Variational Satisfiability Solving Inference Rules

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$$\frac{r \notin dom(\Delta) \qquad \texttt{spawn}(\Delta, r) = (\Delta', s)}{(\Delta, r) \mapsto (\Delta', s)}\ \textsc{Ac-Gen} \qquad \frac{\Delta(r) = s}{(\Delta, r) \mapsto (\Delta, s)}\ \textsc{Ac-Ref}$$

$$\frac{\texttt{negate}(\Delta, s) = (\Delta', s')}{(\Delta, \neg s) \mapsto (\Delta', s')}\ \textsc{Ac-Neg}$$

$$\frac{}{(\Delta, D\langle e_1, e_2\rangle) \mapsto (\Delta, D\langle e_1, e_2\rangle)}\ \textsc{Ac-C}$$

$$\frac{}{(\Delta, \neg D\langle e_1, e_2\rangle) \mapsto (\Delta, D\langle \neg e_1, \neg e_2\rangle)}\ \textsc{Ac-Neg-C}$$

$$\frac{}{(\Delta, \bullet) \mapsto (\Delta, \bullet)}\ \textsc{Ac-Unit} \qquad \frac{\texttt{or}(\Delta, s_1, s_2) = (\Delta', s')}{(\Delta, s_1 \vee s_2) \mapsto (\Delta', s')}\ \textsc{Ac-SOr}$$

$$\frac{\texttt{and}(\Delta, s_1, s_2) = (\Delta', s')}{(\Delta, s_1 \wedge s_2) \mapsto (\Delta', s')}\ \textsc{Ac-SAnd}$$

$$\frac{}{(\Delta, \neg(e_1 \vee e_2)) \mapsto (\Delta, \neg e_1 \wedge \neg e_2)}\ \textsc{Ac-DM-Or}$$

$$\frac{}{(\Delta, \neg(e_1 \wedge e_2)) \mapsto (\Delta, \neg e_1 \vee \neg e_2)}\ \textsc{Ac-DM-And}$$

$$\frac{(\Delta, v_1) \mapsto (\Delta_1, v_1') \qquad (\Delta_1, v_2) \mapsto (\Delta', v_2')}{(\Delta, v_1 \wedge v_2) \mapsto (\Delta', v_1' \wedge v_2')}\ \textsc{Ac-VAnd}$$

$$\frac{(\Delta, v_1) \mapsto (\Delta_1, v_1') \qquad (\Delta_1, v_2) \mapsto (\Delta', v_2')}{(\Delta, v_1 \vee v_2) \mapsto (\Delta', v_1' \vee v_2')}\ \textsc{Ac-VOr}$$

Figure 1: Accumulation semantics on IL formulas.

$$\frac{\texttt{assert}((\Gamma, \Delta), t) = \Gamma'}{((\Gamma, \Delta), t) \rightarrowtail ((\Gamma', \Delta), \bullet)}\ \textsc{Ev-Term}$$

$$\frac{\texttt{assert}((\Gamma, \Delta), s) = \Gamma'}{((\Gamma, \Delta), s) \rightarrowtail ((\Gamma', \Delta), \bullet)}\ \textsc{Ev-Sym}$$

$$\frac{}{(\Theta, D\langle e_1, e_2\rangle) \rightarrowtail (\Theta, D\langle e_1, e_2\rangle)}\ \textsc{Ev-Chc}$$

$$\frac{}{(\Theta, \bullet \wedge v) \rightarrowtail (\Theta, v)}\ \textsc{Ev-UL} \qquad \frac{}{(\Theta, v \wedge \bullet) \rightarrowtail (\Theta, v)}\ \textsc{Ev-UR}$$

$$\frac{}{(\Theta, \neg(v_1 \vee v_2)) \rightarrowtail (\Theta, \neg v_1 \wedge \neg v_2)}\ \textsc{Ev-DM-VOr}$$

$$\frac{}{(\Theta, \neg(v_1 \wedge v_2)) \rightarrowtail (\Theta, \neg v_1 \vee \neg v_2)}\ \textsc{Ev-DM-VAnd}$$

$$\frac{(\Delta, \neg v) \mapsto (\Delta', v')}{((\Gamma, \Delta), \neg v) \rightarrowtail ((\Gamma, \Delta'), v')}\ \textsc{Ev-Neg}$$

$$\frac{(\Delta, v_1) \mapsto (\Delta_1, v_1) \qquad (\Delta_1, v_2) \mapsto (\Delta', v_2)}{((\Gamma, \Delta), v_1 \vee v_2) \rightarrowtail ((\Gamma, \Delta'), v_1' \vee v_2')}\ \textsc{Ev-Or}$$

$$\frac{(\Theta, v_1) \rightarrowtail (\Theta_1, v_1') \qquad (\Theta_1, v_2) \rightarrowtail (\Theta', v_2')}{(\Theta, v_1 \wedge v_2) \rightarrowtail (\Theta', v_1' \wedge v_2')}\ \textsc{Ev-And}$$

Figure 2: Evaluation semantics on IL formulas.

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$$\frac{\texttt{genModel}(\Phi) = m}{(\Phi, \bullet) \Downarrow_i m}\ \textsc{Gen-M} \qquad \frac{((\Gamma, \Delta), s) \rightarrowtail ((\Gamma', \Delta'), \bullet)}{((C, \Gamma, \Delta), s) \Downarrow_i ((C, \Gamma', \Delta'), \bullet)}\ \textsc{Cr-Sym}$$

$$\frac{(\Phi, v_1) \Downarrow_i (\Phi_1, v_1') \qquad (\Phi_1, v_2) \Downarrow_1 (\Phi', v_2')}{(\Phi, v_1 \wedge v_2) \Downarrow_i (\Phi', v_1' \wedge v_2')}\ \textsc{Cr-And}$$

$$\frac{(\Delta, v_1 \vee v_2) \mapsto (\Delta', v)}{((C, \Gamma, \Delta), v_1 \vee v_2) \Downarrow_i ((C, \Gamma, \Delta'), v)}\ \textsc{Cr-Or}$$

$$\frac{C(D) = \mathit{true} \qquad ((C, \Gamma, \Delta), v \wedge \texttt{toIR}(e_1)) \Downarrow_i m}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m}\ \textsc{Cr-And-TR}$$

$$\frac{C(D) = \mathit{true} \qquad ((C, \Gamma, \Delta), \texttt{toIR}(e_1) \wedge v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m}\ \textsc{Cr-And-TL}$$

$$\frac{C(D) = \mathit{false} \qquad ((C, \Gamma, \Delta), v \wedge \texttt{toIR}(e_2)) \Downarrow_i m}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m}\ \textsc{Cr-And-FR}$$

$$\frac{C(D) = \mathit{false} \qquad ((C, \Gamma, \Delta), \texttt{toIR}(e_2) \wedge v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m}\ \textsc{Cr-And-FL}$$

$$\frac{C(D) = \mathit{true} \qquad ((C, \Gamma, \Delta), v \vee \texttt{toIR}(e_1)) \Downarrow_i m}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m}\ \textsc{Cr-Or-TR}$$

$$\frac{C(D) = \mathit{true} \qquad ((C, \Gamma, \Delta), \texttt{toIR}(e_1) \vee v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m}\ \textsc{Cr-Or-TL}$$

$$\frac{C(D) = \mathit{false} \qquad ((C, \Gamma, \Delta), v \vee \texttt{toIR}(e_2)) \Downarrow_i m}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m}\ \textsc{Cr-Or-FR}$$

$$\frac{C(D) = \mathit{false} \qquad ((C, \Gamma, \Delta), \texttt{toIR}(e_2) \vee v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m}\ \textsc{Cr-Or-FL}$$

$$\frac{D \notin C \qquad \begin{array}{l} ((C \cup \{(D, \mathit{true})\}, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_1 \\ ((C \cup \{(D, \mathit{false})\}, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_2 \end{array}}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m_1 \oplus m_2}\ \textsc{Cr-CAnd-R}$$

$$\frac{D \notin C \qquad \begin{array}{l} ((C \cup \{(D, \mathit{true})\}, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_1 \\ ((C \cup \{(D, \mathit{false})\}, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_2 \end{array}}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m_1 \oplus m_2}\ \textsc{Cr-COr-R}$$

$$\frac{D \notin C \qquad \begin{array}{l} ((C \cup \{(D, \mathit{true})\}, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_{i+1} m_1 \\ ((C \cup \{(D, \mathit{false})\}, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_{i+1} m_2 \end{array}}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m_1 \oplus m_2}\ \textsc{Cr-CAnd-L}$$

$$\frac{D \notin C \qquad \begin{array}{l} ((C \cup \{(D, \mathit{true})\}, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_{i+1} m_1 \\ ((C \cup \{(D, \mathit{false})\}, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_{i+1} m_2 \end{array}}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m_1 \oplus m_2}\ \textsc{Cr-COr-L}$$

Figure 3: Variational solving semantics on cores.