Applied Numerical Methods - Math 151B

Homework 4

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April 25, 2016

For this assignment we were to use the Euler method, the Fourth Order Runge-Kutta Method, the Adams Fourth-Order Predictor-Corrector method, the Milne-Simpson Predictor-Corrector method to approximate the given IVP. The solution to the IVP, shown on the first for plots, initially decreases monotonically due to domination from the exponential term and then increases monotonically due to domination of the parabolic term. This change occurs around $t_i = 1.5$.

We can determine whether or not a solution is stable if the absolute error $|w_i - y_i|$ for solution y_i and approximation w_i approaches zero as time goes to infinity. Compared to the Adams and Milne-Simpson method, the Euler method appears fairly stable. However closer inspection of its solution reveals oscillations about the exact solutions that are never removed for small h. Its absolute error plot clearly shows that the error will grow towards infinity as time goes to infinity. Thus the Euler method is unstable for all values of h. Looking as the absolute error plots for the Adams and Milne-Simpson method shows that their errors grow without bound for all values of h except $h = 0.02$, where they approximate the solution exactly. Thus the Adams and Milne-Simpson methods are only stable for time step values less than or equal to $h = 0.02$ and are unstable for time step values greater than that. Finally, the Rk4 method is only unstable for the time step $h = 0.2$ and is stable for all others. Looking at the solution plot reveals that RK4 approximations follow the general shape of the exact solution more closely than any other method for time steps $h = 0.125$ and smaller. Looking at the absolute error plot for the RK4 method shows that the absolute error clearly decreases to zero as time increase for all time steps except for $h = 0.2$. Therefore, RK4 appears to be the most stable method.

From our plots and based on the values of h that were chosen, the region of absolute stability for the Runge-Kutta method is larger than that for the Euler method. The RK4 method is stable for 3 values of time steps h whereas the Euler method is stable for none, and no value of constant λ can make it stable for this particular IVP.

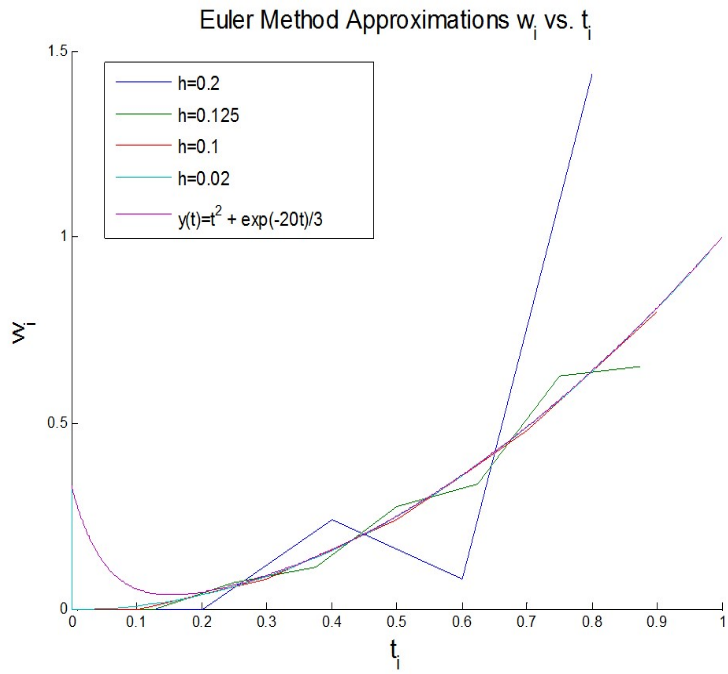

Figure 1: Figure of approximation to the given IVP using the Euler method for four different time steps. None of the approximations are very good and athe approximations oscillate about the actual solution.

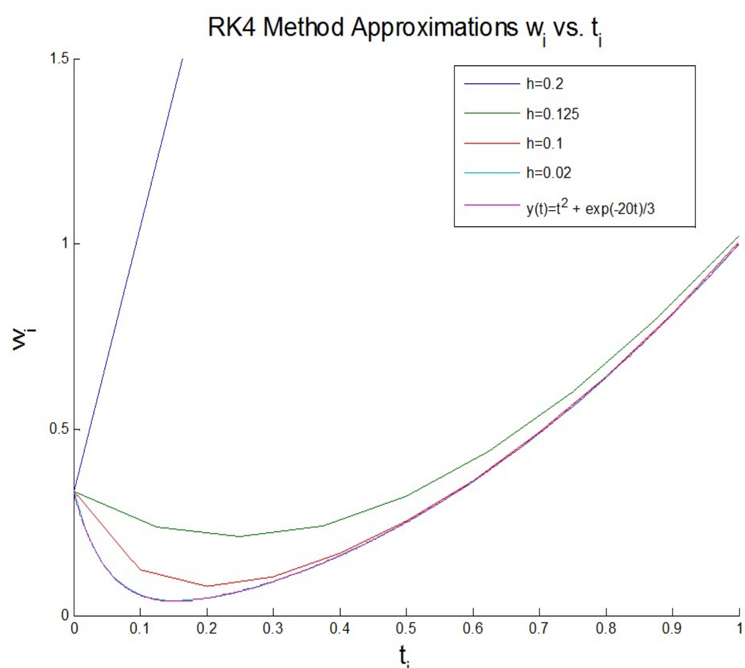

Figure 2: Figure of approximation to the given IVP using the 4th Order Runge-Kutta Method for four different time steps. The RK4 method is relatively stable and small changes in h after $h = 0.2$ lead to correspondingly changes in the approximation. Except for $h = 0.2$, all approximations initially decrease then increase monotonically just like the actual solution. For $h = 0.02$, the RK4 appears to approximate the solution quite well.

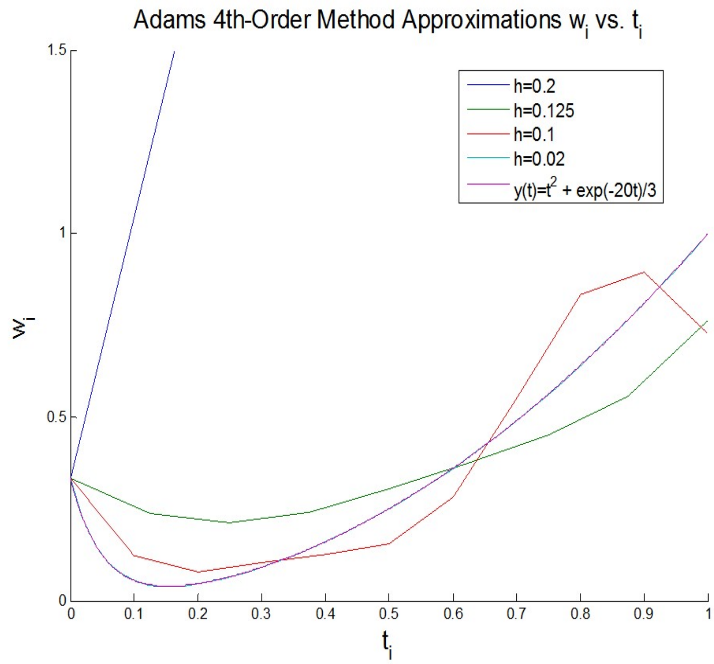

Figure 3: Figure of approximation to the given IVP using the Adams 4th Order Predictor-Corrector method. Like all other methods, the Adams method is unstable for $h = 0.2$ but becomes more stable as h is decreased. Although it oscillates about the actual solution, it is not as wild at for the Euler method. For $h = 0.02$, the Adams method appears to approximate the solution exactly.

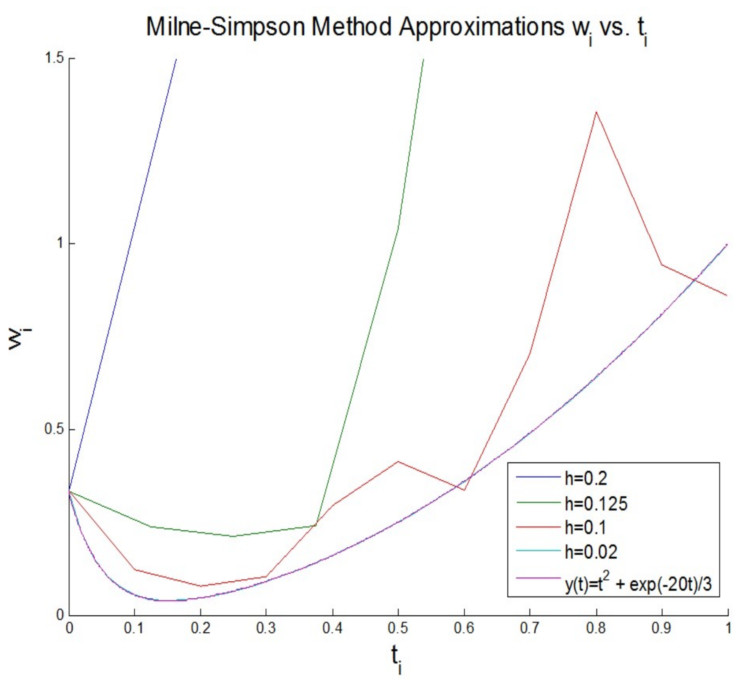

Figure 4: Figure of approximations to the given IVP using the Milne-Simpson method. This method appears unstable for all values of h, having incredibly off and oscillatory shapes, except for $h = 0.02$ where the method approximates the actual solution exactly.

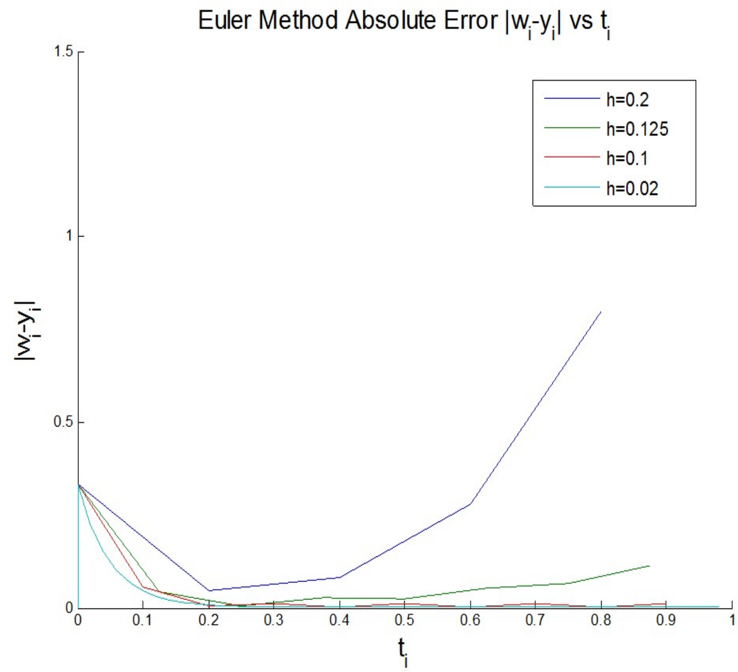

Figure 5: Figure of the absolute error $|w_i - y_i|$ from using the Euler method for various time steps h. While small changes in the time step h lead to correspondingly small changes in the error, the error in Euler's method is always growing and will go towards infinity as t_i increases. This indicates that the Euler method is not stable.

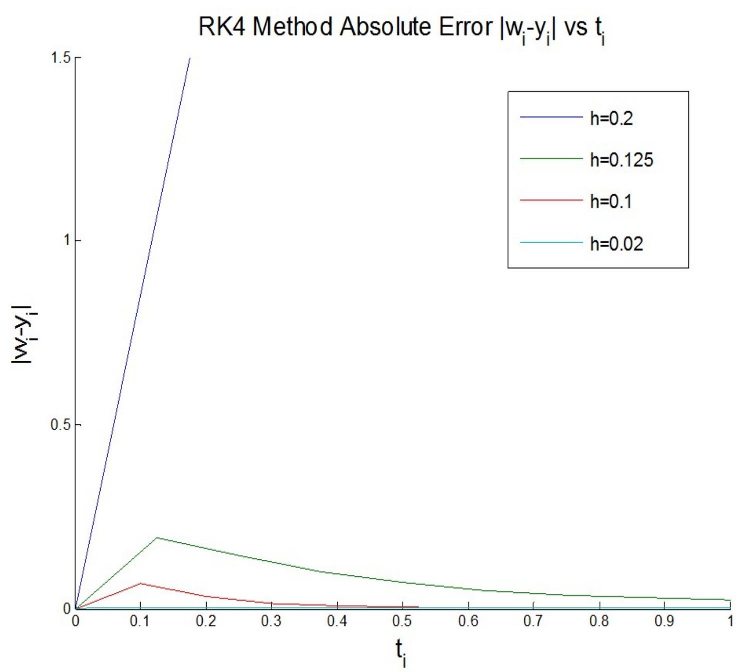

Figure 6: Figure of the absolute error $|w_i - y_i|$ from using the RK4 method for various time steps h. Clearly, this method is not stable for $h = 0.2$ as the error grows without bound. It is, however, very stable for all other time steps and the error approaches zeros for larger times t_i and appears to be zero for all time for $h = 0.02$.

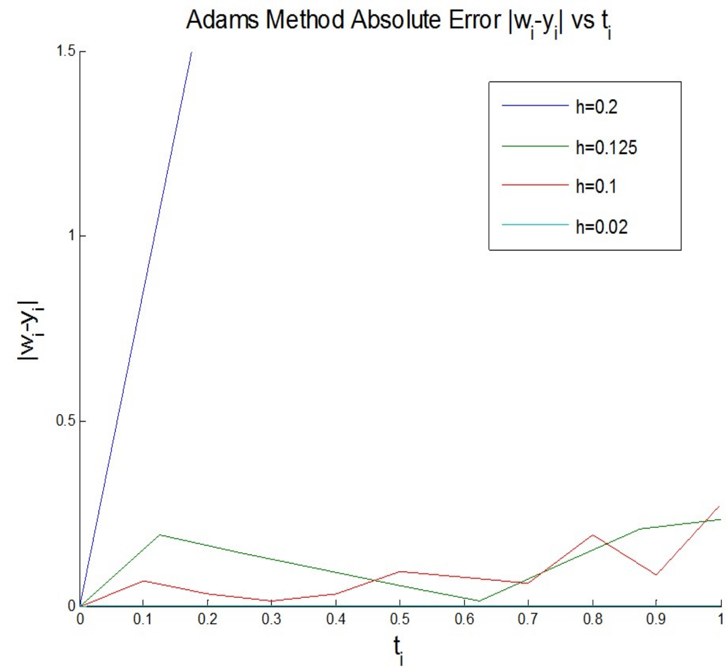

Figure 7: This figure is of the absolute error $|w_i - y_i|$ from using the Adams method for various time steps. Like for RK4, the Adams method is unstable for $h = 0.2$ but also for $h = 0.125$ and $h = 0.1$ as the error grows with t_i. The method is stable, however, for $h = 0.02$, where the error between the approximation and the solution appears to be zero.

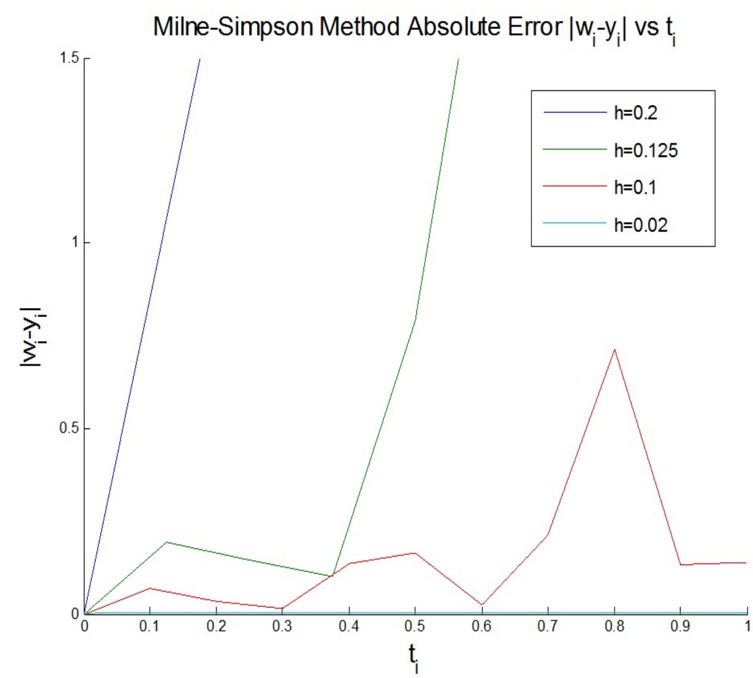

Figure 8: Finally, this figure is for the absolute error $|w_i - y_i|$ from using the Milne-Simpson method for various time steps. This method is similar to the Adams method in that it is unstable for all values of h, more so than any other method, but is stable for $h = 0.02$ where the error appears to zero for all time.