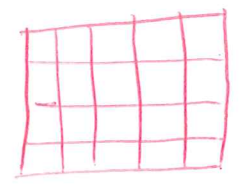


①

Chalmers:  
 Prof. E. M. Stein  
 Peter Sogge  
 Spectral Theory & Harmonic Analysis  
 Probability Theory  
 $\Rightarrow$  Probability



Riemann integrals  
 Lebesgue integrals

11111111

$$0 < f(x) \leq M$$

Riemann  
 $f \in C[0,1]$



$\sum_{i=1}^n f(x_i^*) \Delta x_i$   
 $\approx \int_a^b f(x) dx$   
 $\sum_{i=1}^n f(x_i^*) \Delta x_i$   
 $\approx \int_a^b f(x) dx$

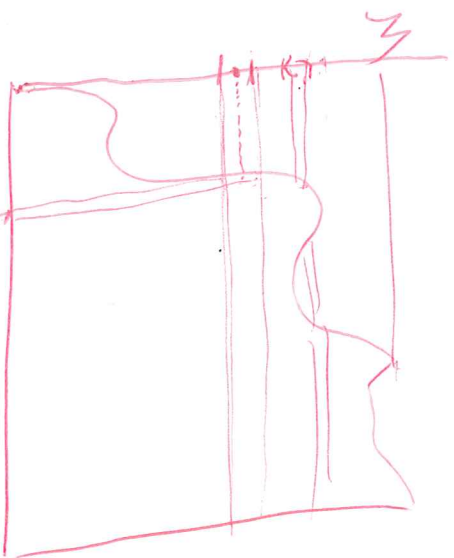
$\sum_{i=1}^n f(x_i^*) \Delta x_i$   
 $\approx \int_a^b f(x) dx$

$\sum_{i=1}^n f(x_i^*) \Delta x_i$   
 $\approx \int_a^b f(x) dx$

Need high order  
 convergence  
 Apply

Lebesgue (= Borel-Stieltjes)

(2)



all that  
works  
convergence is  $M$   
such that for all  $f < t$   
 $0 \leq f(x) \leq M$

Luc Taulin (Moore Jrk)

SIVIA

Set-inversion via I.A.

$$\int_a^b f(x) dx = \sum_{k=1}^n f(\xi_k) (x_k - x_{k-1})$$

interval  $x_{k-1} \leq x \leq x_k$

$\xi_k$   
 $\xi_k = x_{k-1} + \theta_k(x_k - x_{k-1})$   
 $\theta_k \in [0, 1]$

$$\left| \int_a^b f(x) dx - \sum_{k=1}^n f(\xi_k) (x_k - x_{k-1}) \right| \leq \frac{M}{N}$$

Riemann limit

different way  
independent of  
number of  
continuity

h

A hand-drawn diagram of a square with a smaller square inside. The top-left corner of the inner square is labeled  $\frac{9}{10}$  and the bottom-left corner is labeled  $2$ . The top-right corner of the inner square is labeled  $3$  and the bottom-right corner is labeled  $1$ .

Handwritten notes in red ink:

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7655 (2)

[illegible]

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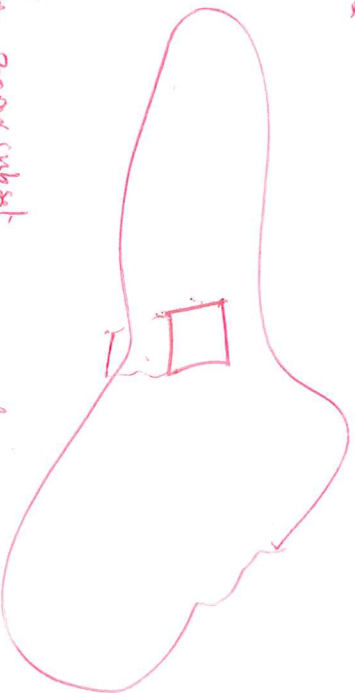
24  
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$\frac{1}{2} \epsilon$

1302

$\vec{r} = r \hat{r}$

compact  
 open  
 proper subset  
 open set



$\square$

With dyadic cubes one can obtain all  $\sigma$ -additive measures

# Lebesgue

occure

$$S(x)$$

Lieschitz continuous + Darb's Monoton.

$$d\mu_s \in L^1$$

General S

$$\{f_n\}$$

Lip + Darb's Monotone

## Program

$$f_1 = S$$

See the case  $N=1$  Lebesgue

continuous

$$S(x) = 0$$

$$S(x) = 1$$

$$d\mu_s$$

Harder fact

more difficult

Not Hols

$$a + c_1 x + c_2 x^2$$

max over function

$$S(x)$$

can be

$$S(x)$$

singular

to Lebesgue

condition with respect

$$d\mu_s$$

But Subject

"Bigger" than

$$d\mu_s \in L^1 \Rightarrow \int_0^\infty e^{ix^2} d\mu_s(x) \rightarrow 0$$

# Exciting Result

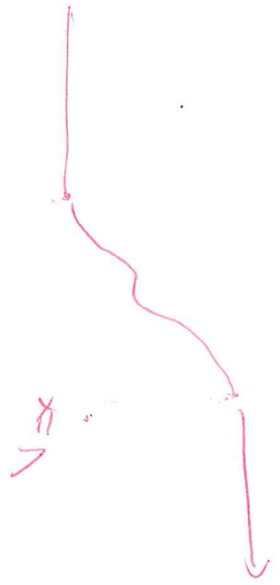
Correlation / Best info  
 Fitchman /  
 1990s. A < small / m

(5)

Prob. Measure.

$$S_{\text{red}} [0, A]$$

$E_{\text{hp}}$



$$A \int_0^x e^{-ix} dx(x)$$

Horizontal axis  
 Reflect zeros

Reflect zeros

active  
 C = plane of

$$c_p(z)$$