Tile or Stare? Cadence and Sky Monitoring Observing Strategies that Maximize the Number of Discovered Transients

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ABSTRACT

To maximize the number of transients discovered on the sky, should sky-monitoring projects stare at one location or continually jump from location to location, tiling the sky? If tiling is preferred, what cadence maximizes the discovery rate? As sky monitoring is a growing part of astronomical observing, utilized to find such phenomena as supernovae, microlensing, and planet transits, well thought out answers to these questions are increasingly important. Answers are sky, source, and telescope dependent and should include information about the source luminosity distribution near the observation limit, the duration of variability, the nature of the dominant noise, and the magnitude of down and slew times. Usually, a critical slope of the effective cumulative transient apparent luminosity distribution (Log N - Log S) at the limiting magnitude will define when "tile" or "stare" is superior. For shallower slopes, when "tile" is superior, optimal cadences and pointing algorithms are discussed. For transients discovered on a single exposure or time-contiguous series of exposures, when down and slew times are small and the character of the noise is unchanged, the most productive cadence for isotropic power-law luminosity distributions is the duration of the transient – faster cadences waste time re-discovering known transients, while slower cadences neglect transients occurring in other fields. A "cadence creep" strategy might find an optimal discovery cadence experimentally when one is not uniquely predetermined theoretically. Guest investigator programs might diversify previously dedicated sky monitoring telescopes by implementing bandpasses and cadences chosen to optimize the discovery of different types of transients. Example analyses are given for SuperMACHO, LSST, and GLAST.

Subject headings: techniques: photometry – telescopes – surveys

1. Introduction

Humanity has monitored the sky by eye at least as long as history has been recorded. Ancient records include, for example, bright supernovae and bright comets. The idea of

automated machine monitoring of the nighttime sky can be traced to several independent origins. Significant early efforts occurred in the X-Ray and gamma ray bands, including the Vela satellites that discovered gamma-ray bursts, first reported in 1972 (Klebesadel, Strong, and Olson 1972). Particularly noteworthy was the Burst and Transient Source Experiment (BATSE) deployed on board the Compton Gamma-Ray Observatory from 1991-2002 that kept a continuous monitor of the entire sky in the gamma-ray band, discovering over 2000 gamma-ray bursts and the phenomena now known as Terrestrial Gamma Flashes (TGFs). A continuously changing armada of satellites and instruments continues to monitor the entire sky in the gamma-ray band (see, for example, Cline et al. 1999).

The idea of continuous machine monitoring in *optical* of large portions of the nighttime sky can also be traced to several independent origins. Paczynski (1996) discussed possible scientific returns from monitoring the entire optical sky to detect different forms of variability. Coincidentally, the GROSCE (Akerlof et al. 1993) project started automated epochal monitoring of a large fraction of the sky in September 1996, continuing on as the LOTIS (Park et al. 1997, Williams et al. 1997) and Super-LOTIS projects, which keep an archive. Nemiroff and Rafert (1999) discussed practical limitations of continuously monitoring and recording the entire sky. The value of such a record would be the ability to discover transience at a later time, a possible advantage given large amounts of storage space and limited amounts of real-time computing power. In Nemiroff and Rafert (1999), distinctions were delineated between projects that continuously record the entire sky and epochal recording which involve observations that return to any one sky location only after a given epoch.

Other notable projects that have monitored pieces of the optical sky include MACHO (Alcock et al. 1993), OGLE (Udalski et al. 1992), EROS (Aubourg et al. 1993), AGAPE (Ansari et al. 1997), TASS (Richmond et al. 1998), ASAS (Pojmanski 1997, 1998), Stardial (McCullough and Thakkar 1997), and ROTSE (Marshall et al. 1997). In the infrared, a night sky monitor sensitive to (almost exclusively) cirrus clouds has been deployed to the Apache Point Observatory (Hogg et al. 2001). Although this instrument returns images in near real time, almost no stars are visible.

In the past few years, the number of sky monitoring projects has blossomed. Reasons for the increase in sky-monitoring popularity likely include dramatic increases in digital storage, transfer, and analysis capabilities, while the price for CCD cameras has continued to drop. Reasons for sky monitoring include discovering distant supernovae, eclipsing binary stars, planetary occultations, Earth-crossing asteroids, distant comets, meteors, and microlensing. A project list is maintained on a web page by Paczynski (http://www.astro.princeton.edu/faculty/bp.html). An abridged version is given below as Table 1, edited to include three other programs deemed relevant. These projects are best known by, and listed by, their acronym in column 1. If

a Principal Investigator could be identified, this person is listed in column 2. Typically, these monitoring projects do not have a paper, preprint, or even abstract published about their capabilities. Significant information can be found from each of the project's web pages, however, and so this is listed in column 3 as it appeared in November 2002.

Even more ambitious sky monitoring projects are being planned for future years. For brevity, only three example projects toward the high end of the cost spectrum are mentioned: LSST, Pan-STARRS, and GLAST, as listed below in Table 2.

The need for efficient sky monitoring pointing algorithms is therefore becoming increasingly important. Sometimes the same sky monitor will use different pointing algorithms, exercising both a "tile" and "stare" mode (e.g. ROTSE: Kehoe et al. 2002). Rarely, however, does a monitoring survey give detailed analysis explaining their chosen cadence or time allocation algorithm. (The term "cadence" here will be taken to mean the average frequency of return to image the same field.) This paper is therefore an attempt to begin a discussion of an attribute common to many sky-monitoring surveys – a desire to maximize the number of transients discovered. In Section 2 some background will be given discussing common to many sky-monitoring telescopes. Section 3 will discuss maximizing quiescents while Section 4 will discuss some general principles of monitoring for transients including the case of transients where the luminosity distribution is described by a power law near and well below the survey limiting magnitude. Section 6 will give some concluding discussion.

2. Background

A telescope will detect a source only if its signal peaks above the noise. For a source of apparent luminosity l (here "apparent luminosity" is used synonymously with "flux"), the signal-to-noise ratio S/N during a single observation can be described by

$$S/N \propto \frac{lt_e}{\sqrt{lt_e + bt_e + c + dl^2 t_e}} \qquad (1)$$

where t_e is the duration of the exposure, b is the equivalent apparent luminosity of the background skyglow that accumulates with exposure time (for a good discussion of detection rates in the face of backgrounds, see, for example, Nemiroff and Rafert 1999), c is a constant background term not affected by exposure time, for instance read-noise, and d is a site, time, telescope, and sky position dependent amplitude for scintillation noise (see, for example, Young 1967 and Dravins, Lindegren, Mezey, and Young 1998).

Given a fixed S/N threshold, b, c, and d at the source detection limit, equation (1) can be inverted to solve for l_{dim} as a function of t_e. The type of noise that dominates observations

Table 1: Current Sky Monitoring Projects

Project	PI	Web Page
CONCAM	Nemiroff, R. J.	http://concam.net
KAIT	Filippenko, A.	http://astron.berkeley.edu/ bait/kait.html
LINEAR	LINEAR team	http://www.ll.mit.edu/LINEAR/
LONEOS	Bowell, T.	http://www.lowell.edu/users/elgb/loneos/
MEGA	Crotts, A.	http://www.astro.columbia.edu/ arlin/MEGA/
NEAT	Helin, E.	http://neat.jpl.nasa.gov/
RAPTOR	Vestrand, W. T.	http://www.raptor.lanl.gov/
Spacewatch	McMillan, R. S.	http://spacewatch.lpl.arizona.edu/
STARE	Brown, T. M.	http://www.hao.ucar.edu/public/research/stare/stare.html
SuperMACHO	Stubbs, C.	http://www.ctio.noao.edu/ macho/
TAOS	Alcock, C.	http://taos.asiaa.sinica.edu.tw/index.html
YSTAR	Byun, Y.-I.	http://csaweb.yonsei.ac.kr/ byun/Ystar/

Table 2: Example Future Sky Monitoring Projects

Project	PI	Web Page
LSST	Tyson, A.	http://www.lsst.org/
Pan-STARRS	Kaiser, N.	http://www.ifa.hawaii.edu/pan-starrs/
GLAST	Michelson, P. F.	http://www-glast.stanford.edu/

divides important regimes in this function. Although the relation is expressible analytically, it will be convenient to write it as a single power law such that

$$l_{dim} \propto t_e^{\beta}. \tag{2}$$

In a common case where c and d are small, β can be found instantaneously at any l_{dim} such that

$$\beta = -\frac{l_{dim} + b}{l_{dim} + 2b}. \tag{3}$$

Now if the background skyglow level b is small compared to l_{dim}, β tends toward -1 so that $l_{dim} \propto t_e^{-1}$. The other extreme case is when background skyglow b is high compared to l_{dim}, so that the b term dominates, β tends toward $-1/2$, and so $l_{dim} \propto t_e^{-1/2}$.

A frequent assumption used in these analyses will be that that the effective cumulative apparent luminosity distribution of sources ("Log N - Log S", hereafter just referred to as "brightness distribution") is a power law such that the number of interesting objects accumulated during a single exposure of duration t_e would be simply $N \propto l_{dim}^{\alpha}$. Non-power law brightness distributions can frequently be approximated by a power law at (and below) the apparent luminosity cut-off l_{dim}, an approximation that makes the following discussion particularly relevant to realistic programs.

At the limit of observation, sources may be so numerous that their point spread functions begin to significantly overlap. When this happens, source confusion will create a practical limit on the faintest source detectable. In practice, source confusion can be incorporated into the above formalism by allowing it to change the brightness distribution $N(l)$ for the given object type, telescope, and sky survey. In fact, since $N(l)$ is an *effective* distribution, a host of practical limitations can be incorporated into it.

3.   Counting Quiescents

Although this paper is primarily interested in transients, it is relevant and instructive to analyze the simpler case of quiescent sources first. A canonical telescope and camera is assumed, with a given field of view of solid angle Ω_{field}, and limiting apparent luminosity l_{dim} observable over the telescopes bandpass. Following Peebles (1993) and Hogg (2000), the number of observable quiescent sources that would be visible in a single field to limiting apparent luminosity l_{dim}, found during an exposure of effective duration t_e would be

$$N_{quiescent}(l > l_{dim}) = \int_{z=0}^{\infty} \int_{L=L_{min}(l_{dim},z)}^{\infty} \frac{\Phi(L,z)K(L,z)D_H(1+z)^2 D_A^2}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}} \, \Omega_{field} \, dL \, dz, \tag{4}$$

where z is redshift, L is absolute luminosity (also sometimes known as intrinsic luminosity or just "luminosity", although it may be normalized), Φ is the luminosity function of candidate quiescent sources, K is the k- correction for the telescope bandpass, $D_H = c/H_o$, c is the speed of light in vacuum, H_o is the Hubble parameter, D_A is angular diameter distance, Ω_M is the energy density of the universe in matter in units of the critical energy density, Ω_k is the energy density due to the curvature of space, and Ω_Λ is the energy density due to the cosmological constant. The integrand L_{min} can be computed from l_{dim} and z from $L_{min} = l_{dim} D_L^2 / K(z)$ where D_L represents luminosity distance.

A perhaps familiar case is that of detecting quiescents in fields of such low background that Poisson noise dominates the counting statistics. This will be referred to as the "low background" case. Given a canonical telescope and set amount of time t_c for an observing campaign, should this time be divided tiling all available fields, or spent staring at a limited number of fields? It will be assumed here that data taken from the same parts of the sky can be efficiently co-added.

To make matters simple, it will be assumed that the brightness distribution can be simplified from equation (4) to $N_{quiescent} \propto l_{dim}^\alpha$. From equation (2) $l_{dim} \propto t_e^\beta$ so that equation (4) simplifies to

$$N_{quiescent} \propto t_e^{\alpha\beta}. \tag{5}$$

Studying this simple equation will give significant insight into the "tile" or "stare" question. To calibrate intuition, let's consider the case of $\alpha\beta = 1$, so that the number of detected sources is just linear with the exposure time. An example case is when the background is low, $\beta \sim -1$, and so $\alpha \sim -1$. Assuming it takes little time to slew to a new field, it then does not matter if one stares at the same location, or tiles the sky: the same number of long duration (and hence quiescent) transients will be detected. Here the answer to "tile or stare" is a formal tie.

If the power-law index $\alpha\beta$ is less than unity, however, equation (5) indicates that "tile" will detect more sources per unit exposure time than "stare". This is because, when staring at a single field, new sources are appearing over the limiting brightness horizon at an increasingly slower rate. Higher rates are found by starting over on a new field. Given a total observing campaign time, the most sources will be found by dividing the time equally between all the available fields.

Similarly, if the power-law index $\alpha\beta$ of equation (5) is greater than unity, "stare" will detect more sources per unit exposure time than "tile". This is because, when staring at a single field, new sources are pouring over the limiting brightness horizon at an increasingly fast rate. Lower rates would be found by starting over on a new field. Therefore, in general, given a total observing campaign time, the most sources will be found by staring at one field.

Stated differently, two identical observations of statistically identical fields should yield twice the number of sources than in a single field. For a steep brightness distribution where $\alpha\beta > 1$, however, spending twice the time on the first field will detect *more* than twice the number of sources. So "stare" is preferred to maximize sources observed or monitored.

Can $\alpha > 0$? Since the brightness distribution $N_{quiescent}$ is a cumulative distribution, it cannot decrease, an equivalent statement to not having $\alpha > 0$ over any part of its length.

For non-power law brightness distributions, the situation is a more complex. Cases likely include where the luminosity function is not a power law (α changes) and cases where noise terms are not constant over the course of observations (β changes). For $\alpha\beta$ decreasing monotonically with increasing t_e in identical fields, one should always observe in the field where the rate of source accumulation is instantaneously highest. Therefore, one should stare at one field only until the rate of source accumulation falls below that in a fresh field. This is certainly true when $\alpha\beta$ falls through unity, although the transition will likely occur in many cases when $\alpha\beta$ is still in excess of unity.

Note that the tile/stare divide for brightness distributions well-characterized by a power-law α is $\alpha = 1/\beta$ which depends on the level of the background at the limiting apparent luminosity of the single field exposure l_{dim}. When sky flux b is negligible, a case here referred to as "low background", equation (3) indicates that $\beta = -1$ so that the divide in terms of the brightness distribution comes at $\alpha = 1/\beta = -1$. When the sky flux dominates, a case here referred to as "high background", then $\beta = -1/2$ so that the divide in terms the brightness distribution comes at $\alpha = -2$. In general, the critical brightness distribution power-law index at the tile/stare divide is

$$\alpha_{critical} = 1/\beta = -\frac{l_{dim} + 2b}{l_{dim} + b}. \tag{6}$$

When $\alpha > \alpha_{critical}$, tiling will optimize source counts, otherwise staring will optimize source counts.

The longer an instrument observes a particular field, the fainter the source detectable at the limiting apparent luminosity (l_{dim} decreases), the smaller the limiting source brightness will be compared to the background sky brightness. Stated differently, even if a field observation started at "low background", it naturally migrates toward "high background" as exposures lengthen. This means that for long exposures, β naturally migrates toward higher values, so that $\alpha_{critical}$ will migrate toward a lower value. Therefore, a switch from "stare" to "tile" might become advantageous even were the brightness distribution power-law α to remain constant.

For non-power law, non-monotonically decreasing brightness distributions, maximizing

source counts becomes similar to a chess game. Fields that start with low source accumulation rates might ramp up quickly at a later time, when, for example, a cluster might become resolved. Therefore, choosing which field to image next and for how long in order to maximize source counts might require a complex Monte-Carlo program, possibly one that operates in real time including topical information about how seeing and weather affect the (effective apparent) brightness distributions in fields across the sky.

4. Counting Transients

In this paper a practical distinction will be made between quiescent sources and long duration transients. Here, increasingly faint quiescent sources can be detected by co-adding images of the same part of the sky at any time, whereas transients will need to be detected on a single exposure or co-added time-consecutive series of sky exposures. For transients of any duration, the fleeting nature of the source makes the "tile" or "stare" question more complicated when trying to maximize the discovery rate.

If telescope fields are easily aligned and relevant data are easily available, it will be possible to discover transients on time-separated exposures, possibly by co-adding frames taken at different times during the transient. For simplicity, however, only the relatively standard paradigm of discovering transience in a time-contiguous series of exposures will be considered here.

Discovering a transient with a single exposure is particularly susceptible to false triggers by non-astronomical phenomena. Transience verification is usually necessary for a practical sky-monitoring algorithm. False triggers are usually a single-frame phenomenon, however, and reality verification can be built into a time-contiguous series of exposures. When these check observations occur time-contiguous with the initial observation, together they can be considered as part what is necessary for transient "discovery."

Given that "tile" is desired, the tiling cadence should of course be chosen for what science return is expected. In general, a "discovery cadence" will be distinguished from a "tracking cadence." Discovery cadence, for example, should maximize the number of transients discovered. Tracking cadence, however, should maximize the scientific return from a single transient. It is possible – even likely – that a non-uniform cadence would better address both discovery and tracking for those monitors not reporting triggers to follow-up instruments. However, unless explicitly stated, uniform cadence rates will be assumed in this paper. In addition, in this paper an attempt will be made to maximize the number of discovered transient objects, expecting that important transient event will be handed off (in

a timely fashion) to a telescope dedicated to following them up specifically.

The way the apparent brightness distribution is defined for quiescents and transients might differ. In particular, l_{dim} for a transient used in equation (9) can be defined a number of ways. Useful definitions include l_{dim} as the apparent luminosity during a quiescent phase, as the apparent luminosity at the peak of an outburst, or as the average apparent luminosity over a given duration. Given a corresponding distance, apparent luminosity l and absolute luminosity L can be directly related.

Each transient will have amplitude A, which can run from less than unity (sources become dimmer, such as during a planetary transit) to greater than unity. For explosive sources, of course, the A is expected to be much greater than unity. Transients will have an amplitude probability density such that the probability of a transient of absolute luminosity L and redshift z having an amplitude between A and $A + dA$ at any time is given by $\psi(A, L, z)dA$. For each L and z this probability is normalized to unity so that $\int_0^\infty \psi(A, L, z)dA = 1$.

Similarly, each transient will have a duration of t_{dur}. Transients will have a duration probability density such that the probability of a transient of absolute luminosity L and redshift z having an duration between t and $t + dt$ at any time is given by $\xi(t, L, z)dt$. This probability is also normalized to unity so that $\int_0^\infty \xi(t, L, z)dt = 1$. It is assumed here that all $t_{dur} >> t_e$, so that these terms do not appear explicitly in the following analyses.

Including these factors, the number of transients detected in a single field down to limiting apparent luminosity l_{dim} in a single exposure (or a single consecutive series of exposures) of (total) duration t_e would be

$$N_{transient}(l > l_{dim}) = \int_{z=0}^\infty \int_{A=0}^\infty \int_{L=L_{min}(l_{dim}, A, z)}^\infty$$
$$\frac{\Phi(L, z)\psi(A, L, z)K(L, z)D_H(1+z)^2 D_A^2}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}} \, \Omega_{field} \, dL \, dA \, dz. \tag{7}$$

Here $N_{transient}$ is *not* a rate – it is the number of transients discovered during exposure time t_e, assuming each transient's source counts accumulate during the entire t_e. During exposure, the sky is considered to be static in the sense that extending t_e will not increase the number of bright transients, but allow more faint transients (visible during the entire exposure) to come over the detection threshold. In this light, $\Phi(L, z)$ is also *not* a rate and describes the space density of transients that happen to be observable at absolute luminosity L and redshift z when integrated over the entire length of the exposure. Although t_e does not appear explicitly in equation (7), it will enter later through l_{dim}.

Equation (7) has two realizations involving L_{min}, the minimum absolute luminosity of a transient that is seen at the apparent detection threshold l_{dim}. When known quiescent sources

are being inspected for transience, L_{min} is determined as it was below equation (4). It is possible, however, that transient sources will be detected *only* because they show transience. This might happen, for example, were a quiescent source originally below the detection threshold of the telescope to experience a high amplitude event that brought it above the detection threshold. This would affect the minimum absolute luminosity in equation (9) such that $L_{min}K(z)A = l_{dim}D_l^2$. An example of this type of transient detection is pixel lensing (Crotts 1992; Gould 1995).

When re-inspecting a given field for transience, the probability of finding new transience may have changed. Suppose, for example, that the same field is inspected twice for transience, one time shortly after the other. The two exposures are not co-added. Given that the exposure time t_e is the same for each field but significantly less than transient duration t_{dur}, the likelihood of finding *new* transients in the second exposure is likely reduced. This is because there has only been a short time during which a new transient could have gone off.

More generally, if N_1 is the number of transients discovered in the first field exposure, and the same field is inspected for an equal exposure time t_e after time t_{return}, then N_2, the number of transients discovered in the second exposure is related to N_1 by

$$N_2 = \min(t_{return}/t_{dur}, 1)N_1. \tag{8}$$

It is assumed that the transient rate remains constant in each region of space. Note that as t_{return} becomes small, so does the number of new transients discovered. When t_{return} becomes larger than t_{dur}, then $N_2 = N_1$ so that field has effectively been "reset" and all discovered episodes of transience at duration t_{dur} are again new.

The number of new transients discovered in a previously observed field is therefore given by

$$
\begin{aligned}
N_{transient}(l > l_{dim}) = \int_{z=0}^{\infty} \int_{A=0}^{\infty} \int_{L=L_{min}(l_{dim},A,z)}^{\infty} \\
\frac{\Phi(L,z)\psi(A,L,z)K(L,z)D_H(1+z)^2 D_A^2}{\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}} \\
\Omega_{field} \min(t_{return}/t_{dur}(L,z,A), 1) \, dL \, dA \, dz.
\end{aligned}
\tag{9}
$$

Equation (9) has realizations that are quite complicated. As with the general quiescent case, for non-power law, non-monotonically decreasing transient brightness distributions, maximizing source counts becomes similar to a chess game. Choosing which field to image next and for how long in order to maximize discovered transients might require a complex Monte-Carlo program, possibly one that operates in real time including topical information

about how seeing and weather affect the apparent brightness distributions in fields across the sky. Significant insight can be gleaned, however, from studying relatively simpler theoretical regimes.

4.1. Isotropic Power-Law Brightness Distributions

In the cases considered here, fields are identical, noise sources are unique and unchanging over observations of interest (β is a constant), and the effective apparent brightness distribution is a single power law over its entire length (α is a constant). Additionally, it will be assumed that transients attain amplitude greater than A for duration t_{dur}. Mathematically, this means $\psi(A') = \delta(A' - A)$ in equation (9).

Suppose an observing campaign is limited to a set amount of total observing time t_c. Given that the time is divided between N_{field} identical fields, the time between returning to re-image the same field is given by

$$t_{return} = N_{field}(t_e + t_d) + (N_{field} - 1)t_s \approx N_{field}(t_e + t_d + t_s) \qquad (10)$$

where t_d is the down time it takes for the imager to reset before taking the next image, and t_s is the time it takes to slew to the next field. The approximation is true when $N_{field} >> 1$. Note that t_d will likely include the time it takes to read out the data and take dark frames.

Given that t_c is large compared to a single exposure time, the total number of exposures taken in the campaign will be

$$N_c = \frac{t_c}{t_e + t_d + t_s}. \qquad (11)$$

The grand total number of sources observed in the campaign will then be

$$N_g = N_c N_{transient}. \qquad (12)$$

Note that these equations are valid even for dedicated telescopes – t_c is then proportional to the lifetime of the telescope.

The number of transients discovered upon return to a single field can then found from equation (9), which simplifies to

$$N_{transient} \propto t_e^{\alpha\beta} \, \min(t_{return}/t_{dur}, 1). \qquad (13)$$

Therefore the grand total number of transients discovered during the campaign is found by combining Eqs. (13) with equation (11) and equation (12) so that

$$N_g \propto \frac{t_e^{\alpha\beta}}{t_e + t_d + t_s} \, (t_{return} > t_{dur}) \qquad (14)$$

$$N_g \propto t_e^{\alpha\beta} \ (t_{return} < t_{dur}). \tag{15}$$

Stated explicitly for the extreme cases of high and low background,

$$N_g \propto \frac{t_e^{-\alpha}}{t_e + t_d + t_s} \ (t_{return} > t_{dur}, \ \text{low background}) \tag{16}$$

$$N_g \propto \frac{t_e^{-\alpha/2}}{t_e + t_d + t_s} \ (t_{return} > t_{dur}, \ \text{high background}) \tag{17}$$

$$N_g \propto t_e^{-\alpha} \ (t_{return} < t_{dur}, \ \text{low background}) \tag{18}$$

$$N_g \propto t_e^{-\alpha/2} \ (t_{return} < t_{dur}, \ \text{high background}). \tag{19}$$

For the cases where $t_{return} > t_{dur}$ one can set $dN_g/dt_e = 0$ and solve for t_e yielding

$$t_e^{best} = \frac{\alpha\beta(t_d + t_s)}{1 - \alpha\beta}. \tag{20}$$

Note that these equations are consistent with many of the conclusions derived above for quiescent sources. When $\alpha\beta$ nears 1, the denominator nears zero so that the t_e^{best} goes to infinity, indicating that "staring" is becoming better than tiling. When $\alpha\beta$ approaches 0, the numerator and hence t_e^{best} also approach zero, indicating an increasingly rapid tiling rate.

Why can't a t_e^{best} be found for cases when $t_{return} < t_{dur}$? Solving $dN_g/dt_e = 0$ formally indicates that N_g is best maximized for the longest values of t_e. Therefore the minimum critical cadence rate is $t_{return}^{critical} = t_{dur}$ so that

$$t_e^{critical} = \frac{t_{dur}}{N_{field}} - t_d - t_s. \tag{21}$$

When $t_{return} \geq t_{dur}$ the actual "best" cadence rate that maximizes the number of transients discovered can be found by substituting the above t_e^{best} equations into $t_{return} = N_{field}(t_e + t_d + t_s)$ which become

$$t_{return}^{best} = \max(t_{dur}, N_{field}\frac{t_d + t_s}{1 - \alpha\beta}). \tag{22}$$

The situation is shown graphically in Figures 1-4. These figures show plots of N_g versus t_{return}. Here N_g is normalized to the number of transients discovered when $t_{return} = t_{dur}$. The value of N_{field} is taken to be large compared to unity, while t_c is taken to be large

compared to all other durations. Their precise values are not important and do not affect the plots. On each of the Figures three curves are drawn, corresponding to α values of -2.5, -1.5, and -0.5.

Figure 1 depicts the transient discovery rate during low background ($\beta = -1$) and negligible down and slew times. In other words, Poisson counts dominate the noise and $t_d + t_s << t_{dur}/N_{field}$. Formally, $t_d + t_s = 0$. Inspection of Figure 1 indicates that for a shallow brightness distribution such as $\alpha = -0.5$, "tile" is the most productive strategy, and the cadence that maximizes transient discovery is t_{dur}, the duration of the transient. For steep brightness distributions such as $\alpha = -1.5$ and $\alpha = -2.5$, longer return rates result in a greater number of transients recovered, indicating that "stare" is the best policy. From the above analysis, note that $\alpha = -1$ is the dividing line between "tile" and "stare."

Figure 2 similarly depicts the transient discovery rate during high background ($\beta = -1/2$) and negligible down and slew times. In other words, a constant sky background dominates the noise while $t_d + t_s << t_{dur}/N_{field}$. Formally, again, $t_d + t_s = 0$. Inspection of Figure 2 indicates that for shallow brightness distributions such as $\alpha = -0.5$ and $\alpha = -1.5$, "tile" is the most productive policy, and the cadence that maximizes transient discovery is t_{dur}, the duration of the transient. For the steepest brightness distribution $\alpha = -2.5$, slower cadences result in greater transients recovered, indicating that "stare" is the best policy. From the above analysis, note that $\alpha = -2$ is the dividing line between "tile" and "stare."

Figure 3 similarly depicts the transient discovery rate during low background and significant down and/or slew times. Specifically, the case where $t_d + t_s = (2/3)(t_{dur}/N_{field})$ is assumed. Inspection of Figure 3 indicates that for a shallow brightness distribution such as $\alpha = -0.5$, "tile" is again the most productive policy. However, now the best cadence is slightly longer than t_{dur}, and is given by equation (22). For steeper brightness distributions such as $\alpha = -1.5$ and $\alpha = -2.5$, longer return rates again result in greater transients recovered, indicating that "stare" is the best policy. From the above analysis, note that $\alpha = -1$ is the dividing line between "tile" and "stare."

Figure 4 similarly depicts the transient discovery rate during high background and significant down and/or slew times. Specifically, the case where $t_d + t_s = (2/3)t_{dur}/N_{field}$ is assumed. Inspection of Figure 4 indicates that for shallow brightness distributions such as $\alpha = -0.5$ and $\alpha = -1.5$, "tile" is the most productive policy, and the cadence that maximizes transient discovery is given by equation (22). For the steepest brightness distribution $\alpha = -2.5$, slower cadences result in greater transients recovered, indicating that "stare" is again the best policy. From the above analysis, note that $\alpha = -2$ is the dividing line between "tile" and "stare."

Transient durations can be so short that they are less than the exposure time. In that case, an effective apparent luminosity should be used that incorporates the amount of actual integrated light over the entire exposure, instead of the peak apparent luminosity of the transient. A more complicated paradigm not considered here are transients with durations longer than t_{return} that are detected by co-adding counts during each return exposure.

The results of this section can be summed up as follows: If, during exposure, the rate that transients come over the limiting magnitude horizon is increasing fast enough ($\alpha\beta > 1$), then "stare" is preferred. If, on the other hand, the rate that transients come over the limiting magnitude horizon is not increasing fast enough ($\alpha\beta \leq 1$), then "tile" should be preferred. Usually the best tiling cadence is the duration of the transient, since a faster tiling cadence will waste effort on transients that have been previously discovered, while a slower tiling cadence will miss transients occurring in other fields. If, however, the duration of the transient is comparable to the cumulative read-out and/or slew times during a sky-tiling, then a mathematical maximization as described above in equation (22) will find the most productive cadence.

5.   Example Applications

5.1.   Local Uniform Isotropic Standard Candle Quiescents

Perhaps the most intuitive example is also the most instructive: that of uniform and isotropic standard candles in a local Newtonian universe. Given that cumulative source numbers increase as the cube of their distance and their apparent luminosity falls as the square of their distance, many an introductory text book correctly states that $N_{source} \propto l_{dim}^{-1.5}$, meaning that $\alpha = -1.5$.

Suppose further that these sources are quiescents and that a campaign of time t_c on a given telescope is dedicated to observing as many of them as possible. Is "tile" or "stare" the best observing strategy? As indicated above in Section 4, the answer is "tile" if $\alpha\beta < 1$, and "stare" otherwise. Since $\alpha = -1.5$, β becomes the determining factor. If $\beta < -2/3$, tiling will maximize quiescent counts, otherwise staring will. Note that this β is between the above discussed cases of low and high background, so that when the background is low, $\beta = -1$, and staring is the best strategy. Alternatively, when the background is high, $\beta = -1/2$, so that tiling wins. Since quiescents can be discovered in frames co-added at any delay, the exact tiling rate is not important, and so can be set to minimize the total slew time, for example. A tiling campaign should best proceed by dividing the time equally between observable identical fields, with fields having the least exposure time getting the highest

priority.

5.2.   Local Uniform Isotropic Standard Candle Transients

Suppose now that the above uniform, isotropic sources are transients with duration t_{dur} and characteristic apparent luminosity l. All transients will be assumed to have the same duration. The fraction of sources that show transience at any one time will turn out to be unimportant for optimization. Again consider the search a campaign of time t_c on a given telescope.

Here, again, the "tile" or "stare" decision depends on the predominant source of noise. Again "stare" will be preferred when $\alpha\beta > 1$, equivalent to $\beta < -2/3$ since $\alpha = -1.5$. In the quiescent case, cadence was not important since sources could be discovered on fields co-added with any time delay. Here the finite duration of transience will indicate a best cadence. Suppose first that $t_{return} < t_{dur}$. Equation (16) indicates that N_g increases monotonically with t_e, pushing us into the regime where $t_{return} \geq t_{dur}$. Equation (16), however, has N_g decreasing monotonically with t_e at large t_e. Equation (22) then gives t_{return}^{best}. The best cadence is seen to be t_{dur} for small down and slew times.

5.3.   Maximizing Microlensing With SuperMACHO

The SuperMACHO project inspects the LMC for microlensing (Stubbs et al. 2002). The LMC, however, shows an anisotropic and non-uniform sky distribution, indicating that the above detailed cadence calculations made for isotropic, uniform distributions are of mainly didactic value. An analysis of the SuperMACHO observing algorithm is given by (Gould 1999). According to their web page, SuperMACHO employs 60 fields each having $\Omega_{field} = 0.36$ deg^2. The web page also indicates a canonical magnitude limit of around $V = 23$. It will be assumed that high background dominates the noise in any exposure, so that $\alpha_{critical} = -2$.

According to Figure 4 of Alcock et al. (2000), a canonical LMC field (Field 13 in their work) has a cumulative luminosity function where $N_g \sim l_{dim}^{-0.9}$ from visual magnitude 20 to visual magnitude 22, and $N_g \sim l_{dim}^{-0.6}$ from visual magnitude 22 to visual magnitude 24. A combined average power-law index from 20 to 24 is about $\alpha = -0.7$.

Since the LMC stars are all at approximately the same distance, the cumulative luminosity function will be the effective cumulative apparent brightness distribution. Therefore, since $\alpha > \alpha_{critical}$, the above analysis indicates that "tile" will discover more transients than

"stare." Now a canonical duration of a microlensing event is about one month. To obtain good coverage, however, one might want to record the event on the rise, so a duration of interest is about two weeks. Since t_d and t_s are on the order of seconds, it will be assumed that they are negligible compared to t_{dur}/N_{field} and t_e. The above analysis then indicates that for each field, the optimal t_{return} time is t_{dur}.

This indicates that SuperMACHO should return to each field after two weeks. The SuperMACHO web page notes, however, that each field is returned to twice a night, "in order to maximize the number of stars inspected for microlensing." Given that each star has a constant probability of being microlensed, the above quoted maximization scheme of maximizing stars would also maximize the transients discovered. Therefore, how can these two cadences be consistent?

One reason may be that the effective cumulative apparent luminosity distribution ($N_{transient}$) of LMC transients is dropping rapidly after a given exposure time (C. Stubbs 2003, private communication). The optimized cadence of two weeks assumed that a constant $\alpha\beta$ continued indefinitely.

Now since each LMC SuperMACHO field is different, N_g is likely different for each field. As indicated above, for complex cases like these, a real-time Monte-Carlo routine might be run planning each night's observing campaign based on present and predicted sky conditions that could best maximize N_g for that night.

5.4.   Maximizing Type IA Supernovae Discovered with LSST

Suppose one wants to maximize the number of Type IA supernovae discovered with the planned Large-aperture Synoptic Survey Telescope (Angel et al. 2001). According to modeling in a Simple Cold Dark Matter universe by Porciani & Madau (2000), the integral number count rate of these transients is approximately $N_g \propto l_{dim}^{-2}$ for $21 < I < 24$, while $N_g \propto l_{dim}^{-0.5}$ for $24 < I < 27$. Now the LSST web page states a design goal of magnitude 24 in a single 10 second exposure over 7 deg^2, with a readout time is estimated to be about 5 seconds. Further suppose that LSST can tile 25% of the sky per night (π steradians; $\sim 10,000$ deg^2). This indicates that on a clear moonless night that $N_{field} \sim 1400$, LSST can point to about 1400 independent fields. A supernova might be perceived to have the most value if caught on the rising part of its light curve, which has duration of about $t_{dur} \sim 15(1 + z)$ days.

This case is simpler than the SuperMACHO/microlensing case since Type IA supernovae can be assumed distributed isotropically in the universe. Also, supernovae should not crowd

each other on the sky, so that we would not expect source confusion to flatten the effective brightness distribution.

At brighter magnitudes, the steep $\alpha = -2$ brightness distribution would place $\alpha\beta > 1$ for any β, indicating that LSST should stare at any field until the brightness distribution breaks. At fainter magnitudes, the shallow α indicates that $\alpha\beta < 1$ for any β, indicating that LSST should tile in this regime.

Given that $\alpha\beta$ is indeed a constant in this regime, the analysis given above in Section 4 can determine the most productive cadence. Assuming the down and slew times are small, the cadence should be the duration of the interesting part of the transient: 15 days for a low-redshift supernova. Given 1400 fields and a 25 percent duty cycle due to the Sun and Moon, the best exposure time comes out to be about 230 seconds. Shorter exposures would lead to returning to a field too rapidly and hence re-discovering known supernovae, while longer exposures would miss supernovae occurring elsewhere.

If it is found that $\alpha\beta$ flattens significantly, tiling will still be preferred, but a shorter cadence than the transient duration may be needed to avoid observing in increasingly barren fields. The actual cadence would need to be found by noting the new transient accumulation rate in a fresh field, and switching to a new field when the rate drops below that in an old field.

The above cadence would *only* be valuable for maximizing local supernova detections during the rise. LSST has several other proposed scientific uses, however. Once could length the cadence to optimize for supernovae at higher redshifts. In this light, an LSST Guest Investigator Program might be of valuable. In such a program, scientists outside the LSST collaboration might be invited to propose different cadence rates and/or bandpasses so as to optimize the detections of sources of different types and/or transients of different durations.

5.5. Maximizing Blazars Discovered with GLAST

The Gamma Ray Large Area Telescope (GLAST; see, for example, Michelson 2002) will surely sample more faint blazers and flares from blazars than ever before in the energy ranges from 10 MeV to 100 GeV. The question has come up, however, as to the most productive algorithm for pointing GLAST (J. Bonnell 2002, private communication). The telescope has a planned constraint of pointing away from the Earth, so that if the zenith angle of the telescope is not changed, GLAST will re-observe the same part of the sky every 90 minutes. What zenith angle rocking algorithm would best maximize the discovery of blazars and blazar flares?

From inspection of Stecker & Salamon (1996) Figure 2, the power-law slope of the cumulative approximated brightness distribution for quiescent blazars is expected to be about -1.3 below integrated flux in (> 100 MeV photons) of 10^{-6} cm^{-2} sec^{-1}. For flaring blazars, this same power-law slope is about -1.0. These estimations are extrapolated from results from the EGRET instrument that flew on the Compton Gamma Ray Observatory (Fichtel 1996).

Now GLAST's field of view is about two steradians, and the likely point spread function of sources is expected to be highly energy dependent. The below analysis will assume that no matter the energy, sources will not significantly overlap, so that source confusion will not significantly flatten the brightness distribution. The energy range for which this will be true may need to be determined by actual GLAST observations, but it is assumed valid here through most of the GLAST energy band.

First addressed here will be the question of maximizing the number of quiescent blazars discovered. It will be assumed that the background in the gamma-ray range of GLAST is dominated by Poisson noise everywhere but in the plane of the Galaxy, a relatively small angular region. The exact boundaries of this region, too, will be energy dependent. Given that $\alpha\beta > 1$ in this region, "stare" mode is to be preferred in maximizing the discovery of new blazars. This could mean that some zenith angles should be relatively ignored since time is better spent re-observing previously observed fields. Alternatively, a "GLAST Deep Field" (GDF) might be created where a significant amount of observing time is spent accumulating the relative abundance of dim blazars.

It is possible, even probable, that $\alpha\beta$ is not constant and will flatten significantly for the dimmer quiescents. In fact, $\alpha\beta$ may dip below unity for different exposure times at different galactic latitudes and for different energies. When this happens, tiling becomes preferred, and GDFs become inefficient in discovering new blazars. The most efficient tiling algorithm might need to await determination of the actual brightness distribution for the fields of interest by GLAST itself.

Next addressed here will be the question of maximizing the number of transient blazar flares discovered. For flares, since $\alpha = -1$, only in the lowest noise regimes can $\alpha\beta \geq 1$. In regions of the sky where the noise is entirely dominated by Poisson, the case is a formal tie so that it does not matter where in these regions GLAST points. For everywhere else, however, $\alpha\beta < 1$ and so some sort of tiling algorithm will maximize the number of flares discovered.

For regions where tiling is to be preferred, we now address the question of the optimal cadence. Here the situation is complicated by several factors. The first is that different

regions of the (direction, energy, exposure duration) matrix will be best characterized by a different $\alpha\beta$. The above analyses in Section 4 assumed a constant $\alpha\beta$, so that it can only be rigorously applied to similar regions. As indicated above, to determine if it is beneficial to jump to a region of different $\alpha\beta$, one should determine whether the rate of new transients discovered in the old field has dropped below that rate in a fresh field.

For regions of similar $\alpha\beta$, given that down and slew times are negligible, the above analyses indicate that the optimal cadence is the duration of interest in the blazar flare. The total duration of blazar flares can be from hours to weeks. The duration of interest may be shorter than this, however, if blazar flares will need to be discovered relatively early on, so that instruments in other bands can be triggered to monitor the event during the flare.

Here the cadence can be used as tool to isolate blazar flares of a given duration – faster cadences will isolate faster flares. To best discover the fastest blazar flares, GLAST might be put into a mode in which it changes its zenith angle rapidly, effectively sampling the entire sky every few hours. A caveat occurs for fields away from the spin poles of the Earth. There a cadence of less than 90 minutes is not possible for fields since this is less than the revolution time of GLAST around the Earth.

6. Discussion

This paper is not meant to be the final word in the determination of pointing algorithms for telescope monitors. Indeed, pointing algorithms will likely need to incorporate more practical considerations that are not formally considered here. First, as mentioned above, given a plethora of potential noise sources that include cosmic ray hits and satellite glints, it is clearly not assured that any single-frame transient is of astronomical importance. Verification observations can and should be built into observing algorithms to assure that triggered transients have a reasonable chance of being of astronomical interest. When these check observations occur time-contiguous with the initial observation, together they can be considered as part what is necessary for transient "discovery."

Next, the idiosyncrasies of different telescopes, observing sites, CCDs, control hardware, control software and observers themselves can also have a large and even deterministic effect on the design and implementation of a practical observing algorithm. An example of this could be the inability for a telescope to slew faster than a certain rate, the need to dither successive observations to minimize pixel inequities, or the occurrence of certain fields at certain times in areas above cities that create relatively bright sky-glow.

Next, the idiosyncrasies of different transient types can drive practical observing algo-

rithms. Some transients might only be found only in certain sections of the sky or superposed on certain types of backgrounds that create specific observing challenges. An example of this is supernova searches, where the transient frequently occurs superposed on its host galaxy. Although potentially difficult, this information could be included in the *effective* $N_{transient}(l)$ function.

Next, the goal of transient observing might not be to discover the maximum raw number of transients but the most transients with a certain attribute. An example of this might be an effort to find particularly bright cases of microlensing (Nemiroff 1998) or planet transits (Pepper, Gould, and Depoy 2002). In these cases, observing deeper would not help discover brighter sources. Here the above formalism might be augmented with a weighting function emphasizing sources in the desired magnitude range. Again, alternatively, the effective $N_{transient}(l)$ could be adapted to incorporate this information, for example not counting transients too dim to be of interest.

Next, it might be preferred that transients *not* be detected in a single exposure or time-contiguous series of exposures, as, for example, discussed in Gould (1999). This would greatly affect the discussion given above. In fact, the drive toward separating observing times by the transient duration is fueled by the single exposure premise. If frames can be routinely aligned and co-added then one can spread the detection observations out over t_{dur} with any distribution at all, only demanding that enough observations be carried out during t_{dur} so that transient detection is assured.

Next, transients do not all have the same durations. Optimizing for a single duration might indeed cause an observing algorithm to miss transients of shorter duration. Robust observing algorithms attacking a distribution of durations might try to optimize toward the peak of the duration distribution, or use the above analysis as the basis for a more sophisticated one that optimizes transient discovery rates over the entire distribution of durations.

Nevertheless, even given all of these caveats, transient detection algorithms need to be determined more by hard logic and mathematical optimization than by whim. A map of how effective $\alpha\beta$ changes with the accumulated exposure time in each field might indeed be useful in matching cadence with scientific return. At minimum, key pieces of information that should be considered include the duration and the brightness distributions of the transients. An example of how they interact in a relative clean set of theoretical but didactic cases is given above.

Even a valiant effort to predetermine a cadence that optimizes discovery rates might fail, given inaccurate knowledge of relevant parameters such as the duration distribution

function. Therefore, a pointing algorithm might deploy "cadence creep" (M. Kowalski 2003, private communication). The idea is to slightly change cadence over time to see if transient detection rates increase. To be effective, enough transients would have to be detected for a statistically meaningful comparison. A search phase for an optimal cadence, between estimated boundaries, might be mandated as an early phase of a transient search program.

A natural extension of the idea that different observing algorithms optimize different scientific return is the implementation of a diverse array of observing algorithms on any given sky monitoring telescope. Guest investigator programs might diversify previously dedicated sky monitoring telescopes by implementing bandpasses and cadences chosen to optimize the discovery of different types of transients.

Last, the decision to "tile" or "stare" and how fast to tile are influenced by more than the ability to discover the maximum amounts of sources and/or transients. The schema discussed above implicitly assumed that other telescopes can be deployed for follow-up observations, and that these follow-up telescopes will maximize the science uncovered per transient. If follow-up time is not expected for discovered transients, one may want to code follow-up observations directly into the timing of the observations. For this reason, a non-uniform cadence, one that combines attributes of both discovery and tracking, such as returning to sources in logarithmically increasing time intervals, might be preferable.

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Fig. 1.— A plot of number of transients discovered, N_g, versus the time taken to return to inspect the same field, t_{return}. N_g has been normalized to the number discovered when $t_{return} = t_{dur}$, the duration of the transient, while t_{return} is given as a fraction of t_{dur}. Here the dominant source of noise is Poisson ("low background", $\beta = -1$), while $t_d + t_s$, the down and slew times for the telescope, are taken to be negligible. Three power laws of the effective cumulative luminosity distribution are depicted: $\alpha = -0.5$ as the solid line, $\alpha = -1.5$ as the dashed line and $\alpha = -2.5$ as the dot-dashed line. For the first two power laws, N_g increases monotonically with t_{return}, indicating that longer exposures detect more transients so that a telescope that "stares" would discover more transients than a similar telescope that "tiles" the sky. For the last power- law, "tiling" is optimal, while a cadence of $t_{return} = t_{dur}$ maximizes the number of transients discovered.

Fig. 2.— Similar to Figure 1 with the exception that the dominant source of noise is considered to be sky-glow, the "high-background" case ($\beta = -0.5$). Here, for the middle $\alpha = -1.5$ case, the most transients are recovered in "tile" mode, with the most productive cadence equal to the duration of the transient.

Fig. 3.— Similar to Figure 1 with the exception that significant down plus slew times are incurred. Specifically, $t_d + t_s = (2/3)(t_{dur}/N_{field})$. Here the $\alpha = -0.5$ case recovers the most transients in "tile" mode, but the most productive cadence t_{return} is greater than t_{dur} and determined by equation (22).

Fig. 4.— Similar to Figure 1 with the exceptions that the dominant source of noise is considered to be sky-glow, the "high-background" case, and significant down plus slew times are incurred. Specifically, $t_d + t_s = (2/3)(t_{dur}/N_{field})$. Here, for the middle $\alpha = -1.5$ case, the most transients are recovered in "tile" mode, with the most productive cadence t_{return} being greater than t_{dur} and determined by equation (22).