Type theory behind GHC internals

Leap Workshop at LambdaConf 2018

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June 3, 2018

Boulder, CO
https://github.com/bravit/tt-ghc-exercises

Review of GHC and its development process

The Glasgow Haskell Compiler

- Haskell compiler
- Website: www.haskell.org/ghc/
- GHC Steering Committee and GHC proposals: github.com/ghc-proposals/ghc-proposals
- The Glasgow Haskell Team (committers & contributors): ghc.haskell.org/trac/ghc/wiki/TeamGHC
- Developed since 1990s

Simon Peyton Jones

Simon Marlow

The Glasgow Haskell Compiler, Marlow and Peyton Jones, 2012:

> *While the ultimate goal for us, the main developers of GHC, is to produce research rather than code, we consider developing GHC to be an essential prerequisite: the artifacts of research are fed back into GHC, so that GHC can then be used as the basis for further research that builds on these previous ideas.*

ghc.haskell.org/trac/ghc/

What you can find in the Developers Wiki

- Newcomers info
- Building
- How to work on GHC
- Implementation details: The GHC Commentary
- Advice on debugging, testing, and profiling
- Issues (bugs, feature-requests, tasks)

Other GHC developer online resources

- The GHC Reading List
- A Haskell Implementation Reading List
  (Stephen Diehl)
- Source code:
  - git.haskell.org/ghc.git
  - github.com/ghc/ghc (mirror)
- Phabricator, phabricator.haskell.org:
  - Sending patches, reviewing
  - Remote building and validating
    (Harbormaster)

bootstrapping

compiler

distrib

docs

driver

ghc

hadrian

includes

inplace

iserv

libffi

libffi-tarballs

libraries

mk

nofib

rts

rules

testsuite

utils

Haskell packages for building GHC itself (`libraries`)

- `ghc-prim`

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- `integer-gmp`, `integer-simple`

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- `integer-gmp`, `integer-simple`
- `base`

Haskell packages for building GHC itself (`libraries`)

- `ghc-prim`
- `integer-gmp`, `integer-simple`
- `base`
- array, binary, bytestring, Cabal, containers, deepseq, directory, dph, filepath, ghc-boot, ghc-boot-th, ghc-compact, ghci, haskeline, hpc, mtl, parallel, parsec, pretty, primitive, process, random, stm, template-haskell, terminfo, text, time, transformers, unix, vector, Win32, xhtml

Runtime system (`rts`**)**

- Memory management (garbage collection, STM primitives)
- Threads management
- Implementation of primitive operations
- Bytecode interpreter and GHCi dynamic linker

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- Threads management
- Implementation of primitive operations
- Bytecode interpreter and GHCi dynamic linker

The most of these components are implemented in C.

```
bravit@:ghc$ inplace/bin/ghc-stage2 -V
The Glorious Glasgow Haskell
Compilation System,
version 8.5.20180603


https://ghc.haskell.org/trac/ghc/
wiki/Newcomers#Firststeps
```

Building steps

1. Prepare workstation (ghc, perl, gcc, make, happy, alex, autoconf, automake, python3, python-sphinx, libedit, ncurses, …)
2. Get sources
3. `./configure`
4. `make` (this builds stage1, stage2 [, stage3] GHC compilers)

Building steps

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4. `make` (this builds stage1, stage2 [, stage3] GHC compilers)

The rule of GHC directory tree reading:

if you see unknown file type, then it comes from build system (99%-guaranteed)

Alternative build system Hadrian

- Andrey Mokhov (Newcastle University, UK)
- github.com/snowleopard/hadrian
- Video: skillsmatter.com/skillscasts/8722-meet-hadrian-a-new-build-system-for-ghc
- Implemented in Haskell, uses Shake (Neil Mitchell, shakebuild.com/)

- First steps
- How to rebuild modified code quickly
- How to find an issue ("low-hanging fruits")
- Advice and links

Don't get scared. GHC is a big codebase, but it makes sense when you stare at it long enough!

Haskell module compilation pipeline

- Compilation manager
- The Haskell Compiler, Hsc
- Driver (composing Hsc with C preprocessor, assembler, linker, etc)
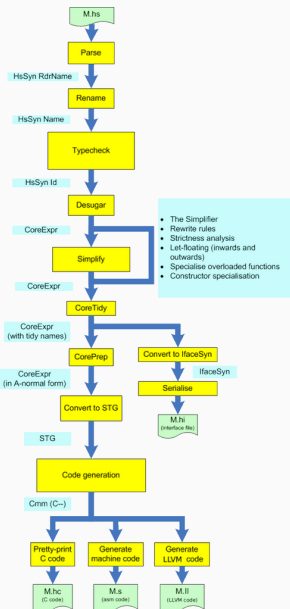
1. Parser (lexical and syntax analysis)
2. Renamer
3. Type checker
4. Desugarer
5. Optimizer
6. Code generator
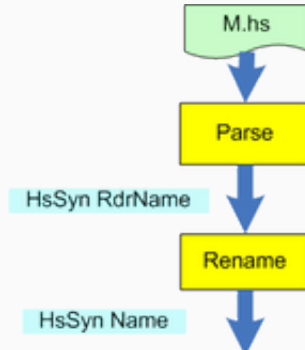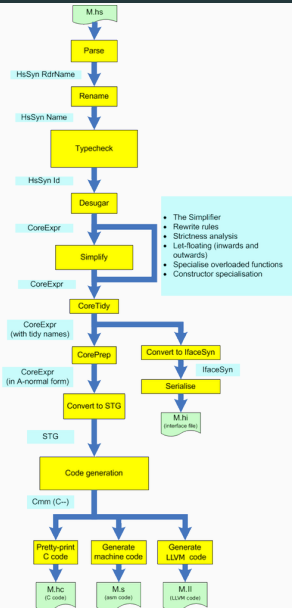
- HsSyn — Abstract Syntax Tree (AST).

- RdrName, Name — names with additional info.

AST for Haskell source code (HsSyn), module

```haskell
data HsModule name
  = HsModule {
    hsmodName :: Maybe
                 (Located ModuleName),
    hsmodExports :: Maybe
                 (Located [LIE name]),
    hsmodImports :: [LImportDecl name],
    hsmodDecls :: [LHsDecl name],
    -- ...
    }
```

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```haskell
type LHsDecl id = Located (HsDecl id)

data HsDecl id
  = TyClD      (TyClDecl id)
  | InstD      (InstDecl  id)
  | DerivD     (DerivDecl id)
  | ValD       (HsBind id)
  | SigD       (Sig id)
  | DefD       (DefaultDecl id)
  | ForD       (ForeignDecl id)
  ...
```

```
...
| WarningD    (WarnDecls id)
| AnnD        (AnnDecl id)
| RuleD       (RuleDecls id)
| VectD       (VectDecl id)
| SpliceD     (SpliceDecl id)
| DocD        (DocDecl)
| RoleAnnotD  (RoleAnnotDecl id)
```
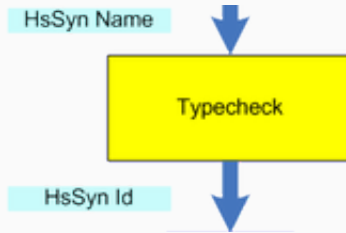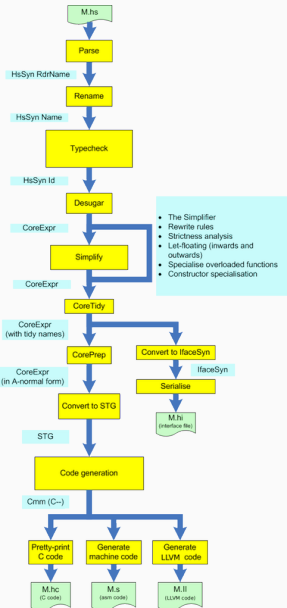
- Id—name with type (every expression has type at this step—either explicitly specified, or inferred)

Compilation pipeline (3): Core and optimisations

- The Simplifier
- Rewrite rules
- Strictness analysis
- Let-floating (inwards and outwards)
- Specialise overloaded functions
- Constructor specialisation
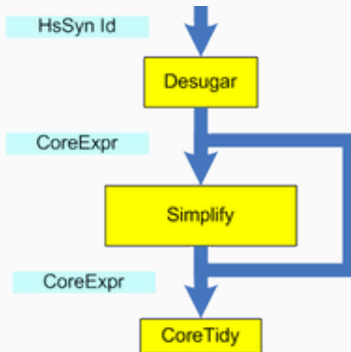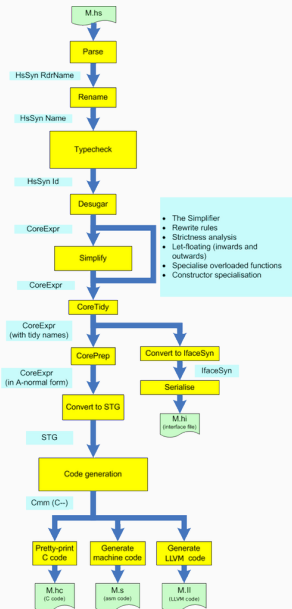
- Desugaring = converting to intermediate language (GHC Core)

- Optimisation

```haskell
type CoreBndr = Var
type CoreExpr = Expr CoreBndr

data Expr b
  = Var    Id
  | Lit    Literal
  | App    (Expr b) (Arg b)
  | Lam    b (Expr b)
  | Let    (Bind b) (Expr b)
  | Case   (Expr b) b Type [Alt b]
  | Cast   (Expr b) Coercion
  | Tick   (Tickish Id) (Expr b)
  | Type   Type
  | Coercion Coercion
```

```haskell
type Arg b = Expr b
type Alt b = (AltCon, [b], Expr b)
data AltCon = DataAlt DataCon
            | LitAlt  Literal
            | DEFAULT
data Bind b = NonRec b (Expr b)
            | Rec [(b, (Expr b))]
```

GHC Core syntax: expressions and patterns

$$
\begin{array}{llll}
t, e, u & ::= & x & \text{Variable} \\
& | & k & \text{Literal} \\
& | & \lambda x : \sigma.e & \text{Abstraction} \\
& | & e\ u & \text{Application} \\
& | & \text{let } \overline{x : \tau = e} \text{ in } u & \text{Local binding} \\
& | & \text{case } e \text{ of } \overline{p \rightarrow u} & \text{Case expression} \\
& | & e \triangleright \gamma & \text{Type casting} \\
& | & \tau & \text{Type} \\
& | & \lfloor \gamma \rfloor & \text{Type coercion} \\
p & ::= & K\ \overline{c : \eta}\ \overline{x : \tau} & \text{Patterns}
\end{array}
$$

- Overline — list of subterms

$$\tau, \ \kappa, \ \sigma, \ \phi \qquad ::=$$
$$| \quad n$$
$$| \quad \tau_1 \ \tau_2$$
$$| \quad T \ \overline{\tau_i}^{\ i}$$
$$| \quad \tau_1 \to \tau_2$$
$$| \quad \forall n.\tau$$
$$| \quad \textsf{lit}$$
$$| \quad \tau \triangleright \gamma$$
$$| \quad \gamma$$

- Simplifier
- Term rewriting rules
- Strictness analysis
- Overloading specialisation
- …

```
f a b = a + b
main = print $ f 2 3
```

GHC Core example

```
f a b = a + b
main = print $ f 2 3

$ ghc -c ex.hs -ddump-simpl
```

```
main :: IO ()
[GblId, Str=DmdType]
main =
 print
  @ Integer
  GHC.Show.$fShowInteger
  (+ @ Integer
      GHC.Num.$fNumInteger
      2 3)
```

Marlow and Peyton Jones, 2012:

*In practice Core has been incredibly stable: over a 20-year time period we have added exactly one new major feature to Core (namely coercions and their associated casts). Over the same period, the source language has evolved enormously. We attribute this stability not to our own brilliance, but rather to the fact that Core is based directly on foundational mathematics: **bravo Girard!***
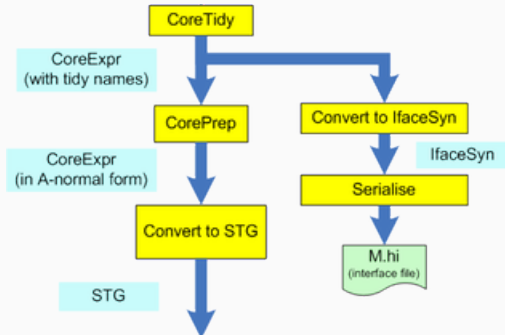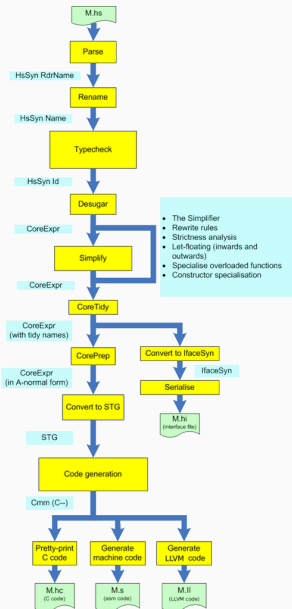
- Constructing *normal form*
- Generating interface files (for cross-module optimisations).
- STG — Spineless Tagless G-machine

STG code fragment

```
sat_s10R :: GHC.Integer.Type.Integer
[LclId, Str=DmdType] =
    \u srt:SRT:[rp0 :-> GHC.Num.$fNumInteger] []
        let {
          sat_s10Q [Occ=Once] ::
                        GHC.Integer.Type.Integer
          [LclId, Str=DmdType] =
              NO_CCS GHC.Integer.Type.S#! [3#]; } in
        let {
          sat_s10P [Occ=Once] ::
                        GHC.Integer.Type.Integer
          [LclId, Str=DmdType] =
              NO_CCS GHC.Integer.Type.S#! [2#];
            } in   GHC.Num.+ GHC.Num.$fNumInteger
                            sat_s10P sat_s10Q;
```

```
Main.main :: GHC.Types.IO ()
[GblId, Str=DmdType] =
  \u srt:SRT:[0B :-> System.IO.print,
              rLs :-> GHC.Show.$fShowInteger,
              s10R :-> sat_s10R] []
  System.IO.print GHC.Show.$fShowInteger sat_s10R;
```
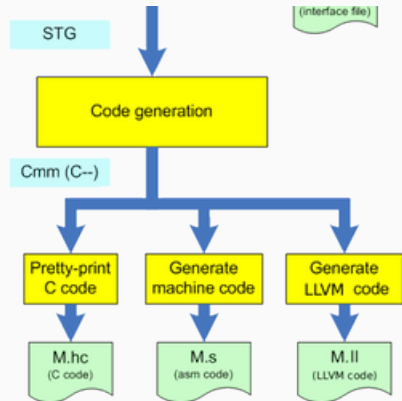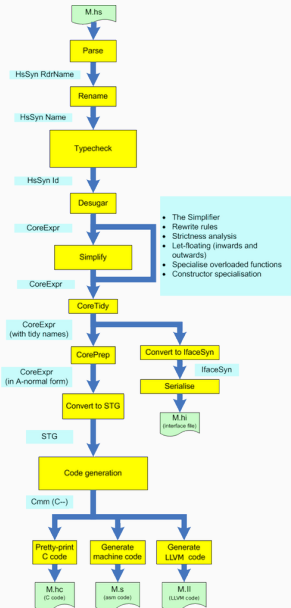
- Cmm — low-level imperative language with explicit stack

Cmm code fragment

```
I64[(old + 24)] = stg_bh_upd_frame_info;
I64[(old + 16)] = _c10Y::I64;
I64[Hp - 24] = GHC.Integer.Type.S#_con_info;
I64[Hp - 16] = 3;
_c111::P64 = Hp - 23;
I64[Hp - 8] = GHC.Integer.Type.S#_con_info;
I64[Hp] = 2;
_c112::P64 = Hp - 7;
R2 = GHC.Num.$fNumInteger_closure;
I64[(old + 48)] = stg_ap_pp_info;
P64[(old + 40)] = _c112::P64;
P64[(old + 32)] = _c111::P64;
call GHC.Num.+_info(R2) args: 48, res: 0, upd: 24;
```

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- Bytecode
- Native code
- C-code
- LLVM IR (LLVM intermediate representation)

- User-defined rewriting rules
- Compiler plugins
- GHC as a library (GHC API)

Term rewriting rules

```
{-# RULES "fold/build"
  forall k z (g::forall b.
              (a->b->b) -> b -> b) .
      foldr k z (build g) = g k z
 #-}
```

- Semantically correct transformation (should
  be proved somehow outside GHC)

- Plugin is a single optimisation step iteration—a function from Core to Core
- Enabling by ghc option or source code pragma
- Plugin annotations to specify exactly when to run transformation

- Steps modularity
- Every step is a function
- Compiler can be part of a user application

Simon L. Peyton Jones, The implementation of functional programming languages. 1987. 445 pp.

- Part 1. Compiling high-level functional languages
- Part 2. Graph reduction
- Part 3. Advanced graph reduction

Untyped λ-calculus

Types and programming languages, Pierce, 2002

Syntax

t	::=		*terms:*
	x		*variable*
	λx.t		*abstraction*
	t t		*application*

| v | ::= | | *values:* |
| | λx.t | | *value-abstraction* |

Evaluation (operational semantics) $\boxed{t \rightarrow t'}$

$$\frac{t_1 \rightarrow t_1'}{t_1 \; t_2 \rightarrow t_1' \; t_2} \qquad \text{(E-App1)}$$

$$\frac{t_2 \rightarrow t_2'}{v_1 \; t_2 \rightarrow v_1 \; t_2'} \qquad \text{(E-App2)}$$

$$(\lambda x.t_{12}) \; v_2 \rightarrow [x \mapsto v_2]t_{12} \qquad \text{(E-AppAbs)}$$

- Boolean constants with operations
- Natural numbers with operations
- Pairs, records, lists

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Options

- Coding these notions in λ-calculus itself
- Adding new constructs

Repository:
https://github.com/bravit/
tt-ghc-exercises

Untyped λ-calculus: Exercises

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```
$ stack build
$ stack exec untyped -- untyped/test.f
```

Simply typed λ-calculus (STLC, $\lambda\rightarrow$)

Syntax

t	::=	*terms:*
	x	*variable*
	λ x :T .t	*abstraction*
	t t	*application*
v	::=	*values:*
	λ x :T .t	*value-abstraction*
T	::=	*types:*
	T → T	*function type*
Γ	::=	*contexts:*
	∅	*empy context*
	Γ,x :T	*binding term variable*

Evaluation $\boxed{t \to t'}$

$$\frac{t_1 \to t'_1}{t_1 \ t_2 \to t'_1 \ t_2} \qquad \text{(E-App1)}$$

$$\frac{t_2 \to t'_2}{v_1 \ t_2 \to v_1 \ t'_2} \qquad \text{(E-App2)}$$

$$(\lambda \ x : T_{11} . t_{12}) \ v_2 \to [x \mapsto v_2]t_{12}$$
$$\text{(E-AppAbs)}$$

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Typing $\boxed{\Gamma \vdash t \, : \quad T}$

$$\frac{x \, : \quad T \in \Gamma}{\Gamma \vdash x \, : \quad T} \qquad \text{(T-Var)}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 \, : \quad T_2}{\Gamma \vdash \lambda x : T_1 . t_2 \, : \quad T_1 \, \to \, T_2} \qquad \text{(T-Abs)}$$

$$\frac{\Gamma \vdash t_1 \, : \quad T_{11} \, \to \, T_{12} \qquad \Gamma \vdash t_2 \, : \quad T_{11}}{\Gamma \vdash t_1 \, t_2 \, : \quad T_{12}}$$

(T-App)

- Basic types (Bool, Nat,…)
- Constants and operations for basic types (true, false, if/then/else, 0, succ, pred, iszero)
- Evaluation and typing rules

New typing rules $\boxed{\Gamma \vdash \mathtt{t} \; : \quad \mathtt{T}}$

$$\vdash \mathtt{true} \; : \quad \mathtt{Bool} \qquad \text{(T-True)}$$

$$\vdash \mathtt{false} \; : \quad \mathtt{Bool} \qquad \text{(T-False)}$$

$$\frac{\Gamma \vdash \mathtt{t_1} \; : \quad \mathtt{Bool} \qquad \Gamma \vdash \mathtt{t_2} \; : \quad \mathtt{T} \qquad \Gamma \vdash \mathtt{t_3} \; : \quad \mathtt{T}}{\Gamma \vdash \mathtt{if} \; \mathtt{t_1} \; \mathtt{then} \; \mathtt{t_2} \; \mathtt{else} \; \mathtt{t_3} \; : \quad \mathtt{T}}$$
$$\text{(T-If)}$$

$$\vdash 0 : \mathtt{Nat} \qquad \text{(T-Zero)}$$

$$\frac{\Gamma \vdash t_1 : \mathtt{Nat}}{\Gamma \vdash \mathtt{succ}\ t_1 : \mathtt{Nat}} \qquad \text{(T-Succ)}$$

$$\frac{\Gamma \vdash t_1 : \mathtt{Nat}}{\Gamma \vdash \mathtt{pred}\ t_1 : \mathtt{Nat}} \qquad \text{(T-Pred)}$$

$$\frac{\Gamma \vdash t_1 : \mathtt{Nat}}{\Gamma \vdash \mathtt{iszero}\ t_1 : \mathtt{Bool}} \qquad \text{(T-IsZero)}$$

Type inference trees: Example

$$\cfrac{\cfrac{\cfrac{x\!:\!\mathsf{Bool} \in x\!:\!\mathsf{Bool}}{x\!:\!\mathsf{Bool} \vdash x\!:\!\mathsf{Bool}}\ \text{T-Var}}{\vdash \lambda x\!:\!\mathsf{Bool}.x\ :\ \mathsf{Bool} \to \mathsf{Bool}}\ \text{T-Abs} \qquad \cfrac{}{\vdash \mathsf{true}\ :\ \mathsf{Bool}}\ \text{T-True}}{\vdash (\lambda x\!:\!\mathsf{Bool}.x)\ \mathsf{true}\ :\ \mathsf{Bool}}\ \text{T-A}$$

Safety = progress + preservation

Safety = progress + preservation

- Progress: correctly typed term is either value or can be further evaluated
- Preservation: if correctly typed term is evaluated then resulting term is correctly typed

- Inversion lemmas (reversing typing rules)
- Type is uniquely identified
- Canonical forms (`succ (succ (succ 0))`)
- Preserving types in substitution
- Shuffling and weakening contexts
- Normalization
- Explicit and implicit typing

Types are not used during evaluation so they can be erased

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$$erase\,(\mathtt{x}) \qquad\qquad = \quad \mathtt{x}$$
$$erase\,(\lambda \mathtt{x}\!:\!\mathtt{T}_1.\mathtt{t}_2) \;=\; \lambda \mathtt{x}. \quad \mathtt{erase}\,(\mathtt{t}_2)$$
$$erase\,(\mathtt{t}_1\ \mathtt{t}_2) \qquad = \quad erase\,(\mathtt{t}_1)\,erase\,(\mathtt{t}_2)$$

Types are not used during evaluation so they can be erased

$$erase(x) = x$$
$$erase(\lambda x{:}T_1.t_2) = \lambda x.\ \ \texttt{erase}(t_2)$$
$$erase(t_1\ t_2) = erase(t_1)\,erase(t_2)$$

Property: evaluation commutes with erasing

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```
$ stack build
$ stack exec stlc -- stlc/test.f
```

The problem: code duplication in STLC

```
doubleNat  = λf:Nat → Nat.
                 λx:Nat. f (f x)
doubleBool = λf:Bool → Bool.
                 λx:Bool. f (f x)
doubleFun  =
  λf:(Nat → Nat) → (Nat → Nat).
     λx:Nat →  Nat. f (f x)
```
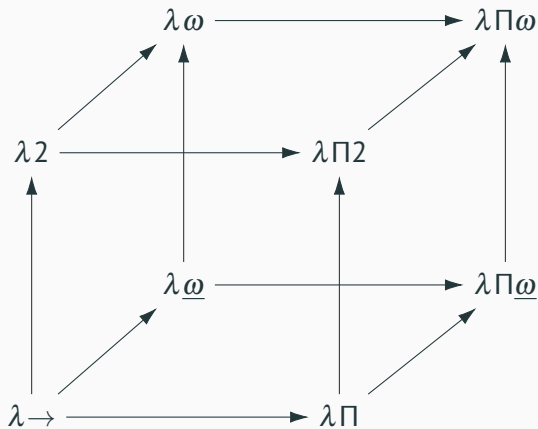
System F

polymorphic λ-calculus, $\lambda 2$

Polymorphic functions

```
id : ∀X. X ->  X
id = ΛX. λx:X. x
idNat = id [Nat]
```

Polymorphic functions

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id : ∀X. X -> X
id = ΛX. λx:X. x
idNat = id [Nat]

double : ∀X. (X -> X) -> X -> X
double = ΛX. λf:X -> X. λa:X. f (f a)
doubleNat = double [Nat]
```

Polymorphic functions

```
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quadruple = ΛX. double [X -> X]
                       (double [X])
```

Polymorphic functions

```
id : ∀X. X ->  X
id = ΛX. λx:X. x
idNat = id [Nat]

double : ∀X. (X ->  X) -> X -> X
double = ΛX. λf:X -> X. λa:X. f (f a)
doubleNat = double [Nat]

quadruple = ΛX. double [X -> X]
                     (double [X])
```

- Parametric polymorphism

Syntax (extends STLC)

t	::=	*terms:*
	x	*variable*
	λx:T.t	*abstraction*
	t t	*application*
	ΛX.t	*type abstraction*
	t [T]	*type application*

v	::=	*values:*
	λx:T.t	*value-abstraction*
	ΛX.t	*value-type abstraction*

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T	::=	*types:*
	X	*type variable*
	T→T	*function type*
	∀X.T	*universal type*
Γ	::=	*contexts:*
	∅	*empty context*
	Γ, x : T	*binding term variable*
	Γ, X	*binding type variable*

Evaluation $\boxed{t \rightarrow t'}$

$$\frac{t_1 \rightarrow t_1'}{t_1 \ t_2 \rightarrow t_1' \ t_2} \qquad \text{(E-App1)}$$

$$\frac{t_2 \rightarrow t_2'}{v_1 \ t_2 \rightarrow v_1 \ t_2'} \qquad \text{(E-App2)}$$

$$(\lambda x{:}T_{11}.t_{12}) \ v_2 \rightarrow [x \mapsto v_2]t_{12} \quad \text{(E-AppAbs)}$$

$$\frac{t_1 \to t_1'}{t_1 \ [T_2] \to t_1' \ [T_2]} \qquad \text{(E-TApp)}$$

$$(\Lambda X. t_{12}) \ [T_2] \to [X \mapsto T_2]t_{12} \qquad \text{(E-TappTabs)}$$

Typing $\boxed{\Gamma \vdash t : \quad T}$

$$\frac{x : \quad T \in \Gamma}{\Gamma \vdash x : \quad T} \quad \text{(T-Var)}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \lambda x : T_1 . t_2 : \quad T_1 \rightarrow T_2} \quad \text{(T-Abs)}$$

$$\frac{\Gamma \vdash t_1 : \quad T_{11} \rightarrow T_{12} \qquad \Gamma \vdash t_2 : \quad T_{11}}{\Gamma \vdash t_1 \ t_2 : \quad T_{12}}$$
$$\text{(T-App)}$$

$$\frac{\Gamma, X \vdash t_2 \; : \quad T_2}{\Gamma \vdash \Lambda X.t_2 \; : \quad \forall X.T_2} \qquad \text{(T-TAbs)}$$

$$\frac{\Gamma \vdash t_1 \; : \quad \forall X.T_{12}}{\Gamma \vdash t_1 \; [T_2] \; : \quad [X \mapsto T_2]T_{12}} \qquad \text{(T-TApp)}$$

- Progress and preservation

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- Normalization

System F properties

- Progress and preservation
- Normalization
- We need `fix` to define non-normalized functions

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- Possibility of type erasure

System F properties

- Progress and preservation
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- We need `fix` to define non-normalized functions
- Possibility of type erasure
- (!!!) type reconstruction is undecidable

- Limited System F: prenex polymorphism (type variables range over types without quantifiers), rank 2 polymorphism

- Limited System F: prenex polymorphism (type variables range over types without quantifiers), rank 2 polymorphism
- Impredicativity (quantified variable can be defined object itself)

Practising with System F: Haskell

```
double : ∀X. (X ->  X) -> X -> X
double = ΛX. λf:X -> X. λa:X. f (f a)

quadruple = ΛX. double [X -> X]
                       (double [X])
```

```
double : ∀X. (X ->  X) -> X -> X
double = ΛX. λf:X -> X. λa:X. f (f a)

quadruple = ΛX. double [X -> X]
                       (double [X])

{-# NOINLINE double #-}
double :: (a -> a) -> a -> a
double f x = f (f x)

{-# NOINLINE quadruple #-}
quadruple = double double
```

Practising with System F: Core

```
double_rpX :: forall a. (a -> a) -> a -> a
double_rpX
  = \ (@ a_ayq) (f_as1 :: a_ayq -> a_ayq)
                (x_as2 :: a_ayq) ->
      f_as1 (f_as1 x_as2)

quadruple_rrY :: forall a. (a -> a) -> a -> a
quadruple_rrY
  = \ (@ a_ayv) ->
        double_rpX @ (a_ayv -> a_ayv)
                     (double_rpX @ a_ayv)
```

System F$_\omega$ ($\lambda\omega$)

Functions over types, kinds

```
Id = ΛX. X
```

Functions over types, kinds

```
Id = ΛX. X

Id = ΛX::*. X
```

Functions over types, kinds

```
Id = ΛX. X

Id = ΛX::*. X

Pair = ΛA::*. ΛB::*.∀X.
            (A ->  B ->  X) -> X
```

Functions over types, kinds

```
Id = ΛX. X

Id = ΛX::*. X

Pair = ΛA::*. ΛB::*.∀X.
          (A ->  B ->  X) -> X

Pair :: * -> * -> *
```

Functions over types, kinds

```
Id = ΛX. X

Id = ΛX::*. X

Pair = ΛA::*. ΛB::*.∀X.
         (A ->  B ->  X) -> X

Pair :: * -> * -> *

PairNB = Pair [Nat] [Bool]
```

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- Kinds: $*$, $* \to *$, $* \to * \to *$, ...
- Kinding relation (kind assignment) $\Gamma \vdash T :: K$
- Definitional equality ($S \equiv T$):

  Id Nat $\to$ Id Bool $\equiv$ Nat $\to$ Bool
- Parallel reduction: directed version of definitional equality

Sample rules

$$\frac{\Gamma, X::K_1 \vdash T_2::K_2}{\Gamma \vdash \Lambda X::K_1 . T_2 \ :: \ K_1 \rightarrow K_2} \qquad \text{(K-Abs)}$$

$$\frac{\Gamma, X::K_1 \vdash T_2::*}{\Gamma \vdash \forall X::K_1 . T_2 \ :: \ *} \qquad \text{(K-All)}$$

$$\frac{S_2 \equiv T_2}{\forall X::K_1 . S_2 \equiv \forall X::K_1 . T_2} \qquad \text{(Q-All)}$$

Modifications in typing rules (fragment)

$$\frac{\Gamma, X :: K_1 \vdash t_2 \ : \ T_2}{\Gamma \vdash \lambda X :: K_1 . t_2 \ : \ \forall X :: K_1 . T_2} \quad \text{(T-TAbs)}$$

$$\frac{\Gamma \vdash t_1 \ : \ \forall X :: K_{11} . T_{12} \qquad \Gamma \vdash T_2 \ :: \ K_{11}}{\Gamma \vdash t_1 \ [T_2] \ : \ [X \mapsto T_2] T_{12}} \quad \text{(T-TApp)}$$

$$\frac{\Gamma \vdash t \ : \ S \qquad S \equiv T \qquad \Gamma \vdash T \ :: \ *}{\Gamma \vdash t \ : \ T} \quad \text{(T-Eq)}$$

- Untyped λ-calculus
- Simply typed λ-calculus
- System F
- System F$_\omega$

For many years, GHC's intermediate language was essentially:

- *System Fw, plus*
- *algebraic data types (including existentials)*

But that is inadequate to describe GADTs and associated types. So in 2006 we extended GHC to support System FC, which adds

- *equality constraints and coercions*

GHC Commentary: The Compiler

```haskell
data Point x = Point x x

{-# NOINLINE xCoord #-}

xCoord (Point x _) = x
yCoord (Point _ y) = y

main = print $ xCoord (Point 2 3)
```

Algebraic data types: GHC Core

```
xCoord_rq1 :: forall x. Point x -> x
xCoord_rq1
  = \ (@ x_a1dk) (ds_d1y8 :: Point x_a1dk) ->
        case ds_d1y8 of { Point x1_as6 ds1_d1ye
                            -> x1_as6 }

main :: IO ()
main
  = print
      @ Integer
      GHC.Show.$fShowInteger
      (xCoord_rq1 @ Integer
                  (Main.Point @ Integer 2 3))
```

Intermediate language GHC Core in System F_ω period

$$
\begin{array}{llll}
t, e, u & ::= & x & \text{Variables} \\
& | & K & \text{Data constructors} \\
& | & k & \text{Literals} \\
& | & \lambda x : \sigma.e \mid e\, u & \text{Abstraction and application (values)} \\
& | & \Lambda a : \eta.e \mid e\, \varphi & \text{Abstraction and application (types)} \\
& | & \text{let } \overline{x : \tau = e} \text{ in } u & \text{Local binding} \\
& | & \text{case } e \text{ of } \overline{p \to u} & \text{Case expression} \\
p & ::= & K\, \overline{c : \eta}\ \overline{x : \tau} & \text{Patterns} \\
\end{array}
$$

Generalized algebraic datatypes and associated types, System FC

[Peyton Jones et al., 2006; Schrijvers et al., 2009]

```
data Exp a where
  Zero :: Exp Int
  Succ :: Exp Int -> Exp Int
  Pair :: Exp b -> Exp c -> Exp (b, c)
```

```
eval :: Exp a -> a
eval Zero = 0    -- Int
eval (Succ e) = eval e + 1    -- Int
eval (Pair x y) = (eval x, eval y)
                                -- (b, c)
```

```
eval :: Exp a -> a
eval Zero = 0    -- Int
eval (Succ e) = eval e + 1    -- Int
eval (Pair x y) = (eval x, eval y)
                            -- (b, c)

res = eval (Pair (Succ Zero) Zero)
```

```
eval :: Exp a -> a
eval Zero = 0    -- Int
eval (Succ e) = eval e + 1    -- Int
eval (Pair x y) = (eval x, eval y)
                              -- (b, c)

res = eval (Pair (Succ Zero) Zero)

      -- (Int, Int)
```

System FC: basic ideas

[Sulzmann et al., 2007, rewritten in 2009–2011]

- Current context contains two syntactically different types (say, a and **Int**) which are in fact the same types (are *coercible*)

[Sulzmann et al., 2007, rewritten in 2009–2011]

- Current context contains two syntactically different types (say, a and **Int**) which are in fact the same types (are *coercible*)

- We introduce *coercion*, some evidence of *coercibility*, into the context

System FC: basic ideas

[Sulzmann et al., 2007, rewritten in 2009–2011]

- Current context contains two syntactically different types (say, a and **Int**) which are in fact the same types (are *coercible*)
- We introduce *coercion*, some evidence of *coercibility*, into the context
- Now it is safe to do type *cast*

```
data Exp a where
  Zero : ∀a.(a ~ Int) => Exp a
  Succ : ∀a.(a ~ Int) =>
                  Exp Int -> Exp a
  Pair : ∀abc.(a ~ (b,c)) =>
            Exp b -> Exp c -> Exp a
```

- **Zero** has additional argument—evidence of coercibility of a to **Int**
- **Exp** is regular ADT now

```
zero : Exp Int
zero = Zero ???
```

```
zero : Exp Int
zero = Zero ???
```

- reflexivity: any type is a coercion to itself

```
eval : Exp a -> a
eval =
  Λa:*.λx:Exp a.
   case x of
    Zero (co : a~Int) -> 0 ▶ sym co
    ...
```

- cast operator: ▶
- symmetry in coercion:
  (sym co : Int ∼ a)
- ▶ casts to the right hand side of ∼

```
eval : Exp a -> a
eval =
  Λa:*.λx:Exp a.
   case x of
    Zero (co : a~Int) -> 0 ▶ sym co
    ...
```

- cast operator: ▶
- symmetry in coercion:
  (sym co : Int ∼ a)
- ▶ casts to the right hand side of ∼
- Why coercion is a type and not a value?

```haskell
class Collects c where
  type Elem c
  empty  :: c
  insert ::  Elem c -> c -> c

instance Eq e => Collects [e] where
  type Elem [e] = e
  ...
instance Collects BitSet where
  type Elem BitSet = Char
  ...
```

```
foo :: Char -> BitSet
foo x = insert x empty
```

Original type classes and instances implementation

- Type class is translated into record (ADT with one constructor, dictionary)
- Every class method becomes record's field
- Instance is a value of this record, fields are methods' implementations
- Every function with this constraint receives additional argument—the dictionary with an implementation

Original type classes and instances implementation

- Type class is translated into record (ADT with one constructor, dictionary)
- Every class method becomes record's field
- Instance is a value of this record, fields are methods' implementations
- Every function with this constraint receives additional argument—the dictionary with an implementation
- Record cannot contain types!

Type class (Haskell)

```
class Collects c where
  type Elem c
  empty  :: c
  insert ::  Elem c -> c -> c
```

Abstract type constructor and type of dictionary (Core)

```
type Elem : * -> *
data CollectsDict c =
  Collects {empty : c;
      insert : Elem c -> c -> c}
```

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Instance (Haskell)

```
instance Collects BitSet where
  type Elem BitSet = Char
  ...
```

Coercion axiom and dictionary (Core)

```
axiom elemBS : Elem BitSet ~ Char

dCollectsBS : CollectsDict Bitset
dCollectsBS = ...
```

Method call (Haskell)

```
foo :: Char -> BitSet
foo x = insert x empty
```

Function call (GHC Core)

```
foo : Char -> BitSet
foo =
  λx:Char. insert [BitSet] dCollectBS
          (x ► sym elemBS)
          (empty [BitSet] dCollectBS)

axiom elemBS : Elem BitSet ~ Char
```

System FC [Sulzmann et al., 2007]

- Coercions as types and arguments passing in System F style, erasability
- Operators over coercions (sym etc.) and their normalization
- Abstract type constructors and axioms
- Typing and kinding rules
- Progress and preservation
- Operational semantics
- Erasure properties
- Rules for translation from Haskell

Bugs in type-level computations and System FC$_2$

GHC extension GeneralizedNewtypeDeriving

```haskell
newtype Age = MkAge { unAge :: Int }
class Idea a where
  good :: a -> a
instance Idea Int where
  good = (+1)
deriving instance Idea Age
```

GHC extension GeneralizedNewtypeDeriving

```haskell
newtype Age = MkAge { unAge :: Int }
class Idea a where
  good :: a -> a
instance Idea Int where
  good = (+1)
deriving instance Idea Age

axiom CoMkAge : Age ~ Int
```

```
axiom CoMkAge : Age ~ Int
```

Question

Let T is a function over types. What about
`T Age ~ T Int`?

```haskell
newtype Age = MkAge { unAge :: Int }

type family Inspect x
type instance Inspect Age = Int
type instance Inspect Int = Bool

class BadIdea a where
  bad :: a -> Inspect a
instance BadIdea Int where
  bad = (> 0)
deriving instance BadIdea Age
```

Which type is it?

```
bad (MkAge 5)
```

```
bad (MkAge 5) :: Int
```

Which type is it?

```
bad (MkAge 5) :: Int
```

What GND builds?

```
axiom CoMkAge : Age ~ Int
```

Which type is it?

```
bad (MkAge 5) :: Int
```

What GND builds?

`axiom CoMkAge : Age ~ Int`

GND would use implementation (>0) from instance for **Int**

Which type is it?

```
bad (MkAge 5) :: Int
```

What GND builds?

axiom CoMkAge : Age ~ Int

GND would use implementation (>0) from instance for **Int** so we'd access internal **Bool** representation! Congrats!

Which type is it?

```
bad (MkAge 5) :: Int
```

What GND builds?

axiom CoMkAge : Age ~ Int

GND would use implementation (>0) from instance for **Int** so we'd access internal **Bool** representation! Congrats!

Inspect Age ≁ Inspect Int

Which type is it?

```
bad (MkAge 5) :: Int
```

What GND builds?

```
axiom CoMkAge : Age ~ Int
```

GND would use implementation (>0) from instance for **Int** so we'd access internal **Bool** representation! Congrats!

```
Inspect Age ≁ Inspect Int
```

But `Maybe Age ~ Maybe Int`

```
bad (MkAge 5) :: Int
```

```
axiom CoMkAge : Age ~ Int
```

GND would use implementation (>0) from instance for **Int** so we'd access internal **Bool** representation! Congrats!

```
Inspect Age ≁ Inspect Int
```

```
But Maybe Age ~ Maybe Int
```

Problem
When is it OK to lift ~?

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[Weirich et al., 2011; *Roles (GHC Dev wiki)*]

Type can play different roles:

- encoding
- representation

Roles idea

[Weirich et al., 2011; *Roles (GHC Dev wiki)*]

Type can play different roles:

- encoding
- representation

Here come different equalities:

- nominal equality (up to type synonyms)
- representational equality

Parametric and non-parametric type var occurences

```haskell
data List a = Nil | Cons a (List a)
data GADT a where
  GAge :: GADT Age
  GInt :: GADT Int
class C1 a where
  foo :: a -> List a
class C2 a where
  bar :: a -> GADT a
class BadIdea a where
  bad :: a -> Inspect a
```

Lifting $\sim$ and roles

- If type variable has parametric occurence then we may lift $\sim$ in case of representational equality

- Otherwise, lifting is allowed only in case of nominal equality

Lifting $\sim$ and roles

- If type variable has parametric occurence then we may lift $\sim$ in case of representational equality

- Otherwise, lifting is allowed only in case of nominal equality

Question
What is the right place for type roles?

Types and kinds in System FC$_2$

```
k   ::=  * | k' -> k           kinds
R   ::=  n | r                 roles
k'  ::=  k/R
```

Types and kinds in System FC$_2$

```
k   ::=  * | k' -> k          kinds
R   ::=  n | r               roles
k'  ::=  k/R
```

Example

```
Maybe   ::  */r -> *    parametric
Inspect ::  */n -> *    non-parametric
axiom CoMkAge : (Age ~ Int)/r
```

Example

```
Maybe   ::  */r -> *  parametric
Inspect ::  */n -> *  non-parametric
axiom CoMkAge : (Age ~ Int)/r
```

$$\frac{\begin{array}{l} \Gamma \vdash \gamma_1 : \varphi_1 \sim \varphi_2 \in (\eta_1/R_2 \to \eta_2)/R_1 \\ \Gamma \vdash \gamma_2 : \psi_1 \sim \psi_2 \in \eta_1/min(R_1, R_2) \end{array}}{\Gamma \vdash \gamma_1 \, \gamma_2 : \varphi_1 \, \psi_1 \sim \varphi_2 \, \psi_2 \in \eta_2/R_1} \quad \text{CAPP}$$

- $n \leqslant r$
- With Maybe we can use whatever we like
- With Inspect—nominal role types only

- Roles for differing type application contexts
- Roles as a kind component
- Progress and preservation
- Updated translation rules
- Simplifying System FC

"Typing" type-level computations and System F$_C^\uparrow$

Traditional implementation

```
data Zero
data Succ n

data Vec :: * -> * -> * where
  VNil :: Vec a Zero
  VCons :: a -> Vec a n ->
                Vec a (Succ n)
```

- `Zero :: *`
- `Succ :: * -> *`
- `Vec  :: * -> * -> *`

- `Zero :: *`
- `Succ :: * -> *`
- `Vec :: * -> * -> *`

- No control at the type level!
- `Succ Bool` or `Vec Zero Int` are allowed!

[Giving Haskell a Promotion: Yorgey et al., 2012]

```haskell
data Nat = Zero | Succ Nat
data Vec :: * -> Nat -> * where
  VNil :: Vec a Zero
  VCons :: a -> Vec a n ->
                  Vec a (Succ n)
```

[Giving Haskell a Promotion: Yorgey et al., 2012]

```haskell
data Nat = Zero | Succ Nat
data Vec :: * -> Nat -> * where
  VNil :: Vec a Zero
  VCons :: a -> Vec a n ->
                   Vec a (Succ n)
```

- DataKinds, PolyKinds GHC extensions
- Type Nat becomes a kind
- Data constructors Zero and Succ become types

Computations over types (literally)

Addition over types

```haskell
type family Plus (a :: Nat) (b :: Nat)
type instance Plus Zero b = b
type instance Plus (Succ a) b =
                    Succ (Plus a b)
```

Function over vectors

```
vconcat :: Vec a n -> Vec a m ->
                      Vec a (Plus n m)
vconcat VNil ys = ys
vconcat (VCons x xs) ys =
          VCons x (vconcat xs ys)
```

Type equality *

```
data EqRefl a b where
  Refl :: EqRefl a a
```

- How to define equality for type constructors, say Maybe?

Type equality *

```
data EqRefl a b where
  Refl :: EqRefl a a
```

- How to define equality for type constructors, say Maybe?

PolyKinds implementation

```
data EqRefl (a :: X) (b :: X) where
  Refl :: ∀X . ∀(a :: X). EqRefl a a
```

$$
\begin{array}{lll}
e, \ u & ::= & \text{Expressions} \\
& | \ x & \text{Variables} \\
& | \ \lambda x{:}\,\tau. \ e \ | \ e_1 \ e_2 & \text{Abstraction/application} \\
& | \ \Lambda a{:}\,\kappa. \ e \ | \ e \ \tau & \text{Type abstraction/application} \\
& | \ \lambda c{:}\,\tau. \ e \ | \ e \ \gamma & \text{Coercion abstraction/application} \\
& | \ \Lambda \mathcal{X}. \ e & \text{Kind abstraction} \\
& | \ e \ \kappa & \text{Kind application} \\
& | \ K & \text{Data constructors} \\
& | \ \mathbf{case} \ e \ \mathbf{of} \ \overline{p \to u} & \text{Case analysis} \\
& | \ e \rhd \gamma & \text{Casting}
\end{array}
$$

Syntax of System F$_C^\uparrow$ [Yorgey et al., 2012]

$$
\begin{array}{llll}
e,\ u & ::= & & \text{Expressions} \\
& | & x & \text{Variables} \\
& | & \lambda x{:}\tau.\ e \mid e_1\ e_2 & \text{Abstraction/application} \\
& | & \Lambda a{:}\kappa.\ e \mid e\ \tau & \text{Type abstraction/application} \\
& | & \lambda c{:}\tau.\ e \mid e\ \gamma & \text{Coercion abstraction/application} \\
& | & \Lambda \mathcal{X}.\ e & \text{Kind abstraction} \\
& | & e\ \kappa & \text{Kind application} \\
& | & K & \text{Data constructors} \\
& | & \textbf{case}\ e\ \textbf{of}\ \overline{p \rightarrow u} & \text{Case analysis} \\
& | & e \triangleright \gamma & \text{Casting} \\
\end{array}
$$

$$
bnd \quad ::= q{:}\tau \mid w{:}\kappa \mid \boxed{\mathcal{X}{:}\square} \qquad \text{Bindings}
$$

ι	$::=$		Base kinds
	$\mid \star$		Star
	$\mid$ Constraint		Constraint kind

$\kappa,\ \eta$	$::=$		Kinds
	$\mid \mathcal{X}$		Kind variables
	$\mid \iota$		Base kinds
	$\mid \kappa_1 \to \kappa_2$		Arrow kinds
	$\mid \forall\, \mathcal{X}.\, \kappa$		Kind polymorphism
	$\mid T\, \overline{\kappa}$		Promoted type constant

- Kinds can't be classified, there is only one "sort" BOX
- Only *-kinded type constructors are promoted
- No promotion for types with promoted types inside them
- No promotion for types with polymorphic kinds
- No promotion for functions!

Giving Haskell a Promotion [Yorgey et al., 2012]

- Enriching kind system
- Extending GHC Core to System $F_C^{\uparrow}$
- Every required type-theoretic property is guaranteed
- More optimisations available
- Changes in type inference algorithm for Haskell

Explicit kind equality

System FC with explicit kind equality

[Weirich, Hsu, and Eisenberg, 2013]

- Unifying types and kinds: $* :: *$
- Kind equality
- Kinds may depend on types
- Same type-theoretic properties
- Weakening System $F_C^{\uparrow}$ limitations
- Extension TypeInType (GHC 8.0, `Type :: Type`)

GHC Core by 2015

[System FC, as implemented in GHC 2015]

$e,\ u$	$::=$		Expressions, $coreSyn/CoreSyn.lhs$:`Expr`
	$\|$	n	`Var`: Variable
	$\|$	lit	`Lit`: Literal
	$\|$	$e_1\ e_2$	`App`: Application
	$\|$	$\lambda n.e$	`Lam`: Abstraction
	$\|$	$\textbf{let}\ binding\ \textbf{in}\ e$	`Let`: Variable binding
	$\|$	$\textbf{case}\ e\ \textbf{as}\ n\ \textbf{return}\ \tau\ \textbf{of}\ \overline{alt_i}^{\,i}$	`Case`: Pattern match
	$\|$	$e \triangleright \gamma$	`Cast`: Cast
	$\|$	$e_{\{tick\}}$	`Tick`: Internal note
	$\|$	τ	`Type`: Type
	$\|$	γ	`Coercion`: Coercion

$$\tau, \ \kappa, \ \sigma, \ \phi \qquad ::=$$

$$\begin{array}{ll}
| & n \\
| & \tau_1 \ \tau_2 \\
| & T \ \overline{\tau_i}^{\ i} \\
| & \tau_1 \to \tau_2 \\
| & \forall n.\tau \\
| & \text{lit} \\
| & \tau \rhd \gamma \\
| & \gamma
\end{array}$$

- Typing rules
- Kinding rules
- Coercion operators
- Roles
- Core linting

1. Adam Gundry. Type Inference, Haskell and Dependent Types. PhD thesis, University of Strathclyde, 2013 (advisor: Conor McBride).
2. Richard A. Eisenberg. Dependent Types in Haskell: Theory and Practice. PhD Thesis, University of Pennsylvania, 2016 (advisor: Stephanie Weirich).

Theses structure

- Outer language (inch — Dependent Haskell)
- Type inference in the outer language
- Inner language (language of evidence — PiCo)
- Translating outer language to inner language
- Programming with dependent types
- Implementation details

A Specification for Dependently-Typed Haskell

STEPHANIE WEIRICH, University of Pennsylvania

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RICHARD EISENBERG, Bryn Mawr College

Presented in ICFP '2017

Design Haskell extension such that:

- no difference between types and terms
- sharing semantics between compile-time and runtime computations

- System DC as backward compatible with System FC
- System D as implicitly typed version of System DC:
  - erasure
  - reconstruction
  - source of ideas for type inference in outer language
- Fully formalized and proved in Coq (SSReflect).

Fixed-size vector: Example

```
data Nat :: Type where
  O :: Nat
  S :: Nat -> Nat

data Vec :: Type -> Nat -> Type where
  Nil  :: Vec a O
  (:>) :: a -> Vec a m -> Vec a (S m)
```

Type for vector with full info about types

```
data Vec (a :: Type) (n :: Nat) :: Type where
  Nil :: (n ~ 0) => Vec a n
  (:>) :: ∀ (m :: Nat). (n ~ S m)
             => a -> Vec a m -> Vec a n
```

Type for vector with full info about types

```
data Vec (a :: Type) (n :: Nat) :: Type where
  Nil :: (n ∼ 0) => Vec a n
  (:>) :: ∀ (m :: Nat). (n ∼ S m)
            => a -> Vec a m -> Vec a n
```

Function over vectors

```
zip :: ∀ n a b. Vec a n -> Vec b n -> Vec (a,b)
zip Nil Nil = Nil
zip (x :> xs) (y :> ys) = (x, y) :> zip xs ys
```

- ∀ for erasable components
- It is enough to have two clauses

Function zip in SystemDC

```
zip =
 \-n:Nat. \-a:Type. \-b:Type.
 \xs:Vec a n. \ys:Vec b n.
    case xs of
      --ALT1
      --ALT2
```

```
--ALT1

Nil -> /\c1:(n ~ 0).
 case ys of
  Nil -> /\c2:(n ~ 0). Nil [a][n][c1]
  (:>) -> \-m:Nat. /\c2:(n ~ S m).
           \y:b. \ys:Vec b m.
               absurd [sym c1; c2]
```

```
--ALT2
(:>) ->
  \m:Nat. /\c1:(n ~ S m).
  \x:a. \xs:Vec a m. case ys of
   Nil -> /\c2:(n ~ 0).
                  absurd [sym c1; c2]
   (:>) -> \-m:Nat. /\c2:(n ~ S m).
           \y:b. \ys:Vec b m.
             (:>) [a][n][m][c1]
                 ((,) [a][b] x y)
                 (zip [m][a][b] xs ys)
```

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```
zip = \-n. \-a. \-b. \xs. \ys.
 case xs of
  Nil -> /\c1. case ys of
    Nil -> /\c2. Nil [][][]
    (:>) -> \-m. /\c2. \y. \ys.
                            absurd []
  (:>) -> \m. /\c1. \x. \xs. case ys of
    Nil -> /\c2. absurd []
    (:>) -> \-m. /\c2. \y. \ys.
            (:>) [][][][]
                  ((,) [][] x y)
                  (zip [][][] xs ys)
System D
```

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terms, types	a, b, A, B	::=	$\star \mid x \mid F \mid \lambda^\rho x.b \mid a\, b^\rho \mid \square$
		$\mid$	$\Pi^\rho x{:}A \to B \mid \Lambda c.a \mid a[\gamma] \mid \forall c{:}\phi.A$
propositions	ϕ	::=	$a \sim_A b$
relevance	ρ	::=	$+ \mid -$
coercions	γ	::=	$\bullet$
values	v	::=	$\lambda^+ x.a \mid \lambda^- x.v \mid \Lambda c.a \mid \star \mid \Pi^\rho x{:}A \to B \mid \forall c{:}\phi.A$
contexts	Γ	::=	$\varnothing \mid \Gamma, x : A \mid \Gamma, c : \phi$
available set	Δ	::=	$\varnothing \mid \Delta, c$
signature	Σ	::=	$\varnothing \mid \Sigma \cup \{F \sim a : A\}$

E-AppAbs

$$\overline{\vDash (\lambda^\rho x.b)\ a^\rho \rightsquigarrow b\{a/x\}}$$

E-CAppCAbs

$$\overline{\vDash (\Lambda c.b)[\gamma] \rightsquigarrow b\{\gamma/c\}}$$

E-Axiom

$$\frac{F \sim a : A \in \Sigma_0}{\vDash F \rightsquigarrow a}$$

E-AbsTerm

$$\frac{\vDash a \rightsquigarrow a'}{\vDash \lambda^- x.a \rightsquigarrow \lambda^- x.a'}$$

E-AppLeft

$$\frac{\vDash a \rightsquigarrow a'}{\vDash a\ b^\rho \rightsquigarrow a'\ b^\rho}$$

E-CAppLeft

$$\frac{\vDash a \rightsquigarrow a'}{\vDash a[\gamma] \rightsquigarrow a'[\gamma]}$$

Typing rules in System D

$$\boxed{\Gamma \vDash a : A}$$

E-STAR
$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star}$$

E-VAR
$$\frac{\vDash \Gamma \qquad x : A \in \Gamma}{\Gamma \vDash x : A}$$

E-PI
$$\frac{\Gamma, x : A \vDash B : \star}{\Gamma \vDash \Pi^\rho x{:}A \to B : \star}$$

E-ABS
$$\frac{\Gamma, x : A \vDash a : B \qquad \rho \vee x \notin a}{\Gamma \vDash \lambda^\rho x.a : \Pi^\rho x{:}A \to B}$$

E-APP
$$\frac{\Gamma \vDash b : \Pi^+ x{:}A \to B \qquad \Gamma \vDash a : A}{\Gamma \vDash b\, a^+ : B\{a/x\}}$$

E-IAPP
$$\frac{\Gamma \vDash b : \Pi^- x{:}A \to B \qquad \Gamma \vDash a : A}{\Gamma \vDash b\, \Box^- : B\{a/x\}}$$

E-CONV
$$\frac{\Gamma \vDash a : A \qquad \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star}{\Gamma \vDash a : B}$$

E-FAM
$$\frac{\vDash \Gamma \qquad F \sim a : A \in \Sigma_0}{\Gamma \vDash F : A}$$

E-CPI
$$\frac{\Gamma, c : \phi \vDash B : \star}{\Gamma \vDash \forall c{:}\phi.B : \star}$$

E-CABS
$$\frac{\Gamma, c : \phi \vDash a : B}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B}$$

E-CAPP
$$\frac{\Gamma \vDash a_1 : \forall c{:}(a \sim_A b).B_1 \qquad \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}}$$

Typing rules in System D

$$\boxed{\Gamma \vDash a : A}$$

E-Star
$$\frac{\vDash \Gamma}{\Gamma \vDash \star : \star}$$

E-Var
$$\frac{\vDash \Gamma \qquad x : A \in \Gamma}{\Gamma \vDash x : A}$$

E-Pi
$$\frac{\Gamma, x : A \vDash B : \star}{\Gamma \vDash \Pi^\rho x{:}A \to B : \star}$$

E-Abs
$$\frac{\Gamma, x : A \vDash a : B \qquad \rho \vee x \notin a}{\Gamma \vDash \lambda^\rho x.a : \Pi^\rho x{:}A \to B}$$

E-App
$$\frac{\Gamma \vDash b : \Pi^+ x{:}A \to B \qquad \Gamma \vDash a : A}{\Gamma \vDash b \, a^+ : B\{a/x\}}$$

E-IApp
$$\frac{\Gamma \vDash b : \Pi^- x{:}A \to B \qquad \Gamma \vDash a : A}{\Gamma \vDash b \, \square^- : B\{a/x\}}$$

E-Conv
$$\frac{\Gamma \vDash a : A \qquad \Gamma; \widetilde{\Gamma} \vDash A \equiv B : \star}{\Gamma \vDash a : B}$$

E-Fam
$$\frac{\vDash \Gamma \qquad F \sim a : A \in \Sigma_0}{\Gamma \vDash F : A}$$

E-CPi
$$\frac{\Gamma, c : \phi \vDash B : \star}{\Gamma \vDash \forall c{:}\phi.B : \star}$$

E-CAbs
$$\frac{\Gamma, c : \phi \vDash a : B}{\Gamma \vDash \Lambda c.a : \forall c{:}\phi.B}$$

E-CApp
$$\frac{\Gamma \vDash a_1 : \forall c{:}(a \sim_A b).B_1 \qquad \Gamma; \widetilde{\Gamma} \vDash a \equiv b : A}{\Gamma \vDash a_1[\bullet] : B_1\{\bullet/c\}}$$

$$\boxed{\rho \vee x \notin A}$$

Rho-Rel
$$\frac{}{+ \vee x \notin A}$$

Rho-IrrRel
$$\frac{x \notin \mathsf{fv}A}{- \vee x \notin A}$$

- definitional equality (many rules)
- Progress and preservation

- System D plus explicit types in abstractions proofs for coercions
- Erasing to System D
- Decidable syntax-directed typing
- Uniqueness of typing
- Evaluation
- Progress and preservation
- No full System FC implementation

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