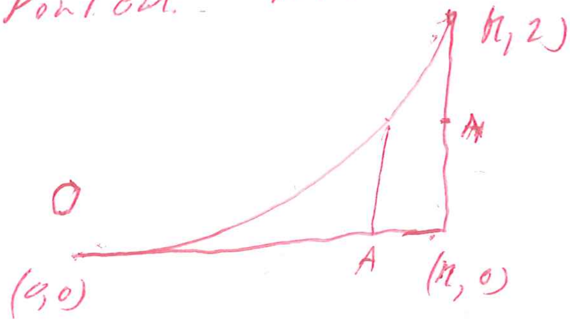
Cycloid

Refer to Wik Rolling wheel

+ animation.

Portood. Turn $\mathcal{C}$ around.



Parametric equation

$$x = \varphi + \sin\varphi \qquad 0 \leq \varphi \leq \pi$$

$$y = 1 - \cos\varphi$$

Recall $\cos\pi = -1 \Rightarrow y(\pi) = 2$!

$\sin\pi = 0 \Rightarrow$ Reach $x = \pi$

Recall Pythagoras to measure arc-length

$x \longrightarrow s(x)$ ~~by~~ satisfies

$$\frac{ds}{dx} = \sqrt{1 + y'(x)^2}$$ local differential equation

$$s(A) = \int_0^A \sqrt{1+y'(x)^2}\, dx.$$ In particular

$$L = s(\pi) = \int_0^\pi \sqrt{1 + y'(x)^2}\, dx =$$ Total length of the curve $O\mathcal{C}$

We are going to prove: Note $L > \pi$!

1° $L = \cancel{2\sqrt{2}}\ 4$ (*)

To get (*) we shall regard S as a function of y and prove

(**) $S(y) = \sqrt{2}\cdot 2\sqrt{y}$ $\quad 0 \le y \le 2$??

To get (**) we shall use several ~~differential~~ derivatives such as

$$\dot{x} = \frac{dx}{d\varphi} \quad \& \quad \dot{y} = \frac{dy}{d\varphi}$$

and then $y'(x) = \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$!!

Calculus about Trigonometric sine- and cosine functions tell us

$$\begin{cases} \dot{x} = 1 + \cos\varphi \\ \dot{y} = \sin\varphi \end{cases} \Longrightarrow$$

$$1 + y'(x)^2 = 1 + \frac{\sin^2\varphi}{(1+\cos\varphi)^2} = \frac{1 + 2\cos\varphi + \cos^2\varphi + \sin^2\varphi}{(1+\cos\varphi)^2}$$

$$= \frac{2}{1+\cos\varphi}$$

Conclusion

$$\frac{ds}{dx} = \sqrt{1+y'(x)^2} = \sqrt{2}\,\frac{1}{\sqrt{1+\cos\varphi}}$$

Now $\dfrac{ds}{dx} = \dfrac{ds}{dy}\cdot\dfrac{dy}{dx} = \dfrac{ds}{dy}\cdot\dfrac{\dot{y}}{\dot{x}}$.

$$\frac{ds}{dx} = \frac{\dot{y}}{\dot{x}}\,\frac{ds}{dy} = \frac{\sin\varphi}{1+\cos\varphi}\cdot\frac{ds}{dy} = \sqrt{2}\,\frac{1}{\sqrt{1+\cos\varphi}}$$

$$\Rightarrow \boxed{\frac{ds}{dy}} = \sqrt{2}\,\frac{\sqrt{1+\cos\varphi}}{\sin\varphi}$$

$$\sqrt{2}\cdot\frac{\sqrt{1+\cos\varphi}}{\sqrt{1-\cos^2\varphi}} = \sqrt{2}\,\frac{\sqrt{1+\cos\varphi}}{\sqrt{(1-\cos\varphi)(1+\cos\varphi)}}$$

$$= \sqrt{2}\cdot\frac{1}{\sqrt{1-\cos\varphi}} = \sqrt{2}\,\frac{1}{\sqrt{y}}\ ! \quad (**)$$

~~He~~ Now $\dfrac{d}{dy}(\sqrt{y}) = \dfrac{1}{2}\,\dfrac{1}{\sqrt{y}}$

$$\Rightarrow \boxed{s(y) = 2\sqrt{2}\cdot\sqrt{y}}$$

$V =$ Initial speed to the Puck

Normal cycloid $T(H)$ is small!

$v =$ Initial velocity

Reach altitude ("height")

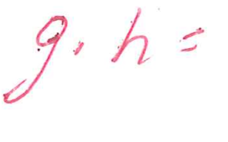
$g \cdot h = \frac{V^2}{2}$ Galilei [falling time T is to moon]

$h_{max} = H = 2$ peak   critical V $\boxed{g = 9{,}8}$

$V^2 = 4g \Rightarrow V \approx 2\sqrt{g} \approx 6{,}2$ m/sec

Rather Modest   Large cycloid $x = R(\varphi + \sin\varphi)$

Roll



changed the scale

Peak $2R$

V_{max} $2\sqrt{g} \cdot \sqrt{R}$

"cycloid ↔ Hill of a mountain"

(5)

$R = \cancel{\ldots} 400$

$V_{max} = \cancel{2\sqrt{g}\,\sqrt{\ldots} \cdot 10\sqrt{10} \simeq 200}$ m/sec

$V_{max} = 2\sqrt{g} \cdot \cancel{20} = \cancel{40} \approx 20$

$\simeq 120$ m/s fastest Ishockey!!

$= R = 100$ V_{max} $2\sqrt{g} \cdot 10 = 20 \cdot \sim 3$

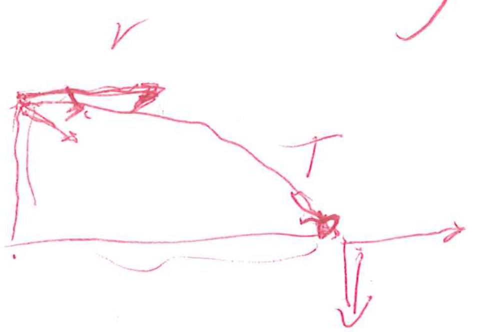
60 m/sec $\simeq \geq 200$ km/hour

Scale Important Related to

$T = T(h_{max})$ Time!!

Formula for T

as function of R!



we know $\frac{ds}{dt} \downarrow$ $\frac{ds}{dt}$ at height h

$h = 0$

$\frac{V^2}{2}$ by Galilei: $\left(\frac{ds}{dt}\right)^2 = \frac{V^2}{2} - gh$ Pres. of energy

V_{max} $\left(\frac{ds}{dt}\right)^2 = \cancel{g(\ldots)} = 2g(H-y)$ $y = h$ varies find $\underline{\underline{T(H)}}$ (6)

Cycloid normal
" R

$$\left(\frac{ds}{dt}\right)^2 = 2g(H-y) \qquad (y=h)$$

$$\frac{ds}{dt} = \sqrt{2g}\sqrt{H-y}$$

$$\Rightarrow \quad \frac{ds}{\sqrt{H-y}} = \sqrt{2g}\cdot dt$$

$$\sqrt{2g}\,T(H) = \int_0^H \frac{s'(y)}{\sqrt{H-y}}\,dy =$$

$$\sqrt{2g}\,T(H) = \sqrt{2}\int_0^H \frac{dy}{\sqrt{y(H-y)}} \leftarrow \text{by } (**)$$

variable substitution

~~$\xi = Hy$~~

$\underline{y = H\xi}$

$dy = H\,d\xi$

$$\sqrt{2}\int_0^1 \frac{d\xi}{\sqrt{\xi(1-\xi)}} \qquad \xi = \sin^2\theta$$

$$\sqrt{2}\int_0^{\pi/2} \frac{2\sin\theta\cos\theta}{\sin\theta\cdot\cos\theta\sqrt{1-\sin^2\theta}} = \sqrt{2}\cdot 2\int_0^{\pi/2} d\theta$$

$$= \underline{\sqrt{2}\cdot\pi}$$

$$T(H) = \frac{\pi}{\sqrt{g}} \;!!$$

$R > 1$ $\quad T(R \cdot 2)$ increase

falling Time

$$\boxed{\frac{\pi}{\sqrt{g}} \cdot \sqrt{R} \quad ??}$$

$\simeq 7$

$R = 100$

$\simeq 20$ seconds!!

Pipi on The top of The block

$\times$

200

A tricky bird's play

Pippi with hockey stick.

(Abel's ...

speed of a stone slipped from H by a bird at the same time when pipi's puck just reaches ~~the~~ its peak which is below H = hmax.

$$gh = \frac{v^2}{2}$$

$$w^2 = (\overset{2*g}{2 \cdot 200}) = 400 \times (\sim 20) = 8000$$

$$\boxed{w = \sqrt{8000} \text{ m/sec}}$$

This is speed at which the stone will hit the puck as Nielsen times his clock & he'll be shocked.

8 ....

Write original notes