Tutorial 05 Notes

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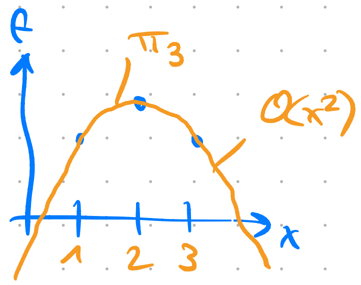
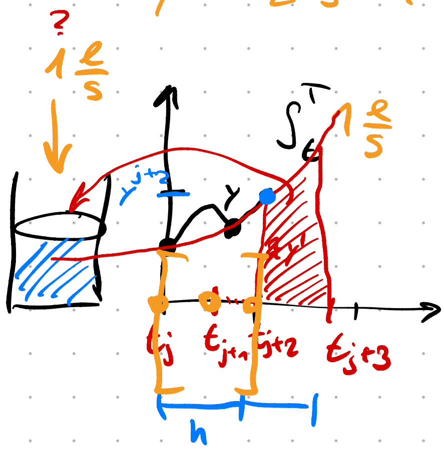
$$y^{j+3} = y^{j+2} + \int_{t_{j+2}}^{t_{j+3}} \overset{y'}{\Pi_3(s)}\, ds \qquad (*) \qquad y' = f(t, \boxed{y})$$



I) $\Pi_3(\underbrace{t_{j+3-1}}_{t_{j+2}}) \overset{!}{=} f(t_{j+2}, y^{j+2})$

II) "$\Pi_3(t_{j+1}) \overset{!}{=} f(t_{j+1}, y^{j+1})$

III) $\Pi_3(t_j) \overset{!}{=} f(t_j, y^j)$



$$\int \Pi_3(s) = \int f(t_j, y^j) \cdot L_1(s) + \int f(t_{j+1}, y^{j+1}) \cdot L_2(s) + \int f(t_{j+2}, y^{j+2})\ L_3(s)$$

$$L_j(t) := \prod_{i=1, i\neq j}^{k} \frac{(t - t_i)}{(t_j - t_i)}.$$

$$L_1(s) = \left(\frac{s - t_2}{\underbrace{t_1 - t_2}_{-h}}\right) \cdot \left(\frac{s - t_3}{\underbrace{t_1 - t_3}_{-2h}}\right) = \frac{s - t_{j+1}}{} \cdot \frac{s - t_{j+2}}{2h^2}$$

$$L_2(s) = \frac{s - t_1}{\underbrace{t_2 - t_1}_{h}} \cdot \frac{s - t_3}{\underbrace{t_2 - t_3}_{h}} = \frac{(s - t_j)(s - t_{j+2})}{-h^2}$$

$$L_3(s) = \frac{(s - t_j) \cdot (s - t_{j+1})}{2h^2}$$

$$\int_{t_{j+2}}^{t_{j+3}} \Pi_3(s)\, ds \overset{\Sigma}{=} \int f$$

$3-1=2$
$k-1$

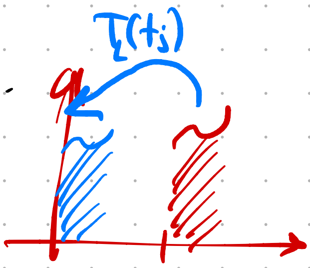
$$y^{j+3} = \underbrace{y^{j+3-1}}_{y^{j+2}} + h \sum_{\ell=0} b_{k,\ell} f(t_{j+\ell}, y^{j+\ell})$$

$$y^{j+3} = y^{j+2} + \int_{t_{j+2}}^{t_{j+3}} \Pi_3(s)\,ds$$

Linear $\Pi_3(s)$

$$= y^{j+2} + h\int \underbrace{f(t_j, y^j)}_{\text{const}} \frac{L_1(s)}{h} + h\int f(t_{j+1}, y^{j+1}) \cdot \frac{L_2(s)}{h}\,ds + h\int f(t_{j+2}, y^{j+2}) \frac{L_3(s)}{h}\,ds$$

$$= y^{j+2} + h \cdot f(t_j, y^j) \cdot \underbrace{\int_{t_{j+2}}^{t_{j+3}} \frac{L_1(s)}{h}\,ds}_{b_0} + \dots$$



$$b_0 = \int_{t_{j+2}}^{t_{j+3}} \frac{L_1(s)}{h}\,ds = \int_{t_{j+2}}^{t_{j+3}} \frac{(s - t_{j+1})(s - t_{j+2})}{2h^2 \cdot h}\,ds = \int^{h} \frac{((\hat{s} + t_{j+2}) - t_{j+1})((\hat{s} + t_{j+2}) - t_{j+2})}{2h^2 \cdot h}\,d\hat{s}$$

sub $\hat{s} = \boxed{s} - t_{j+2}$

$\hat{s}(t_{j+2}) = t_{j+2} - t_{j+2}$

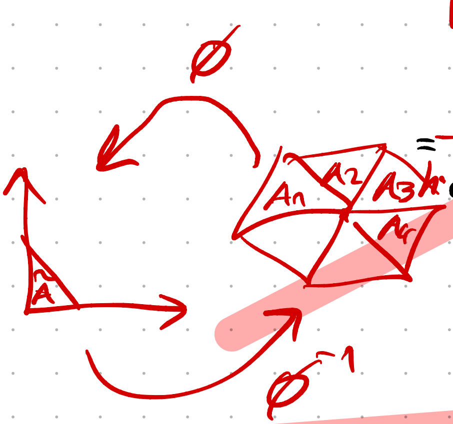
$$= \int_0^h \frac{(\hat{s}+h)(\hat{s})}{2h^3}\,d\hat{s}$$

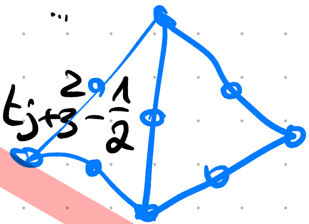
$\frac{d\hat{s}}{ds} = 1 \iff ds = d\hat{s}$

$$= \frac{1}{h \cdot 2h^2}\left(\int s^2 \cdot 1 + \int s(-t_{j+2} - t_{j+1}) + \int t_{j+1} t_{j+2}\right)$$

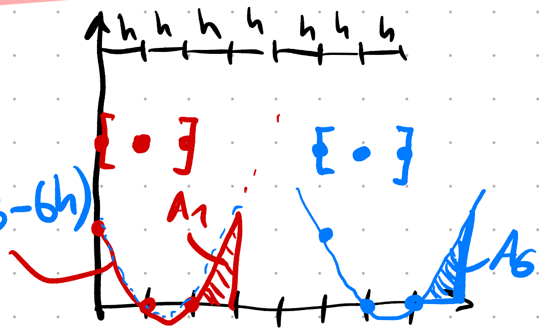
$$= \frac{1}{h \cdot 2h^2}\left[\left[\frac{1}{3}s^3\right]_{t_{j+2}}^{t_{j+3}} - \left[\frac{1}{2}s^2\right](t_{j+2} + t_{j+1}) + [s](t_{j+1} t_{j+2})\right.$$

$$= \frac{1}{2h^2}\left[\frac{1}{3}t_{j+3}^3 - \frac{1}{3}t_{j+2}^3 - \left[\frac{1}{2}t_{j+3}^2 - \frac{1}{2}\right.\right.$$





$L_1(s) = L_1(s - 6h)$



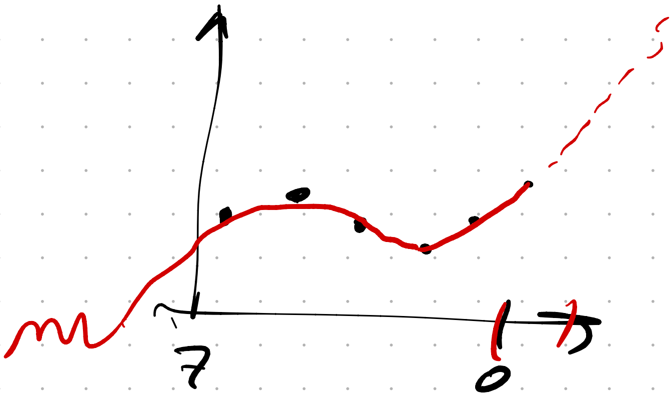
$$b_0 = \int_0^h \frac{(s+h)\,\dot{s}}{2h^3} = \frac{1}{2h^3}\left\{ \frac{1}{3}h^3 + h\cdot\frac{1}{2}h^2 \right\}$$

$$= \frac{1}{2h^3}\cdot\frac{5}{6}h^3 = \frac{5}{12} \quad ✓$$

sub

$$b_1 = -\int_0^h \frac{s(s+2h)}{h^3} = -\frac{4}{3}$$

$$b_2 = \int_0^h \frac{(s+h)(s+2h)}{2h^3} = \frac{23}{12}$$



AB3

$$y^{j+3} = y^{j+2} + \frac{5}{12}\cdot h\cdot f(t_j, y^j) - \frac{4}{3}\cdot h\cdot f^{j+1} + \frac{23}{12}\cdot h\, f^{j+2}$$

EXPLICIT

AM2

$$y^{j+2} = y^{j+1} + \dots\dots \quad y^{j+2}$$

IMPLIZIT