Visualizing Data using t-SNE

Taran Lynn, Xiaoli Yang, Xiaoxin Chen

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Data Visualization and Manifold Learning

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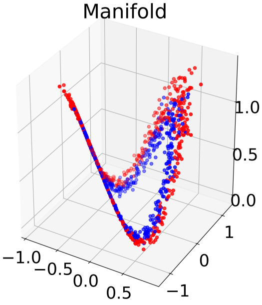
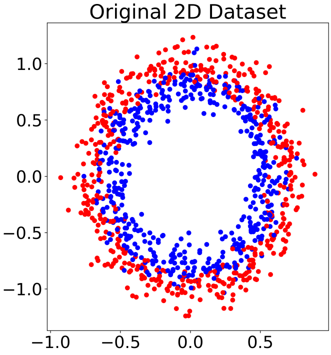
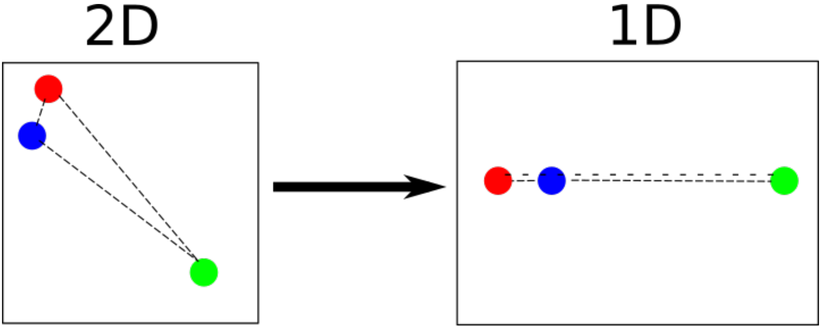




Stochastic Neighbor Embedding (SNE)

Spring Metaphor

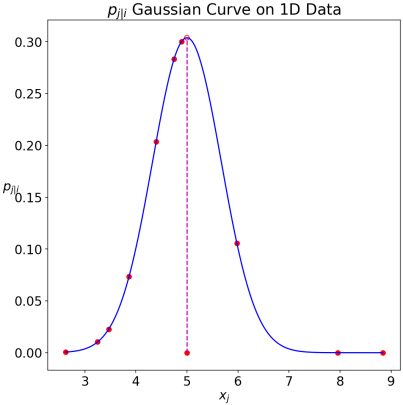
1. Consider putting a spring between each point in higher-dimensional space, and then squashing the points down into a lower dimension.
2. Spring tension is given by neighbor locality.



Stochastic Neighbor Embedding (SNE)

What is $p_{j|i}$?

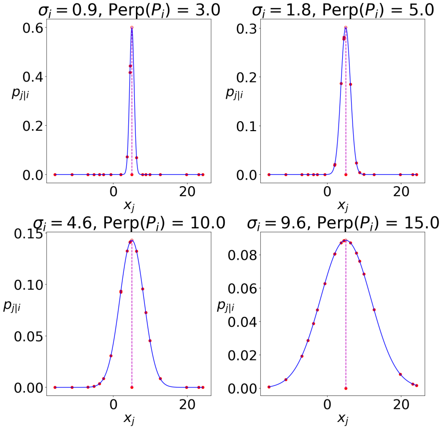
1. $p_{j|i} = \frac{\exp\left(-||x_i - x_j||^2/2\sigma_i^2\right)}{\sum_{k \neq i} \exp\left(-||x_i - x_k||^2/2\sigma_i^2\right)}$
2. Probability datapoint x_i views x_j as its neighbor given Gaussian with variance σ_i



Stochastic Neighbor Embedding (SNE)

Determining σ_i

Perplexity	Smooth approximation of number of neighbors $Perp(P_i) = 2^{H(P_i)}$
Shannon Entropy	Information present in probability space $H(P_i) = -\sum_j p_{j\|i} \log_2 p_{j\|i}$



Stochastic Neighbor Embedding (SNE)

Gradient Descent Optimization

1. $q_{j|i}$ is analogous to $p_{j|i}$ for lower dimension points y_i and y_j. $\sigma = 1/\sqrt{2}$ for all $q_{j|i}$
2. Cost function is $C = \sum_i KL(P_i||Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$
3. $\frac{\partial C}{\partial y_i} = 2\sum_j \left(p_{j|i} - q_{j|i} + p_{i|j} - q_{i|j}\right)(y_i - y_j)$
4. $Y^{(t)} = Y^{(t-1)} + \nu\frac{\partial C}{\partial Y} + \alpha(t)\left(Y^{(t-1)} - Y^{(t-2)}\right)$
5. Simulated annealing through decaying Gaussian noise

Problems with SNE

- Computational inefficiency
- Overcrowding