## Math 456/556: Networks and Combinatorics

### Solutions to HW #8

**4.** A transitive tournament on three vertices has a closed Eulerian trail, but not a directed one.

**22.** Here is a core allocation:

$$\rho(1) = 3, \quad \rho(2) = 4, \quad \rho(3) = 5, \quad \rho(4) = 2, \quad \rho(5) = 1, \quad \rho(6) = 6, \quad \rho(7) = 7.$$

Note that this is core even though $t_6$ got stuck with his least favorite item!

**24.** In the first network, there is a flow of value 9 (ignore the edge in the middle, flow with weights 4 and 5 out of the source, and flow to capacity on all other edges) and a cut of capacity 9 (the two edges going into the target).

In the second network, there is a flow of value 12 (flow with weight 1 on the highest edge with capacity 2, and flow to capacity on all other edges) and a cut of capacity 12 (the three edges going into the target).

In the third network, there is a cut of capacity 7 (the left-most and right-most edge of the large square and the bottom edge of the small square) and a flow of value 7 that I won't attempt to describe in words.

**25.** This problem essentially asks you to redo the three examples in problem 24, but with all capacities equal to 1. This makes all three examples much easier. The numbers that come out are 2, 3, and 3.

**26.** See hint.

**29.** We did the first example in class, finding a matching of size $7 = |X|$ (therefore $X$ is an optimal cover). In the second example, there is a cover of size 6 consisting of the first two and last three elements of $X$ along with the third element of $Y$. It is easy to draw a matching of size 6, as well.