# Models of statistical mechanics and cooperative game theory

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- Cooperative games: games with interaction (exchange of information) among players
- The ideal player: completely rational!
- The real player: behaves in another way, more difficult to be modeled.

A possible approach: try to make a statistical model of the behavior

The model:

- Each agent $i$ can invest $\sigma_i \in \{0, 1, ..., q-1\}$
  (the states $\{0, 1, \ldots, q-1\}$ can be replaced by $\{d_0, d_1, \ldots, d_{q-1}\}$)

- The agent $i$ talks with $i+1$

- $P(\sigma_i|\sigma_{i+1}) = \frac{1}{Z} \exp -\beta J(\sigma_i)\delta_{\sigma_i, \sigma_{i+1}}, \quad \beta \geq 0$
  (with $J(0) = J_0,\ J(1) = J_1\ ...$)

The global state is $\sigma = (\sigma_1, \ldots, \sigma_N)$.

Normalization (=partition function):

$$Z_N(\beta) = \sum_{\sigma} \exp\left(-\beta \sum_{i=1}^{n} J(\sigma_i)\delta_{\sigma_i \sigma_{i+1}}\right)$$

We can rewrite the function above in order to obtain an easy expression to the mean investment:

$$Z_N(\beta, D) = \sum_{\sigma} \exp\left(-\beta \sum_{i=1}^{n} J(\sigma_i)\delta_{\sigma_i \sigma_{i+1}} - \frac{\beta D}{2} \sum_{i=1}^{N} (\sigma_i + \sigma_{i+1})\right)$$

Now, the mean investment per agent is

$$m(\beta) = -\frac{1}{\beta N} \frac{\partial}{\partial D} \log Z_N(\beta, D)|_{D=0}$$

If $N$ is very large, the expression of $m(\beta)$ takes the form

$$m(\beta) = -\frac{1}{\beta\lambda_1}\frac{\partial}{\partial D}\log\lambda_1(\beta, D)|_{D=0}$$

where $\lambda_1$ is the largest eigenvalue of the corresponding transfer matrix.

For $q = 2$ the matrix is

$$M = \begin{pmatrix} \exp(-\beta J_0) & \exp(-(\beta/2)D) \\ \exp(-(\beta/2)D) & \exp(-\beta J_1 - \beta D) \end{pmatrix}$$

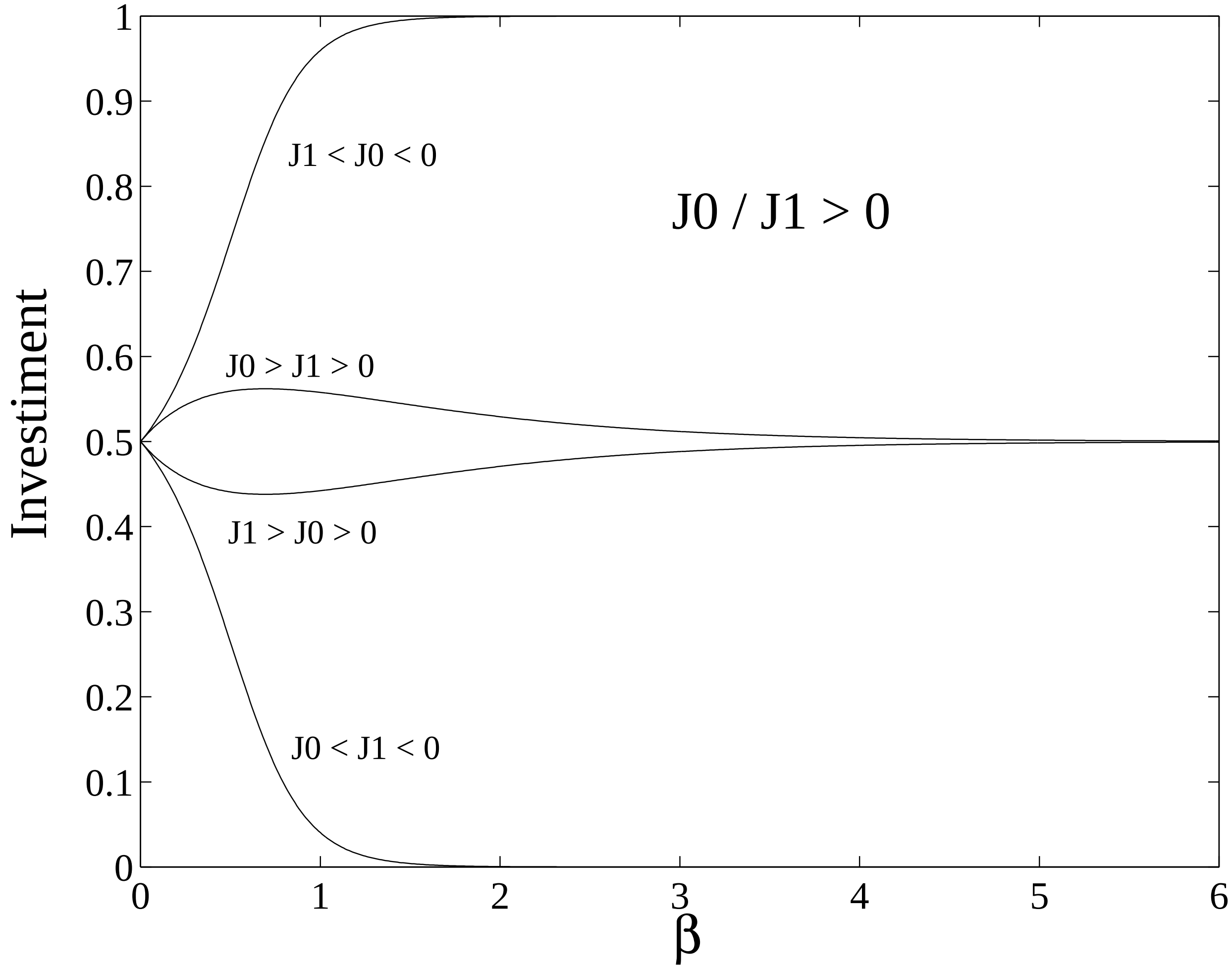


Figure 1: $m(\beta)$

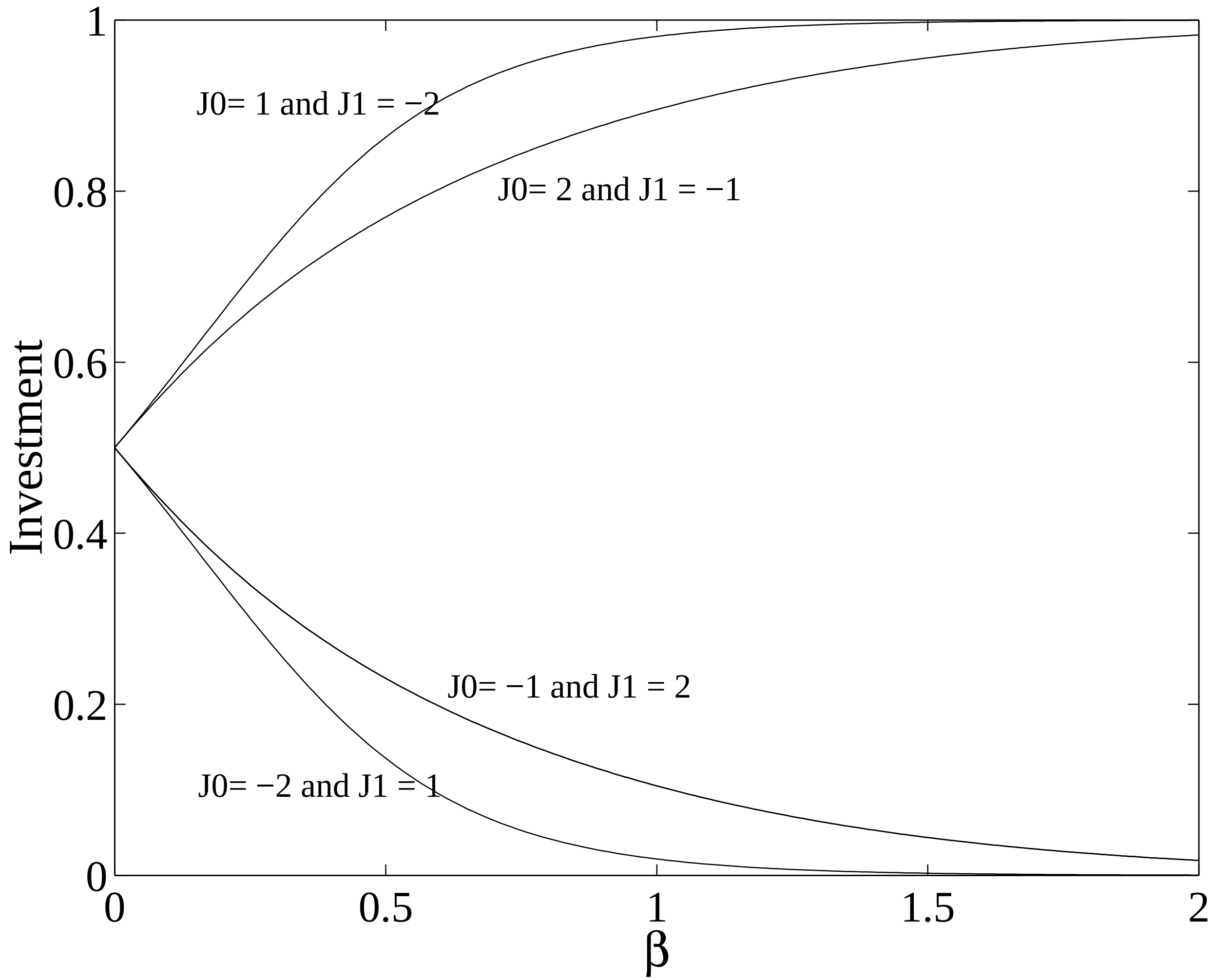


Figure 2: $m(\beta)$