Special Topics

Introduction.

Above are headlines for the sections. There detailed contents are listed in the next pages and further comments appear in the individual sections. The level of the material changes from fairly elementary facts to results whose proofs are quite demanding. The first three sections are essential for much of the subsequent material, especially facts about functions in the Jensen-Nevanlinna class which together with the Brothers Riesz Theorem from Section 1 are used to study Hardy spaces an they appear frequently in later sections as well. The topics treat both analytic function theory and harmonic analysis where the interplay lead to a powerful theory such as in Section 14 where Beurling-Wiener algebras are studied. A veritable high-light is the notion and properties of Carleson measures which are to establish the interpolation theorem for bounded analytic functions in section 12. Section 19-20 are a bit apart from analytic function theory but have been included since the methods are of interest. For example, symmetrizations is often used in analytic function theory and the proof in Section 20 illustrates one application of symmetrization methods and the non-linear solution to a PDE-equation in section 19 is proved via succesive analytic series which eventually reduce the proof of existence to solving a family of linear Neumann problems.

Guidance to the sections.

Below follows a more detailed list of material from some of the extensive sections. For the shorter sections we refer to the individual introductions.

I. The disc algebra $A(D)$

1. Theorem of Brothers Riesz.

2. Ideals in the disc algebra

3. A maximality theorem for uniform algebras.

2. The Jensen-Nevanlinna class and Blaschke products.

1. The Herglotz integral.

2. The class JN(D)

3. Blaschke products

4. Invariant subspaces of $H^2(T)$

5. Beurling's closure theorem

6. The Helson-Szegö theorem.

3.A: The Hardy-Littlewood maximal function

1. The weak type estimate

2. An L^2-inequality

3 Harmonic functions and Fatou sectors

4. Application to analytic functions

5. Conformal maps and the Hardy space $H^1(T)$

3:B. The Hardy space H^1

1. Zygmund's inequality

2. A weak type estimate.

3. Kolmogorov's inequality.