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Course: Algorithmic Coding Theory

Assignment - 1
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Problem 1 Chapter 1

Ex 1.18

Solution:

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Problem 2 Chapter 2

Ex 2.13

Solution:

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Problem 3 Chapter 2

Ex 2.16

Solution:

(a) Since G has full rank, $rank(G) = k$. Therefore in the reduced column echelon form of G the first k columns forms a identity matrix I_k. We denote the matrix formed by the rest $n-k$ columns by A. Since the reduced column echelon form of a matrix and the matrix generate the same vector space they are equivalent. And since the reduced column echelon form can be obtained through the Gaussian elimination method we can convert G to a matrix G' of the form $G' = [I_k|A]$ in polynomial time where G' and G are equivalent.

(b) We should have $GH^T = 0$ where G is of the form $G = [I_k|A]$. where A is a $k \times (n-k)$ matrix. Take $H = [-A^T|I_{n-k}]$. Suppose we denote $G = (g_{i,j})_{\substack{1\leq i\leq k \\ 1\leq j\leq n}}$ and $H = (h_{i,j})_{\substack{1\leq i\leq n \\ 1\leq j\leq n-k}}$. Let $C = GH^T = (c_{i,j})_{\substack{1\leq i\leq k \\ 1\leq j\leq n-k}}$

$$c_{i,j} = \sum_{m=1}^{n} g_{i,m}h_{m,j} = \sum_{m=1}^{k} \delta_{i,m}h_{m,j} + \sum_{m=k+1}^{n} g_{i,m}\delta_{m-k,j} = h_{i,j} + g_{i,k+j} = -a_{i,j} + a_{i,j} = 0$$

So we get every entry of C is 0. Hence $GH^T = 0$. Therefore H is the parity check matrix of G and since H is of the form $H = [-A^T|I_{n-k}]$ so it has full rank $n-k$. Hence H is a parity check matrix.

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Problem 4 Chapter 2

Ex 2.17

Solution:

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Problem 5 Chapter 5

Ex 5.8

Solution:

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Problem 6 Chapter 5

Ex 5.15

Solution:

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Problem 7 Chapter 5

Ex 5.16

Solution:

1. We have $f(X+Z) = \sum_{i=0}^{t} r_i(X)Z^i$. Now differentiating f with respect to Z we have

$$f'(X+Z) = \sum_{i=0}^{t-1}(i+1)r_{i+1}(X)Z^i$$

Let for $n = k-1$ we have

$$f^{(k-1)}(X+Z) = \sum_{i=0}^{t-k+1} \frac{(i+k-1)!}{i!} r_{i+k-1}(X)Z^i$$

Denote $\frac{(i+k-1)!}{i!} r_{i+k-1}(X) = g_i(X)$. Then for $n = k$ we have

$$\begin{aligned} f^{(k)}(X+Z) &= \sum_{i=0}^{t-k}(i+1)g_{i+1}Z^i = \sum_{i=0}^{t-k} \frac{((i+1)+k-1)!}{i!} r_{(i+1)+k-1}(X)Z^i \\ &= \sum_{i=0}^{t-k} \frac{(i+k)!}{i!} r_{i+k}(X)Z^i \end{aligned}$$

Hence by mathematical induction we have

$$f^{(n)}(X+Z) = \sum_{i=0}^{t-n} \frac{(i+n)!}{i!} r_{i+n}(X)Z^i$$

Therefore

$$f^{(n)}(X) = f^{(n)}(X+0) = \sum_{i=0}^{t-n} \frac{(i+n)!}{i!} r_{i+n}(X)0^i = \frac{n!}{0!} r_n(X) = n! r_n(X)$$

2. Let $char(\mathbb{F}_q) = m$. So $j \geq m$. Hence $j! = j(j-1)\cdots(m+1)m(m-1)! = mk$ where $k = j(j-1)\cdots(m+1)(m-1)!$. Since $f^{(j)}(X) = j!r_j(X) = m\left(kr_j(X)\right) \equiv 0$.

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Problem 8 Chapter 5

Ex 5.17

Solution:

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