CS70–Spring 2022 — Disc00b Solutions

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Collaborators: NONE

1. Propositional Practice

(a) $(\exists x \in \mathbb{R})(x \notin \mathbb{Q})$ is true with $x = \sqrt{2}$

(b) $(\forall x \in \mathbb{Z})(x \in \mathbb{N} \vee x \leq 0)$ is false with $x = 0$

(c) Let $P(x)$ be "x is divisible by 6", $Q(x)$ be "x is divisible by 2", $R(x)$ be "x is divisible by 3". We have $P(x) \implies Q(x) \vee R(x)$ that is true because 2 and 3 are factors of 6.

(d) For all x in the set of integers, x is in the set of rational numbers, which is true because $\mathbb{Z}$ is a proper subset of $\mathbb{Q}$.

(e) For all x in the set of integers, if x is divisible by 2 or x is divisible by 3 is true, then x is divisible by 6, which is false with $x = 2$.

(f) For all x in the set of natural numbers, if x is greater than 7, then there exists a and b in the set of natural numbers such that the sum of a and b is equal to x, which is true with $x = 8, a = b = 4$.

2. Truth Tables

(a) The two proposition forms are not logically equivalent.

P	Q	$P \wedge (Q \vee P)$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	F	F
F	F	F	F

(b) The two proposition forms are logically equivalent.

P	Q	R	$(P \vee Q) \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(c) The two proposition forms are logically equivalent.

P	Q	R	$(P \wedge Q) \vee R$	$(P \vee R) \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	F

3. Implication

N.B. An implication $P \implies Q$ is only false when P is true and Q is false.

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$ is true.

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$ is false with the predicate $P(x,y)$ being "the number x is greater than y".

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$ is true (source: StackExchange).