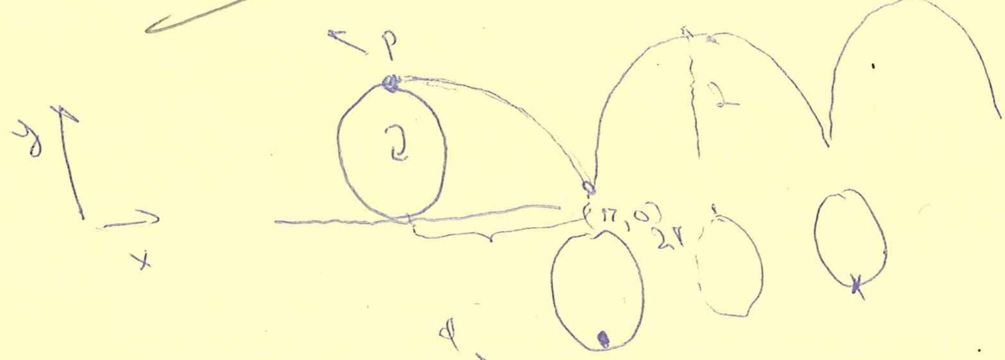
Rough Work

$\boxed{\text{Cycloid}}$



(1)

$$\begin{cases} y = \cos\varphi + 1 \\ x = \varphi + \sin\varphi \end{cases}$$

$y(0) = 2$

$y(\pi) = 0$

$\cos\pi = 0$

$\sin\pi = 0$

$x(\pi) = \pi$

$x(0) = 0$

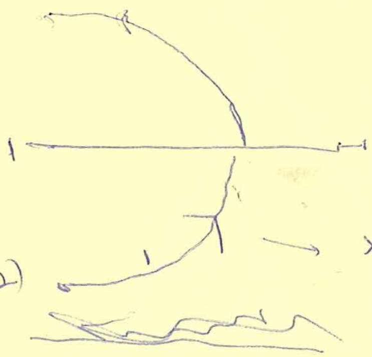
$\delta/2$

$1 + \cos(\pi - \delta) \approx 1 + \delta^2/2$

$\sin(\pi - \delta) = \delta$

$\frac{dy}{dx} = \frac{\sin\varphi}{1 + \cos\varphi}$

$\frac{2}{\delta} \uparrow \infty$



$\dot{x}(\pi) \neq \pi$

$\dot{y} = \sin\varphi = 0 \quad \varphi = \pi$

$y(0) = -2$

$x = \varphi + \sin\varphi$

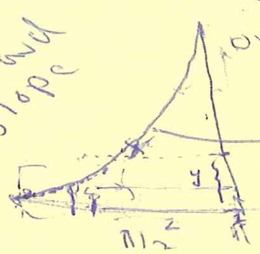
$y = -1 - \cos\varphi$

$\dot{x} = 1 + \cos\varphi$

$\dot{y} = \sin\varphi$

(0;−2)

upward slope



$x = \varphi + \sin\varphi$

$y = 1 - \cos\varphi$

"stupid" imperative

insist to write "ugly formulae"

$y = y(x)$

(2nd) express

S = arclength as

function of y!!

Takes a while

before y↑ compared

to s.

$y \to s(y)\uparrow$ much faster than y!

$y = ax^2$

$\dot{x} = 1 + \cos\varphi$

$\dot{y} = \sin\varphi$

$(\dot{x} - 1)^2 = \cos^2 = 1 - \sin^2\varphi$

$= 1 - \dot{y}^2$ "ack!"

$\dot{y}^2 = 2\dot{x} - \dot{x}^2$ "ugly"

$\underline{S(y) = c_0 \cdot \sqrt{y}}$!! $c_0 > 0$

~~}; this ok!~~

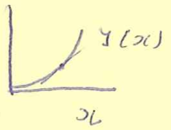
$\boxed{\frac{ds}{dy} = \frac{c_0}{2\sqrt{y}}}$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\boxed{\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2}$$

$$\boxed{\sqrt{1 + y'(x)^2} = s'(x)}$$

Pythagoras

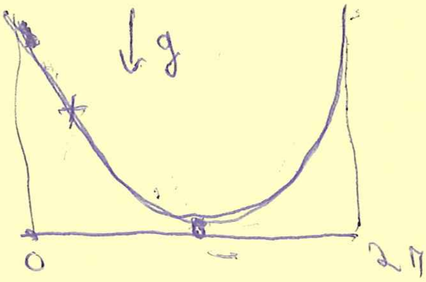


$$\boxed{\frac{dy}{dx} = \frac{\dot y}{\dot x}}$$

$$\left(\frac{ds}{dx}\right)^2 = 1 + \frac{\sin^2\varphi}{(1+\cos\varphi)^2}$$

$$= \frac{1 + \cos^2\varphi + 2\cos\varphi + \sin^2\varphi}{(1+\cos\varphi)^2} = 2\,\frac{1+\cos\varphi}{(1+\cos\varphi)^2} = \frac{2}{1+\cos\varphi} \;!!$$

Fermat
Descartes } 1640
(Hughens)



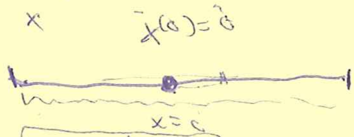
$$\frac{s'(y)^2}{\left(\frac{dx}{dy}\right)^2} = \frac{s'(y)^2}{\dot x^2/\dot y^2} = \frac{\dot y^2\, s'(y)^2}{\dot x^2}$$

$$= \frac{\sin^2\varphi \cdot s'(y)^2}{(1+\cos\varphi)^2} \Rightarrow \frac{2}{1+\cos\varphi}$$

$$\sin^2\varphi \cdot s'(y)^2 = 2(1+\cos\varphi)$$

Try it

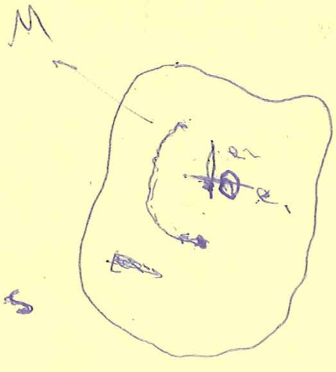
Tautochronom



$$\boxed{\ddot x + k^2 x = 0}$$

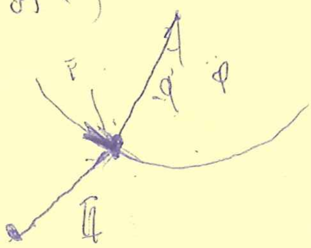
$$x(t) = a\sin(kt)$$

Apell want $s'(y) = \dfrac{c_0^2}{4y}$



$$\frac{c_0^2}{4y}\sin^2\varphi = \cancel{2y(1+\cos\varphi)} = 2(1+\cos\varphi)$$

$$\frac{c_0^2}{4}\sin^2\varphi = 2y(1+\cos\varphi) = 2\,\frac{(1-\cos^2\varphi)}{2\sin^2\varphi}$$



m ant

$$\boxed{c_0^2 = 2}$$

$$\boxed{s(y) = \sqrt{2}\cdot\sqrt{y}}$$

Find Motion of the plate.