

# Polkadot Runtime Environment Spec Glossary (WIP)

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## Basics

Symbol	Description	Defined
$b$	a sequence of bytes of length $n$	$b := (b_0, b_1, \dots, b_{n-1})$ such that $0 \leq b_i \leq 255$
$\mathbb{B}_n$	the set of all byte arrays of length $n$	$\mathbb{B} := \bigcup_{i=0}^{\infty} \mathbb{B}_i$
$I$	little-endian representation of a non-negative integer	$I = (B_n \dots B_0)_{256}$
$B$	byte array	$B = (b_0, b_1, \dots, b_n)$ such that $b_i := B_i$
$Enc_{LE}$		$Enc_{LE} : \mathbb{Z}^+ \rightarrow \mathbb{B}$ $(B_n \dots B_0)_{256} \rightarrow (B_0, B_1, \dots, B_n)$
$C$	a blockchain is a directed path graph. Each node of the graph is called Block and indicated by $B$	
$P(B)$	the parent of block $B$	$B_{n+1} := P(B_n)$
$\mathcal{N}$	the set of the nodes of the Polkadot state trie	$N \in \mathcal{N}$
$N$	an individual node in the trie	$\mathcal{N}_b := \{N \in \mathcal{N} \mid N \text{ is a branch node}\}$
$\mathcal{N}_b$	a branch node which has one child or more (max 16)	$\mathcal{N}_l := \{N \in \mathcal{N} \mid N \text{ is a leaf node}\}$
$\mathcal{N}_l$	a leaf node is a childless node	$SV_N :=$
$SV_N$	the subvalue of the given node	$\begin{cases} Enc_{SC}(StoredValue(k_N)) & N \text{ is a leaf node} \\ ChildrenBitmap(N) \parallel Enc_{SC}(H(NC_1)) \dots Enc_{SC}(StoredValue(k_N)) & N \text{ is a branch node} \end{cases}$ $StoredValue \quad \mathcal{K} \rightarrow \mathcal{V}$
$StoredValue$	function to retrieve the value stored under a specific key in the state storage	$k \rightarrow \begin{cases} v & \text{if } (k, v) \text{ exists in state storage} \\ \phi & \text{otherwise} \end{cases}$

where  $K \subset \mathbb{B}$  and  $\mathcal{V} \subset \mathbb{B}$  are respectively the set of all keys and values stored in the state storage

Symbol	Description	Defined
$KeyEncode(k)$	function to encode keys for labeling brnaches of the Trie	$k_{enc} := (k_{enc1}, ..., k_{enc2n}) := KeyEncode(k)$
$k_{enc}$		such that:
		$KeyEncode(k) : \begin{cases} \mathbb{B} & \rightarrow Nibbles^4 \\ k := (b_1, ..., b_n) := & \rightarrow (b_1^1, b_1^2, b_2^1, b_2^2, ..., b_n^1, b_n^2) \\ & \rightarrow := (k_{enc1}, ..., k_{enc2n}) \end{cases}$
		where $Nibble^4$ is the set of all nibbles of 4-bit arrays and $b_i^1$ and $b_i^2$ are 4-bit nibbles, which are the big endian representation of $b_i$ :
		$(b_i^1, b_i^2) := (b_i/16, b_i, mod16)$
		where mod is the remainder and / is the integer division operators.
$pk_N$	TODO	
$pk_N^{Agr}$	TODO	
$HeadN$	the node header of node $N$	
$v_N$	the node value which is stored by the node $N \in \mathcal{N}$	$v_N := HeadN \parallel Enc_{HE}(pk_N) \parallel SV_N$
$ChildrenBitmap$		$ChildrenBitmap : \begin{cases} \mathcal{N}_b \rightarrow \mathbb{B}_2 \\ N \rightarrow (b_{15}, ..., b_8, b_7, ...b_0)_2 \end{cases}$
		where
		$b_i := \begin{cases} 1 & \exists N_c \in \mathcal{N} : k_{N_c} = k_{N_b} \parallel i \parallel pk_{N_c} \\ 0 & otherwise \end{cases}$
$H(N)$	the Merkle value of $N$	$H : \mathbb{B} \rightarrow \mathbb{B}_{32}$ $H(N) : \begin{cases} v_N & \parallel v_N \parallel < 32 \\ Blake2b(v_N) & \parallel v_N \parallel \geq 32 \end{cases}$
		Where $v_N$ is the node value of $N$ and $0_{32-\parallel v_N \parallel}$ an all zero byte array of length $32 - \parallel v_N \parallel$ . The Merkle hash of the Trie is defined as: $Blake2b(H(R))$ where $R$ is the root of the Trie.

## Block Format

Symbol	Description	Defined
$H_p$	the 32-byte Blake2b hash of the header of the parent of the block	
$H_i$	the interger representing the index of the current block in the chain. It is equal to the number of the ancestor blocks. The genesis block has number 0	
$H_r$	the root of the Merkle trie, whose leaves implement the storage for the system	

Symbol	Description	Defined
$H_e$	the field which is reserved for the Runtime to validate the integrity of the extrinsics composing the block body. The extrinsics_root is set by the runtime and its value is opaque to Polkadot RE	
$H_d$	used to store any chain-specific auxiliary data	$H_d(B) := H_d^1, \dots, H_d^n$ where $H_d^i$ 's are digest items
$H_h(B)$	the hash of the header of block $B$ by codec	$H_h(B) := \text{Blake2b}(\text{Enc}_{SC}(\text{Head}(B)))$
$H_h(B)$	Block hash	
$H_i(B)$	Block number	
$Body(B)$	the body of block $B$	$Body(B) := \text{Enc}_{SC}(E_1, \dots, E_n)$ where each $E_i \in \mathbb{B}$ is a SCALE encoded extrinsic

## SCALE Codec

Symbol	Description	Defined
$A$	Byte array	$A := b_1, b_2, \dots, b_n$
$T$	Tuple where $A_i$ 's are values of different types	$T := (A_1, \dots, A_n)$
$S$	Sequence where $A_i$ 's are values of the same type (and the decoder is unable to infer value of $n$ from the context)	$S := A_1, \dots, A_n$
$\tau$	Varying data type (TODO)	$T = \{T_1, \dots, T_n\}$
$\text{Enc}_{SC}(A)$	SCALE encoding of byte array $A$ such that $n < 2^{256}$	$\text{Enc}_{SC}(A) := \text{Enc}_{SC}^{Len}(\  A \ ) \  A$
$\text{Enc}_{SC}(T)$	SCALE encoding of tuple $T$	$\text{Enc}_{SC}(T) := \text{Enc}_{SC}(A_1) \  \text{Enc}_{SC}(A_2) \  \dots \  \text{Enc}_{SC}(A_n)$
$\text{Enc}_{SC}(S)$	SCALE encoding of sequence $S$	$\text{Enc}_{SC}(S) := \text{Enc}_{SC}^{Len}(\  S \ ) \text{Enc}_{SC}(A_1)   \text{Enc}_{SC}(A_2)   \dots   \text{Enc}_{SC}(A_n)$
$\text{Enc}_{SC}^{Len}$	SCALE length encoding aka. compact encoding of non-negative interger numbers of varying sized prominently in an encoding length of arrays	$\text{Enc}_{SC}^{Len} : \mathbb{N} \rightarrow \mathbb{B}$ $n \rightarrow b \begin{cases} l_1 & 0 \leq n < 2^6 \\ i_1 i_2 & 2^6 \leq n < 2^{14} \\ j_1 j_2 j_3 & 2^{14} \leq n < 2^{30} \\ k_1 k_2 \dots k_m & 2^{30} \leq n \end{cases}$

in where the least significant bits of the first byte of byte array  $b$  are defined as follows:

$$\begin{aligned} l_1^1 l_1^0 &= 00 \\ i_1^1 i_1^0 &= 01 \\ j_1^1 j_1^0 &= 10 \\ k_1^1 k_1^0 &= 11 \end{aligned}$$

and the rest of the bits of  $b$  store the value of  $n$  in little-endian format in base-2 as follows:

$$\left. \begin{aligned} &l_1^7 \dots l_1^3 l_1^2 && n < 2^6 \\ &i_2^7 \dots i_2^0 i_1^7 \dots i_1^2 && 2^6 \leq n < 2^{14} \\ &j_4^7 \dots j_4^0 j_3^7 \dots j_1^7 \dots j_1^2 && 2^{14} \leq n < 2^{30} \\ &k_2 + k_3 2^8 + k_4 2^{2 \times 8} + \dots + k_m 2^{(m-2) \times 8} && 2^{30} \leq n \end{aligned} \right\} := n$$

such that:

$$k_1^7 \dots k_1^3 k_1^2 := m - 4$$

# GRANDPA

Symbol	Description	Defined
$v$	GRANDPA Voter	
$k_v^{pr}$	ED25519 private key of $v$	
$v_{id}$	ED25519 public key of $v$	
$\mathbb{V}$	set of all GRANDPA voters	
$\mathbb{V}_B$	set of all GRANDPA voters for a given block	
$\mathbb{V}_{id}$	is an incremental counter tracking membership, which changes in $V$	
$GS$	GRANDPA state	$GS := \{\mathbb{V}, id_{\mathbb{V}}, r\}$
$V(B)$	GRANDPA vote	$V(B) := (H_h(B), H - I(B))$
$V_v^{r,pv}$	pre-vote	
$V_v^{r,pc}$	pre-commit	
$r$	Voting round number	
$V_{id}$	Incremental counter tracking membership	
$V_v^{r,stage}(B)$	equivocatory vote	
$\mathcal{E}^{r,stage}$	set of all equivocators voters in sub-round “stage” of round $r$	
$\mathcal{E}_{obs(v)}^{r,stage}$	set of all equivocators voters in sub-round “stage” of round $r$ observed by voter $v$	
$VD_{obs(v)(B)}^{r,stage}$	the set of observed direct votes for block $B$ in round $r$	
$V_{obs(v)}^{r,stage}$	the set of total votes observed by voter $v$ in sub-round “stage” of round $r$	
$V_{obs(v)}^{r,stage}(B)$	set of all observed votes by $v$ in the sub-round stage of round $r$ for block $B$	$V_{obs(v)}^{r,stage}(B) := \bigcup_{v_i \in \mathbb{V}, B \geq B'} VD_{obs(v)}^{r,stage}(B')$
$B_v^{r,pv}$	The current pre-voted block	$H_n(B_v^{r,pv}) = \text{Max}(H_n(B) \parallel \forall B : \#V_{obs(v)}^{r,pv}(B) \geq 2 \setminus 3 \parallel \mathbb{V} \parallel)$

## Voting Messages Specification

Symbol	Description	Defined
$M_v^{r,stage}$	A broadcasted message by the voter $v$ casting his vote to the network	$M_v^{r,stage} := \text{Enc}_{SC}(r, id_{\mathbb{V}}, \text{Enc}_{SC}(\text{stage}, V_v^{r,stage}, \text{Sig}_{ED25519}(\text{Enc}_{SC}(\text{stage}, V_v^{r,stage}, r, V_{id}), v_{id})))$
$J^r(B)$	The justification for block $B$ in round $r$	The justification is a vector of pairs of the type $(V(B'), (\text{Sign}_{vi}^{r,pc}(B'), v_{id}))$ in which either $B' \geq B$ or $V_{vi}^{r,pc}(B')$ is an equivocatory vote
$\text{Sign}_{vi}^{r,pc}(B')$	The signature of voter $v$ , broadcasted during the pre-commit sub-round of round $r$	
$M_v^{r,Fin}(B)$	The finalizing message broadcasted by voter $v$ to the network indicating that voter $v$ has finalized bock $B$ in round $r$	$M_v^{r,Fin}(B) := \text{Enc}_{SC}(r, V(B), J^r(B))$

## Cryptographic keys

Symbol	Description	Defined
Account key $(sk^a, pk^a)$	A keypair of type of either SR25519, ED25519, secp256k1	

## Hex encoding

Symbol	Description	Defined
$Enc_{HE}(PK)$	hex encoding	$Enc_{HE}(PK) := \begin{cases} Nibbles_4 \rightarrow \mathbb{B} \\ PK = (k_1, \dots, k_n) \rightarrow \begin{cases} (16k_1 + k_2, \dots, 16k_{2i-1} + k_{2i}) & n = 2i \\ (k_1, 16k_2 + k_3, \dots, 16k_{2i} + k_{2i+1}) & n = 2i + 1 \end{cases} \end{cases}$