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Course: Parallel Algorithms and Complexity

Assignment - 6
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Problem 1

Find Parallel Implementation of Boruvka's Algorithm of Minimum Spanning Tree

Solution: We will describe a parallel implementation of boruva's algorithm of minimum spanning tree

Step 1: From each of the vertex the edge with minimum weight is taken, parallely

Step 2: In each of the tree formed the one with the smaller number is designated as the root of the component. The component is the tree. Each vertex finds the root of the tree it belongs, parallely in each component and renames it with the root vertices.

Step 3: The edges of all vertices in a component are merged ie each component is now contracted to a vertex.

Step 4: The in each component the edges inside the component are removed and new edge list is performed.

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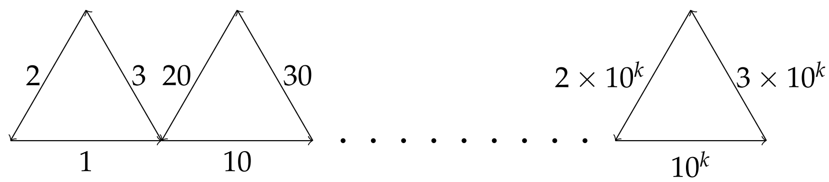
Problem 2

Show a counter example that Edmond's Algorithm is not Parallelizable.

Solution: This process is recursed again and again until one vertex is remained.



In each iteration of Edmond's algorithm get exactly one cycle which is a triangle. First we will get the left most one. Since at each step we get only one cycle we have no hope for parallely computing for all disjint cycles.

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Problem 3

In 3-connected planar graphs faces are precisely nonseparating induced cycles of the graph.

Solution: Let G is a 3-connected planar graoh. Let $C \subseteq G$ is a non-separating induced cycle. Since it is a non separating cycle all the points in $G - C$ appear in the same side of the cycle. Since C is induced cycle it created 2 faces and only one of them contains all the points in $G - C$. Therefore C bounds a face of G.
Lemma : In a 2-connected planar graph every face is bounded by a cycle.
Proof: Assume that there is a face a which is not bounded by a cycle. Choose a vertex v so that the boundary walk of a passes through the vertex v twice. Then we may draw a closed curve starting and ending at v with interior contained in a. This curve separates the plane into two components each of which must contain a vertex of G, so we find that v is a cut vertex. Thus, G is not 2-connected.

Suppose C is bounds a face f. By the Lemma C is a cycle. Let C has a chord $e = (u, v)$. Since G is 3-connected $G - \{x, y\}$ is connected. There exists a C-path P between the components of $C - \{u, v\}$ since G is 3-connected. But this is not possible as both the edge e and the path P runs thriugh the same face created by C. Which is not possible because if $P_!, P_2, P_3$ are three arcs between the same endpoints then if P is an arc between a point in P_1 and a point in P_3 whose interior lies in the region $\mathbb{R}^2 - (P_1 \cup P_3)$ that contains P_2 then P contains a interior point of P_2. Here P_1, P_3 are the path between the components along the cycle and P_2 is the chord. Hence we have that a face in a 3-connected planar graph is induced.

Now wwe have to shoe that face is non separating. Let $x, y \in G - C$. By Menger's Theorem x and y are linked in G by three independent paths since G is 3-connected. Therefore f must completely lie in one of 3 faces described by the paths. f can be bounded by only two paths among them. Therefore the third avoids f and its boundary C.

So every face is is precisely its non separating induced cycles.

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Problem 4

Prove that Fundamental Cycles form a cycle basis.

Solution: Suppose $\mathcal{C}$ denote the cycle space. Let the fundamental cycles, $\mathcal{F}$ are obtained from the spanning tree T. So for each edge $e \notin E(T)$ the fundamental cycle created by joining e is C_e. So the fundamental cycles are linearly independent because for each C_e is the only fundamental cycle obtained from e because

$$\mathop{\triangle}_{C_e \in \mathcal{F}} C_e = \{e \mid e \in E(G) - E(T)\}$$

Hence the fundamental cycles are linearly independent.

Now let $H \in \mathcal{C}$.Let $e_1, e_2, \ldots, e_k$ are the edges in H which are not in T. Then let

$$H' = H \triangle \left(\mathop{\triangle}_{i \in [k]} C_{e_i} \right)$$

This H' is a elements in the cycle space. Now each e_i appear one time in H and one time in C_{e_i}. Hence in H' none of the e_i edges are remaining. Since apart from the edges e_i all the edges of H and C_{e_j}'s are the edges of the tree T we can say H' is a subgraph of T. Since H' is a subgraph of T it is not a cycle. But it is also an element of the cycle space. Hence $E(H') = \phi$. Therefore $H = \mathop{\triangle}_{i \in [k]} C_{e_i}$. Therefore the fundamental cycles form a cycle basis of the cycle space.

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