# Finding a dominance order most consistent with a linear hierarchy: a new procedure and review

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**Abstract.** A procedure for ordering a set of individuals into a linear or near-linear dominance hierarchy is presented. Two criteria are used in a prioritized way in reorganizing the dominance matrix to find an order that is most consistent with a linear hierarchy: first, minimization of the numbers of inconsistencies and, second, minimization of the total strength of the inconsistencies. The linear ordering procedure, which involves an iterative algorithm based on a generalized swapping rule, is feasible for matrices of up to 80 individuals. The procedure can be applied to any dominance matrix, since it does not make any assumptions about the form of the probabilities of winning and losing. The only assumption is the existence of a linear or near-linear hierarchy which can be verified by means of a linearity test. A review of existing ranking methods is presented and these are compared with the proposed method.

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An important topic in social ethology is the analysis of dominance relationships in a social group of individuals. A recent paper by Drews (1993) presented an extensive review of the literature for the purpose of elucidating the concept of dominance. On the basis of the original definition of dominance given by Schjelderup-Ebbe (1922), Drews proposed the following structural definition: 'Dominance is an attribute of the pattern of repeated, agonistic interactions between two individuals, characterized by a consistent outcome in favour of the same dyad member and a default yielding response of its opponent rather than escalation. The status of the consistent winner is dominant and that of the loser subordinate.'

Of particular interest in the analysis of social dominance is whether the individuals in the group form a linear dominance hierarchy. In a linear hierarchy the dominance relation is transitive. This means that for every three individuals A, B and C in the group the following holds: if A dominates B and B dominates C then A also dominates C. Landau (1951) and Kendall (1962) each developed independently a linearity index (ranging between 0 and 1) which expresses the

strength of the linearity present in a set of dominance relationships. A value of 1 indicates complete linearity and a value of 0 indicates that each individual dominates an equal number of other individuals. Appleby (1983) presented a statistical test of linearity originally developed by Kendall (1962). I have extended this linearity test to situations where the set of dominance relationships may include tied and/or unknown relationships (de Vries 1995). For this purpose I introduced a linearity index $h'$, which is based on Landau's $h$ index, but is corrected for the number of unknown relationships. Whenever the linearity in a set of dominance relationships is significantly stronger than expected by chance (that is: the linearity test yields a significant outcome), ordering the individuals into a linear or near-linear dominance hierarchy is worth while and justified. If the linearity is complete ($h$ or $h'$ equals 1), it is not difficult to rank the individuals. However, when the linearity is incomplete but significant, finding the optimal near-linear order of the individuals may not be so easy. In fact, this problem is one of a class of very hard combinatorial problems, for which it is likely that a generally successful and efficient algorithm cannot be found (Roberts 1990).

Many different methods have been developed for ranking individuals on the basis of their wins

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and losses of dyadic dominance encounters. These methods differ with respect to the restrictiveness of the assumptions to be satisfied by the data and the type of data used in the ranking procedure. Below I present a survey of these methods ordered according to these two aspects. Many of these methods have originally been developed in the context of a paired comparisons paradigm (David 1988). In a paired comparisons experiment several objects are compared with each other, one pair at a time, by a judge (or judges) with regard to some specific aspect. An important research question is to find out whether these paired comparisons can be combined into a consistent preference ranking of the objects. To make this paired comparisons paradigm applicable to dominance behaviour, the outcome of a dominance encounter (a win or loss) is considered as the analogue of the outcome of a comparison between two objects (Boyd & Silk 1983; McMahan & Morris 1984). In a paired comparisons experiment the pair comparison is the observational (or experimental) unit of analysis. Some of the conditions necessary for applying a ranking method are fulfilled by the design of the experiment. In particular, care is taken to ensure that replicated comparisons on the same pair of objects are independent, either by using independent judges, or, if one is interested in the choice preferences of each judge separately (as in the study of Tversky (1969) on intransitivity of preferences), the memory of earlier choices is minimized by intermingling the presentation of the relevant pairs of objects with 'irrelevant' pairs. One also tries to keep the number of comparisons per each pair of objects constant. That is, the experimenter prefers a balanced design. The more the number of comparisons differ the more the results are due to those pairs of objects that have been compared most.

In contrast to this experimental situation, the analogous conditions for the situation where one observes animals living in a social group are often not fulfilled. Specifically, it is doubtful whether one can assume that each dominance encounter between a pair of individuals represents an independent trial with a constant probability of winning. It is not unlikely that each win or loss changes the probability of winning or losing the next encounter (e.g. Fagen 1977; Chase et al. 1994). Also, it is unlikely that the number of encounters is independent of the dyad. For example, Freeman et al. (1992) showed that the

agonistic encounters among seven stags, *Cervus elaphus* (original data from Appleby 1983) occur primarily between pairs of stags that have one animal in between them in the hierarchy. This result allowed them to clarify the rather counter-intuitive finding of Iverson & Sade (1990) that the probability of a stag defeating a lower ranking one is constant, regardless of how far apart they are in the hierarchy. Iverson & Sade arrived at this conclusion by applying a statistical test that uses the agonistic encounter as the observational unit. Freeman et al. (1992, page 248) wrote: 'It is reasonable to suppose that, among the type of pairings, in which stags are separated by exactly two steps in the hierarchy, the probability of victory of the dominant animal will be more or less constant. And, since fights of this sort are greatly overrepresented, they can be expected to be the major attributors to the constant probability of victory of the dominant animal reported by Iverson & Sade (1990).'

Of course, the departure from these independence conditions depends on the particular species studied and the circumstances under which the individuals are living, but in general one must be careful with carrying over the paired comparisons paradigm to socially interacting animals, because animals are not static objects to be compared, but are dynamic creatures which can and often do change their relationships. Strictly speaking, only if the investigator experimentally controls each pair of animals to have only one dominance encounter, can the paired comparisons paradigm be applied straight away. In other cases a careful check of the assumptions with regard to the independence of the dominance encounters is necessary. A similar caveat was expressed by Kramer & Schmidhammer (1992), who equally pointed out that frequency-type ethological data, such as the number of encounters, are likely to violate the assumption of independence, and who therefore advise against the use of the chi-square test of significance for this type of data.

For individuals living in a social group it is desirable to have a ranking method that does not require these assumptions to hold. For this purpose it is appropriate to analyse the dominance data at the level of the dyad (the dominance relationship) rather than at the level of the dominance encounter. In other words, the dyad, not the dominance encounter, is to be used as the observational unit of analysis. The outcomes of

|  | | Loser | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
|  |  | a | v | b | h | g | w | e | k | c | y |
| Winner | a | * | 5 | 4 | 6 | 3 | 0 | 2 | 2 | 3 | 1 |
|  | v | 0 | * | 0 | 0 | 2 | 1 | 2 | 0 | 7 | 7 |
|  | b | 0 | 0 | * | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
|  | h | 0 | 3 | 0 | * | 0 | 0 | 6 | 0 | 2 | 5 |
|  | g | 0 | 0 | 0 | 1 | * | 2 | 4 | 0 | 3 | 0 |
|  | w | 2 | 0 | 0 | 3 | 0 | * | 0 | 0 | 2 | 1 |
|  | e | 0 | 0 | 0 | 0 | 0 | 0 | * | 0 | 0 | 4 |
|  | k | 0 | 0 | 0 | 0 | 0 | 0 | 0 | * | 2 | 1 |
|  | c | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | * | 6 |
|  | y | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | * |

**Figure 1.** Matrix containing frequencies of wins and losses in dyadic dominance interactions.

the encounters in a period of observation are used to determine the dominance relationships in that period. This can be done in several ways. A simple and straightforward way, which is more or less standard in social dominance studies, is to call A dominant to B if A wins more encounters from B than B from A. If A and B have an equal number of wins and losses the dominance relationship is tied: A and B are said to be equidominant. If no encounters have been observed their relationship is unknown. Alternative ways to derive the dominance relationship from the numbers of wins and losses are discussed at the end of the paper.

The method I develop in this paper for ordering the individuals into a linear or near-linear hierarchy is based on this definition of dominance relationship. It is applicable to any set of dominance relationships without making any assumption about the probability distribution of the wins and losses of the dominance encounters. In this sense this method can be called non-parametric. The only assumption that must be satisfied is that the linearity in the set of relationships is statistically significant. This assumption of the existence of a (near-)linear hierarchy can be verified by means of the linearity test. In accordance with this test the ranking procedure uses only information at the level of the dyad (the individual dominance encounters are used only for determining the dyadic dominance relationship, but are not taken into account in the ranking procedure itself), and allows for tied and/or unknown relationships. The method is feasible for matrices of up to 80 individuals; it finds an optimal or near-optimal ranking within a reasonable time. The procedure is implemented in the program MatMan version 3.1

or later. Previous versions of MatMan (de Vries et al. 1993) implement a rank-ordering algorithm which is essentially the one proposed by Roberts (1990): first, order the individuals according to the number of individuals dominated; next, flip all pairs of individuals adjacent to each other in the ranking, for which the one with the lower position dominates the higher one. Iterate this 'flipping process' until the rank order is Hamiltonian, that is, until no pair of adjacent individuals is inconsistent with the rank order finally obtained.

In this paper I first define some essential concepts and explain the proposed linear ordering procedure; next I present an ordered survey of existing ranking methods in which the proposed method has also been given its appropriate place. In the next section I present several examples illustrating the characteristic features of the proposed ranking method compared with other existing methods. The final section discusses alternative definitions of dominance and contains some remarks with regard to the applicability of the method.

## THE I & SI METHOD OF LINEAR ORDERING

### Some Essential Concepts

Before explaining the method proposed in this paper for finding a (near-)linear order, it is necessary to define some essential concepts. It is convenient to introduce these concepts by means of a fictitious dominance interaction matrix (Fig. 1),

containing the numbers of wins and losses in dyadic dominance interactions among 10 individuals. Note that: (1) the number of dominance interactions per dyad is not necessarily fixed to a number that is the same for all dyads; (2) there may be dyads without any dominance interactions, that is, unknown relationships; and (3) there may be dyads with an equal number of losses and wins, that is, tied relationships. For reasons of illustration only, the order of the individuals in this example matrix is not too far off from the optimal near-linear rank order.

Individuals a and v have had five dominance encounters, which were all won by a: $s_{av} = s_{12} = 5$ and $s_{va} = s_{21} = 0$. $s_{av}$ (or $s_{12}$) indicates the number of wins of row individual a against column individual v. The number of dominance encounters between a and v is $n_{12} = s_{12} + s_{21}$. We will say that individual $i$ is dominant to $j$ (and $j$ subordinate to $i$) if $s_{ij} > s_{ji}$. Two individuals $i$ and $j$ will be said to have an inconsistency relative to the current rank order, if $j$ dominates $i$ ($s_{ji} > s_{ij}$) and $j$ is below $i$ in the current ordering of the individuals. Slater (1961) introduced this term in the context of a paired comparisons schedule in which every pair of objects was compared exactly once. Note that the term inconsistency is defined as a property of a dyad relative to some ranking. The above matrix contains four inconsistencies: (w, a), (h, v), (g, h) and (w, h). The first task is to find an ordering of the individuals such that the number of inconsistencies called I is minimal. If there are circular triads (or circular polyads) present in the set of dominance relationships not all inconsistencies can be resolved. The individuals w and a are involved in the circular triad: a>b, b>w and w>a; where a>b means: a dominates b. Therefore, in any one ordering of the individuals at least one of these three dyads will be inconsistent. The dyad (w, h), on the other hand, is inconsistent with this ordering of the individuals, but w and h are actually not involved in a circular triad. Therefore, this inconsistency can be resolved without introducing a new one (see the example below). A ranking in which no individual dominates the individual just above him is called Hamiltonian. The current ranking is not Hamiltonian because g dominates h just above him. This inconsistency can be resolved simply by switching their positions (this is the so-called 'flipping rule' for adjacent individuals). The fourth inconsistency (h, v) cannot be resolved without introducing a new one,

because h and v are involved in the circular polyad h>v, v>g, g>w and w>h.

The absolute difference between the ranks of the two individuals that are involved in an inconsistency will be called the strength of that inconsistency (denoted by SI). For instance, the inconsistency (w, a) has a strength of 5 in the current ranking. The second task is to find an ordering such that the total strength of all inconsistencies is minimized after the number of inconsistencies has been minimized, and subject to the condition that the number of inconsistencies does not increase. If there exists a circular polyad involving $n$ individuals there will in any ordering always be an inconsistency with a strength of at least $n - 1$. So the second task is to find an ordering such that for each two individuals having an unresolvable inconsistency because they are involved in a circular $n$-polyad, the strength of this inconsistency is $n - 1$, being the minimum value possible. It will turn out that the strength of the unresolvable inconsistency (w, a) can be reduced to the minimum value of 4 by a suitable ranking of the individuals (see the example below).

The ranking method which is based on first minimizing I and then, subsequently, minimizing SI, subject to the condition that I does not increase, will be called the I & SI method.

## The Linear Ordering Procedure in Detail

The algorithm used in the I & SI method is somewhat inspired by an algorithm proposed by Johnson et al. (1982), and generalizes the 'flipping rule' for adjacent individuals suggested by Roberts (1990), to include non-adjacent individuals as well. The iterative matrix reorganization procedure consists of a main phase and a final phase.

In the main phase two criteria are used in reordering the individuals. These criteria are applied in the following prioritized order: (1) minimization of the number of inconsistencies in the matrix, I; and (2) minimization of the total strength of the inconsistencies in the matrix, SI. The procedure starts off with an iterative loop which is the main part of the whole algorithm (see Iteration 2 in Appendix 1). This loop implements the generalized swapping rule: switch the positions of adjacent as well as non-adjacent individuals if this results in fewer inconsistencies. There is no guarantee that this algorithm yields the optimal

solution, unless all inconsistencies can be resolved, that is, unless there are no circular triads/polyads. Therefore, if the linearity is not complete, this initial ordering is followed up by a search for a better near-linear order by generating a series of promising initial orders through randomly repositioning those individuals that are most likely to have a wrong position in the order found thus far. Each of these partly random orders is generated by repositioning each lowest individual involved in an inconsistency to a randomly chosen higher position. Starting from this new, partly random order the main iterative loop is run again and the ordering produced by this procedure is compared with the best order found thus far. In this comparison the two criteria are used as follows. First, if the number of inconsistencies in the newly found order is less than the number of inconsistencies in the best order found thus far, this best order found is replaced by the newly found order. Second, replacement also occurs if the number of inconsistencies does not differ between the two orders, but if the total strength of the inconsistencies in the newly found order is less than that in the best order found thus far.

It turns out that in practice a series of 100 sequential tries (i.e. newly generated partly random orders) is broadly sufficient to find an ordering in which the number of inconsistencies is minimized. Minimizing the total strength of the inconsistencies without increasing I turns out to be somewhat harder in difficult cases. In Appendix 1 the iterative algorithm used in this main phase is presented in a semi-formal programming language. The essential part of the algorithm is the generalized swapping rule in the inner iteration loop (Iteration 2 in lines 6–20). The outer iteration loop (Iteration 1) controls the process of trying to improve on the currently found ordering by randomly repositioning the lower individuals of the inconsistencies.

In the final phase of the procedure the individuals that have an undecided relationship with other individuals adjacent to them are ordered. Two individuals $i$ and $j$ have an undecided relationship if $s_{ij}=s_{ji}$. They can have a tied relationship ($s_{ij}>0$) or an unknown one ($s_{ij}=0$). Each pair of adjacent individuals $i$ and $j$ with an undecided dominance relationship is ordered according to the following rule. If $D_i - S_i > D_j - S_j$ then put $i$ above $j$ in the rank order (unless this increases the total strength of inconsistencies), where $D_i$ is the number of individuals dominated by $i$, and $S_i$ is the number of individuals by which $i$ is dominated.

## REVIEW OF RANKING METHODS

Table I presents a survey of ranking methods ordered according to the number and restrictiveness of the assumptions to be satisfied and the type of data used in the ranking procedure. The first two methods are based on the dyad as the observational unit of analysis and belong to the class of so-called combinatorial methods. Slater (1961) originally formulated his method for a paired comparisons schedule in which every pair of objects is compared once and only once. Translated into behavioural terms this means that dominance relationship and dominance encounter coincide in this case. He denoted the minimum number of inconsistencies by **i** and a ranking that realizes this minimum is called a nearest adjoining order. He argued that this statistic **i** is to be used in testing the null hypothesis of random choices against the alternative that the choices are based on one underlying dimension of preference. For this purpose he presented a probability distribution of **i** under the null hypothesis for paired comparisons schedules with at most eight objects. Often the nearest adjoining orders for a specific outcome of a paired comparisons schedule is not unique; that is, there are different orders all having the same minimum number of inconsistencies **i**. A further selection from these different nearest adjoining orders is possible by introducing the extra criterion of minimizing the total strength of the inconsistencies SI, subject to the condition that the number of inconsistencies I does not increase. If the two criteria are applied in this prioritized way the Hamiltonian property of the resulting ranking is secured. If minimizing SI would be the only criterion, the resulting order is not necessarily Hamiltonian (see the example on chickens below). Moreover, application of this criterion without requiring that I be minimum would necessitate the extra assumption that the probability of an individual defeating a lower ranked individual is an increasing function of the difference in ranks (see below: Crow's method).

If none of the relationships is undecided and all relationships are transitive, a unique, perfect linear order exists. In practice, some relationships

**Table I.** Ranking methods ordered according to the assumptions to be satisfied and the type of data used in the ranking procedure

| Method | Assumptions | Type of data used | Hamiltonian |
|---|---|---|---|
| Minimize the number of inconsistencies; i.e. minimize $\Sigma\text{Sign}(s_{ji}-s_{ij})$ Slater (1961)* | Choices are based on one underlying dimension of preference. This can be statistically verified by inspecting the probability of Slater's **i** under randomness | Binary, i.e. the asymmetry information within a dyad | Yes |
| The I&SI method: first minimize the number of inconsistencies and, subsequently, the total strength of the inconsistencies This paper | A (near-)linear dominance hierarchy exists. This can be statistically verified by means of the linearity test (Appleby 1983; de Vries 1995) | Binary The dyad is the observational unit of analysis | Yes |
| Maximize the likelihood under the assumption of a paired comparisons WST model† McMahan & Morris (1984) Thompson & Remage (1964) This method appears to yield quite similar rankings as, but is not equivalent to, the well-known method described by Brown (1975): minimize the proportion of entries below the diagonal, $\Sigma s_{ji}/N$† | Each encounter represents an independent trial with, for each different dyad (i,j), a constant probability of winning $P_{ij}$. This means: the strength of each dominance relationship is constant in time Each encounter between two animals is independent of encounters between other animals The number of encounters between two animals is independent of the identity of these animals The binomial probabilities $P_{ij}$ satisfy the weak stochastic transitivity condition | Frequency of wins and losses The dominance encounter is the observational unit of analysis | Yes (See McMahan & Morris 1984, page 376) |
| Maximize the likelihood under the assumption of a paired comparisons SST model Bossuyt (1990) | Same as above and: the more i and j are separated in the hierarchy, the larger is $P_{ij}$. A test of SST is described in Bossuyt 1990 (page 110 ff) | Frequency of wins and losses | No (See Table 4.5 in Bossuyt 1990) |
| Maximize the likelihood under the assumption of a rank difference function (which is stronger than SST); this is equivalent to: maximize $\Sigma(s_{ij}-s_{ji})$ (j-i) Crow (1990)‡ | Same as above and: $P_{ij}$ is an increasing function of the difference in ranks of i and j | Frequency of wins and losses | No (See example) |
| Maximize the likelihood under the assumption of a Bradley–Terry model Boyd & Silk (1983) | Same as above and: cardinal dominance indices $D_i$ exist, such that $P_{ij}$ depends logistically on the difference between the dominance indices $D_i$ and $D_j$ | Frequency of wins and losses | No (See example) |

See text for explanation. Everywhere summation is over i and j with i<j. $s_{ij}$ is cell value above the diagonal, $s_{ji}$ below the diagonal. The Sign function is defined as: $\text{Sign}(x)=1$ if $x>0$; $\text{Sign}(x)=0$ if $x=0$; $\text{Sign}(x)=-1$ if $x<0$.

*Slater (1961) originally formulated his method for a paired comparisons schedule in which every pair of objects is compared once and ties are not allowed.

†When only the binary information is used, this method is identical to minimizing the number of inconsistencies. Thompson & Remage (1964, page 739) show that the maximum likelihood weak stochastic ranking yields Slater's criterion when every pair of objects is compared exactly once.

‡When only the binary information is used, this method is identical to minimizing the total strength of the inconsistencies (without the condition that I be kept at its minimum value).

are often unknown, so that only a partial order exists. If there are tied relationships and/or intransitive triads, a perfect linear order does not exist. When the order is not fully linear, it could still be possible that a substantial part of the structure present in the set of relationships is explained by the linearity (transitivity) property. This possibility can be tested statistically by means of the linearity test (Appleby 1983; de Vries 1995). If this test yields significance, obtaining the rank order that is most consistent with a completely linear hierarchy is worth while and justified.

The other four methods, which are ordered into a hierarchy of models of stochastic (or probabilistic) transitivity, use the dominance encounter as the statistically independent unit of analysis. The least restrictive model is the weak stochastic transitivity (WST) model. This model holds if the binomial dominance probabilities $P_{ij}$ ($=s_{ij}/n_{ij}$) satisfy the following condition:

$$P_{ij} \geqslant 0.5 \text{ and } P_{jk} \geqslant 0.5 \text{ imply } P_{ik} \geqslant 0.5.$$

Thompson & Remage (1964) developed a method for deriving from the frequencies of wins and losses a ranking that is most likely to be the true one if one assumes that weak stochastic transitivity holds. The method involves a search for the ranking that maximizes the logarithm of the likelihood (less a constant)

$$\Sigma (s_{ij} \ln \hat{P}_{ij} + s_{ji} \ln \hat{P}_{ji}) \text{ (summation is over } i \text{ and } j; \\ i < j)$$

where $\hat{P}_{ij}$ is the maximum likelihood estimator of $P_{ij}$ under WST. Bossuyt (1990) showed that a slightly less restrictive property (stochastic acyclicity: $P_{ij} > 0.5$ and $P_{jk} > 0.5$ and ... and $P_{uv} > 0.5$ imply $P_{iv} \geqslant 0.5$) is already sufficient and necessary for a weak stochastic ranking to exist. He also presented a statistical test (which improves upon a testing procedure presented by Tversky in 1969) involving a Monte Carlo approach to test the hypothesis that the binomial probabilities $P_{ij}$ satisfy the stochastic acyclicity property for paired comparisons experiments with a fixed number of comparisons per pair of objects. McMahan & Morris (1984) translated the method of Thompson & Remage (1964) into dominance behavioural terms. An advantage of their method (which is also shared by the I & SI method, but not by the other, more restrictive stochastic

transitivity models) is that the obtained ranking is Hamiltonian.

McMahan & Morris' method appears to yield quite similar rankings as, but is not equivalent to, the often used ordering method described by Brown (1975): minimize the proportion of entries below the diagonal $\Sigma s_{ji}/N$. Also note that these two methods are identical to Slater's method when the matrix is binary or when only the binary information is used. Below I present an example matrix to compare these methods with the I & SI method.

The next model, strong stochastic transitivity (SST), requires that the dominance probabilities satisfy the following condition:

$$P_{ij} \geqslant 0.5 \text{ and } P_{jk} \geqslant 0.5 \text{ imply } P_{ik} \geqslant \max\{P_{ij}, P_{jk}\}.$$

This model has, as far as I know, not been applied in dominance hierarchy analyses, although it is probably the one that is assumed implicitly by many researchers of animal social behaviour. Iverson & Sade (1990) give an apt description of SST: 'SST captures the idea that the $P_{ij}$ increase with increasing separation between ranks $i$ and $j$, or that the tendency for one animal to dominate another grows with increasing separation in the hierarchy'. Bossuyt (1990) has also developed for this stochastic transitivity model a Monte Carlo based test procedure, as well as a method involving a branch and search algorithm for finding the optimal ranking under this model.

Crow's (1990) method was developed in the context of sports contests. He showed that under certain conditions such as: (1) the contestants have played a series of independent contests and (2) the probability of a player defeating a lower-ranked player in any contest is an increasing function of the difference in ranks, maximizing the likelihood is equivalent, to first-order approximation, to maximizing the net difference in ranks, NDR, defined as $\Sigma(s_{ij} - s_{ji})(j - i)$ $(i < j)$, where $s_{ij}$ is the number of contests won by player $i$ from player $j$. Note that this method reduces to minimizing the total strength of inconsistencies (without the requirement that I takes on its minimum value) if only the asymmetry information in the won and lost contests played between the members of a dyad is used.

Finally, the method that is based on the most restrictive model, but that also yields the most informative results, is the Bradley–Terry model,

which has been adapted for the rank analysis of dominance interactions by Boyd & Silk (1983). They presented a procedure for ranking the individuals through assigning cardinal dominance ranks $D_i$ to them. The Bradley–Terry model defines a scaling of the individual dominance indices $D_i$ in such a way that the binomial probabilities $P_{ij}$ depend logistically on the difference between the dominance indices $D_i$ and $D_j$. Besides the general requirements imposed by the paired comparison method, this method also requires that the wins–losses matrix satisfies certain conditions in order to be able to apply the method. For instance, the method often cannot be applied when the dominance matrix contains very few entries below the diagonal (Boyd & Silk, page 49).

Another set of methods is based on generating a non-parametric individual dominance index. There is the well-known index, number of individuals dominated, which requires that all dominance relationships are known. Alternatively, the proportion of individuals dominated could be calculated for each animal. This index has the drawback, however, that animals that avoid encounters with dominant ones obtain artificially high values. Clutton-Brock et al. (1979) devised an index of fighting success that takes the strength of the animals beaten into account as well as the strength of the animals lost to. For each animal $i$, one calculates the score $(B+\Sigma b+1)/(L+\Sigma l+1)$, where $B$ is the number of animals beaten by $i$ (irrespective of how often) and $\Sigma b$ the total number they beat excluding $i$; $L$ is the number of animals $i$ lost to (irrespective of how often) and $\Sigma l$ the total number to which they lost excluding $i$. A somewhat similar score was proposed by David (1987) for ranking players in an incomplete or unbalanced round-robin tournament with at most one match per pair. His score equally reflects the strength of players defeated by a player $i$ and the weakness of players by whom $i$ was defeated: $w+w^{(2)}-l-l^{(2)}$, where $w$ is $i$'s wins, $l$ is $i$'s losses, $w^{(2)}$ is the wins of players defeated by $i$, $l^{(2)}$ is the losses of players to whom $i$ lost. A tied match counts as half a win plus half a loss. The same formula can also be used in larger tournaments. The cells of the wins–losses matrix must then contain the $P_{ij}$ values; if $n_{ij}=0$ both $P_{ij}$ and $P_{ji}$ are zero. David (1988, page 108) noted that it must be realized, however, that this method cannot be entirely satisfactory when the $n_{ij}$ differ greatly. This might

be remedied by using, instead of the $P_{ij}$, a dyadic dominance index $d_{ij}$ (presented in Appendix 2) that is corrected for chance and for differences in the $n_{ij}$.

These individual score measures are certainly useful if the main interest is in the total winning or fighting success of each individual rather than in finding an ordering of the individuals that is most consistent with a linear dominance hierarchy. To obtain this last goal the prevailing criterion should be the actual dominance relationship between two individuals rather than each individual's overall fighting, winning or dominance success, which may result in rankings that are not Hamiltonian. This recommendation has also been put forward by Roberts (1990), who proposed for this purpose his 'flipping rule': flip two individuals adjacent in the current ranking if the lower individual dominates the one above him, thus securing a Hamiltonian ranking. Initially, a ranking had been obtained simply by ordering the individuals according to the number of individuals dominated. Digby & Kempton (1987) also suggested applying this flipping rule; but instead of starting from the initial ranking based on the number of dominated individuals, they determined an initial ranking of the individuals by means of a method developed by Gower (1977).

The I & SI method, which can start from any initial order, includes this flipping rule for adjacent individuals as part of a more general swapping rule. In fact, this general swapping rule can be seen as a rather straightforward generalization of Roberts' flipping rule to both adjacent and non-adjacent individuals.

## EXAMPLES

### An Illustrative Example

As a first example the matrix in Fig. 1 will be submitted to the linear ordering procedure. First, the strength and the significance of the linearity are determined by means of the linearity test described in de Vries (1995). The value of the linearity index $h'$ (which takes the number of unknown relationships into account) equals 0.64. A randomization test with 10 000 randomizations gives a right-tailed probability $P_r$ of 0.009, which indicates significant linearity. Next, the matrix is submitted to the linear ordering procedure which

Initial order of the individuals: avbhgwekcy
Number of inconsistencies = 4. Strength of the inconsistencies = 10

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Try: 0. Number of inconsistencies = 3. Strength of the inconsistencies = 12
ahbvgwekcy
Try: 1. Number of inconsistencies = 2. Strength of the inconsistencies = 8
avbgwhekcy
Try: 5. Number of inconsistencies = 2. Strength of the inconsistencies = 7
abvgwhekcy

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Final order of the individuals: abvgwhkecy
Inconsistencies (each with its strength):
  (w,a): 4   (h,v): 3
Number of inconsistencies: 2
Total strength of the inconsistencies: 7

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|  |  | Loser |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | a | b | v | g | w | h | k | e | c | y |
|  | a | * | 4 | 5 | 3 | 0 | 6 | 2 | 2 | 3 | 1 |
|  | b | 0 | * | 0 | 1 | 1 | 0 | 2 | 1 | 2 | 2 |
|  | v | 0 | 0 | * | 2 | 1 | 0 | 0 | 2 | 7 | 7 |
|  | g | 0 | 0 | 0 | * | 2 | 1 | 0 | 4 | 3 | 0 |
|  | w | 2 | 0 | 0 | 0 | * | 3 | 0 | 0 | 2 | 1 |
| Winner | h | 0 | 0 | 3 | 0 | 0 | * | 0 | 6 | 2 | 5 |
|  | k | 0 | 0 | 0 | 0 | 0 | 0 | * | 0 | 2 | 1 |
|  | e | 0 | 0 | 0 | 0 | 0 | 0 | 0 | * | 0 | 4 |
|  | c | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | * | 6 |
|  | y | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | * |

**Figure 2.** The result of the linear ordering procedure applied to the matrix in Fig. 1, and the reorganized matrix.

yields the following results (see Fig. 2). Note that in the final step the individuals e and k, which have an undecided dominance relationship, change position because $D_k - S_k > D_e - S_e$.

## Comparing the I & SI Method with Other Ranking Methods

To get insight into the effects of the different criteria used by the currently proposed method and other ranking methods, I compare the rankings obtained by applying these methods to some selected example matrices. Table II presents different rankings obtained by applying four ranking methods to the aggressive interaction matrix presented by McMahan & Morris (1984, their table I). McMahan & Morris' method of maximizing the log likelihood under weak stochastic transitivity (WST) yielded three rankings which are equally likely under the assumption of WST. Applying the I & SI method to this matrix yielded the second ranking as the optimal one. The first two rankings both have an I and SI of 2 and 6; for the third ranking, however, I is 2 and SI is 7. The second ranking is the optimal one, because in the final phase of the procedure h is ranked above g since $D_h - S_h > D_g - S_g$. Note that the linearity test gives a *P*-value of 0.061 and a *h'*-value of 0.52, so application of the I & SI method is only marginally justified.

**Table II.** Different optimal rankings obtained by applying four ranking procedures to the data in Table I of McMahan & Morris (1984)

| Rank | McMahan & Morris' method | | | Clutton-Brock's index§ | | Crow's method§ |
|---|---|---|---|---|---|---|
| | I&SI method | | | | | |
| 1 | X907=a | a | a | 33.00 | a | a |
| 2 | X915=b | b | b | 3.50 | c | e |
| 3 | X912=c | c | c | 3.10 | b | c |
| 4 | X910=d | d | d | 0.91 | g | b |
| 5 | X917=e | e | e | 0.90 | d | f |
| 6 | X898=f | f | f | 0.86 | e | g |
| 7 | X897=g | h | h | 0.82 | f | d |
| 8 | X911=h | g | i | 0.53 | h | h |
| 9 | X904=i | i | g | 0.20 | i | i |
| 10 | X902=j | j | j | 0.03 | j | j |
| I and SI* | 2; 6 | 2; 6 | 2; 7 | 4; 8 | | 8; 17 |
| LogLH† | −9.70 | −9.70 | −9.70 | −14.03 | | −13.34 |
| NDR‡ | 261 | 257 | 245 | 225 | | 298 |

*I is the number of inconsistencies; SI is the total strength of the inconsistencies.
†LogLH is the log likelihood under WST (McMahan & Morris 1984).
‡NDR is the net difference in ranks (Crow 1990).
§These rankings are not Hamiltonian.

I also applied the methods of Clutton-Brock et al. (1979) and Crow (1990) to this matrix. Inspection of the obtained rankings shows that they differ substantially from the rankings obtained with the I & SI method and the LogLH method (i.e. maximize the log likelihood under WST). Both are non-Hamiltonian; the I and SI values are larger than the minimum values, and the value of LogLH is smaller than the maximum value possible.

It appears that in many cases the rankings found by the LogLH method and the I & SI method do not differ very much. However, the more the frequencies of the encounters differ between the dyads, the more the optimal rankings yielded by these two methods may differ from each other. I illustrate this by an example. I slightly changed two values in the data matrix: the value of cell (g, e) was changed from 2 to 4, and cell (h, d) was changed from 1 to 10. The ranking obtained by the current method is the same as before, but McMahan & Morris' LogLH method now yields a different one (see Table III). This last method puts h above d, e and f (thereby introducing an extra inconsistency), because the 10 wins by h against d are weighted stronger than the four and three wins by e and f against h. In larger matrices with larger differences in frequencies, the optimal rankings obtained by these two methods can become rather different.

Comparison of the last two columns shows that maximizing the log likelihood under WST is not equivalent to the well-known method of minimizing the proportion of entries (reversals) below the diagonal (Brown 1975).

The next two examples illustrate the difference between the I & SI method and two other methods both of which provide an individual dominance score, the first one non-parametric, and the second, a parametric score. David (1987) illustrated his non-parametric scoring method with an example of an incomplete tournament shown here in Fig. 3 with the players ranked according to his score $w + w^{(2)} - l - l^{(2)}$. Note that this ranking is clearly non-Hamiltonian; it has four inconsistencies with a total strength of 7. Applying the I & SI method results in the ranking b, a, c, e, d, g, f which has only one inconsistency with a strength of 6.

| | | | Loser | | | | | David's |
|---|---|---|---|---|---|---|---|---|
| | | a | b | c | d | e | f | g | score |
| | a | * | 0 | 1 | 1/2 | 1 | | | 4.5 |
| | b | 1 | * | 1/2 | | 1 | 0 | | 3.0 |
| | c | 0 | 1/2 | * | | | 1 | 1 | 2.0 |
| Winner | d | 1/2 | | | * | 0 | 1 | 1 | 1.5 |
| | e | 0 | 0 | | 1 | * | | | 0.5 |
| | f | | 1 | 0 | 0 | | * | 0 | −5.5 |
| | g | | | 0 | 0 | 0 | 1 | * | −6.0 |

**Figure 3.** An incomplete tournament (from David 1987).

**Table III.** Different optimal rankings obtained by applying three ranking methods to the data in Table I of McMahan & Morris (1984) with cell (g,e) changed to 4 and cell (h,d) changed to 10

| Rank | The present method: minimize first I, and then SI | McMahan's method: maximize LogLH under WST | Brown's method: minimize PEBD* |
|---|---|---|---|
| 1 | a | a | a |
| 2 | b | b | b |
| 3 | c | c | c |
| 4 | d | h | h |
| 5 | e | d | d |
| 6 | f | e | g |
| 7 | h | f | e |
| 8 | g | g | f |
| 9 | i | i | i |
| 10 | j | j | j |
| I and SI | 2; 6 | 3; 7 | 3; 9 |
| LogLH | −17.33 | −15.25 | −15.67 |
| PEBD | 0.176 | 0.147 | 0.137 |

*PEBD is the proportion of entries below the diagonal.

|   | d | e | f | h | g |
|---|---|---|---|---|---|
| d | * | 1 | 1 | 0 | 1 |
| e | 0 | * | 7 | 4 | 1 |
| f | 0 | 0 | * | 3 | 2 |
| h | 10 | 0 | 0 | * | 0 |
| g | 0 | 4 | 0 | 0 | * |

|   | h | d | e | f | g |
|---|---|---|---|---|---|
| h | * | 10 | 0 | 0 | 0 |
| d | 0 | * | 1 | 1 | 1 |
| e | 4 | 0 | * | 7 | 1 |
| f | 3 | 0 | 0 | * | 2 |
| g | 0 | 0 | 4 | 0 | * |

|   | h | d | g | e | f |
|---|---|---|---|---|---|
| h | * | 10 | 0 | 0 | 0 |
| d | 0 | * | 1 | 1 | 1 |
| g | 0 | 0 | * | 4 | 0 |
| e | 4 | 0 | 1 | * | 7 |
| f | 3 | 0 | 2 | 0 | * |

Boyd & Silk (1983) illustrated their method of assigning cardinal dominance ranks with nine data sets of dominance interactions among five captive male cockroaches, *Nauphoeta cinerea*, reported by Bell & Gorton (1978). The third data set is shown here in Fig. 4 with the animals ranked according to their cardinal dominance rank. This ranking is not Hamiltonian; it has two inconsistencies each of strength 1. Applying the I & SI method yields the ranking A, D, E, B, C without inconsistencies which accords to the one presented by Bell & Gorton.

It should be noted that both matrices are not significantly linear according to the linearity test described in de Vries (1995) (Fig. 3: $h'=0.23$, $P_r=0.77$; Fig. 4: $h=h'=1$, $P_r=0.12$), so applying the I & SI method is not justified statistically.

**Two More Examples**

The next two examples illustrate the effectiveness of the I & SI method. The first involves a

|   |   | Loser |   |   |   |   |
|---|---|---|---|---|---|---|
|   |   | D | A | E | C | B | Cardinal rank |
| Winner | D | * | 9 | 12 | 6 | 27 | 1.0 |
|   | A | 10 | * | 9 | 12 | 12 | 1.2 |
|   | E | 2 | 5 | * | 0 | 2 | 2.1 |
|   | C | 3 | 3 | 0 | * | 2 | 2.5 |
|   | B | 2 | 3 | 0 | 4 | * | 3.0 |

**Figure 4.** Dominance interactions among cockroaches (from Bell & Gorton 1978).

matrix containing the dominance relationships among 32 chickens, *Gallus gallus domesticus* (Guhl 1953). Fig. 5a shows the original matrix as presented by Guhl, who ordered the birds according to the number of birds dominated. This ranking contains 45 inconsistencies with a total strength of 158. The value of the linearity index $h$ (or $h'$) equals 0.89 which has a right-tailed probability $P_r$ of 0.0001. When the flipping rule is applied to the adjacent pairs of individuals only, already a considerable improvement is obtained, giving an ordering with 30 inconsistencies and a

(a)

| | bB | 2Y | nG | GG | RY | G | R | RB | bb | n | nn | RG | BG | Rb | GR | BY | YR | Gn | B | Gb | nb | YG | bG | nB | BB | Yn | Y | Yb | bR | bY | nR | YY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| bB | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2Y | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1/2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| nG | – | 1 | * | – | 1 | 1 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| GG | – | – | 1 | * | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RY | – | – | – | 1 | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| R | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RB | – | – | – | – | – | – | – | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| bb | – | – | – | – | – | – | – | – | * | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| n | – | – | – | – | – | – | – | – | 1 | * | 1 | 1 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| nn | – | 1/2 | 1 | – | – | – | 1 | – | – | – | * | – | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RG | – | – | – | – | – | – | – | – | – | – | 1 | * | – | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| BG | – | – | – | – | – | – | – | – | – | – | 1 | 1 | * | 1 | 1 | – | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Rb | – | – | – | – | – | – | – | – | – | – | 1 | – | – | * | 1 | 1 | 1 | – | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| GR | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | * | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| BY | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | 1 | * | 1 | 1 | 1 | 1 | 1/2 | 1 | 1 | – | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 |
| YR | – | – | – | – | – | – | – | – | – | – | – | 1 | – | 1 | – | * | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Gn | – | – | – | – | – | – | – | – | – | – | 1 | 1 | 1 | – | 1 | * | 1 | * | 1 | 1 | – | – | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| B | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | – | 1 | – | – | * | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Gb | – | – | – | – | – | – | 1 | 1 | – | – | – | – | – | – | – | – | – | – | 1 | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 |
| nb | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | 1 | * | – | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| YG | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | 1/2 | – | 1 | – | – | 1 | * | – | 1 | – | – | 1 | 1 | 1 | 1 | 1 | 1 |
| bG | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | 1 | * | 1 | 1 | 1 | 1 | – | 1 | – | 1 |
| nB | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| BB | – | – | – | – | – | – | – | – | – | – | – | 1 | – | 1 | – | – | – | 1 | – | – | 1 | – | – | * | – | – | – | 1 | 1 | 1 | 1 |
| Yn | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | * | – | 1 | 1 | 1 | – | 1 |
| Y | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | 1 | * | 1 | – | 1 | 1 | 1 |
| Yb | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | * | 1 | – | 1 | 1 |
| bR | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | 1 | – | – | 1 | – | – | 1 | – | – | 1 | – | * | – | – | – | – |
| bY | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | 1 | * | 1 | – |
| nR | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | – | * | 1 |
| YY | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | 1 | – | * |

(b)

| | bB | nG | 2Y | RY | GG | G | R | RB | n | bb | RG | nn | BY | Gn | BG | Rb | YR | GR | Gb | nb | B | bG | YG | nB | Y | Yn | Yb | BB | bY | nR | YY | bR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| bB | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| nG | – | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2Y | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1/2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RY | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| GG | – | 1 | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| R | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RB | – | – | – | – | – | – | – | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| n | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| bb | – | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| RG | – | – | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | – | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| nn | – | 1 | 1/2 | – | – | – | 1 | – | – | – | 1 | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | – |
| BY | – | – | – | – | – | – | – | – | – | – | – | – | * | 1 | 1 | – | 1 | – | 1 | 1 | 1 | 1 | 1/2 | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | – |
| Gn | – | – | – | – | – | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | – | 1 | 1 | – | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 |
| BG | – | – | – | – | – | – | – | 1 | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Rb | – | – | – | – | – | – | – | – | – | – | 1 | 1 | – | – | – | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| YR | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| GR | – | – | – | – | – | – | – | 1 | – | 1 | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Gb | – | – | – | – | – | – | 1 | – | – | 1 | – | – | – | – | – | – | – | – | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | – |
| nb | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | * | 1 | 1 | – | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| B | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| bG | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | – |
| YG | – | – | – | – | – | – | – | – | – | – | – | 1/2 | 1 | – | – | – | – | 1 | – | – | 1 | – | * | 1 | 1 | – | 1 | – | 1 | 1 | 1 | 1 |
| nB | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Y | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | * | 1 | 1 | 1 | 1 | 1 | 1 | 1 | – |
| Yn | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | * | 1 | 1 | 1 | – | 1 | 1 | 1 |
| Yb | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | * | 1 | – | 1 | 1 | 1 |
| BB | – | – | – | – | – | – | – | – | – | 1 | 1 | – | – | – | – | – | – | 1 | – | – | – | * | 1 | 1 | 1 | 1 |
| bY | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | * | 1 | 1 |
| nR | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | – | 1 | – | * | 1 | 1 |
| YY | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | 1 | – | * | 1 |
| bR | – | – | – | – | – | – | – | – | – | – | 1 | – | – | – | – | 1 | – | – | 1 | – | – | 1 | – | – | – | – | – | – | – | – | – | * |

**Figure 5.** (a) Matrix showing dominance relationships among 32 chickens, with the individuals ordered according to the number of birds dominated. This matrix contains 45 inconsistencies. (b) The same matrix reorganized by means of the I & SI linear ordering procedure. This matrix contains only 27 inconsistencies.

total strength of the inconsistencies of 168. After submitting the matrix to the I & SI procedure a ranking is obtained that contains only 27 inconsistencies with a total strength of 181 (Fig. 5b). This example shows that the lower number of inconsistencies is 'bought' at the cost of making some other inconsistencies stronger, resulting in a greater total strength. For this dominance matrix a ranking exists (bB, 2Y, nG, RY, GG, G, R, RB, bb, n, nn, RG, BG, Rb, GR, Gn, YR, BY, Gb, B, nb, bG, YG, nB, BB, Y, Yn Yb, bR, bY, nR, YY) with a total strength of inconsistencies of 156, which appears to be the minimum. This ranking which has 42 inconsistencies is not Hamiltonian, however.

It is worth noting that applying McMahan & Morris' method or Brown's method to Guhl's matrix both yield rankings that have the minimum number of 27 inconsistencies, (since for binary matrices these two methods are equivalent to Slater's method), but that these methods do not further discriminate between these rankings. The I & SI method on the other hand, tries to improve on this result by using the extra criterion of minimizing SI, subject to the condition that I does not increase.

Lott (1979) applied two techniques to rank order a dominance matrix containing aggressive interaction outcomes in a group of 26 mature male American bison, *Bison bison*. The first technique, described by Brown (1975) and dating back to Schein & Fohrman (1955), involves minimization of the proportion of entries below the diagonal, PEBD, that is: $\Sigma s_{ji}/N$ ($i<j$; $N$ is the matrix total), or, equivalently, $\Sigma s_{ji}$, because $N$, being a constant, is irrelevant. In the second technique the entries below the diagonal are multiplied by their distance from the diagonal. So, here the following measure is minimized: $\Sigma s_{ji}(j-i)$. Note that this measure shows some resemblance to Crow's criterion, who proposed to maximize $\Sigma(s_{ij}-s_{ji})(j-i)$, which is of course equivalent to minimizing $\Sigma(s_{ij}-s_{ji})(j-i)$. (Everywhere summation is over $i$ and $j$ with $i<j$.) The order obtained by Lott with the first technique (his table 1) has 29 inconsistencies with a total strength of 252; the PEBD equals 0.22. The order obtained by the second technique (his table 2) has 29 inconsistencies with a total strength of 261; the PEBD also equals 0.22.

The linearity test applied to Lott's data gives a value of 0.32 for the linearity index $h'$ with a

$P_r$-value of 0.0004. Note that the linearity in this set of relationships is rather low ($h'=0.32$); yet, the test indicates that the null hypothesis of randomly distributed dominance relationships can be rejected at a $P$-level of 0.0004 in favour of the alternative hypothesis that the hierarchy is linear. It might be questioned whether in a case such as this (with such a low linearity index) it is justified to search for an optimal near-linear ranking. The situation here is quite analogous to a linear regression situation, with a low but highly significant correlation. Just as it is justified to fit a straight line through the data points, thereby estimating the values of the dependent variable $y$ under the assumption that $y$ is linearly dependent on the $x$ variable, it is also justified to search for the rank order that is most consistent with a linear hierarchy.

Application of the I & SI procedure to Lott's data yields a ranking that contains only 23 inconsistencies with a total strength of 217; the PEBD of this ranking is 0.23. So, this ranking is a better estimate of the linear hierarchy, according to the combined I and SI criterion, than the two rankings obtained by Lott. This ranking is as follows: 26, 25, 24, 23, 01, 02, 03, 12, 04, 05, 06, 10, 07, 18, 08, 13, 09, 20, 14, 17, 21, 19, 11, 15, 16, 22.

## DISCUSSION

### Defining Dominance

I started this paper with a definition of dominance that points out that dominance is a relational feature. It is an attribute of the pattern of agonistic interactions between two individuals. So, dominance is a measure determined at the level of the dyad. In this paper I argue that in the analysis of dominance and rank of animals living in a social group one should take the dyad as the observational unit of analysis rather than the dominance encounter. Probabilistic models that are considered in the analysis of paired comparisons experiments are based on the statistical independence of each pair comparison, which is the unit of analysis in this type of experiment.

In this paper I used the asymmetry information of the wins and losses of the dyad members to determine the dominance relationship. As soon as A has won more often from B than B from A, A is called dominant to B. A stronger criterion could

be that the number of wins by A should differ significantly from the wins by B before A is to be called dominant to B. A drawback of this approach is that a dyad should have at least five encounters that are all won by the same animal to obtain a decided dominance relationship. It is clear that many of the reported dominance matrices would then contain a disproportionally great number of undecided relationships making it almost impossible to do a rank analysis. Moreover, in many cases, observers often feel justified to call A dominant to B if A has won only three out of three encounters (and, sometimes, depending on previous knowledge, two out of two or one out of one is already sufficient to conclude safely that A is dominant to B). A somewhat related point is that not all types of agonistic behaviour patterns are equally suitable for determining the dominance relationship. As a rule, submissive behaviour, by which one animal recognizes another as dominant, is a clearer indication of the dominance relationship than aggressive behaviour. Therefore, it is necessary to investigate the suitability of different types of behaviour patterns as dominance parameters. See for example van Dierendonck et al. (1995), who presented an extensive analysis of dominance and its behavioural parameters in a herd of Icelandic horses, *Equus caballus*.

Instead of a binary dominance measure, one could devise a dyadic dominance index (ranging between 0 and 1) based on the numbers of wins and losses. A straightforward index is $P_{ij} = s_{ij}/n_{ij}$. In Appendix 2 I present another normalized index, which has the advantage that it controls for chance wins or losses (supposing an equal chance to win or lose), and also takes the number of encounters into account. In this respect, it resembles an index like Cohen's kappa (Cohen 1960). This dyadic dominance index could, for instance, be used in the formula of David's score.

### Remarks about the Method

In applying the I & SI method for finding an order that is most consistent with a linear hierarchy the following points should be noted. First, searching for an optimal (near-)linear rank-order is justified only if the linearity, as measured by the index *h* (Landau 1951) or, if unknown

relationships are present, the index *h′* (de Vries 1995), is statistically significant. Although, of course, the ordering procedure can also be applied if there is non-significant linearity, and could in such cases still give a useful insight into the dominance structure in the group, the linear ranking thus obtained under-represents the information present in the full set of dominance relationships.

Second, if there is incomplete (but significant) linearity there may be more than one optimal (or near-optimal) solution possible. Deciding between these solutions in favour of one particular optimal solution is somewhat arbitrary.

Third, the ordering procedure does not provide an explicit dominance score for each individual. However, an ordinal rank number can be assigned to each individual on the basis of the ordering obtained. Subsequently, the relationship between this dominance rank and other individual-level variables can then be investigated. It should be noted, however, that assigning rank numbers requires inspection of the inconsistencies and the tied or unknown relationships around the diagonal of the matrix. It is not possible to give clearcut rules in this respect. For instance, in the case of a genuine circular triad it is best to assign the average rank number to each of the three individuals. But if there is a higher order circular polyad, such a rule cannot be given. As an alternative to deriving individual dominance ranks from dyadic dominance relationships and then investigating the relation between rank and other individual-level variables, one could also investigate the relation between the dominance relationships and other (dyad-level) variables straight away by means of a 'dyad-wise' matrix correlation method. (Individual-level variables, such as age, can also be converted into dyadic ones by taking for each pair of individuals their difference value.) Starting from a generalized matrix correlation coefficient

$$\Gamma = \frac{\Sigma \gamma(X_{ij},X_{kl})\gamma(Y_{ij},Y_{kl})}{(\Sigma \gamma(X_{ij},X_{kl})^2 \Sigma \gamma(Y_{ij},Y_{kl})^2)^{1/2}},$$

(for rows *i, k* and columns *j, l*) de Vries (1993) defined a row-wise matrix correlation by restricting the summation to all those pairs of cells that have a row individual in common, that is, those pairs that belong to the same row:

$$\Gamma_{\text{rw}} = \frac{\Sigma\gamma(X_{ij},X_{ik})\gamma(Y_{ij},Y_{ik})}{(\Sigma\gamma(X_{ij},X_{ik})^2\Sigma\gamma(Y_{ij},Y_{ik})^2)^{1/2}},$$

for rows *i* and columns *j, k*. In a similar fashion a dyad-wise matrix correlation coefficient can be defined by restricting the summation to those pairs that have a row individual *and* a column individual in common, that is, those pairs of cells that belong to the same dyad:

$$\Gamma_{\text{dw}} = \frac{\Sigma\gamma(X_{ij},X_{ji})\gamma(Y_{ij},Y_{ji})}{(\Sigma\gamma(X_{ij},X_{ji})^2\Sigma\gamma(Y_{ij},Y_{ji})^2)^{1/2}},$$

for rows and columns *i, j*. If $\gamma(X_{ij},X_{ji})=X_{ij}-X_{ji}$ a Pearson product–moment correlation coefficient results. Before computing the Pearson form of a dyad-wise matrix correlation it is appropriate to normalize the dyadic frequency counts first by transforming these into dyadic dominance index values. When $\gamma(X_{ij}X_{ji})=\text{sign}(X_{ij}-X_{ji})$, we have a Kendall's $\tau_b$-type correlation coefficient. When the denominator is replaced by $n(n-1)/2$, the number of dyads, a Kendall's $\tau_a$-type correlation coefficient is obtained, which is more appropriate if there are ties or zeros (in the dominance relationships and/or in the other dyadic variables) and one is interested in the question: to what extent do these other dyadic variables agree with a (supposedly or actually existing) linear rank order (cf. Kendall 1962, chapter 3).

A decisive advantage of the I & SI procedure is its general applicability. It does not make any assumptions about the form of the probabilities of the winning/losing of dominance interactions, and it allows for the presence of unknown and tied dominance relationships. The only assumption is the existence of a linear or near-linear hierarchy, which can be tested for by means of the linearity test. So the procedure does not ask too much of the data, a point also stressed by Roberts (1990). In many cases the collected socio-ethological data do not justify the adoption of more strict assumptions and therefore preclude the application of other methods. Obviously, if the rather severe conditions necessary for the application of Boyd & Silk's (1983) method for assigning cardinal dominance ranks can be assumed to hold, their method may be preferred, because with their method more detailed information can be extracted from the dominance interaction matrix.

Another feature of the method is that the final ranking obtained is always Hamiltonian. This is appropriate for animals living in a social group, where the interest is in obtaining a ranking for which the dyadic dominance relationships are most consistent with a linear hierarchy rather than in a ranking that is based on each individual's total dominance (or winning or fighting) success.

Finally, a practical advantage of the method is that it can be used for relatively large matrices of up to 80 individuals.

## APPENDIX 1

### The Iterative Algorithm of the Main Phase of the Linear Ordering Procedure

For the matrix M with the current ordering:
Compute number of inconsistencies I, and set Imin:=I;
Compute total strength of the inconsistencies SI, and set SImin:=SI;
*t*:=0; {The counter *t* counts the number of tries}
REPEAT
  REPEAT
    StopIteration2:=TRUE;
    FOR each dyad (*i, j*) {*j>i*} DO:
      IF *j* dominates *i* {that is: the dyad *i, j* is not consistent with a linear hierarchy} THEN
        NetIncs:=0;
        FOR *k*:=*i* TO *j*−1 DO
          NetIncs:=
          NetIncs+Sgn(M[*j, k*]−M[*k, j*]);
        END;
        IF NetIncs>0 THEN
          Swap *i* and *j*;
          StopIteration2:=FALSE
        END;
      END;
    END;
  UNTIL StopIteration2=TRUE;
Compute number of inconsistencies I;
Compute total strength of the inconsistencies SI;
IF (I<I min) OR (I=I min AND SI<SI min) THEN
  Best-Order-Found-Thus-Far:=Current Order;
  Imin:=I; SImin:=SI;
  StopIteration1:=FALSE;
ELSE

```
t: = t+1;
IF (SImin>0) AND (t<nTries) THEN
   FOR each individual j DO
      IF j dominates some individual higher in
      the current ranking THEN
         Choose randomly an individual i
         above j in the current ranking;
         Swap j and i;
         StopIteration1:=FALSE;
      END;
   ELSE
      {Optimal or near-optimal linear ranking is
      found!}
      StopIteration1:=TRUE;
   END;
END;
UNTIL StopIteration1=TRUE;
```

Explanatory notes:

The algorithm consists of two nested iterative loops. The outer loop is called Iteration1, the inner loop is called Iteration2.

Sgn is the sign function: $\mathrm{Sgn}(x)=1$ if $x>0$; $\mathrm{Sgn}(x)=0$ if $x=0$; $\mathrm{Sgn}(x)=-1$ if $x<0$.

Swap $i$ and $j$ means: In the matrix M row $i$ and row $j$, as well as column $i$ and column $j$, are exchanged.

$n$Tries is the number of tries, that is, the number of partly random orders to be generated from the ranking that is initially found after a first run through the inner iterative loop. (By default $n$Tries is set to 100).

## APPENDIX 2

### A Normalized Dyadic Dominance Index Corrected for Chance

The degree of dominance of individual $i$ over individual $j$ can be defined as the proportion of wins by $i$ relative to the total number of dominance interactions between $i$ and $j$. Thus the degree of dominance is $P_{ij}=s_{ij}/n_{ij}$. Next, define the degree of dominance of $i$ over $j$ corrected for chance under the assumption of equal wins and losses as:

$d_{ij}$ = observed proportion
   − {(observed proportion
       − expected proportion)
   × Prob[observed proportion
           − expected proportion]}; that is,

$d_{ij}=P_{ij}-(P_{ij}-0.5)\times\mathrm{Prob}[P_{ij}-0.5]$,

where $\mathrm{Prob}[P_{ij}-0.5]$ is the probability that this deviation of the observed proportion from the expected proportion will occur by chance. For example, if $s_{ij}=4$ and $s_{ji}=1$, then

$$P_{ij}=4/5=0.8 \text{ and } P_{ji}=1/5=0.2,$$

while

$$d_{ij}=4/5-\{4/5-0.5)\times\mathrm{Prob}[s_{ij}=4|n_{ij}=5]\}$$
$$=0.8-0.3\times0.1562=0.753$$

and

$$d_{ji}=1/5-\{1/5-0.5)\times\mathrm{Prob}[s_{ji}=1|n_{ij}=5]\}$$
$$=0.2+0.3\times0.1562=0.247.$$

It can easily be seen that $d_{ij}=1-d_{ji}$, and that for $s_{ij}=s_{ji}$ (not zero) the value of $d_{ij}$ equals 0.5. If $n_{ij}=0$, $d_{ij}$ is undefined; when using the dyadic dominance index $d_{ij}$ instead of $P_{ij}$ in the calculation of David's scoring formula, both $d_{ij}$ and $d_{ji}$ should be set to zero. The value of $d_{ij}$ approaches 1.0 if the difference between $s_{ij}$ and $s_{ji}$ approaches infinity. For example, $s_{ij}=1$ and $s_{ji}=0$ gives a $d_{ij}$ of 0.75, while $s_{ij}=5$ and $s_{ji}=0$ gives a $d_{ij}$ of 0.98.

The use of this index is of course not restricted to dominance interactions. A completely similar dyadic index can be defined for approaches and leaves by two individuals to and from each other, and may be a useful alternative to the approach and leave indices (and thus also the index expressing the difference between these two) described by Hinde & Atkinson (1970).

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