MATH 325 - Lecture 2

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1 Logical Operators

Negation: NOT, ˜, $\neg$
Conjunction: AND, $\wedge$
Disjunction: OR, $\vee$
Implication: IF... THEN..., $\implies$
Equivalence: ...IF AND ONLY IF..., IFF $\iff$
Precedence: (1^{st}) NOT, AND, OR, IF... THEN..., IFF (Last)

2 Example 4

2.1 Logical Analog of DeMorgan's Laws (p.33)

Logical Definition of DeMorgan's Laws

$$\neg(P \vee Q) \iff \neg P \wedge \neg Q$$
$$\neg(P \wedge Q) \iff \neg P \vee \neg Q$$

Truth Table proving $\neg(P \wedge Q) \iff \neg P \vee \neg Q$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
F	F	T	T	F	**T**	**T**
F	T	T	F	F	**T**	**T**
T	F	F	T	F	**T**	**T**
T	T	F	F	T	**F**	**F**

2.2 The Negation of $P \implies Q$

$P \wedge \neg Q \iff \neg(P \implies Q)$

P	Q	$\neg Q$	$P \wedge \neg Q$	$P \implies Q$	$\neg(P \implies Q)$
T	T	F	**F**	T	**F**
T	F	T	**T**	F	**T**
F	T	F	**F**	F	**F**
F	F	T	**F**	T	**F**

Note:

$\neg(P \implies Q) \iff P \wedge \neg Q$	Negation of an Implication
$\neg[\neg(P \implies Q)] \iff \neg(P \wedge \neg Q)$	Double Negation
$P \implies Q \iff \neg P \vee Q$	Equivalent Form of Implication

3 Definition 5 (p. 34):

Suppose P and Q are statements.

1. Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$
2. Converse of $P \implies Q$ is $Q \implies P$
3. Inverse of $P \implies Q$ is $\neg P \implies \neg Q$
4. Negation of $P \implies Q$ is $P \wedge \neg Q$

3.1 Remark 6:

- The contrapositive is always equivalent ot the original implication.
- The converse and inverse are not equivalent to an implication.
- The converse and inverse are contrapositives.
- To prove equivalence, once ust prove implication and its converse.

4 Variables and Quanitifiers (sec 1.4)

Make statements that depend on an unspecified parameter, a variable, but a context is needed to determine the truth value.

4.1 Example 7

$P(x) = x^2 - 5x + 6 = 0$
$P(x)$ is a sentence, but not a statement. You need information about x to determine its truth value.
Need to know what “universe” x lives in.

$P(0), P(2)$ are statements: $P(0) = 0^2 - 5 * (0) + 6 = 0$
$6 \neq 0$
False

$P(2) = 2^2 - 2 * (5) + 6 = 0$
$0 = 0$ True

Quantifiers are used to provide a larger context for variables

4.2 Definition 8 (p 33)

- Phrases such as "For all...", "For every..." are <u>universal quantifiers</u>: $\forall$
- Phrases such as "There exists...", "There is at least one..." are <u>existential quantifiers</u>: $\exists$

4.2.1 More Notation:

$\ni$, s.t.: "such that"
$\exists!$: "there exists a unique (exactly one)..."

WARNING: Unless otherwise stated, all variables are real numbers.

4.2.2 Convention:

If a variable appears in an antecedent of an implication without a quantifier, then we assume there is a universal quantifier. $x > 1 \implies x^2 > 1$ really means $\forall x \in \mathbb{R}, x > 1 \implies x^2 > 1$

4.2.3 Example:

$P(x) = x^2 - 5x + 6 = 0$
(Assuming x is a real number)
$\forall x \in \mathbb{R}, P(x) = x^2 - 5x + 6 = 0$ means...
"For all real numbers x, $x^2 - 5x + 6 = 0$"
(False statement because some real numbers x don't satisfy $P(x)$)

$\exists x \ni P(x)$ means...
"There is a real number x s.t. $x^2 - 5x + 6 = 0$
(True statement since $P(2)$ evalulates to true)

$\exists! x \ni P(x) = 0$ means...
"There exactly one real s.t. $x^2 - 5x + 6 = 0'$"
(False statement since $P(2)$ and $P(3)$ both evaluate to true)

WARNING: $\forall x, \exists y$ is not the same as $\exists y, \forall x$

4.2.4 Example 8: (Prob 4)

Suppose that $P(x, y)$ is a statement for each x and y.
$\forall x, y, P(x, y) \iff \forall x, \forall y P(x, y) \iff \forall y, \forall x, P(x, y)$
Means $P(x, y)$ is true regardless of what x and y are (order does not matter for the same quantifier).
$\exists x, y \ni P(x, y) \iff \exists x \ni \exists y \ni P(x, y) \iff \exists y \ni \exists x \ni P(x, y)$
Means there's atleast one x and one y that satisfies $P(x, y)$ (order does not matter for the same quantifier).
$\forall x, \exists y \ni P(x, y) \not\iff \exists \ni \forall x, P(x, y)$
$\forall x, \exists y \ni P(x, y)$ y can depend on x.
$\exists y \ni \forall x, P(x, y)$ y cannot depend on x. There is some y s.t. no matter what x, $P(x, y)$ holds.