MATH 325 - Lecture 1

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1 Logic and Proof Construction

Statement: a sentence with a truth value (True or False)
Ex: are these statements?

1. It is windy: No, 'windy' is not well-defined.
2. Life is good: no, 'good' is not well-defined.
3. Every continous function is differentiable: Yes (This statement is false).
4. There is a continous function that is differentiable: Yes (This statement is true).
5. $x^2 - 5x + 6 = 0$: No (x is not defined).
6. This sentence is false: No, cannot determine a truth value.

Compound Statement: formed from simpler component statements. A table of truth values for a statement and all its components is a truth table.

1.1 Common Connectives:

- Negation: not ˜or $\neg$
- Conjunction: and $\wedge$
- Disjunction: or $\vee$
- Implication: if...then... $\implies$
- Biconditional: iff... $\iff$

P	Q	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \iff Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	T

1.1.1 Precendence Rules:

$$\neg \to \wedge \to \vee \to \Longrightarrow \to \Longleftrightarrow$$

1.1.2 Example:

$P \iff Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T