

Variational Satisfiability Solving Inference Rules

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$$\begin{array}{c}
 \frac{r \notin \text{dom}(\Delta) \quad \text{spawn}(\Delta, r) = (\Delta', s)}{(\Delta, r) \mapsto (\Delta', s)} \text{AC-GEN} \quad \frac{\Delta(r) = s}{(\Delta, r) \mapsto (\Delta, s)} \text{AC-REF} \quad \frac{\text{assert}((\Gamma, \Delta), t) = \Gamma'}{((\Gamma, \Delta), t) \succ ((\Gamma', \Delta), \bullet)} \text{EV-TERM} \\
 \\
 \frac{\text{negate}(\Delta, s) = (\Delta', s')}{(\Delta, \neg s) \mapsto (\Delta', s')} \text{AC-NEG} \quad \frac{\text{assert}((\Gamma, \Delta), s) = \Gamma'}{((\Gamma, \Delta), s) \succ ((\Gamma', \Delta), \bullet)} \text{EV-SYM} \\
 \\
 \frac{}{(\Delta, D\langle e_1, e_2 \rangle) \mapsto (\Delta, D\langle e_1, e_2 \rangle)} \text{AC-C} \quad \frac{}{(\Theta, D\langle e_1, e_2 \rangle) \succ (\Theta, D\langle e_1, e_2 \rangle)} \text{EV-CHC} \\
 \\
 \frac{}{(\Delta, \neg D\langle e_1, e_2 \rangle) \mapsto (\Delta, D\langle \neg e_1, \neg e_2 \rangle)} \text{AC-NEG-C} \quad \frac{}{(\Theta, \bullet \wedge v) \succ (\Theta, v)} \text{EV-UL} \quad \frac{}{(\Theta, v \wedge \bullet) \succ (\Theta, v)} \text{EV-UR} \\
 \\
 \frac{}{(\Delta, \bullet) \mapsto (\Delta, \bullet)} \text{AC-UNIT} \quad \frac{\text{or}(\Delta, s_1, s_2) = (\Delta', s')}{(\Delta, s_1 \vee s_2) \mapsto (\Delta', s')} \text{AC-SOR} \quad \frac{}{(\Theta, \neg(v_1 \vee v_2)) \succ (\Theta, \neg v_1 \wedge \neg v_2)} \text{EV-DM-VOR} \\
 \\
 \frac{\text{and}(\Delta, s_1, s_2) = (\Delta', s')}{(\Delta, s_1 \wedge s_2) \mapsto (\Delta', s')} \text{AC-SAND} \quad \frac{}{(\Theta, \neg(v_1 \wedge v_2)) \succ (\Theta, \neg v_1 \vee \neg v_2)} \text{EV-DM-VAND} \\
 \\
 \frac{}{(\Delta, \neg(e_1 \vee e_2)) \mapsto (\Delta, \neg e_1 \wedge \neg e_2)} \text{AC-DM-OR} \quad \frac{(\Delta, \neg v) \mapsto (\Delta', v')}{((\Gamma, \Delta), \neg v) \succ ((\Gamma, \Delta'), v')} \text{EV-NEG} \\
 \\
 \frac{}{(\Delta, \neg(e_1 \wedge e_2)) \mapsto (\Delta, \neg e_1 \vee \neg e_2)} \text{AC-DM-AND} \quad \frac{(\Delta, v_1) \mapsto (\Delta_1, v_1) \quad (\Delta_1, v_2) \mapsto (\Delta', v_2)}{((\Gamma, \Delta), v_1 \vee v_2) \succ ((\Gamma, \Delta'), v'_1 \vee v'_2)} \text{EV-OR} \\
 \\
 \frac{(\Delta, v_1) \mapsto (\Delta_1, v'_1) \quad (\Delta_1, v_2) \mapsto (\Delta', v'_2)}{(\Delta, v_1 \wedge v_2) \mapsto (\Delta', v'_1 \wedge v'_2)} \text{AC-VAND} \quad \frac{(\Theta, v_1) \succ (\Theta_1, v'_1) \quad (\Theta_1, v_2) \succ (\Theta', v'_2)}{(\Theta, v_1 \wedge v_2) \succ (\Theta', v'_1 \wedge v'_2)} \text{EV-AND} \\
 \\
 \frac{(\Delta, v_1) \mapsto (\Delta_1, v'_1) \quad (\Delta_1, v_2) \mapsto (\Delta', v'_2)}{(\Delta, v_1 \vee v_2) \mapsto (\Delta', v'_1 \vee v'_2)} \text{AC-VOR}
 \end{array}$$

Figure 1: Accumulation semantics on IL formulas.

Figure 2: Evaluation semantics on IL formulas.

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$$\begin{array}{c}
\frac{\text{genModel}(\Phi) = m}{(\Phi, \bullet) \Downarrow_i m} \text{ GEN-M} \quad \frac{((\Gamma, \Delta), s) \rightsquigarrow ((\Gamma', \Delta'), \bullet)}{((C, \Gamma, \Delta), s) \Downarrow_i ((C, \Gamma', \Delta'), \bullet)} \text{ CR-SYM} \\
\\
\frac{(\Phi, v_1) \Downarrow_i (\Phi_1, v'_1) \quad (\Phi_1, v_2) \Downarrow_i (\Phi', v'_2)}{(\Phi, v_1 \wedge v_2) \Downarrow_i (\Phi', v'_1 \wedge v'_2)} \text{ CR-AND} \\
\\
\frac{(\Delta, v_1 \vee v_2) \mapsto (\Delta', v)}{((C, \Gamma, \Delta), v_1 \vee v_2) \Downarrow_i ((C, \Gamma, \Delta'), v)} \text{ CR-OR} \\
\\
\frac{C(D) = \text{true} \quad ((C, \Gamma, \Delta), v \wedge \text{toIR}(e_1)) \Downarrow_i m}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m} \text{ CR-AND-TR} \\
\\
\frac{C(D) = \text{true} \quad ((C, \Gamma, \Delta), \text{toIR}(e_1) \wedge v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m} \text{ CR-AND-TL} \\
\\
\frac{C(D) = \text{false} \quad ((C, \Gamma, \Delta), v \wedge \text{toIR}(e_2)) \Downarrow_i m}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m} \text{ CR-AND-FR} \\
\\
\frac{C(D) = \text{false} \quad ((C, \Gamma, \Delta), \text{toIR}(e_2) \wedge v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m} \text{ CR-AND-FL} \\
\\
\frac{C(D) = \text{true} \quad ((C, \Gamma, \Delta), v \vee \text{toIR}(e_1)) \Downarrow_i m}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m} \text{ CR-OR-TR} \\
\\
\frac{C(D) = \text{true} \quad ((C, \Gamma, \Delta), \text{toIR}(e_1) \vee v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m} \text{ CR-OR-TL} \\
\\
\frac{C(D) = \text{false} \quad ((C, \Gamma, \Delta), v \vee \text{toIR}(e_2)) \Downarrow_i m}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m} \text{ CR-OR-FR} \\
\\
\frac{C(D) = \text{false} \quad ((C, \Gamma, \Delta), \text{toIR}(e_2) \vee v) \Downarrow_i m}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m} \text{ CR-OR-FL} \\
\\
\frac{D \notin C \quad \frac{((C \cup \{(D, \text{true}\}), \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_1 \quad ((C \cup \{(D, \text{false}\}), \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_2}{((C, \Gamma, \Delta), v \wedge D\langle e_1, e_2 \rangle) \Downarrow_i m_1 \oplus m_2}} \text{ CR-CAND-R} \\
\\
\frac{D \notin C \quad \frac{((C \cup \{(D, \text{true}\}), \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_1 \quad ((C \cup \{(D, \text{false}\}), \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_{i+1} m_2}{((C, \Gamma, \Delta), v \vee D\langle e_1, e_2 \rangle) \Downarrow_i m_1 \oplus m_2}} \text{ CR-COR-R} \\
\\
\frac{D \notin C \quad \frac{((C \cup \{(D, \text{true}\}), \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_{i+1} m_1 \quad ((C \cup \{(D, \text{false}\}), \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_{i+1} m_2}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \wedge v) \Downarrow_i m_1 \oplus m_2}} \text{ CR-CAND-L} \\
\\
\frac{D \notin C \quad \frac{((C \cup \{(D, \text{true}\}), \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_{i+1} m_1 \quad ((C \cup \{(D, \text{false}\}), \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_{i+1} m_2}{((C, \Gamma, \Delta), D\langle e_1, e_2 \rangle \vee v) \Downarrow_i m_1 \oplus m_2}} \text{ CR-COR-L}
\end{array}$$

Figure 3: Variational solving semantics on cores.