Chalmers.

Prof. Emeretus

Peter Sjögren

Specialist

Potential Theory + Harmonic analysis

$\Longleftrightarrow$ Probability Theory

Riemann.

$f \in C^0[0,1]$ $\quad 0 \le f(x) \le M$

$N >> 1$

$$S^*(N) = \frac{1}{N}\sum_{\nu=0}^{N-1} \cancel{\max} \sup_{\frac{\nu}{N} \le x \le \frac{\nu+1}{N}} f(x)$$

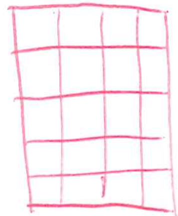
inf

$$S_* = \frac{1}{N}$$

$$\lim S^*(N) = \lim S_*(N) = \int_0^1 f(x)\,dx$$

Convergence is slow

Need Uniform continuity



N

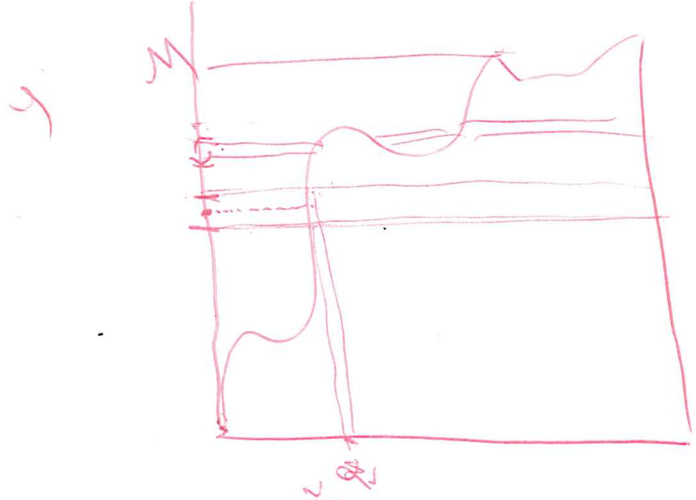
Riemann intervall
Lebesgue int.

Lebesgue (= Borel-Stieltjes

all that matters Rate of convergence is M !!

Same speed for all f s.t.

$0 \le f(x) \le M$



Luc Jaulin (Moore Prize)

SIVIA

↑

set-inversion via I.A.

Slice y-axis

$\left[M\cdot\frac{v}{N}, M\frac{v+1}{N}\right] = J_v \quad f^{-1}[J_v]$

N. $\{\alpha_v\}$ intervals on x-axis

Total length of

$SB_N = \frac{1}{N}\sum_{1}^{N} v\,\alpha_v \longrightarrow \int f(x)\,dx$ efficient way

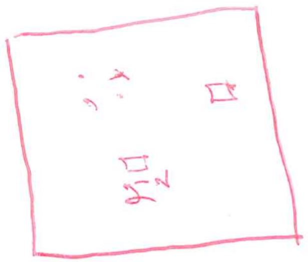
$\left|SB_N - \int_0^N f(x)\,dx\right| \le \frac{M}{N}$ independent of Modulus of continuity

Robust Limit

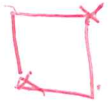
Basic Mea. Th.

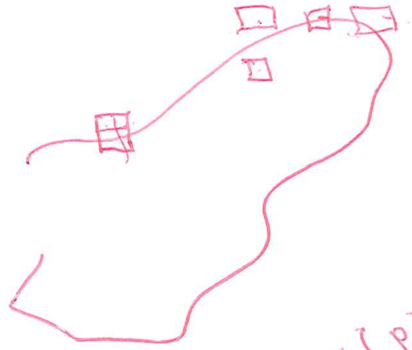
Let S be a

$n=2$



$S(x,y)$
Doubly
Monotone

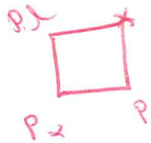




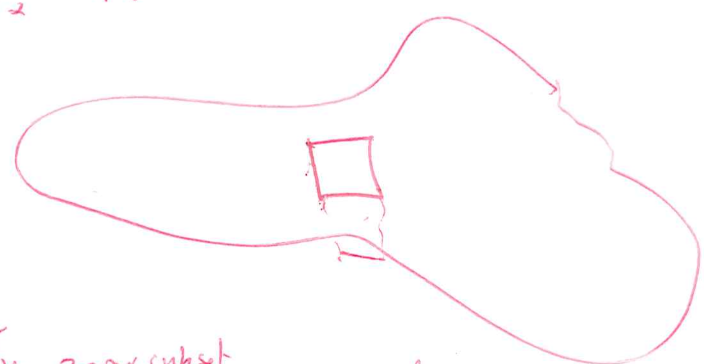
K compact $[0,1]$

$\mathcal{D}_N(K) = \{\delta \in \mathcal{D}_N : \delta \cap K \neq \phi\}$

$S(p^*) + S(p_*) - S(p_l) - S(p_r)$



$S_N(K) = \sum_{\delta \in \mathcal{D}_N(K)} \operatorname{var}_S(\delta)$



$N \to S_N(K) \downarrow$

$\lim_{N \to \infty} S_N(K) = \mu_S(K)$

compact dyadic cube — proper subset — open set

$\delta \subset U\}$

U open $\mathcal{D}_N(U) = \{\delta \in \mathcal{D}_N :$

$S_N(U) = \sum_{\delta \in \mathcal{D}_N(U)} \operatorname{var}_S(\delta)$

$\lim S_N(U) = \mu_S(U)$

$\circ\circ$ with dyadic cubes one can obtain all σ-additive measures

4

$$d\mu_s \in L^1 \Rightarrow$$
$$\xi \to \int_0^1 e^{ix\xi}\, d\mu_s(x) \to 0$$
$$|\xi| \to \infty$$

$\boxed{\text{Lebesgue}}$

occur $S(x,y)$ Lieschitz continuous
+ Doubly Monoton.
"Nice"

$d\mu_s$ L^1

General S given $d\mu_s \in L^1$

$\Longleftrightarrow$ $\{f_v\}$ Lip + Doubly Monotone

$f_v(x,y) \le f_v(x,$

$\lim f_v = S$

$\boxed{\text{Problem}}$

Point: Mothers $S(x,y)$ can be

$= \max(a + c_1x + c_2y,\ a' + c_1'x + c_2'y)$

Sums max linear function
pair.

Specific case

$\boxed{n=1 \quad \text{Lebesgue}}$

$S(x)$ continuous $\uparrow [0,1]$.

$S'(x) = 0$ a.e.

$S(0) = 0$
$S(1) = 1$

$d\mu_s$ singular.

Harder fact

More difficult.

To produce $S(x,y)$ continuous Mothers
s.t. $d\mu_s$ singular Mass with respect to Lebesgue

But Support "Bigger" than $\bigcup$ of lines

(5)

$\boxed{\text{Exciting Result}}$

Carleman / Beurling
Titschmarsh / M 1960
1920-30. $A \in \operatorname{supp}(\mu)$

μ Prob. Measure. supp $[0, A]$

$F_{\mu}(x)$

x

$$\hat{\mu}(z) = \int_0^A e^{ixz}\, d\mu(x)$$

$\underline{\text{entire}}$

$\mathbb{C}$ - plane of z

How does $\underline{A}$ (size of)

Reflect Zeros of $\hat{\mu}(z)$