

1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

1. Dadas las siguientes expresiones en sintaxis concreta de nuestro lenguaje MiniLisp (a) obtener su sintaxis abstracta, (b) evaluarlas usando las reglas de semántica natural y (c) evaluarlas usando las reglas de semántica estructural. Todas las reglas las podrán consultar en la Nota de Clase 6 y la Nota de Clase 7.

- $(- (+ 20 3) (- -18 (+ 50 20)))$

- (a) *Syntax abstracta* :

Sub (Add (Num(20), Num(3)) , Sub (Num (-18), Add (Num(50), Num(20))))

- (b) Evaluación por semántica natural :

$$\frac{\frac{\frac{Num(50) \Rightarrow Num(50) \quad Num(70) \Rightarrow Num(70)}{Num(20) \Rightarrow Num(20) \quad Num(3) \Rightarrow Num(3)}}{Num(-18) \Rightarrow Num(-18) \quad Add(Num(50), Num(20)) \Rightarrow Num(70)}}{Add(Num(20), Num(3)) \Rightarrow Num(23)} \\ \frac{Sub(Num(-18), Add(Num(50), Num(20))) \Rightarrow Num(-88)}{Sub(Add(Num(20), Num(3)), Sub(Num(-18), Add(Num(50), Num(20)))) \Rightarrow Num(111)}$$

- (c) Evaluación por semántica estructurada :

$$Sub(Add(Num(20), Num(3)), Sub(Num(-18), Add(Num(50), Num(20))))$$
$$\longrightarrow Sub(Num(23), Sub(Num(-18), Add(Num(50), Num(20))))$$
$$\longrightarrow Sub(Num(23), Sub(Num(-18), Num(70)))$$
$$\longrightarrow Sub(Num(23), Num(-88))$$
$$\longrightarrow Num(111)$$

- (not (+ 1 (- 3 (+ -8 1))))

- a) $(not(add(Num(1), sub(Num(3), (Add(Num(-8), Num(1)))))))$

- b)

$Num(11) \Rightarrow Num(11)$
$Not(11) \Rightarrow Boolean(False)$
$Num(-8) \Rightarrow Num(-8) \quad Num(1) \Rightarrow Num(1)$
$Num(3) \Rightarrow Num(3) \quad (Add(Num(-8), Num(1))) \Rightarrow (-8 + 1) \Rightarrow -7$
$(Add(Num(1), Sub(Num(3), (Add(Num(-8), Num(1))))) \Rightarrow (1 + 10) \Rightarrow 11$
$(Not(Add(Num(1), Sub(Num(3), (Add(Num(-8), Num(1))))) \Rightarrow (Not(11)) \Rightarrow Boolean(False)$

- c) $(not(add(Num(1), sub(Num(3), (Add(Num(-8), Num(1)))))))$

$$\rightarrow (not(add(Num(1), sub(Num(3), (Num(-7))))))$$
$$\rightarrow (not(add(Num(1), Num(10))))$$
$$\rightarrow (not(11))$$
$$\rightarrow (Boolean(False))$$

- (not (not (+ 3 5)))

- a) `not(not(Add(Num(3),Num(5))))`

1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

b)

Not(<i>False</i>) \Rightarrow Boolean(<i>True</i>)
Num(8) \Rightarrow Num(8)
Not(8) \Rightarrow Boolean(<i>False</i>)
Num(3) \Rightarrow Num(3)
Num(5) \Rightarrow Num(5)
Add(Num(3), Num(5)) \Rightarrow Num(8)
not(not(Add(Num(3), Num(5)))) \Rightarrow Not(8) \Rightarrow Boolean(<i>True</i>)

c) not(not(Add(Num(3), Num(5))))

$$\rightarrow not(not(Num(8)))$$
$$\rightarrow \text{not}(\text{not}(8))$$
$$\rightarrow not(Boolean(False))$$
$$\rightarrow Boolean(True)$$

2. Como segundo ejercicio deberán extender la batería de operaciones de MiniLisp, para ello deberán (a) dar la gramática libre de contexto modificada (en notación EBNF) añadiendo las nuevas construcciones del lenguaje, (b) modificar las reglas de sintaxis abstracta para considerar los nuevos constructores y finalmente (c) extender las reglas de semántica natural y estructural. En los tres casos, deberás usar la notación formal que vimos en clase.

- Veamos como es la Gramática Libre de Contexto:

```
< SinCero > ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

$\langle \text{Expresiones} \rangle ::= \langle \text{Mult} \rangle \mid \langle \text{Div} \rangle \mid \langle \text{Incr} \rangle \mid \langle \text{Decr} \rangle \mid \langle \text{Raiz} \rangle$

```
< Mult > ::= "(" "*" < Num > < Num > ")"
```

```
< Div > ::= "(" "/" < Num > < SinCero > ")"
```

```
< Incr > := "(" "add1" < Num > ")"
```

$$\langle \text{Decr} \rangle := "(" \text{ "sub1" } \langle \text{Num} \rangle ") "$$

```
< Raiz > ::= "(" "sqrt" < Num > ")"
```

- Especificar un nuevo constructor $*$ para la multiplicación binaria de expresiones aritméticas.
Por ejemplo:

$i \quad (* \quad 20 \quad 2) \quad 40$

- Regla de sintaxis abstracta :

$Mult(i, d)$ es un ASA si tanto i como d son ASAs, entonces también :

$$\frac{iASA \quad dASA}{Mult(i, d)ASA}$$

1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- Reglas de semántica natural :

$$\frac{i \Rightarrow Num(n_1) \quad d \Rightarrow Num(n_2)}{Mult(i, d) \Rightarrow Num(i * d)}$$

- Reglas de semántica estructurada:

$$\frac{i \rightarrow i'}{Mult(i, d) \rightarrow Mult(i', d)}$$

$$\frac{d \rightarrow d'}{Mult(Num(n_1), d) \rightarrow Mult(Num(n_1), d')}$$

$$\overline{Mult(Num(n_1), Num(n_2))} \rightarrow Num(n_1 * n_2)$$

- Especificar un nuevo constructor / para la división binaria de expresiones aritméticas. Consideren que no se pueden realizar divisiones entre cero. Por ejemplo:

- (/ 20 2)

- 10

$$- \begin{pmatrix} / & 10 & 0 \end{pmatrix}$$

- error: División entre cero

- Regla de sintaxis abstracta :

$Div(i, d)$ es un ASA si tanto i como d son ASAs tal que $d \neq Num(0)ASA$, entonces también :

$$\frac{iASA \quad dASA}{Div(i, d)ASA}$$

- Reglas de semántica natural :

$$\frac{i \Rightarrow Num(n_1) \quad d \Rightarrow Num(0)}{Div(i, d) \Rightarrow Error}$$

$$\frac{i \Rightarrow Num(n_1) \quad d \Rightarrow Num(n_2)}{Div(i, d) \Rightarrow Num(i/d)}$$

- Reglas de semántica estructurada:

$$\frac{i \rightarrow i'}{Div(i, d) \rightarrow Div(i', d)}$$

$$\frac{d \rightarrow d'}{\text{Div}(\text{Num}(n_1), d) \rightarrow \text{Div}(\text{Num}(n_1), d')}$$

$$\overline{Div(Num(n_1), Num(n_2))} \rightarrow Num(n_1/n_2)$$

- Especificar un nuevo constructor `add1` que dada una expresión, incrementa en uno su valor. Por ejemplo:

```
(add1 10) 11
```

1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- $$\frac{iASA}{Add1(i)ASA}$$

- $$\frac{i \Rightarrow Num(n_1)}{Add1(i) \Rightarrow Num(n_1 + 1)}$$

- $$\frac{i \rightarrow i'}{Add1(i) \rightarrow Add1(i')}$$

$$\frac{iASA}{sub1(i)ASA}$$

- $$\frac{i \Rightarrow Num(n)}{sub1(i) \Rightarrow Num(n-1)}$$

- $$\frac{i \rightarrow n}{sub1(i) \rightarrow sub1(n)}$$

$$\frac{iASA}{sqrt(i)ASA}$$

- $$\frac{i \Rightarrow Num(-n)}{sqrt(i) \Rightarrow Error}$$

1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	0	0	1	0	1	0	1	0	1	1	0	1	1	1	0	1	0	0	1	1	0	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$$\frac{i \Rightarrow Num(n)}{sqrt(i) \Rightarrow Num(Sqrt(n))}$$

- Reglas de semántica estructurada:

$$\frac{i \rightarrow -n}{\text{sqrt}(i) \rightarrow \text{Error}}$$

$$\frac{i \rightarrow n}{\text{sqrt}(i) \rightarrow \text{Sqrt}(n)}$$

$$\overline{sqrt(Num(i)) \rightarrow Num(Sqrt(i))}$$