Big Proof Trees
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1 Introduction									
				$\{\ \}, [\mathtt{n} \mapsto \mathtt{1}]$	$1] \vdash n \Rightarrow 1 \qquad \{\ \}, [] \vdash 2 \Rightarrow 2 \qquad \qquad \{\ \}, [n \mapsto 0] \vdash n \qquad \qquad \{\ \}, [n$	$n\Rightarrow 0 \qquad \{\ \}, []\vdash 2\Rightarrow 2$			
				{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) { }, [n → 2] ⊢ n ⇒ 2 { }, [] ⊢ 1 ⇒ 1	$ \begin{array}{c} (1,0) \\ (1,0) $	$0 \mid (n < 2) \Rightarrow \text{True}$ { }, $[n \mapsto 0] \vdash n \Rightarrow 0$			
					$+1] \vdash \text{if } (n < 2) \text{ then } n \text{ else } (\text{fib } ((n-1)) + \text{fib } ((n-2))) \Rightarrow 1 \\ \left\{ \right. \left. \right\}, \left[\right] \vdash \text{fib} \Rightarrow (\text{closure } n \rightarrow \text{if } (n < 2) \text{ then } n \text{ else } (\text{fib } ((n-1)) + \text{fib } ((n-2)))) \\ \left. \left\{ \right. \right\}, \left[n \mapsto 2 \right] \vdash (n-2) \Rightarrow 0 \\ \left\{ \right. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left. \right\}, \left[n \mapsto 0 \right] \vdash (n-2) \Rightarrow 0 \\ \left. \left(n \mapsto 0 \right) \vdash (n-2) \Rightarrow 0 \\ \left. \left(n \mapsto 0 \right) \vdash (n-2) \Rightarrow 0 \\ \left. \left(n \mapsto 0 \right) \vdash (n-2) \Rightarrow 0 \\ \left. \left(n \mapsto 0 \right) \vdash (n-2) \Rightarrow 0 \\ \left(n \mapsto$	- if $(n < 2)$ then n else (fib $((n - 1))$ + fib $((n - 2))) \Rightarrow 0$			
			$\{\},[n\mapsto 2]\vdash n\Rightarrow 2\qquad \{\},[]\vdash 2\Rightarrow $	$\{\ \}, [n\mapsto 2] \vdash fib\ ((n-1)) \Rightarrow 1$	$\{\ \}, [n\mapsto 2] \vdash fib\ ((n-2)) \Rightarrow 0$	$\{\ \}, [n\mapsto 1]\vdash n\Rightarrow 1 \qquad \{\ \}, []\vdash 2\Rightarrow 2$		$\{\ \}, [\mathtt{n} \mapsto \mathtt{1}] \vdash \mathtt{n} \Rightarrow \mathtt{1} \qquad \{\ \}, [] \vdash \mathtt{2} \Rightarrow \mathtt{2}$	$\{ \ \}, [n\mapsto 0]\vdash n\Rightarrow 0 \qquad \{ \ \}, []\vdash 2\Rightarrow 2$
			{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) { }, [] ⊢ n ⇒ 3 { }		$\{\ \}, [\mathtt{n} \mapsto 2] \vdash (\mathtt{fib} \ ((\mathtt{n} - 1)) + \mathtt{fib} \ ((\mathtt{n} - 2))) \Rightarrow 1$	{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))		$ \{ \ \}, \ [] \vdash (\texttt{fun n} \rightarrow \texttt{if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))} \Rightarrow (\texttt{closure n} \rightarrow \texttt{if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))} \\ \ \{ \ \}, \ [n \mapsto 2] \vdash \texttt{n} \Rightarrow 2 \\ \ \{ \ \}, \ [] \vdash \texttt{1} \Rightarrow \texttt{1} \\ \ \ \{ \ \}, \ [n \mapsto 1] \vdash (\texttt{n < 2}) \Rightarrow \texttt{True} \\ \ \{ \ \}, \ [n \mapsto 1] \vdash \texttt{n} \Rightarrow \texttt{1} \\ \ \ \{ \ \}, \ [n \mapsto 1] \vdash \texttt{1} \Rightarrow \texttt{1} \\ \ \ \ \{ \ \}, \ [n \mapsto 1] \vdash \texttt{1} \Rightarrow \texttt{1} \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$\vdash n \Rightarrow 1 \hspace{1cm} \{\ \}, \ [] \vdash (\text{fun } n \rightarrow \text{if } (n < 2) \text{ then nelse (fib } ((n - 1)) + \text{fib } ((n - 2)))) \Rightarrow (\text{closure } n \rightarrow \text{if } (n < 2) \text{ then nelse (fib } ((n - 1)) + \text{fib } ((n - 2)))) \\ \ \{\ \}, \ [] \vdash 1 \Rightarrow 2 \hspace{1cm} \{\ \}, \ [] \vdash 2 \Rightarrow 2 \hspace{1cm} \{\ \}, \ [] \vdash (n < 2) \Rightarrow \text{True} \\ \ \{\ \}, \ [] \vdash 2 \Rightarrow 2 \hspace{1cm} \{\ \}, \ [] \vdash 2 $
			$\{\ \}, [] \vdash fib \Rightarrow (closure\ n \rightarrow if\ (n < 2)\ then\ n\ else\ (fib\ ((n - 1)) + fib\ ((n - 2)))) \\ \{\ \}, [n \mapsto 3] \vdash (n - 1) \Rightarrow 2$	$\{\ \}, [n\mapsto 2] \vdash \text{if } (n<2) \text{ then } n \text{ else (fib ((n-1)) + fib ((n-2)))} \Rightarrow 1$		{ }, [] ⊢ fib ⇒ (closure n → if (n < 2) then n else (fib ((n − 1)) + fib ((n − 2)))) { }, [] ⊢ fib ⇒ (closure n → if (n < 2) then n else (fib ((n − 1)) + fib ((n − 2))) ⇒ 1	\overline{a}	$ \left\{ \right. \right\}, \left[\right] \vdash \text{fib} \Rightarrow \text{(closure n -> if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))} \\ \left. \left\{ \right. \right\}, \left[n \mapsto 2 \right] \vdash (n - 1) \Rightarrow 1 \\ \left. \left\{ \right. \right\}, \left[n \mapsto 1 \right] \vdash \text{if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))} \right\} $	$ \begin{array}{c} \vdash n \Rightarrow 1 \\ (n-2)) \Rightarrow 1 \end{array} \begin{array}{c} \left\{ \right\}, \left[\right] \vdash (\text{fun } n \Rightarrow \text{if } (n < 2) \text{ then nelse (fib } ((n-1)) + \text{fib } ((n-2))) \right\} \Rightarrow (\text{closure } n \Rightarrow \text{if } (n < 2) \text{ then nelse (fib } ((n-1)) + \text{fib } ((n-2))) \\ (n-2)) \Rightarrow 1 \end{array} \begin{array}{c} \left\{ \right\}, \left[\right] \vdash (\text{fun } n \Rightarrow \text{if } (n < 2) \text{ then nelse (fib } ((n-1)) + \text{fib } ((n-2))) \right\} \\ \left\{ \right\}, \left[n \mapsto 2 \right] \vdash (n-2) \Rightarrow 0 \end{array} \begin{array}{c} \left\{ \right\}, \left[n \mapsto 0 \right] \vdash (n < 2) \Rightarrow \text{True} \end{array} \begin{array}{c} \left\{ \right\}, \left[n \mapsto 0 \right] \vdash (n < 2) \Rightarrow \text{True} \end{array} \begin{array}{c} \left\{ \right\}, \left[n \mapsto 0 \right] \vdash (n < 2) \Rightarrow \text{True} \end{array} \right. \end{array}$
		$\{\ \}, [n\mapsto 3]\vdash n\Rightarrow 3 \qquad \{\ \}, []\vdash 2$	$\{\ \}, [n\mapsto 3] \vdash fib\ ((n-1)) \Rightarrow$	>1		$\{\ \}, [n \mapsto 3] \vdash fib\ ((n - 2)) \Rightarrow 1$	$ \{\ \}, [n \mapsto 2] \vdash n \Rightarrow 2 \qquad \{\ \}, [] \vdash 2 \Rightarrow 2 $	$\{\},[n\mapsto 2]\vdash fib((n-1))\Rightarrow 1$	$\{\ \}, [n \mapsto 2] \vdash fib\ ((n-2)) \Rightarrow 0$
		{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) { }, [n → 4] ⊢ n ⇒ 4 { }, [] ⊢ 1 ⇒ 1 { }, [n → 3] ⊢ (n < 2) ⇒ False	.so		$\{\ \}, [n\mapsto 3] \vdash (\mathtt{fib}\ ((n-1)) + \mathtt{fib}\ ((n-2))) \Rightarrow 2$		{ }, [] ⊢ (fum n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))		$\{\}, [n \mapsto 2] \vdash (fib ((n - 1)) + fib ((n - 2))) \Rightarrow 1$
		$ \{\ \}, [] \vdash \text{fib} \Rightarrow (\text{closure n} \rightarrow \text{if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2))))} \\ \{\ \}, [n \mapsto 4] \vdash (n - 1) \Rightarrow 3 $		$\{\ \}, [n \mapsto 3] \vdash \text{if } (n < 2) \text{ then n else (fib } ((n - 1)) + \text{fib } ((n - 2))) \Rightarrow 2$			$ \{\ \}, [] \vdash fib \Rightarrow (closure \ n \rightarrow if \ (n < 2) \ then \ n \ else \ (fib \ ((n - 1)) + fib \ ((n - 2)))) $	$\{\ \}, [n\mapsto 2] \vdash \text{if } (n \lessdot 2) \text{ then } n \text{ else } (\text{fib } ((n - 1)) + \text{fib } ((n - 2))) \Rightarrow 1$	
	$\{\ \}, [n\mapsto 4] \vdash n\Rightarrow 4 \qquad \{\ \}, [] \vdash 2\Rightarrow 2$		$\{\ \}, [n\mapsto 4]\vdash fib\ ((n-1))\Rightarrow 2$				$\{\ \}, [n \mapsto 4] \vdash fib\ ((n-2)) \Rightarrow 1$	1	
	{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) { }, [n → 5] ⊢ n ⇒ 5				$\{\ \}, [n\mapsto 4] \vdash (fib\ ((n-1)) + fib\ ((n-2))) \Rightarrow 3$				
	$ \{\ \}, [] \vdash fib \Rightarrow (closure \ n \rightarrow if \ (n < 2) \ then \ n \ else \ (fib \ ((n - 1)) + fib \ ((n - 2)))) $		$\{\ \}, [n \mapsto 4] \vdash \text{if } (n < 2) \text{ then n else } (\text{fib } ((n - 1)) + \text{fib } ((n - 2)) \\ \\$) ⇒ 3					
$\{\ \}, [\mathtt{n} \mapsto \mathtt{F}'$	$0 \mapsto \delta \vdash n \Rightarrow \delta$ { }, $[] \vdash 2 \Rightarrow 2$	$\{\ \}, [n \mapsto 5] \vdash fib\ ((n-1)) \Rightarrow 3$							
{ }, [] ⊢ (fun n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) ⇒ (closure n → if (n < 2) then n else (fib ((n - 1)) + fib ((n - 2)))) { { }, [¬	$\},[\mathtt{n}\mapsto\mathtt{5}]\vdash(\mathtt{n}<2)\Rightarrow\mathtt{False}$					$\{\ \}, [n \mapsto 5] \vdash (fib ((n-1)) + fib ((n-2))) \Rightarrow 5$			
$\{\} \mid \vdash fih \Rightarrow (closure \ n \rightarrow if \ (n < 2) \ then \ n \ else \ (fih \ ((n - 1)) + fih \ ((n - 2))))$			() he'	S = S + S					