Evolutionary Game Theory

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Evolutionary Game Theory (EGT)

Origin in Biology

- Ronald A. Fisher
  - *The Genetic Theory of Natural Selection* 1930
  - Why equal sex ratio?
  - **Frequency Dependent Individual Fitness**
- Richard C. Lewontin
  - *Evolution and the Theory of Games* 1961
  - Explicitly: game theory $\rightarrow$ evolutionary biology
- Taylor and Jonker (1978) and Zeeman (1979)
  - **Replicator Equations** as evolutionary dynamic in EGT
- John Maynard Smith
  - "The Logic of Animal Conflict" - *Nature* 1973 (G. R. Price)
  - *Evolution and the Theory of Games* 1982
  - **Evolutionarily Stable Strategy (ESS)**

- Economics, Sociology, Anthropology, Philosophy, etc.

Appeal:

- No “Rational Player” $\rightarrow$ players learn, adapt, and evolve
- Focus on *Population Dynamics* rather than individual ‘solutions’ based on a priori reasoning
- Evolution can be viewed as *Cultural Evolution* - change in beliefs and norms (learning through imitation)

Evolutionary Game Theory

Maynard Smith's shifts from Classical GT

- Strategy
  - Species have strategy sets (not players)
  - Individuals inherit strategy - possibly mutated
- Equilibrium
  - Evolutionarily Stable Strategy (ESS) in place of NE
  - Population using strategy A cannot be invaded by a small group using strategy B
- Player Interactions
  - Repeated, random pairings of agents in population

Hawk Dove Game

(Chicken Game if $C > G$)

- "The Logic of Animal Conflict"
- Population of birds fighting over food
- *Hawk*: escalate battle
- *Dove*: retreat if opponent escalates

Payoff Matrix

$$\begin{array}{ccc} & H & D \\ H & (G-C)/2 & G \\ D & 0 & G/2 \end{array}$$

G = Payoff from food, C = Cost of injury

Frequency Dependent Fitness

- $W_{\sigma\mu}$ is payoff of σ when playing μ
- F_i is fitness of strategy i - analogous to payoff received

If we have two strategies $\sigma \in S$ and $\mu \in S$, $\mu \neq \sigma$, then

$$F_\sigma = (1 - p) * W_{\sigma\sigma} + p * W_{\sigma\mu}$$

$$F_\mu = (1 - p) * W_{\mu\sigma} + p * W_{\mu\mu}$$

where p is proportion of μ in population.

Evolutionary Stability

A strategy is *evolutionarily stable* if no other strategy can invade it under the influence of natural selection. We say a strategy μ can *invade* a population of σ if $F_\mu \geq F_\sigma$.

Strategy σ is an *evolutionary stable strategy* if, for all strategies $\mu \neq \sigma$,

$$W_{\sigma\sigma} \geq W_{\mu\sigma}$$

and if $W_{\sigma\sigma} = W_{\mu\sigma}$,

$$W_{\sigma\mu} > W_{\mu\mu}$$

Hawk Dove Game Stability

- $S = \{\text{Hawk, Dove}\}$
- Is a strategy evolutionarily stable if $G \geq C$?
  - $W_{HH} \geq W_{DH}$?

Payoff Matrix

	H	D
H	$(G-C)/2$	G
D	0	$G/2$

Payoff Matrix

	H	D
H	1	3
D	0	1.5

Hawk Dove Game Stability

- $S = \{\text{Hawk, Dove}\}$
- Is a strategy evolutionarily stable if $G \geq C$?
  - $W_{HH} \geq W_{DH}$?
- Hawk is evolutionarily stable

Payoff Matrix

	H	D
H	$(G - C)/2$	G
D	0	$G/2$

Payoff Matrix

	H	D
H	1	3
D	0	1.5

Hawk Dove Game Stability

- $S = \{\text{Hawk, Dove}\}$
- Is a strategy evolutionarily stable if $G < C$?
  - $W_{HH} \geq W_{DH}$?
  - $W_{DD} \geq W_{HD}$?

Payoff Matrix

$$\begin{array}{ccc} & H & D \\ H & (G-C)/2 & G \\ D & 0 & G/2 \end{array}$$

Payoff Matrix

$$\begin{array}{ccc} & H & D \\ H & -1 & 2 \\ D & 0 & 1 \end{array}$$

Hawk Dove Game Stability

- $S = \{\text{Hawk, Dove}\}$
- Is a strategy evolutionarily stable if $G < C$?
  - $W_{HH} \geq W_{DH}$?
  - $W_{DD} \geq W_{HD}$?
- Neither is evolutionarily stable
- So, what happens in a pop of H and D ?

Payoff Matrix

	H	D
H	$(G - C)/2$	G
D	0	$G/2$

Payoff Matrix

	H	D
H	-1	2
D	0	1

Replicator Dynamics

Replicator

- Central actor in an evolutionary system
- Means of making approximately accurate copies of itself
- Gene, Organism, Strategy, Belief, Convention, etc.

Evolutionary Dynamic

- Process of change over time in the frequency distribution of replicators
- Darwinian natural selection:
  higher payoff $\rightarrow$ *faster reproduction*
- Replicator Equation is most popular way of specifying dynamic

Replicator Equation

If we express evolutionary success as the difference between the fitness of a replicator (player or strategy in evolutionary game theory) and the average fitness in the population, we obtain the ODE:

$$\dot{\mathbf{x}}_i = \mathbf{x_i}[\mathbf{F_i}(\mathbf{x}) - \theta(\mathbf{x})],$$

where $\mathbf{x}$ is a vector holding the proportions of all player types in the population, $\mathbf{x_i}$ is the proportion of player type i in the population, $\dot{\mathbf{x}}_i$ is the rate of change, $F_i(\mathbf{x})$ is the average fitness of a player of type i (depending on the population make-up $\mathbf{x}$), and $\theta(\mathbf{x})$ is the average fitness in the population.

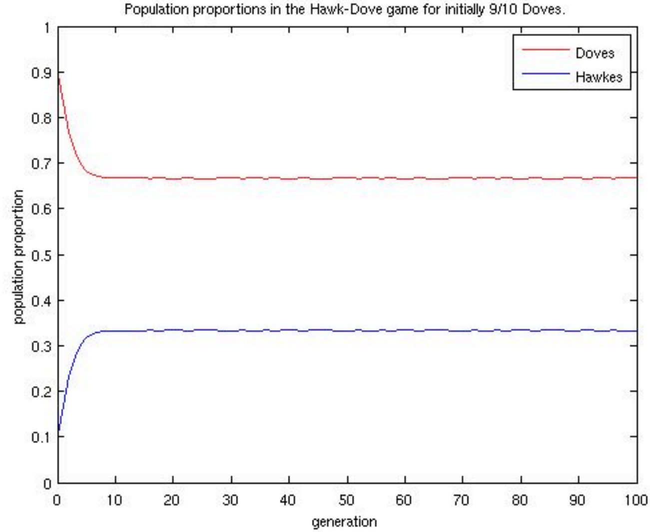
Hawk Dove Game Stability

- $S = \{\text{Hawk, Dove}\}$
- Is a strategy evolutionarily stable if $G < C$?
  - $W_{HH} \geq W_{DH}$?
  - $W_{DD} \geq W_{HD}$?
- Neither is evolutionarily stable
- So, what happens in a pop of H and D ?

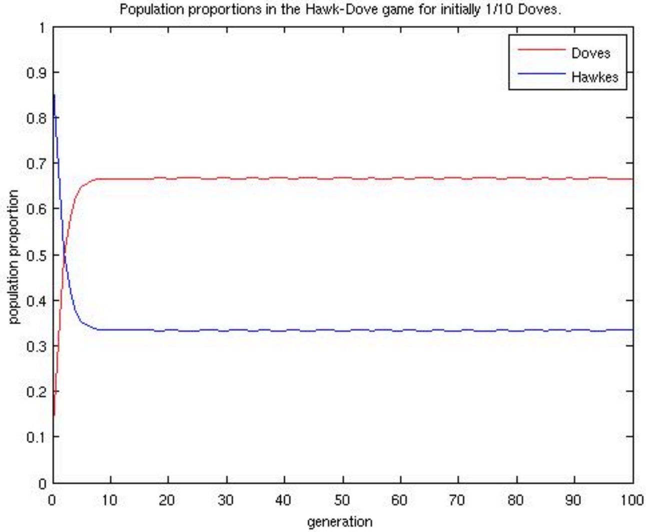
Payoff Matrix

	H	D
H	0	7
D	2	6

Hawk Dove Under Replicator



Hawk Dove Under Replicator



If we have an $n \times n$ matrix $\mathbf{U}$, such that $F_i(\mathbf{x}) = (\mathbf{Ux})_\mathbf{i}$, then the replicator equation

$$\dot{\mathbf{x}}_i = \mathbf{x_i}[\mathbf{F_i}(\mathbf{x}) - \theta(\mathbf{x})],$$

takes the form

$$\dot{\mathbf{x}}_i = \mathbf{x_i}[(\mathbf{Ux})_\mathbf{i} - \mathbf{x} \cdot \mathbf{Ux}],$$

the *rest points* of which are the solutions of

$$(\mathbf{Ux})_\mathbf{1} = ... = (\mathbf{Ux})_\mathbf{n}$$

State of population: $\mathbf{x} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

Payoff matrix: $\mathbf{U} = \begin{bmatrix} 0 & 7 \\ 2 & 6 \end{bmatrix}$

Then,

$$\mathbf{Ux} = \begin{bmatrix} 0 & 7 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} \frac{14}{3} & \frac{14}{3} \end{bmatrix}$$

$$(\mathbf{Ux})_1 = (\mathbf{Ux})_2$$

Theorem

If $\mathbf{x}_*$ is an evolutionarily stable strategy, then $\mathbf{x}_*$ is an evolutionary equilibrium of the replicator dynamic. Moreover, if $\mathbf{x}_*$ uses all strategies with positive probability, then σ is a globally stable fixed point.

Evolutionary equilibrium = asymptotically stable fixed point

Theorem

Under replicator dynamic,

- If $\mathbf{x}_*$ is a Nash equilibrium of the evolutionary game, $\mathbf{x}_*$ is a fixed (rest) point of the replicator dynamic.
- If $\mathbf{x}_*$ is an evolutionary equilibrium of the replicator dynamic, then it is a Nash equilibrium.

Replicator Equation and ESS

Things to Note

- Frequency of strategy increases exactly when it has above average payoff
- Replicator dynamic does NOT mean agents adopt a best reply to the overall frequency distribution of strategies in previous population (bounded rationality?)
- If a strategy does not exist at any point, it will never exist in the future (no incorporation of mutation or innovation)
- Assumes no mistakes (generally does not make a difference in system behavior)
- Idealized version of how agent systems develop, number of players must be sufficiently large
- ESS assumes random pairings

Prisoner's Dilemma in EGT

Payoff Matrix

$$\begin{array}{ccc} & C & D \\ C & 3 & 0 \\ D & 5 & 1 \end{array}$$

- What happens under replicator Dynamics?

Prisoner's Dilemma in EGT

Payoff Matrix

$$\begin{array}{ccc} & C & D \\ C & 3 & 0 \\ D & 5 & 1 \end{array}$$

- What happens under replicator Dynamics?
- D always takes over entire population : (
- D is evolutionarily stable

Cooperation in the IPD

Cooperation is a popular problem in EGT

- Robert Axelrod: The Evolution of Cooperation *Science* 1981
- Iterated games
- Tit for Tat (TFT)
- *Reciprocity* explains cooperation

TFT and evolutionary stability

Strategy σ is an *evolutionary stable strategy* if, for all strategies $\mu \neq \sigma$,

$$W_{\sigma\sigma} \geq W_{\mu\sigma}$$

and if $W_{\sigma\sigma} = W_{\mu\sigma}$,

$$W_{\sigma\mu} > W_{\mu\mu}$$

- TFT evolutionarily stable ?

TFT and evolutionary stability

Strategy σ is an *evolutionary stable strategy* if, for all strategies $\mu \neq \sigma$,

$$W_{\sigma\sigma} \geq W_{\mu\sigma}$$

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$$W_{\sigma\mu} > W_{\mu\mu}$$

- TFT evolutionarily stable ?
- Not under definition above (e.g. TF2T)
- TFT satisfies $W_{\sigma\sigma} \geq W_{\mu\sigma}$
- Bendor et. al.: Types of evolutionary stability and the problem of cooperation 1995

Cooperation in (Non-Iterated) PD

- Riolo et. al.: Evolution of cooperation without reciprocity. *Nature*, 2001
- Hales and Edmonds: Evolving Social Rationality for MAS using "Tags", 2003
- Pure cooperators and defectors
- Agents matched up to play by tag
- Tags as well as strategy are inherited by offspring (with mutation)
- Emerging 'groups' of cooperators

Cooperation in (Non-Iterated) PD

Other explanations for cooperation:

- Kin selection
  - Hamilton 1964
  - Fitness of the behavior induced by a certain gene should include the behavior's effect on kin who might carry the same gene
- Group selection
  - A. Traulsen, M. A. Nowak 2006
  - Interaction within group
  - Agents reproduce proportional to fitness, but into their own group
  - Groups split when large enough
- Punishment
  - Boyd and P. J. Richerson 2005
- ...

References

- Hofbauer and Sigmund, "Evolutionary Games and Population Dynamics", Cambridge University Press, 1998
- Herbert Gintis, "Game Theory Evolving", Princeton University Press, 2000

Thank You !