ISSN 0005-1179, Automation and Remote Control, 2010, Vol. 71, No. 11, pp. 2417–2426.

CONTROL IN SOCIAL ECONOMIC SYSTEMS

Dynamic Models of Informational Control in Social Networks

I. N. Barabanov, N. A. Korgin, D. A. Novikov, and A. G. Chkhartishvili

Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
Received January 11, 2010

Abstract—The dynamic models of informational control in social networks were considered. The problems of analysis and design of the optimal controls were posed and examined.

DOI: 10.1134/S0005117910110135

1. INTRODUCTION

The present paper is devoted to the formulation and consideration of the dynamic problems of informational control in social networks. By the *social network* is meant the social structure consisting of the set of *agents* (subjects—individual or collective members of the network such as individuals, families, groups, or organizations) and the set of *relations* (assemblage of the inter-agent *ties* such as acquaintance, friendship, cooperation, communication) defined on it [1]. In formal terms, the social network is an oriented graph $G(N, E)$ where N is the set of vertices (agents) and E is the set of arcs reflecting the inter-agent ties.

At modeling the social networks, one has to take into account the mutual influence of their members, the dynamics of their opinions. A review of the influence models in the social networks can be found in [2]. A purposeful influence of the members of a social network (or subjects not included in the network, but using it as a tool of informational action) is a special case of the *informational control* lying in forming awareness of the controlled subject (as a rule by communicating the corresponding information) [3] such that their decisions are most beneficial to the controlling subject [4].

The present paper rests upon the model of informational influence described in [1] which considered single informational actions and, in distinction to it, is devoted to the dynamic model of informational control. The second section describes the model of the social network, and the third and fourth sections consider, respectively, the problems of analysis and design of controls. The promising paths of future research are outlined in the conclusions.

2. MODEL OF THE SOCIAL NETWORK

The approach of [1] extending the classical models of [5–7] (see also the reviews in [8, 9]) was used as the basis of the social network model. The paper [1] considered the informational influence of the agents in the social networks on formation of mutual opinions. The network structure is described using the notions of *community* (set of agents that are not influenced by the outsiders), *group* (community of agents where any two agents influence each other), and *companion* (agent that does not influence any group) [1].

Let $N = \{1, \ldots, n\}$ be the set of agents included in a social network. The agents in the network influence each other, the degree of influence being defined by the $n \times n$ matrix of direct influence $A = ||a_{ij}||$, where $a_{ij} \geq 0$ denotes the degree of *confidence* of the ith agent on the jth agent or, which we regard as equivalent, the degree of influence of the jth agent on the ith agent. The *normalization condition*

$$\forall i \in N \quad \sum_{j=1}^{n} a_{ij} = 1 \tag{1}$$

is assumed to be satisfied, that is, A is a row-stochastic matrix.

If the ith agent trusts to the jth agent and the jth agent trusts to the kth agent, this implies that the kth agent influences indirectly the ith one, and so on, that is, that "chains" of indirect influences are possible.

At the initial time instant each agent has an *opinion* about a certain question. The opinions of all influence agents are reflected by the column vector x^0 of dimension n of the real-valued initial ("nonperturbed" in the absence of control) opinions. A classification and numerous examples of the opinions of the members of social networks can be found in [10]. The agents in a social network interact by exchanging opinions which results in variations in the opinion of each agent under the influence of the agents to which it trusts. We assume that at the time instant k the opinion of the ith agent $x_i^k \in \Re^1$ is as follows:

$$x_i^k = \sum_{j \in N} a_{ij} x_j^{k-1}, \quad k = 1, 2, \ldots. \tag{2}$$

We denote by $x^k = (x_1^k, \ldots, x_n^k)$ the state of the social network at the instant k. Equation (2) is put down in the vector–matrix terms as

$$x^k = A x^{k-1}, \quad k = 1, 2, \ldots. \tag{2$'$}$$

Let us assume that each group has at least one agent which trusts himself at least to some extent, that is, $\exists i$: $a_{ii} > 0$. Then (see, for example, [11–14]), owing to the repeated exchange of opinions, in the long run the agents' opinions converge to the resulting (*summarized*) opinion vector $X = \lim\limits_{k \to \infty} x^k$. The general necessary and sufficient convergence conditions such as regularity of the Markov chain and so on which can be found in [11–14] are presented below in a nutshell, that is, finally the opinions of companions are defined by the opinions of groups, and the opinions within the groups are flattened.

Then, we can set down the relation

$$X = A^{\infty} x, \tag{3}$$

where $A^{\infty} = \lim\limits_{k \to \infty} A^k$. At that, (i) the summarized opinions of the agents of each group coincide (consensus is reached) and are independent of the initial opinions of the agents not included in the given group and (ii) the summarized opinions of the companions are completely defined by the opinion of one or more groups. In terms of the theory of Markov finite chains [11, 12], each agent of the group is an *essential state*. This theory states that if a Markov process has more than one group of essential states (which in terms of the social networks correspond to the notion of group), then the transition matrix A is representable as

$$A = \begin{pmatrix} A_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & A_p & 0 \\ Q_1 & \cdots & Q_p & R \end{pmatrix},$$

where A_l, $l = \overline{1,p}$, are the in-group transition matrices (*indecomposable* stochastic matrices), p is the number of groups, Q_l is the matrix describing the influence of group l on the companions. At that, the total influence of the group l on some companion is defined as the sum of elements of the corresponding row of the matrix Q_l, and R is the matrix of influence of the companions on each other.

If the Markov chain finds itself in any state from an arbitrary group of essential states, then only the states from this group are possible further. At that, it is possible to return to the same state after some number of steps. The minimal number of steps after which the process leaving an essential state can return to it is called the *state period*. If we assume that the interaction matrix A_l describes a graph of the group l, then the state period coincides with the length of the minimal cycle passing through the agent corresponding to this state. Obviously, the period of any state from a group does not exceed the number of states in this group.

The greatest common divisor of the periods of all essential states of one group is called the group *cyclicity* d_l [11–13]. This characteristic of the group is extremely important because the necessary and sufficient condition for convergence of opinions within an individual group l is represented by the *acyclicity* (or Kolmogorov *primitivity* [11]) of the interaction matrix of the agents of this group $d_l = 1$ [13].

As was noticed above, availability in a group of at least one agent which trusts itself to some extent suffices for convergence of opinions within the group. One can readily see that the interaction matrix of such group is acyclic because for the given agent the length of the minimal cycle is one. If all groups in the matrix A are acyclic, then it is called the *simple* matrix [11]. If the simple matrix A has only one group, then it is called the *regular* matrix [11, 12].

Sometimes we assume for simplicity (mentioning this in each particular case) that all elements of the stochastic matrix of direct influence A are strictly positive, which suffices for regularity of A, that is, all terms of the social network make up a single acyclic group. We notice that convergence of the agents' opinions generally would require infinite time even within the framework of this sufficiently strong assumption.

It is common knowledge that for the regular matrix A all rows of the matrix A^∞ are equal to the same probabilistic positive vector $\alpha = (\alpha_1, \dots, \alpha_n)$: $\sum\limits_{i=1}^{n} \alpha_i = 1$, $\forall i \in N \quad \alpha_i > 0$. Moreover, this vector is a solution of the equation $\alpha A = \alpha$ which has a single solution in virtue of regularity of the matrix A [11, 12]. The vector α is the *final* (*limiting*) distribution of the regular Markov chain [12].

For the given vector of initial opinions x^0, the summarized opinion of each agent is αx^0. Therefore, from the standpoint of the model under consideration the value of α_i may be treated as the *consequence* of the ith agent because this value defines the impact of its initial opinion on the summarized opinion. It is also interesting that the equality $\alpha x^0 = \alpha x^k$, where $x^k = A^k x^0$ (see (2')), is satisfied for any $k = 1, 2, \dots$. As follows from this fact, it is possible to determine for all $a \in \Re^1$ the *attraction domain* $X(a)$, the set of initial opinions from which the given value is reachable as the consensus of the group—$X = a(1 \dots 1)^{\mathrm{T}}$:

$$X(a) \subseteq \Re^{n-1} = \{x^0 \in \Re^n | \alpha x^0 = a\}.$$

At that, $\forall a, b \in \Re^1$, $a \neq b \quad X(a) \cap X(b) = \varnothing$. The geometrical interpretations of this assertion are closely related with the condition appearing in the formulation of Assertion 2c presented in what follows.

As a digression we note that the problem of determining the relative influence of the agents in a social network (analysis of equations like $\alpha A = \alpha$) is allied to the so-called *PageRank problem* (see, for example, reviews in [15–17]).

A model of the informational control lying in a single generation by the controlling organ, the *principal*, of the initial opinions of the agents was considered in [1]. It is of interest to analyze the potentialities of the informational control realized for at least several time periods. Now we proceed to discussing the corresponding models.

3. DYNAMIC MODELS OF INFORMATIONAL CONTROL. ANALYSIS

"Static" model. We first consider the "static" case where the principal can exert influence only once—at the initial time instant—on the initial sets $M \subseteq N$ of agents called conventionally the "*agents of influence*," their number being $m = |M|$.

We denote by $x(u^0) = (x_1, \dots, x_n)^{\mathrm{T}}$ the vector of initial opinions of the agents with regard for control, $x = (x_M, x_{N\setminus M})$, where $u^0 = (u_j^0)_{j\in M}$ is the control vector, $u_j^0 \in U_j$, U_j are some sets representing the constraints on controls (their particular form in one or another mathematical model is defined by the sense of controls), $j \in M$, $x_M = (x_j(u^0))_{j\in M}$.

We assume that the control—principal's influence on the agents' opinions—is additive, that is, $x_i(u^0) = x_i^0$, $i \in N\setminus M$, $x_j(u^0) = x_j^0 + u_j^0$, $j \in M$. With allowance for controls, (3) goes to

$$X(u^0) = A^\infty x(u^0).$$

Nothing was said until now about the criterion of efficiency of the informational control, that is, the principal's objective function or its preferences (these terms are used below as synonyms). We assume for the time being that the principal's preferences depend on the summarized agents' opinions X. For example, the principal may be interested in one or another value of the average arithmetic of the opinions of the members of a social network, whereas, as was shown in [1], the static problem of determining the optimal control comes to a problem of linear programming. The corresponding numerical examples are presented in [10].

Dynamic models: analysis. Now we turn to the case of the dynamic informational control where the principal can influence the agents not only at the initial, but also at other time instants.

We assume without loss of generality that the agents $1, 2, \dots, m$ are those of influence. We denote by $u^k = (u_j^k)_{j=1}^m$, $k = 0, 1, \dots$ the control vector at the instant k and consider the $n \times m$ matrix $B = \begin{pmatrix} 1 & & \\ & \cdots & \\ & & 1 \\ & 0 & \end{pmatrix}$. It is desired to know the minimal number of the influence agents and the cases where the presence of a single such agent allows one to control the entire social network (also by one-time actions). The following assertion defines the sufficient conditions.

Assertion 1. *Let all elements of the stochastic direct influence matrix A be strictly positive and the single action exerted at the initial time instant be unlimited. Then, any unanimous value of the summarized opinions of the members of a social network is realizable in the presence of at least one (arbitrary) influence agent.*

Validity of Assertion 1 follows from the fact that within the framework of the assumption of strict positiveness of the elements of the matrix A all rows of the matrix A^∞ are identical and have no zero elements (as was noted above, the elements of the matrix A^∞ reflect the *consequence* of the agents [1, 9, 13]).

With the assumption that in each period, including the zero period, control precedes exchange of opinions between the agents, the equation of opinion dynamics is representable in the matrix form as follows (cf. (2)):

$$x^{k+1} = A[x^k + B\,u^k], \quad k = 0, 1, \dots. \tag{4}$$

This expression is a difference equation describing a discrete linear control system [18] with the stochastic matrix A. Its solution under a given initial condition (a counterpart of the solution of the Cauchy problem in the continuous case) is representable as

$$x^k = A^k x^0 + \sum_{\tau=0}^{k-1} A^{k-\tau} B\, u^\tau, \quad k = 1, 2, \ldots. \tag{5}$$

We call $\Phi = [B' \;\; A\,B' \;\; \ldots \;\; A^{n-1}\; B']$, where $B' = A\; B$, the *controllability matrix* of system (4) and for the time being assume that there are no control constraints, that is, $U_j = \Re^1$, $j \in M$. Then, the question of reachability of an arbitrary state x^T of the linear system (4) in T ($T \geq n$) steps comes to the question of nondegeneracy of the pair of matrices A and $A\,B$ or, which is the same, to the question of equality of the rank of the matrix Φ to the number n. An answer to this question may be sought using the well-known results of the theory of discrete control systems (see, for example, [18]).

It is of interest to see to what extent the summarized agents' opinions depend on the time periods when the actions were executed. This question was answered by Assertion 2a. If the principal's preferences depend namely on the summarized agents' opinions, the Assertion 2b enables one to substantially reduce the problem to the static one. Unlimitedness of controls is sufficient for that. If the principal's preferences depend on the agents' opinions at some final time instant, then the additional condition appearing in Assertion 2c must be satisfied to enable reduction of the control problem to the static problem.

Assertion 2a. *Let the principal perform actions $u^0, \ldots, u^l$, $l < +\infty$. The vector of the summarized agent opinions (for $k \to +\infty$) does not change if the same (in magnitude) actions were performed at any other finite time instants.*

Assertion 2b. *Let the control be unlimited. Then, at the initial (zero) time instant for any sequence of the control vectors $u^0, \ldots, u^l$, $l < +\infty$, there exists a control vector ν such that it leads to the same summarized opinions of the agents.*

Assertion 2c. *Let the controls be unlimited and*

$$\operatorname{span}(\Phi) \subseteq \operatorname{span}(A^{l+1}\, B),$$

where $\operatorname{span}(\cdot)$ *is the linear hull of the matrix columns.*

Then, for any sequence of the control vectors $u^0, \ldots, u^l$, $l < +\infty$ and the realized state x^{l+1} of the social network, at the initial (zero) time instant there exists a control vector $\hat{\nu}$ such that it leads to the same state x^{l+1} of the social network at the time instant $l+1$.

Proof of Assertions 2a–2b. We get in virtue of (3) and $A^\infty A = A^\infty$ that

$$\begin{aligned} X &= A^\infty \left[\ldots A\left(A\left(A\left(x^0 + B\,u^0\right) + B\,u^1\right) + B\,u^2\right) + \ldots + B\,u^l\right] \\ &= A^\infty(x^0 + B\,u^0) + A^\infty \sum_{k=1}^{l} B u^k. \end{aligned} \tag{6}$$

By denoting

$$\nu = \sum_{k=0}^{l} u^k, \tag{7}$$

we obtain $X = A^\infty(x^0 + B\nu)$, which is what we set out to prove.

Proof of Assertion 2c. According to (5), it is possible to set down $x^{l+1} = A^{l+1}[x^0 + B\,u^0] + \sum_{k=1}^{l} A^{l-k+1} B\,u^k$. On the other hand, it is desired to determine a vector $\hat{\nu}$ such that $x^{l+1} = A^{l+1}[x^0 + B\hat{\nu}]$. If the conditions of Assertion 2c are satisfied, then by the Kronecker–Capelli theorem it is possible to determine the vector $\hat{\nu}$ which solves the system of linear algebraic equations

$$A^{l+1}B\hat{\nu} = A^{l+1}Bu^0 + \sum_{k=1}^{l} A^{1-k}B\,u^k.$$

Corollary 1. *Let the control be unlimited and the criterion for control efficiency depend only on the summarized agents' opinions and the sum (over the agents and time periods) of controls. Then, for any finite sequence of the control vectors there exists vector* (7) *of the initial controls of at least the same efficiency.*

Therefore, within the framework of the conditions of Corollary 1, the time-dependent control will give anything new as compared with the static case. It should be emphasized that this result may prove to be extremely effective in the models of cognitive cards (see [19] for the discussion of the problems of control on the "cognitive cards"). Therefore, an essential assumption which below is assumed to be satisfied states that the principal's preferences depend on the agents' opinions in a finite number $T < +\infty$ of the first periods of their interaction.

The sum $w_j^k = \sum_{i\in N} (A^k)_{ij}$ is called the *consequence* of the agent j at the instant k. The sum $\sum_{i\in N} x_i^k$ is called the *summary opinion* of the agents at the instant k. Let the principal choose the control $u^0, \ldots, u^l$, $l \le T$. The sum $\sum_{\xi=0}^{l} \sum_{j\in M} u_j^{\xi}$ is called the *summary action* of agents at the instant k.

For convenience of computations we introduce the matrix $C_0 = \underbrace{(1\ldots1)}_{n}$ and set down in the matrix form: $w^k = C_0A^k$ is the row matrix of dimension n consisting of the agents' consequences; $x_{\Sigma}^k = C_0x^k$ is the summary opinion of the agents at the time instant k; $u_{\Sigma} = \sum_{\xi=0}^{l} C_0Bu^{\xi}$ is the summary action.

The question about what agents at what the time instants should be influenced by the principal is answered by the following assertion.

Assertion 3. *Let the controls be nonnegative* ($u_j^k \ge 0,\ j \in M,\ k = 0, 1, \ldots$). *If for the given summary action the principal tries to reach the maximum summary opinion of the agents at the instant T, then it suffices to exert at the instant k^* a single action on a single agent j^* of maximal consequence:*

$$(j^*, k^*) \in \operatorname*{Argmax}_{j\in M,\ k\in\{0,\ldots,T-1\}} w_j^{T-k}. \tag{8}$$

Proof. The vector of the agents' opinions at the instant T is as follows (see (5)):

$$x^T = A^Tx^0 + \sum_{k=0}^{T-1} A^{T-k}Bu^k, \quad T = 1, 2, \ldots.$$

As before, we denote by u_{Σ} the summary action

$$u_{\Sigma} = \sum_{k=0}^{T-1} \sum_{j\in M} u_j^k = C_0B\sum_{k=0}^{T-1} u^k.$$

In virtue of (8), the following chain of relations is valid for the summary opinion of the agents at the instant T:

$$x_\Sigma^T = \sum_{i\in N} x_i^T = \sum_{i\in N}(A^T x^0)_i + \sum_{k=0}^{T-1}\sum_{i\in N}(A^{T-k}Bu^k)_i = C_0 A^T x^0 + \sum_{k=0}^{T-1} C_0 A^{T-k}Bu^k$$

$$= w^T x_0 + \sum_{k=0}^{T-1} w^{T-k}Bu^k \le w^T x_0 + \max_{j\in M,\, t\in\{0,\,1,\ldots,\,T-1\}} w_j^t \sum_{k=0}^{T-1} C_0 Bu^k = w^T x_0 + w_{j^*}^{k^*} u_\Sigma.$$

On the other hand, if one takes the controls $u_{j^*}^{k^*} = u_\Sigma$, $u_j^k = 0$ $(j \ne j^*,\ k \ne k^*)$, then the above inequality goes to equality, which proves Assertion 3.

A result similar to Assertion 3 takes place also if in opinion of the agents the principal's objective function is partially monotone (at any instant of the horizon of planning) and the constraints are defined on the individual controls and not for the summary action. At that, the optimal controls will be singlefold and lie on the boundary of the set of permissible controls.

The situation complicates if the principal's objective function is not partially monotone in the agent actions. Then depending on the structure of the principal's objective function, the dynamic problem of designing the optimal informational control comes to one or another optimization problem which can be solved numerically in each particular case. Linearity of the controlled system (see (5)) is an essential simplifying factor. Now, we formulate the dynamic problem of designing the optimal informational control.

4. DYNAMIC MODELS OF INFORMATIONAL CONTROL. DESIGN

In the general case, the problem of design is formulated as follows. We denote by $y = Y(x) \in \Re^q$ the vector of *observed states* of the social network, $Y:\ \Re^q \to \Re^k$ a certain function, $q \le n$, T the planning horizon, $x^{1,T} = (x^1, \ldots, x^T)$ the trajectory of states of the social network, $y^{1,T} = (y^1, \ldots, y^T)$ the trajectory of the observed states of the social network, $u(y):\ \Re^q \to U$ the *control law*, $u^{1,T} = (u(y^1), \ldots, u(y^T))$ the sequence of controls, and $F(y^{1,T}, u^{1,T})$ the criterion for control efficiency.

Let the initial observed state of the social network be known. In the general form, the dynamic *problem of designing the optimal positional informational control* lies in determining a permissible law of control of the discrete system (4) having the maximum efficiency:

$$F\left(y^{1,T}(x^{1,T}(u(\cdot))), u(\cdot)\right) \to \max_{u(\cdot)}. \qquad (9)$$

In the general form, the dynamic *problem of designing the optimal program informational control* lies in determining a sequence of controls of the discrete system (4) having the maximum efficiency:

$$F\left(y^{1,T}(x^{1,T}(u^{1,T})), u^{1,T}\right) \to \max_{u^{1,T}}. \qquad (10)$$

It is only natural that in the deterministic problem under consideration (in the absence of uncertainty) the sequences of controls that are solutions of problems (9) and (10) coincide. The problem of constructing the optimal control of the discrete-time systems was examined by many authors. Some approaches to it can be found, for example, in [18].

Let us consider some special cases of problems (9) and (10) that are of importance for the social networks from the practical standpoint. Let the vector y^* which is the "aim" of the informational control in the space of observed states of the social network be fixed. The problems

$$||y^T - y^*|| \to \min_{u(\cdot)} \qquad (11)$$

and

$$||y^T - y^*|| \to \min_{u^{1,T}} \tag{12}$$

are called the *problems of positional and program control of the final state of the social network.*

Let us consider problem (12) and assume for simplicity that $y = x$, that is, the states of all system agents are observable. If the conditions of Assertion 2c are satisfied, then the minimum in (12) is equal to zero and it suffices to apply control only once (Assertions 2a and 2b). If the condition $\operatorname{span}(\Phi) \subseteq \operatorname{span}(A^T B)$ is not satisfied, then, generally speaking, the system does not reach the state y^* ($x^* = y^*$ in the case at hand). Then we can only consider how to drive the system to some state on the set $A^T x^0 + \operatorname{span}(\Phi)$ which is as close as possible to y^* in the sense of the Euclidean metric. In this case, the problem of determining the corresponding control comes to the problem of unconditional minimization of the nonnegative definite quadratic form. In this case, solution will not be unique again in virtue of Assertions 2a and 2b, and a single action upon the system is one of the solutions.

Problem (12) comes to the well-known problem also under the condition that u_i is bounded and takes values on some convex set U,—for example, $|u_i| \leq 1$. Then, the problem of determining a program control driving the system from some given initial state x^0 to a state close to x^* comes to the problem of convex programming

$$\left\| A^T x^0 + \sum_{t=1}^{T-1} A^{T-t} B u^t - x^* \right\| \to \min_{u_i^t \in U}$$

that is solvable by the existing methods (see, for example, [20]).

The present section is concluded by an example of formulating a problem of positional control—design of a linear controller *stabilizing the social network*. Let $y = Cx$, where $C \in \Re^{q \times n}$ is some matrix. We select a linear law of control in the form $u = Ky$ (if $u = Kx$, then $C = E_n$, provided that the states of all agents are observable). The equation of the closed-loop control system is as follows:

$$x^{k+1} = (A + A\ B\ K\ C)x^k. \tag{13}$$

In virtue of linearity of the considered system, stabilization of an arbitrary position x^* amounts to stabilization of the zero equilibrium position of the closed-loop system (13). Control will be stabilizing if the spectrum of the matrix of the closed-loop system $A + A\ B\ K\ C$ lies within the unit circle on the complex plane centered at zero. We note that in the case of $C = E_n$ and nondegeneracy of the pair A, $A\ B$ the corresponding matrix K can be always determined.

In the general case, the well-known methods of design of the linear stabilizing controllers for the linear discrete systems can be used for stabilization of the social networks [18].

5. CONCLUSIONS

The main result of the present work lies in reducing the dynamic problems of control of a certain class of the social networks to the canonical control-theoretical problems of controllability study and design of the linear discrete control systems. Therefore, further extension of the results of the control theory to the control in social networks seems promising. The following also seems showing great promise:

(1) rejection of the very strong assumption, if introduced, about the strict positiveness of the influence matrix and, in a more general case, study of the influence of the communication graph on the properties and controllability of the social network;

(2) use of the numerous existing results on the studies of the Markov chains and controllable Markov processes at construction of various models of social influence;

(3) consideration of more general criteria for control efficiency;

(4) detailed examination of the role of constraints on control;

(5) study of the models of social networks with various agents' "susceptibility" to the informational control actions, which may be reflected by the corresponding "weight" coefficients used in the matrix B;

(6) study of the problems of control of the "nonlinear" social networks, that is, networks with nonlinear equations of dynamics of the agents' opinions similar to (2);

(7) allowance for the probabilistic or interval uncertainty, primarily, addition to the right-hand side of (4) of additive (stochastic or bounded) external disturbances;

(8) introduction of more than one control unit controlling some set of agents, the same agent being affected by different control units. Then, assuming that the controls are additive, we obtain an ordinary dynamic game of the control units (the so-called model of informational confrontation [1, 21]) for which one can seek, for example, subgame-perfect equilibrium, and so on [22];

(9) formulation and solution of the problem of control of the structure of communications between the members of the social network [10] with possible extension of the results to the problem of consensus [23, 24] and vice versa;

(10) development of simulation models allowing one to analyze the dynamic processes of informational control.

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This paper was recommended for publication by B.T. Polyak, a member of the Editorial Board