Special Topics

1. *The disc algebra*

2. The Jensen-Nevanlinna class and Blaschke products.

3. The Hardy space $H^1(T)$

4. Nevanlinna-Pick theory

5. Series and analytic functions.

6. Uniqueness theorems for analytic functions.

7. Lindelöf functions.

8. Approximation theorems in complex domains

9. Radial limit of functions with finite Dirichlet integral

10. The Denjoy conjecture and Carleman's differential inequality

11. The Dagerholm series

12. Uniform approximation by Blaschke products

13. Interpolation and solutions to the $\bar{\partial}$-equation.

14. Entire functions of exponential type

15. Beurling-Wiener algebras

16. The Robin constant and harmonic measures

17. An automorphism of product measures

18. The Mellin expansion and the Radon transform

19. A non-linear PDE-equation

20. An isoperimetric problem

20. Doubly periodic meromorphic functions.

Introduction.

Above are headlines for the sections. There detailed contents are listed in the next pages and further comments appear in the individual sections. The level of the material changes from fairly elementary facts to results whose proofs are quite demanding. The first three sections are essential for much of the subsequent material, especially facts about functions in the Jensen-Nevanlinna class which together with the Brothers Riesz Theorem from Section 1 are used to study Hardy spaces an they appear frequently in later sections as well. The topics treat both analytic function theory and harmonic analysis where the interplay lead often to a powerful theory

such as in Section 14 where Beurling-Wiener algebras are studied. A veritable highlight is the notion and properties of Carleson measures which are to establish the interpolation theorem for bounded analytic functions in section 12. Section 19-20 are a bit apart from analytic function theory but have been included since the methods are of interest. For example, symmetrizations is often used in analytic function theory and the proof in Section 20 illustrates one application of symmetrization methods and the non-linear solution to a PDE-equation in section 19 is proved via succesive analytic series which eventually reduce the proof of existence to solving a family of linear Neumann problems.

Guidance to the sections.

Below we have included a more detailed list of material from some of the extensive sections. For the shorter sections we refer to the individual introductions.

I. The disc algebra $A(D)$

1. Theorem of Brothers Riesz.

2. Ideals in the disc algebra

3. A maximality theorem for uniform algebras.

2. The Jensen-Nevanlinna class and Blaschke products.

1. The Herglotz integral.

2. The class JN(D)

3. Blaschke products

4. Invariant subspaces of $H^2(T)$

5. Beurling's closure theorem

6. The Helson-Szegö theorem.

3.A: The Hardy-Littlewood maximal function

1. The weak type estimate

2. An L^2*-inequality*

3 Harmonic functions and Fatou sectors

4. Application to analytic functions

5. Conformal maps and the Hardy space $H^1(T)$

3:B. The Hardy space H^1

1. Zygmund's inequality

2. A weak type estimate.

3. Kolmogorov's inequality.

4. The dual space of $H^1(T)$

5. The class BMO

6. The dual of $\mathfrak{Re}\, H_0^1(T)$

7. Theorem of Gundy and Silver

8. The Hardy space on $\mathbf{R}$.

9. BMO and radial norms on measures in D.

4. Nevanlinna-Pick theory

0. The Nevanlinna-Pick Interpolation

1. The Lindelöf-Pick principle with an application

2. A result by Julia

3. Geometric results by Löwner

5. Series and analytic functions.

1. A theorem by Kronecker.

2. Newton polynomials and the disc algebra.

3. Absolutely convergent Fourier series.

4. Harald Bohr's inequality

5. Theorem of Fatou and M. Riesz

6. On Laplace transforms

7. The Kepler equation and Lagrange series

8. An example by Bernstein

9. Almost periodic functions and additive number theory

5. Uniqueness theorems for analytic functions.

A. A sharp version of the Phragmén-Lindelöf theorem

B. Asymptotic series.

C. A uniqueness theorem for asymptotic series

8. Approximation theorems in complex domains

A. Weierstrass approximation theorem

B. Polynomial approximation with bounds

C. Approximation by fractional powers

D. Theorem of Müntz

13. Interpolation and solutions to the $\bar{\partial}$-equation.

1. Carleson's interpolation theorem

2. Wolff's theorem

3. A class of Carleson measures.

4. Berndtsson's $\bar{\partial}$-solution

5. Hörmander's L^2-estimate

6. The Corona problem.

15. Beurling-Wiener algebras

A: Beurling-Wiener algebras on the real line.

B: A Tauberian theorem

C: Ikehara's theorem

D: The Gelfand space of $L^1(\mathbf{R}^+)$.

19. Homogeneous distributions and the Mellin transform

A. Polar distributions

B. Homogeneous distributions

C. The family $|P(x,y)|^\lambda$

D. The Radon transform

E. The Mellin transform