Introduction to types and type classes in Haskell

Daniel Alonso

- Types and type signatures
- Type classes
  - Functor
  - Applicative

Types and type signatures

Haskell's three levels

Haskell code can be seen at three levels:

Values	Booleans, numbers, lists, functions. . .
Types	"Sets of values."
Kinds	Something else.

Motivation for types

- Static typing is a (lightweight) formal verification method that guarantees the absence of certain classes of errors (e.g. `True + 'c'`).
- The static type of a function is a partial, machine checked specification (e.g. `fst :: (a,b) -> a`).
- Types are a design language.
- Types help with software maintenance.

(Stolen from SPJ's lecture “Adventure with types in Haskell”)

Type signatures

The `::` symbol is read as "is of type."

$$\underbrace{\underset{\text{Value}}{\underset{\uparrow}{\texttt{True}}} \;\; \texttt{::} \;\; \underset{\text{Type}}{\underset{\uparrow}{\texttt{Bool}}}}_{\text{Type signature}}$$

`True` is of type `Bool`

Type signatures

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`True` is of type `Bool`

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []
map f (x:xs) = f x : map f xs
```

Defining types

Capitalized

```
data Language = Haskell
```

Type
constructor

Data
(value)
constructor

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data Language = Haskell
```

Type constructor

Data (value) constructor

In the interpreter

Ask for the type of values with `:t`.

```
Prelude> :t Haskell
Haskell :: Language
```

Defining types

Capitalized

```
data Language = Haskell
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Type constructor

Data (value) constructor

In the interpreter

Ask for the type of values with `:t`.

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Prelude> :t Haskell
Haskell :: Language
```

```
Prelude> :t Language
<interactive>:1:1: error:
Data constructor not in scope:
Language
```

Defining types

Capitalized

`data Language = Haskell`

Type constructor

Data (value) constructor

In the interpreter

Ask for the type of values with `:t`.

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Prelude> :t Haskell
Haskell :: Language
```

```
Prelude> :t Language
<interactive>:1:1: error:
Data constructor not in scope:
Language
```

Same name for type and data constructors is OK:

`data Hask = Hask`

More than one value

```
data Bool = False | True
```

Different data constructors for Bool

In the interpreter

```
Prelude> :t False
False :: Bool

Prelude> :t True
True :: Bool
```

More than one value

```
data Bool = False | True
```

Different data constructors for `Bool`

In the interpreter

```
Prelude> :t False
False :: Bool

Prelude> :t True
True :: Bool
```

Any number of data constructors:

```
data Three = A | B | C
```

$\vdots$

Conceptually:

```
data Int = ... | -1 | 0 | 1 | ...
```

Data constructors with parameters

```
data Point = Point Double Double
```

(Types of the)
parameters to the
data constructor

Data constructors with parameters

```
data Point = Point Double Double
```

(Types of the)
parameters to the
data constructor

In the interpreter

```
Prelude> :t Point
Point :: Double -> Double -> Point

Prelude> :t Point 0.0 0.0
Point 0.0 0.0 :: Point
```

Parametric polymorphism: motivation

Optional value:

```
data MaybeInt = NothingInt | JustInt Int
data MaybeString = NothingString | JustString String
```

$\vdots$

Is there a better way?

Parametric polymorphism

Function definition:

```
positive n = n > 0
```

Function name

parameter declaration (lowercase)

parameter use

Parametric polymorphism

Function definition:

```
positive n = n > 0
```

Function name

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parameter use

Similarly, for types:

```
data Maybe a = Nothing | Just a
```

type parameter declaration

type parameter use

Parametric polymorphism

Function definition:

```
positive n = n > 0
```

Function name

parameter declaration (lowercase)

parameter use

Similarly, for types:

```
data Maybe a = Nothing | Just a
```

type parameter declaration

type parameter use

In the interpreter

```
Prelude> :t Nothing
Nothing :: Maybe a
Prelude> :t Just
Just :: a -> Maybe a
Prelude> :t Just True
Just True :: Maybe Bool
```

Introducing kinds

- "Arity of type constructors:" how many type parameters do they have (and of what kinds)?
- Inhabited types (types with values) have kind `*` (`Bool :: *`).
- Higher kinded type constructors (`Maybe :: * -> *`) have no values.

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In the interpreter

Values $\xrightarrow{\texttt{:t}}$ Types $\xrightarrow{\texttt{:k}}$ Kinds

```
True ::
```

Introducing kinds

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In the interpreter

$$\text{Values} \xrightarrow{\texttt{:t}} \text{Types} \xrightarrow{\texttt{:k}} \text{Kinds}$$

```
True :: Bool
        Bool ::
```

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Values $\xrightarrow{\texttt{:t}}$ Types $\xrightarrow{\texttt{:k}}$ Kinds

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True :: Bool
                Bool :: *
Just ::
```

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Values	$\xrightarrow{:t}$	Types	$\xrightarrow{:k}$	Kinds

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True :: Bool
             Bool :: *
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            Maybe ::
```

Introducing kinds

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Values $\xrightarrow{\texttt{:t}}$ Types $\xrightarrow{\texttt{:k}}$ Kinds

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     True :: Bool
                     Bool :: *
     Just :: a -> Maybe a
                    Maybe :: * -> *
Just True ::
```

Introducing kinds

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In the interpreter

Values $\xrightarrow{\texttt{:t}}$ Types $\xrightarrow{\texttt{:k}}$ Kinds

```
      True :: Bool
                        Bool :: *
      Just :: a -> Maybe a
                       Maybe :: * -> *
 Just True :: Maybe Bool
                  Maybe Bool ::
```

Introducing kinds

- "Arity of type constructors:" how many type parameters do they have (and of what kinds)?
- Inhabited types (types with values) have kind `*` (`Bool :: *`).
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In the interpreter

Values $\xrightarrow{:t}$ Types $\xrightarrow{:k}$ Kinds

```
      True :: Bool
                          Bool :: *
      Just :: a -> Maybe a
                         Maybe :: * -> *
 Just True :: Maybe Bool
                    Maybe Bool :: *
```

Exercises

Given the following types:

```
data Either a b = Left a | Right b
data Maybe a = Nothing | Just a
```

what are the types/kinds of the following values/types?

```
Left
Right
Either
Just (Left True)
```

Exercises

Given the following types:

```
data Either a b = Left a | Right b

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what are the types/kinds of the following values/types?

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Left :: a -> Either a b

Right

Either

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```

Exercises

Given the following types:

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Left :: a -> Either a b
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Either
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Given the following types:

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what are the types/kinds of the following values/types?

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Left :: a -> Either a b
Right :: b -> Either a b
Either :: * -> * -> *
Just (Left True)
```

Exercises

Given the following types:

```
data Either a b = Left a | Right b
data Maybe a = Nothing | Just a
```

what are the types/kinds of the following values/types?

```
Left :: a -> Either a b
Right :: b -> Either a b
Either :: * -> * -> *
Just (Left True) :: Maybe (Either Bool b)
```

Function type

Functions:

Start with	Notation	Example	Convert
letter	prefix	`pAnd True False`	``True `pAnd` False``
non-letter	infix	`True && False`	`(&&) True False`

Function type

Functions:

Start with	Notation	Example	Convert
letter	prefix	`pAnd True False`	``True `pAnd` False``
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The same happens with type constructors!

Start with	Notation	Example	Convert
letter	prefix	`Function Int Bool`	—
non-letter	infix	`Int -> Bool`	`(->) Int Bool`

Function type

Functions:

Start with	Notation	Example	Convert
letter	prefix	`pAnd True False`	``True `pAnd` False``
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The same happens with type constructors!

Start with	Notation	Example	Convert
letter	prefix	`Function Int Bool`	—
non-letter	infix	`Int -> Bool`	`(->) Int Bool`

In the interpreter

```
Prelude> :k (->)
(->) :: * -> * -> *
```

No data constructor for functions:

```
f, g :: Int -> Bool
f n = n > 0
g n = odd n
```

Currying

- Functions only take one parameter (possibly returning other functions):

```haskell
f :: Int -> Int -> Bool
f m n = m < n
```

```haskell
f’ :: Int -> (Int -> Bool)
f’ m = g
    where g n = m < n
```

Currying

- Functions only take one parameter (possibly returning other functions):

```haskell
f :: Int -> Int -> Bool
f m n = m < n
```

```haskell
f' :: Int -> (Int -> Bool)
f' m = g
    where g n = m < n
```

- `(->)` is right associative:
  `a -> b -> c -> d` is the same as `a -> (b -> (c -> d))`.

Currying

- Functions only take one parameter (possibly returning other functions):

```
f :: Int -> Int -> Bool
f m n = m < n
```

```
f’ :: Int -> (Int -> Bool)
f’ m = g
    where g n = m < n
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- `(a -> b) -> c` means the first parameter of the function is a function of type `a -> b`.

Currying

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```haskell
f :: Int -> Int -> Bool
f m n = m < n
```

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f' :: Int -> (Int -> Bool)
f' m = g
    where g n = m < n
```

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In the interpreter

```
Prelude> f :: Int -> Int -> Bool; f m n = m < n
Prelude> :t f
f :: Int -> Int -> Bool
Prelude> :t f 2
f 2 :: Int -> Bool
```

More exercises

Given the functions

```
f :: Int -> Bool -> Int -> Int
f m add n = if add then m + n else m - n

g = f 3 True

g' = f 3 False
```

What is the type of `g`? And of `g'`? What is the value of `g 2`? And of `g' 2`?

Given the type

```
data Either a b = Left a | Right b
```

What is the kind of `Either Int`?

More exercises

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What is the type of `g`? And of `g'`? What is the value of `g 2`? And of `g' 2`?

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g :: Int -> Int
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What is the type of `g`? And of `g'`?

```
g :: Int -> Int
g' :: Int -> Int
```

What is the value of `g 2`? And of `g' 2`?

```
g 2 == 5
g' 2 == 1
```

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Given the type

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```

What is the kind of `Either Int`?

```
Either Int :: * -> *
```

What to remember

- Type definitions consist of type constructors (functions on types) and data constructors (functions on values).
- Types can be seen as sets of values, kinds as the arity of type functions.
- The function type `(->)` takes two type parameters: the domain ("input") and the codomain ("output") types.
- All functions take exactly one parameter, and can return other functions.

Type classes

Type classes: motivation

- Want to test equality with `v1 == v2` for different types: `Bools`, `Ints`, `Strings`... But not for others: `f1 == f2` (equality between functions?).
- `n1 + n2` should work for `Ints`, `Doubles`, `Complex` numbers...
- The executed code will be different for each type.

Type classes: ad hoc polymorphism

- Type classes are sets of types that support certain operations. Think of OOP interfaces, not OOP classes.

```
class Eq a where
    (==) :: a -> a -> Bool
        ⋮
```

```
instance Eq Bool where
    (==) = implementation
        ⋮
```

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```

```
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    (==) = implementation
        ⋮
```

- Subclasses are subsets that support additional operations:

```
class Eq a => Ord a where
    (<=) :: a -> a -> Bool
        ⋮
```

```
instance Ord Bool where
    (<=) = implementation
        ⋮
```

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```

```
instance Ord Bool where
    (<=) = implementation
    ⋮
```

- The compiler can autogenerate instances for some type classes:

```
data Language = Lisp | Haskell
    deriving (Eq, Ord, Show)
```

How to use type classes

In the interpreter

```
Prelude> :k Ord
Ord :: * -> Constraint
```

Constraint

```
sort :: Ord a => [a] -> [a]
sort  = ...
```

We can use `sort` for lists of any type `a` thas is an instance of the `Ord` class.

Multiple constraints possible:

```
f :: (Ord a, Num a) => ...
g :: (Ord a, Ord b) => ...
```

Functor

Functor: motivation

- Lists are polymorphic over the type of its elements (`[Bool]`, `[Int]`...). `map` applies a function to all the elements of a list.
  ```
  map :: (a -> b) -> [a] -> [b]
  ```

Functor: motivation

- Lists are polymorphic over the type of its elements (`[Bool]`, `[Int]`. . .). `map` applies a function to all the elements of a list.
  `map :: (a -> b) -> [a] -> [b]`
- Can we have polymorphism over the type of the container? A `map` that works for `Maybe a`, `[a]`, `Tree a`. . .

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  `map :: (a -> b) -> [a] -> [b]`
- Can we have polymorphism over the type of the container? A `map` that works for `Maybe a`, `[a]`, `Tree a`...
- Not only for containers.

Functor

Function application (space or `($)`) has type:

```
($) :: (a -> b) ->   a ->   b
```

```
f $ x = f x
```

Functor

Function application (space or `($)`) has type:

```
     ($) :: (a -> b) ->   a ->   b
```

```
f $ x = f x
```

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

Functor

Function application (space or `($)`) has type:

```
   ($) :: (a -> b) ->   a ->   b
```

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f $ x = f x
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class Functor f where
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```

In the interpreter

```
Prelude> :k Functor
Functor :: (* -> *) -> Constraint
```

Functor

Function application (space or `($)`) has type:

```
($) :: (a -> b) -> a -> b
```

```
f $ x = f x
```

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

In the interpreter

```
Prelude> :k Functor
Functor :: (* -> *) -> Constraint
```

`Maybe` has kind `* -> *`. Let's see its `Functor` instance:

```
instance Functor Maybe where
    fmap :: (a -> b) -> Maybe a -> Maybe b
    fmap _ Nothing = Nothing
    fmap g (Just x) = Just (g x)
```

Functor laws

`fmap id = id`

> *If the fmapped function does not change the values of type a, nothing changes.*

`fmap (g . h) = (fmap g) . (fmap h)`

> *Fmapping a function all at once or one piece at a time makes no difference.*

Functor laws

`fmap id = id`

> *If the fmapped function does not change the values of type* `a`*, nothing changes.*

`fmap (g . h) = (fmap g) . (fmap h)`

> *Fmapping a function all at once or one piece at a time makes no difference.*

Let's see two rogue instances:

```
instance Functor Maybe where
    fmap _ Nothing = Nothing
    fmap g (Just x) = Nothing
```

Functor laws

`fmap id = id`

If the fmapped function does not change the values of type a, nothing changes.

`fmap (g . h) = (fmap g) . (fmap h)`

Fmapping a function all at once or one piece at a time makes no difference.

Let's see two rogue instances:

```
instance Functor Maybe where
    fmap _ Nothing = Nothing
    fmap g (Just x) = Nothing

instance Functor [] where
    fmap _ [] = []
    fmap g (x:xs) = g x : g x : fmap g xs
```

Functor laws intuition

```
class Functor f where
      fmap :: (a -> b) -> f a -> f b
```

Functor laws intuition

U Can't Touch This!

```
class Functor f where
      fmap :: (a -> b) -> f a -> f b
```

Exercises

Write the `Functor` instance for the following types:

```
data EqPair a =
    EqPair a a
```

```
data DiffPair a b =
    DiffPair a b
```

```
data Validated e a =
    Error e | OK a
```

```
data Phantom a =
    Phantom
```

Exercises

Write the Functor instance for the following types:

```
data EqPair a =
    EqPair a a
```

```
instance Functor EqPair where
    fmap g (EqPair x y) =
        EqPair (g x) (g y)
```

```
data DiffPair a b =
    DiffPair a b
```

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data Validated e a =
    Error e | OK a
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Exercises

Write the Functor instance for the following types:

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data EqPair a =
    EqPair a a
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instance Functor EqPair where
    fmap g (EqPair x y) =
        EqPair (g x) (g y)
```

```
data DiffPair a b =
    DiffPair a b
```

```
instance Functor (DiffPair a) where
```

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data Validated e a =
    Error e | OK a
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data Phantom a =
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```

Exercises

Write the `Functor` instance for the following types:

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data EqPair a =
    EqPair a a
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```
instance Functor EqPair where
    fmap g (EqPair x y) =
        EqPair (g x) (g y)
```

```
data DiffPair a b =
    DiffPair a b
```

```
instance Functor (DiffPair a) where
    fmap g (DiffPair x y) =
        DiffPair x (g y)
```

```
data Validated e a =
    Error e | OK a
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data Phantom a =
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Exercises

Write the Functor instance for the following types:

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data EqPair a =
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instance Functor EqPair where
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```

```
instance Functor (DiffPair a) where
    fmap g (DiffPair x y) =
        DiffPair x (g y)
```

```
data Validated e a =
    Error e | OK a
```

```
instance Functor (Validated e) where
    fmap _ (Error x) = Error x
    fmap g (OK y) = OK (g y)
```

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data Phantom a =
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```

Exercises

Write the `Functor` instance for the following types:

```
data EqPair a =
    EqPair a a
```

```
instance Functor EqPair where
    fmap g (EqPair x y) =
        EqPair (g x) (g y)
```

```
data DiffPair a b =
    DiffPair a b
```

```
instance Functor (DiffPair a) where
    fmap g (DiffPair x y) =
        DiffPair x (g y)
```

```
data Validated e a =
    Error e | OK a
```

```
instance Functor (Validated e) where
    fmap _ (Error x) = Error x
    fmap g (OK y) = OK (g y)
```

```
data Phantom a =
    Phantom
```

```
instance Functor Phantom where
    fmap _ Phantom = Phantom
```

Exercises

Write the Functor instance for the following types:

```
data EqPair a =
    EqPair a a
    deriving (Functor)

data DiffPair a b =
    DiffPair a b
    deriving (Functor)

data Validated e a =
    Error e | OK a
    deriving (Functor)

data Phantom a =
    Phantom
    deriving (Functor)
```

```
{-# LANGUAGE DeriveFunctor #-}
```

What to remember

- The `Functor` type class allows to apply a function through an outer context, without knowing what this context is.
- The context will not change.
- Not only for containers:

In the interpreter

```
capitalize :: String -> String
getLine :: IO String
Prelude> fmap capitalize getLine
"haskell" -- User input
"HASKELL"
```

Applicative

Applicative: motivation (i)

```
data Contact = Contact Phone Email
valPhone :: String -> Maybe Phone
valEmail :: String -> Maybe Email
```

Applicative: motivation (i)

```
data Contact = Contact Phone Email
valPhone :: String -> Maybe Phone
valEmail :: String -> Maybe Email

valContact :: String -> String -> Maybe Contact
valContact sPhone sEmail = case valPhone sPhone of
    Nothing -> Nothing
    Just phone-> case valEmail sEmail of
        Nothing -> Nothing
        Just email -> Just (Contact phone email)
```

Imagine a function with more parameters!

Applicative: motivation (ii)

Why not `fmap` the `Contact` data constructor into a `Maybe Phone`?

Applicative: motivation (ii)

Why not `fmap` the `Contact` data constructor into a `Maybe Phone`?

Remember currying?

```
Contact :: Phone ->  Email -> Contact
```

$$\texttt{Contact :: }\underbrace{\texttt{Phone}}_{\texttt{a}}\texttt{ -> }\underbrace{\texttt{(Email -> Contact)}}_{\texttt{b}}$$

Applicative: motivation (ii)

Why not `fmap` the `Contact` data constructor into a `Maybe Phone`?

Remember currying?

$$\texttt{Contact} :: \texttt{Phone} \to \texttt{Email} \to \texttt{Contact}$$

$$\texttt{Contact} :: \underbrace{\texttt{Phone}}_{\texttt{a}} \to \underbrace{(\texttt{Email} \to \texttt{Contact})}_{\texttt{b}}$$

`fmap` specialized to `Maybe` has this signature:

```
fmap :: (a -> b)
        -> Maybe a
        -> Maybe b
```

Applicative: motivation (ii)

Why not `fmap` the `Contact` data constructor into a `Maybe Phone`?

Remember currying?

$$\texttt{Contact} :: \texttt{Phone} \to \texttt{Email} \to \texttt{Contact}$$

$$\texttt{Contact} :: \underbrace{\texttt{Phone}}_{\texttt{a}} \to \underbrace{(\texttt{Email} \to \texttt{Contact})}_{\texttt{b}}$$

`fmap` specialized to `Maybe` has this signature:

```
fmap :: (a -> b)
     -> Maybe a
     -> Maybe b
```

Let's substitute accordingly:

```
fmap :: (Phone -> (Email -> Contact))
     -> Maybe Phone
     -> Maybe (Email -> Contact)
```

Applicative

```
 ($) ::   (a -> b) ->   a ->   b
fmap ::   (a -> b) -> f a -> f b
 ?? :: f (a -> b) -> f a -> f b
```

Applicative

```
 ($) ::   (a -> b) ->   a ->   b
fmap ::   (a -> b) -> f a -> f b
 ?? :: f (a -> b) -> f a -> f b

class Functor f => Applicative f where
     <*> :: f (a -> b) -> f a -> f b
    pure :: a -> f a
```

Applicative

```
 ($) ::   (a -> b) ->   a ->   b
fmap ::   (a -> b) -> f a -> f b
 ?? :: f (a -> b) -> f a -> f b

class Functor f => Applicative f where
     <*> :: f (a -> b) -> f a -> f b
    pure :: a -> f a

instance Applicative Maybe where
    <*> :: Maybe (a -> b) -> Maybe a -> Maybe b
    (Just f) <*> (Just x) = Just (f x)
    _ <*> _ = Nothing

    pure :: a -> Maybe a
    pure x = Just x
```

Applicative valContact

```haskell
valContact :: String -> String -> Maybe Contact
valContact p e = fmap Contact (valPhone p) <*> (valEmail e)
```

Applicative `valContact`

```haskell
valContact :: String -> String -> Maybe Contact
valContact p e = fmap Contact (valPhone p) <*> (valEmail e)
```

Or better yet, using `<$>` (infix version of `fmap`):

```haskell
valContact p e = Contact <$> valPhone p <*> valEmail e
```

Applicative `valContact`

```
valContact :: String -> String -> Maybe Contact
valContact p e = fmap Contact (valPhone p) <*> (valEmail e)
```

Or better yet, using `<$>` (infix version of `fmap`):

```
valContact p e = Contact <$> valPhone p <*> valEmail e
```

This can scale to any number of arguments:

```
valContact p e a ... = Contact <$> valPhone p
                               <*> valEmail e
                               <*> valAddress a
                                     ⋮
```

Applicative laws

They exist.

Exercises

Write the `Applicative` instance for the following types:

```
data Validated e a = Error e | OK a
```

```
data DiffPair a b = DiffPair a b
```

Exercises

Write the `Applicative` instance for the following types:

```
data Validated e a = Error e | OK a

instance Applicative (Validated e) where
    (OK f) <*> (OK x) = OK (f x)
    (Error y) <*> (OK _) = Error y
    (OK _) <*> (Error y) = Error y
    (Error y1) <*> (Error y2) = Error y1

    pure x = OK x


data DiffPair a b = DiffPair a b
```

Exercises

Write the `Applicative` instance for the following types:

```
data Validated e a = Error e | OK a

instance Applicative (Validated e) where
    (OK f) <*> (OK x) = OK (f x)
    (Error y) <*> (OK _) = Error y
    (OK _) <*> (Error y) = Error y
    (Error y1) <*> (Error y2) = Error y1

    pure x = OK x


data DiffPair a b = DiffPair a b

instance Applicative (DiffPair a) where
    (DiffPair x1 f) <*> (DiffPair x2 y) = DiffPair x1 (f y)

    pure x = DiffPair ?? x
```

Example

`Applicative` works for contexts other than containers, too. Each of the arguments can be an action that produces some effect in the context and returns the appropriate value.

```
parserContact = Contact <$> parserPhone <*> parserEmail
```

What to remember

- `Applicative` allows to "fmap functions with more than one parameter" into a context.
- Each of the arguments to the function can produce effects.
- `pure` allows to "inject" values into a context without producing any effect.

To learn more

Haskell Programming from first principles:
`http://haskellbook.com/`

Typeclassopedia:
`https://wiki.haskell.org/Typeclassopedia`

Thanks