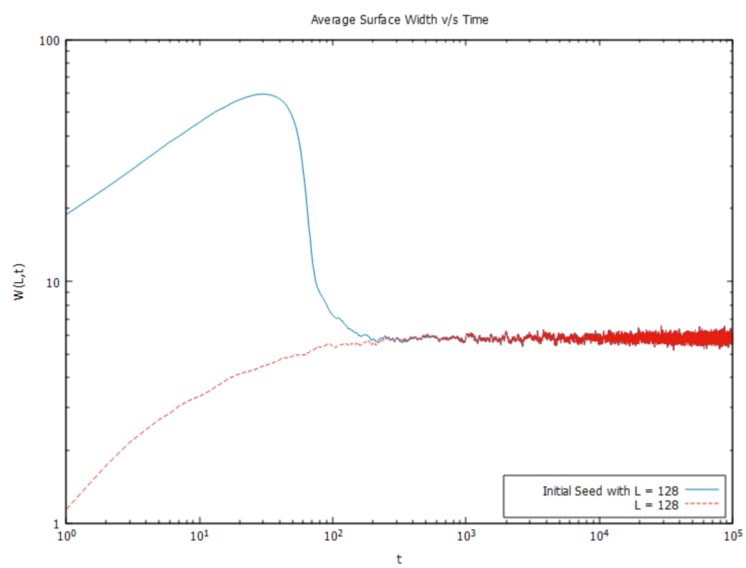

Figure 13: comparison of surface width as a function of time for BD with and without initial seed for a box size L=128

We see that the surface width for the case of initial seed increases due to the presence of a single peak until the average height of the entire system reaches to the height of that peak and later on decreases quickly until it starts saturating at the same width as that for normal BD. This implies that presence of an initial seed does not affect the system saturation width in any way which is expected due to the fact that correlation begins as the average height reaches to the point of the initial seed. Hence, we conclude with the result that the surface on which BD occurs has no impact on the final results of the average surface width achieved after saturation.

3.2.3 Porosity

Another important property of how our surface evolves is to study the 'voids' that are left while the evolution occurs, to be more specific is to study the emptiness of our system as it evolves. This can easily be studied by defining porosity of the system i.e.

$$\phi = \frac{V_v}{V_t}$$

where, ϕ = Porosity, V_v = Void Volume, V_t = Total Volume.

It is easily seen that Porosity (ϕ) takes values from 0 to 1, 0 1 signifying completely empty system and 0 being highly compact system.

On further analysis of our simulations we found the following results.

Random Ballistic Deposition: Theoretically, as it is seen that for RBD there are not neighbouring interactions making it a highly compact system ($\phi \sim 0$). Hence, we are tempted to assume to see similar results in calculating porosity for RBD. The following Fig. 14 Shows the results acquired by calculating porosity for RBD

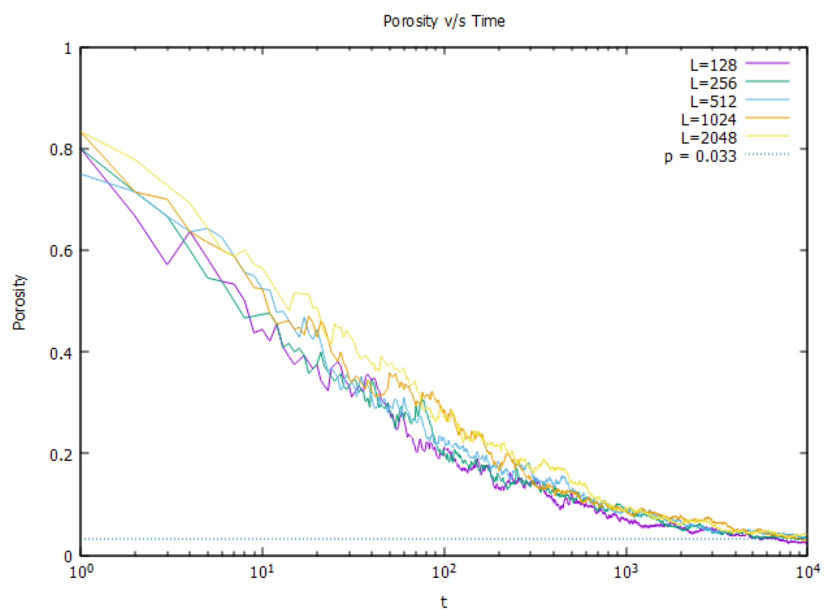

Figure 14: Porosity as a function of time for Random ballistic deposition where, each line represents different box sizes ranging from L = 128 to 2048 on a time scale ranging from t = 0 to 10^4

As we see that the saturation for porosity reaches very close to 0 but not exactly equal to 0 due to the surface effects at maximum height of our simulation. to be more specific, $\phi_{RBD} \sim 0.033$.

Ballistic Deposition: For the case of Ballistic Deposition we see similar curves for different values of the size of our system which after passing through the growth regime start saturating at a point close to 0.5 Fig. 15.

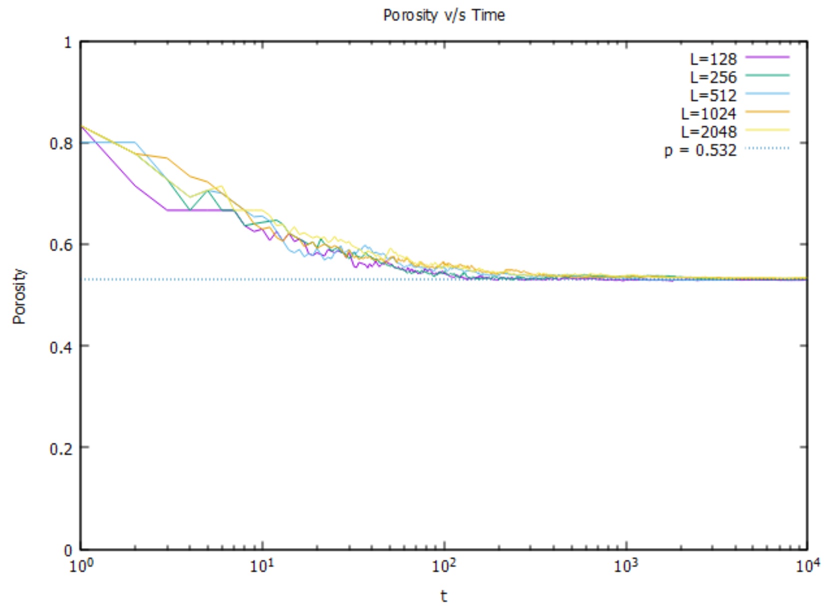

Figure 15: Porosity as a function of time for Ballistic Deposition where, each line represents different box sizes ranging from L = 128 to 2048 on a time scale ranging from t = 0 to 10^4

More specifically, $\phi_{BD} \sim 0.532$. But as it is seen in RBD that was supposed to be highly compact structure with porosity being close to 0, we see surface effects causing our porosity to have a slighly higher value ($\phi_{RBD} = 0.033$), we can assume to have similar effects in BD. We assume the surface effects to be the same for BD and RBD therefore, we consider the value of ϕ_{RBD} = Surface Effect Error.

Hence, the actual value of ϕ_{BD} = 0.532 - ϕ_{RBD} = 0.532 - 0.033 = 0.499. i.e.

$$\phi_{BD} = 0.499 \sim 0.5$$

3.3 Experimental Studies

Experimental studies were conducted for the same where the environmental factors affecting our surface growth were maximally isolated and on further reviewing, the results seemed to match with the simulated ones to a high accuracy which evidently set the model to have a physical significance in the studies of surface growth. for example:

Example 1: In the paper 'Structure and Mechanical Properties of High-Porosity Macroscopic Agglomerates Formed by Random Ballistic Deposition '[2], Jürgen Blum and Rainer Schräpler studied the model of RBD and implemented and experimental model to perform similar surface growth of SiO_2 molecules. The following Fig. 16 shows the basic apparatus of the experimental model set up by Blum and Schräpler to follow RBD.

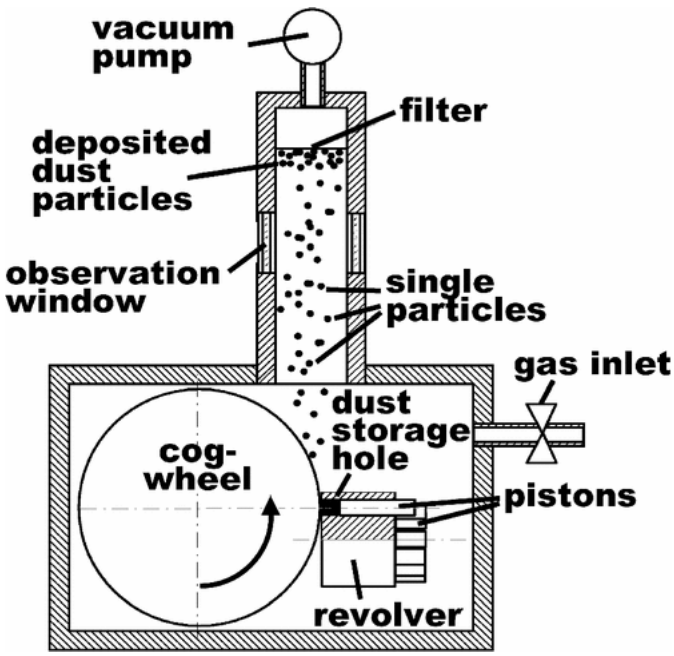

Figure 16: Schematics of the experimental setup for the formation of macroscopic RBD agglomerates. [2]

They presented experimental results on the mechanical properties of macroscopic agglomerates formed by ballistic hit-and-stick deposition. The agglomerates, produced with a new experimental method, consist of mono-disperse SiO_2 spheres with $1.5\mu m$ diameter and have a volume filling factor of $\phi = 0.15$, matching very closely the theoretical value for random ballistic deposition. They are mechanically stable against unidirectional compression of up to 500 Pa. For pressures above that value, the volume filling factor increases to a maximum of $\phi = 0.33$ for pressures above 10^5 Pa. The tensile strength of slightly compressed samples ($\phi = 0.2$) is 1000 Pa. the resulting images of the experiment conducted by Blum and Schräpler are shown in Fig. 17

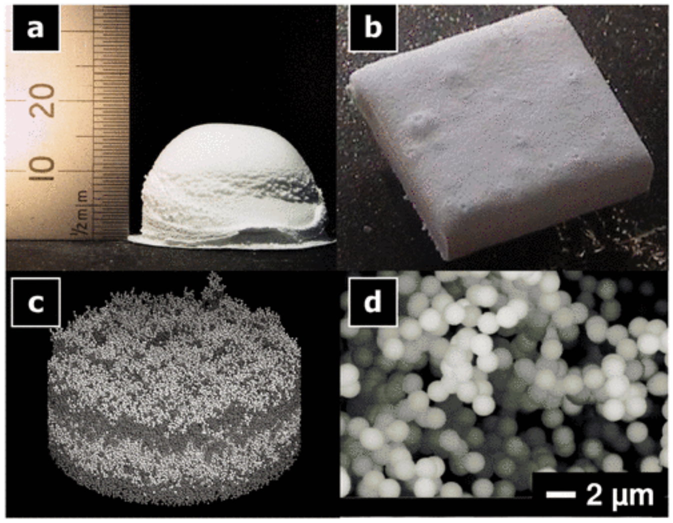

Figure 17: a) An example of an agglomerate with a volume filling factor of $\phi = 0.15$. (b) Specimen of an agglomerate after manual cutting to $\sim 1010mm^2$. (c) Result of a Monte Carlo simulation of ballistic deposition. (d) High resolution scanning electron microscopy (SEM) image of the surface of an agglomerate consisting of SiO_2 spheres with $1.5\mu m$ diameter.[2]

Example 2: Another Example of Experimental Analysis of BD is the paper 'Deposition Kinetics of Particles at a Solid Surface Governed by the Ballistic Deposition Model' [3] where they studied, The deposition kinetics of polymeric micro-spheres i.e melamine formaldehyde particles (average diameter $8.742\mu m$, density $1.50gcm^{-3}$) on smooth silica plates that was determined experimentally.

The experimental system consists of a cell, connected to a suspension reservoir, a microscope, a CCD video camera, and a computer devoted to the image analysis. The measurements were carried out in a sedimentation cell using a direct microscope observation technique supplemented by the image analysis.

The experimental data were interpreted in terms of the ballistic deposition (BD) model, where the irreversibility of the adsorption was assumed. It was found that both deposition kinetics and the jamming coverage (determined to be 0.60 ± 0.02) were in quantitative agreement with the BD model. their experimental data constitute, therefore, a direct proof of the validity of this model for characterizing particle deposition processes driven by a strong gravitational force field.

The Ballistic Deposition model used in the simulation of the experiment was non-lattice oriented and the particles deposited had spherical structure. The simulation model was far more complex than the one studied hence, was a better approximation to natural surface growth. The results are listed as charts in Fig. 18 and 19

"Although the experiments were performed for large particles, it can be postulated that the BD model is valid for smaller particles (colloids), provided that the value of R* characterizing the particles is large and that the Debye length is small compared to the particle size. However, for determining the limiting value of the R* parameter, for which the BD model becomes valid as far as the kinetics is concerned, additional experiments are needed." [3]

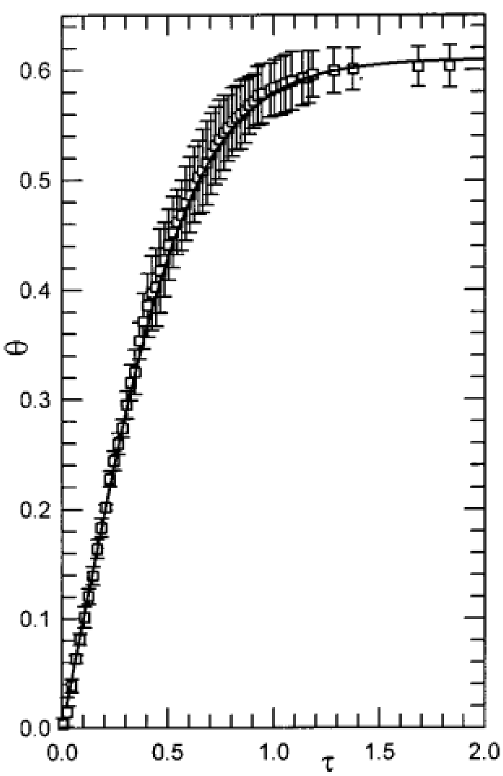

Figure 18: Evolution of the coverage θ as a function of the normalized time τ for particles of diameter d = 8.742μm. The continuous line represents the kinetics derived from the BD model.[3]

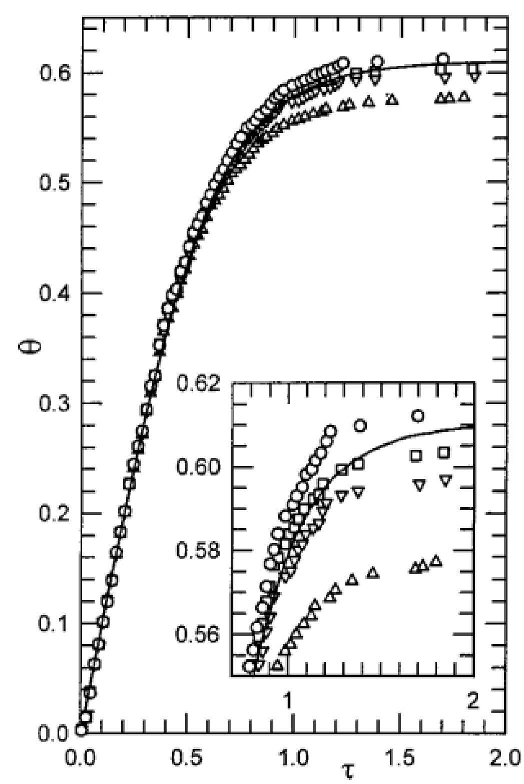

Figure 19: Evolution of the coverage θ as a function of the normalized time τ for particles of diameter d = 8.742μm. Different points correspond to different exclusion radii [3]

4 References

We implemented and made our codes available on GitHub.[7]

[1] A-L Barabási and Harry Eugene Stanley. *Fractal concepts in surface growth.* Cambridge university press, 1995.

[2] Jürgen Blum and Rainer Schräpler. “Structure and mechanical properties of high-porosity macroscopic agglomerates formed by random ballistic deposition”. In: *Physical review letters* 93.11 (2004), p. 115503.

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[6] Marjorie J Vold. “A numerical approach to the problem of sediment volume”. In: *Journal of colloid science* 14.2 (1959), pp. 168–174.

[7] `https://github.com/lalwsank/ballistic-deposition`