

# MATH 325 - Lecture 2

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September 23, 2017

## 1 Logical Operators

Negation: NOT,  $\sim$ ,  $\neg$

Conjunction: AND,  $\wedge$

Disjunction: OR,  $\vee$

Implication: IF... THEN...,  $\implies$

Equivalence: ...IF AND ONLY IF..., IFF  $\iff$

Precedence: ( $1^{st}$ ) NOT, AND, OR, IF... THEN..., IFF (Last)

## 2 Example 4

### 2.1 Logical Analog of DeMorgan's Laws (p.33)

Logical Definition of DeMorgan's Laws

$$\neg(P \vee Q) \iff \neg P \wedge \neg Q$$

$$\neg(P \wedge Q) \iff \neg P \vee \neg Q$$

Truth Table proving  $\neg(P \wedge Q) \iff \neg P \vee \neg Q$

$P$	$Q$	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
F	F	T	T	F	<b>T</b>	<b>T</b>
F	T	T	F	F	<b>T</b>	<b>T</b>
T	F	F	T	F	<b>T</b>	<b>T</b>
T	T	F	F	T	<b>F</b>	<b>F</b>

### 2.2 The Negation of $P \implies Q$

$$P \wedge \neg Q \iff \neg(P \implies Q)$$

$P$	$Q$	$\neg Q$	$P \wedge \neg Q$	$P \implies Q$	$\neg(P \implies Q)$
T	T	F	<b>F</b>	T	<b>F</b>
T	F	T	<b>T</b>	F	<b>T</b>
F	T	F	<b>F</b>	F	<b>F</b>
F	F	T	<b>F</b>	T	<b>F</b>

Note:

$\neg(P \implies Q) \iff P \wedge \neg Q$	Negation of an Implication
$\neg[\neg(P \implies Q)] \iff \neg(P \wedge \neg Q)$	Double Negation
$P \implies Q \iff \neg P \vee Q$	Equivalent Form of Implication

### 3 Definition 5 (p. 34):

Suppose  $P$  and  $Q$  are statements.

1. Contrapositive of  $P \implies Q$  is  $\neg Q \implies \neg P$
2. Converse of  $P \implies Q$  is  $Q \implies P$
3. Inverse of  $P \implies Q$  is  $\neg P \implies \neg Q$
4. Negation of  $P \implies Q$  is  $P \wedge \neg Q$

#### 3.1 Remark 6:

- The contrapositive is always equivalent to the original implication.
- The converse and inverse are not equivalent to an implication.
- The converse and inverse are contrapositives.
- To prove equivalence, one must prove implication and its converse.

## 4 Variables and Quantifiers (sec 1.4)

Make statements that depend on an unspecified parameter, a variable, but a context is needed to determine the truth value.

### 4.1 Example 7

$$P(x) = x^2 - 5x + 6 = 0$$

$P(x)$  is a sentence, but not a statement. You need information about  $x$  to determine its truth value.

Need to know what “universe”  $x$  lives in.

$P(0), P(2)$  are statements:  $P(0) = 0^2 - 5 * (0) + 6 = 0$

$6 \neq 0$

False

$P(2) = 2^2 - 2 * (5) + 6 = 0$

$0 = 0$  True

Quantifiers are used to provide a larger context for variables

## 4.2 Definition 8 (p 33)

- Phrases such as “For all...”, “For every...” are universal quantifiers:  $\forall$
- Phrases such as “There exists...”, “There is at least one...” are existential quantifiers:  
 $\exists$

### 4.2.1 More Notation:

$\ni$ , s.t.: “such that”

$\exists!$ : “there exists a unique (exactly one)...”

**WARNING:** Unless otherwise stated, all variables are real numbers.

### 4.2.2 Convention:

If a variable appears in an antecedent of an implication without a quantifier, then we assume there is a universal quantifier.  $x > 1 \implies x^2 > 1$  really means  $\forall x \in \mathbb{R}, x > 1 \implies x^2 > 1$

### 4.2.3 Example:

$P(x) = x^2 - 5x + 6 = 0$

(Assuming  $x$  is a real number)

$\forall x \in \mathbb{R}, P(x) = x^2 - 5x + 6 = 0$  means...

“For all real numbers  $x$ ,  $x^2 - 5x + 6 = 0$ ”

(False statement because some real numbers  $x$  don't satisfy  $P(x)$ )

$\exists x \ni P(x)$  means...

“There is a real number  $x$  s.t.  $x^2 - 5x + 6 = 0$

(True statement since  $P(2)$  evaluates to true)

$\exists! x \ni P(x) = 0$  means...

“There exactly one real s.t.  $x^2 - 5x + 6 = 0$ ”

(False statement since  $P(2)$  and  $P(3)$  both evaluate to true)

**WARNING:**  $\forall x, \exists y$  is not the same as  $\exists y, \forall x$

#### 4.2.4 Example 8: (Prob 4)

Suppose that  $P(x, y)$  is a statement for each  $x$  and  $y$ .

$$\forall x, y, P(x, y) \iff \forall x, \forall y P(x, y) \iff \forall y, \forall x, P(x, y)$$

Means  $P(x, y)$  is true regardless of what  $x$  and  $y$  are (order does not matter for the same quantifier).

$$\exists x, y \ni P(x, y) \iff \exists x \ni \exists y \ni P(x, y) \iff \exists y \ni \exists x \ni P(x, y)$$

Means there's atleast one  $x$  and one  $y$  that satisfies  $P(x, y)$  (order does not matter for the same quantifier).

$$\forall x, \exists y \ni P(x, y) \not\iff \exists \ni \forall x, P(x, y)$$

$\forall x, \exists y \ni P(x, y)$   $y$  can depend on  $x$ .

$\exists y \ni \forall x, P(x, y)$   $y$  cannot depend on  $x$ . There is some  $y$  s.t. no matter what  $x$ ,  $P(x, y)$  holds.