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Course: Parallel Algorithms and Complexity

Assignment - 8
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Problem 1

Pf' orientation of C_1 and Pf' orientation of C_2 yield a Pf' orientation of $C_1 \oplus C_2$

Solution: For any simple cycle C let f be the number of edges in forward direction and k be the number of vertices strictly inside C. Then we will show if for C_1 and C_2 we have f_1, k_1 and f_2, k_2 respectively and $f_1 + k_1 \equiv 1 \pmod 2$ and $f_2 + k_2 \equiv 1 \pmod 2$ then so is for the cycle $C := C_1 \oplus C_2$.

Now let the path P shared by C_1 and C_2 contains m vertices. So the number of vertices of C is $k = k_1 + k_2 + b$. Since all the edges of P is either in forward direction with respect to C_1 or C_2 if we take $f_1 + f_2$ then this number has all the edges of P. So the number of edges in forward direction of C is $f = f_1 + f_2 - (b+1)$. Hence

$$k + f = (k_1 + k_2 + b) + (f_1 + f_2 - (b+1)) = (k_1 + f_1) + (k_2 + f_2) - 1 \equiv 1 \pmod 2$$

So we obtain a Pf' orientation of $C_1 \oplus C_2$

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Problem 2

Given any matrix of univariate polynomials of degree $\leq n^{O(1)}$ then prove that the coefficent of x^i in the determinant of the matrix is in $GapL$

Solution: Suppose the given matrix is M. Now we replace the entries of M with new variables. So the (i,j)th entry of M is replaced by the variable $x_{i,j}$. Suppose the new matrix obtained is M'. Now using Mahajan-Vinay's method we obtain an arithmetic branching program which computes the determinant of M'. Now in the ABP we replace every $x_{i,j}$ with the (i,j)th polynomial in M. So this new ABP now computed the determinant of M. Let the source vertex of this is s and target vertex is t. Let $\deg(\det M) = d$.

We will do now homogenization of the ABP. We will start from right. Apart from the target vertex of ABP we replace each vertex v of the ABP with $d+1$ many vertices $v^{(0)}, v^{(1)}, \dots, v^{(d)}$ going from right to left. Where $v^{(i)}$ computes the $\deg i$ term of the polynomial obtained from the ABP by making v the source vertex and the target vertex is same as before. Here by $v^{(i)}$ we mean that polynomial also. Thus the polynomial obtained by making $s^{(i)}$ the source vertex and target vertex t same as before we get the coefficient of x^i in $\det M$.

To homogenize let before there was an edge (u,v) with weight $p(x) = a_d x^d + \cdots + a_1 x + a_0$. Since v is on right side of u, v is replaced with $v^{(0)}, v^{(1)}, \dots, v^{(d)}$. Now we first replace u with $u^{(0)}, u^{(1)}, \dots, u^{(d)}$. Now obviously we have $u^{(i)} = \sum\limits_{j=0}^{i} a_j x^j v^{(i-j)}$. So for $0 \leq j \leq i$ we join the edges $(u^{(i)}, v^{(i-j)})$ with weight $a_j x^j$. We keep on doing this from right to left.

In the end the source vertex is splited into $d+1$ vertices. Here $s^{(i)}$ computes the coefficient of x^i in $\det M$ multiplied by x^i. Now if we replace the variable x in this new homogenized ABP with 1 then we can say that $s^{(i)}$ computes the coefficient of x^i in $\det M$. So now we reduced that coefficient of x^i is $\det M$ is the value of the ABP whose source vertex is $s^{(i)}$ and target vertex is t, same as before. Since ABP is in $GapL$ we have coefficient of x^i in $\det M$ is in $GapL$.

□

Problem 3

Give a dicirculation of a bidirected graph G using non-vanishing circulation

Solution: Let the bidirected graph is $G = (V, E)$. From G we create a new graph where for all $(u, v), (v, u) \in E$ we introduce a new vertex $t_{u,v}$ and the edges $\{u, t_{u,v}\}, \{t_{u,v}, v\}$. So this new graph call this $\tilde{G}$.

Now let $\tilde{G}$ has any non vanishing circulation. So for any edge $\{x, y\} \in E(\tilde{G})$ we denote the weight of the edge as $w(x, y)$. Now we define the weights in G such that

$$w'(u, v) = w(u, t_{u,v}) - w(t_{u,v}, v)$$

where w' is the weight of the edge $(u, v) \in E$. We claim this is a dicirculation of G. To prove let C be any cycle in G. Let $C = u_0 u_1 u_2 \cdots u_{2k-1} u_1$. Then

$$w(C) = \sum_{e \in C} w(e) = \sum_{i=0}^{2k-1} w'(u_i, u_{i+1}) = \sum_{i=0}^{2k-1} w(u_i, t_{u_i,u_{i+1}}) - w(t_{u_i,u_{i+1}}, v) \neq 0$$

Since this is true for any cycle we obtain a dicirculation.

$\square$