Math 556 Homework 1

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1.1 How many integers greater than 76,000 have distinct digits, with the digits 1, 2, and 3 not occurring?

case 1: $76,001 - 79,999$.

We have 1 choice for the first number (7), and 3 for the second (6, 8, 9). The rest can be selected from the remaining numbers (4, 5, 6, 8, 9).

$$(1) \cdot (3) \cdot (4 \cdot 3 \cdot 2) = 72$$

case 2: $80,000 - 99,999$

We have 2 choices for the first number (8,9). The rest can be selected from the remaining numbers (8,9,4,5,6,7).

$$(2) \cdot (5 \cdot 4 \cdot 3 \cdot 2) = 240$$

case 3: $100,000 - 999,999$

The first number can be anything except 0, so we have 6 choices (4, 5, 6, 7, 8, 9). The rest can be selected from the remaining numbers (0, 4, 5, 6, 7, 8, 9).

$$(6) \cdot (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2) = 4320$$

case 4: $1,000,000 - 9,999,999$

The first number can be anything except 0, so we have 6 choices (4, 5, 6, 7, 8, 9). The rest can be selected from the remaining numbers (0, 4, 5, 6, 7, 8, 9).

$$(6) \cdot (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 4320$$

case 5: $10,000,000 - \cdots$

Because we are only allowed to use 7 digits (0, 4, 5, 6, 7, 8, 9), we will not have any integers with 8 or more digits.

So by the Addition Principle, in total we have 8952 integers.

$$72 + 240 + 4320 + 4320 = 8952$$

1.2 Ten dishes are offered at the cafeteria, four of which have meat, and the remaining six of which are vegetables. You are allowed to choose any four dishes, and at least one of them has to be a vegetable. Assume the choices are distinct

a) How many ways can you do this?

   We have 4 choices for the vegetable. For the remaining 3 dishes, we can choose from any of the 9 remaining items.

$$(4) \cdot (9 \cdot 8 \cdot 7) = 2016$$

b) Suppose that you really hate the combination of meatloaf and okra (even though you are happy to eat either one individually). How many ways can you make your choice without taking both meatloaf and okra?

   case 1: No meatloaf and no okra

   We have 3 choices for the vegetable. For the remaining 3 dishes, we can choose from any of the items except meatloaf, okra, and the vegetbale we chose..

$$(3) \cdot (7 \cdot 6 \cdot 5) = 630$$

   case 2: Meatloaf and no okra

   We have 3 choices for the vegetable. We have meatloaf. For the remaining 2 dishes, we can choose from any of the items except okra, meatloaf, and the vegetable we've already chosen.

$$(3) \cdot (1 \cdot 7 \cdot 6) = 126$$

   case 3: No meatloaf and okra.

   We choose okra, and thus have a vegetable. For the remaining 3 dishes, we can choose anything except meatloaf and okra.

$$(1) \cdot (8 \cdot 7 \cdot 6) = 336$$

   So by the Addition Principle, in total we have 1092 ways to eat lunch.

$$630 + 126 + 336 = 1092$$