Single-round vs Multi-round Distributed Query Processing in Factorized Databases

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Acknowledgements

Abstract

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3.1 Motivation

Previous work on Factorized Databases (**REFERENCE HERE**) provides searching for a good factorization tree (f-tree) upon a database based on asymptotic bounds and the size of the input. It has been proven to be optimal, many times generating exponentially more compressed representations than normal flat relational databases.

Although complexity bounds are nice, there are a lot of cases where they are not sufficient and we need more explicit properties. For example, given a database Q, the previous work might find that the optimal f-tree has parameter $s(Q) = 2$, where $s(Q)$ is the cost measurement function, and that there are multiple trees with this property. But the question is which of those f-trees having parameter $s(Q) = 2$ is better ? At the moment, the implementation just uses the *first* f-tree that has the optimal parameter.

What we really want to investigate is how to find a good f-tree, using more refined parameters, that will also depend on the *data* we want to factorize and not only on the f-tree structure which ignores data (except relation sizes). The reason why this is an important part of the project is that in a distributed system, **see discussion in experiments Section X.Y**, the biggest bottleneck is communication and data distribution. Therefore, although *s(Q)* provides optimal trees we want to minimize communication cost, thus requiring an f-tree that results in the smallest factorization size possible.

For example, in real-world scenarios it can happen that two f-trees have the same *s(Q)* parameter, let's say 2, but they might differ in size with a factor of 4x. More precisely, f-tree A can produce a factorization with 1 million singletons (value nodes) where f-tree B can produce a factorization of 4 million singletons. Asymptotically, we cannot discriminate the two, but in real life using f-tree B will result in excessive data distribution thus increasing our communication cost a lot, so it does matter in the end-to-end processing.

3.2 Contribution

This chapter's contribution is a *COST* function that given an f-tree and certain statistics (number of unique values per attribute, number of unique values per attribute under any other attribute of the f-tree) returns an estimation of the total factorization size (number of singletons, value nodes) that would occur if our database (factorization) was factorized based on that given f-tree.

3.3 Idea

The requirement is to have a cost function that would take into account the actual values of a database instance in order to be able to compare in a more precise manner f-trees that are asymptotically optimal.

Let's start with some facts about FDB factorizations:

1. each union has its values ordered in ascending order
2. each union has unique values
3. a factorization may have many relation dependencies and each dependency forces its attributes to exist along a single path in the f-tree (like a linear linked list)
4. some attributes belong to many relations, thus have many dependencies

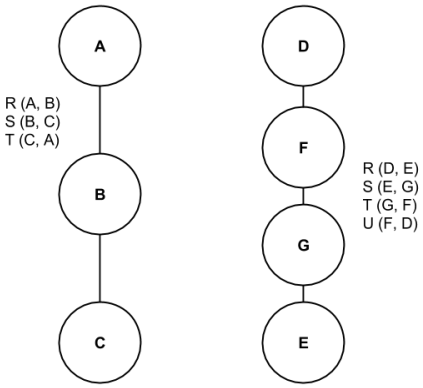


Figure 3.2: Triangle and Square queries

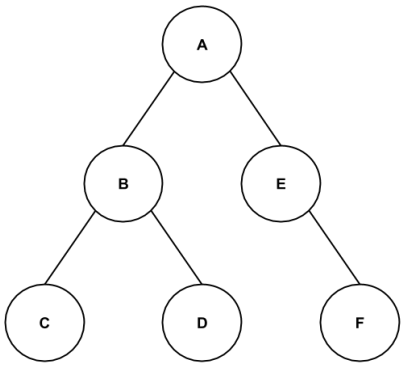


Figure 3.1: Example f-tree

Considering the above facts, we used the number of unique values per union, therefore easily finding unique values per attribute. Additionally, the dependencies matter a lot since in complex queries like *triangles* or *squares*, see Figure 3.2, we have all the attributes in a single path, forming a single linked list and each level down the path affects the factorization size.

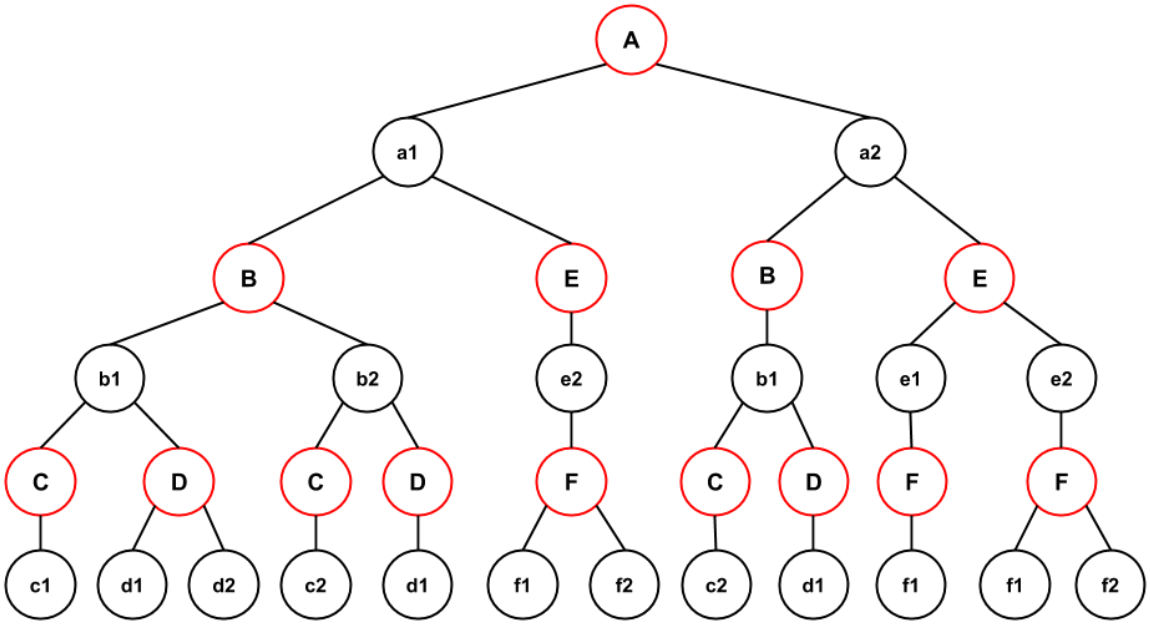


Figure 3.3: Example factorization

We define *cost* of a factorization the total number of value nodes or singletons, thus the sum of the number of value nodes for each attribute. For example the factorization in Figure 3.3 has 20 value nodes (black nodes) so the cost for that f-tree is 20.

3.3.1 Initial thoughts

A first idea was to use an f-tree as a reference tree and based on some statistics calculated on this reference tree we would calculate the factorization estimated size for any other arbitrary f-tree.

Given an f-tree and its factorization, we calculate for each attribute the average number of unique values (children of a union) under any of its ancestor attributes. The average is taken over all the ancestor's children values.

Notation

1. $X u Y$ denotes the average number of unique values of attribute X *under* a single value of attribute Y, where Y is an ancestor of X.

2. *uniq(X)* denotes the average unique number of values among all the unions of attribute X.

For example, assuming the f-tree in Figure 3.1 and its factorization, see Figure 3.3, we have the following statistics:

- *unique values per attribute:* uniq(A), uniq(B), uniq(C), uniq(D), uniq(E), uniq(F)
- *number of unique values per attribute X under an ancestor attribute Y:* BuA, CuA, CuB, DuA, DuB, EuA, FuA, FuE

Having the above statistics calculated, given any other f-tree T the estimated size of the factorization would be calculated by summing the estimated number of nodes for each attribute. To calculate the cost for an attribute X, a path between X and its parent in T should be found inside the reference tree, followed by the multiplication of all the pair-wise averages (XuY) along the path to get an estimation for the number of values of X.
This approach quickly turned out to be wrong and over-estimating because of the excessive usage of estimates when we multiplied them for all the attribute pairs along the path.

3.3.2 Proposed Idea

The final solution is based on the same intuition but in a more precise and more accurate way. Instead of depending on estimates of a reference tree which lead to artificial over-estimation, statistics such that we can use them with any f-tree should be calculated, regardless of the input f-tree. Recall that our *cost* function should be able to accept an arbitrary f-tree and return the estimation size as accurate as possible.

As a result, the following properties (statistics) are used during estimation:

1. *Average number of unique values of attribute X under any attribute Y* (single value of Y), denoted as XuY, where Y is an ancestor of X.

2. *Average unique number of values among all union nodes for each attribute,* denoted as *uniq(X)* where X is an attribute.
3. Flat size of the database (number of tuples).

Another important observation is that the number of nodes for each attribute in the factorization is related to *all* of its ancestor attributes and not only to its parent. For example, in figure 3.1, the number of nodes for attribute C depend both on B *and* A, therefore we somehow have to incorporate them in our estimation for attribute C.

In the following formula *COST(X)* denotes the estimated number of value nodes (singletons) for attribute X in the result factorization.
Input:

1. f-tree T
2. XuY and *uniq(X)*, as described above
3. flat factorization size

Estimation Formula
If attribute X is f-tree root:

$$COST(X) = uniq(X)$$

Else:

$$COST(X) = MIN(\, COST(parent(X)) * MIN\_AVERAGE(X,T), FLAT\_SIZE \,) \quad (1)$$

Where: *MIN_AVERAGE(X, T)* = the minimum average XuY, where Y is an ancestor of X along the path from X to the root of f-tree T. Y should also exist in a common relation with X (dependency).

The above formula gives an estimation for the number of value nodes for a given attribute in a given factorization tree. The total size of the factorization is the sum of the individual cost for each attribute.

It is important that we take into consideration dependencies and only use XuY averages for the ancestor attributes that are in a common relation with attribute X since we do not know the relationship of X with attributes in other relations.

Additionally, we restrict the estimation size of the number of values per attribute to the flat size of the representation since that is the maximum amount of singletons we can have for each attribute, which is the worst case where each tuple is a separate path in the factorization.

3.4 Algorithms

In this section the pseudocode for the complete factorization size estimation procedure is provided that implements the *COST* function described above.

Estimate Factorization Size

The algorithm is an iteration over the attributes in the factorization tree in a BFS-traversal order memoizing the estimations of already visited attributes to use in their descendants cost calculation.

Algorithm 3.1 calculates the estimated size of the representation that will be created based on the input factorization tree. The algorithm assumes that the averages are already calculated and are ready to be used. This is common in the databases-world where some properties are calculated off-line in order to be used during runtime (value histograms, unique values, selectivity, etc.).

The complexity of the algorithm is quadratic to the number of attributes in the factorization tree, $\mathsf{O}(N^2)$ since we visit each attribute exactly once and for each attribute we call the *min_average()* function which has linear complexity, or more precisely its complexity depends on the longest root-to-leaf path (we visit each attribute's ancestor in the f-tree).

Algorithm 3.1: Calculate estimated size for factorization using given f-tree

```
// @fTree: the f-tree to estimate the size for, if used for factorization
// @FLATSIZE: the flat size in number of tuples
double estimate_size(FactorizationTree *fTree, unsigned int FLATSIZE) {
    // queue for BFS - holds pairs of attribute IDs <parentID, childID>
    queue<pair<int, int>> Q;
    // memoization array of costs estimated - size = number of attributes
    vector<double> costs(ftree->num_of_attributes());

    // cost for the root
    rootID = fTree->root->ID;
    costs[rootID] = uniq(rootID);
    // add root's children in queue
    for each child attribute CA in fTree->root->children {
        Q.push_back({rootID, CA->ID});
    }
    while (!Q.empty()) {
        parent_child = Q.pop_front();
        parentID = parent_child->first;
        childID = parent_child->second;

        // calculate the minimum of all averages XuY where X = childID and
        // Y is every ancestor of X in the fTree that belongs to a common
        // relation (dependency) with X.
        double min_est = min_average(fTree, childID);
        // calculate the cost for this attribute
        // COST(X) = min(COST(par(X)) * min(all averages XuY), FLAT_SIZE)
        costs[childID] = min((costs[parentID] * min_est), FLATSIZE);

        // add the attribute's children to the BFS queue
        for each child attribute CA in fTree->node(childID)->children {
            Q.push_back({childID, CA->ID});
        }
    }
    // the total cost estimation is the total number of value nodes
    // which is the sum of all the value nodes for each attribute
    return sum(costs);
}
```

For the sake of completion the code for *min_averages()* function is provided below.

Algorithm 3.2: Find min *XuY* for an attribute

```
// @fTree: the factorization tree we currently estimate the size
// @attributeID: the attributeID we are calculating the estimated number of nodes
double min_average(FactorizationTree *fTree, attributeID) {
    // get the attribute node
    cN = fTree->node(attributeID);

    // the maximum average for each attribute is the unique number of values of it
    double min_est = uniq(attributeID);

    // we now traverse the path from the current attribute up to the root
    // and check the average of children with each ancestor
    // ONLY if it belongs to common relation/dependency (hyperedge)
    while (cN != NULL) {
        if (same_hyperedge(attributeID, cN->ID)) {
            min_est = min(min_est, XuY(attributeID, cN->ID));
        }
        cN = cN->parent;
    }
    return min_est;
}
```

The complexity of the above pseudocode is linear in the longest path from an attribute node to the root and it finds the minimum average number of children (unique values) of the current attribute under any ancestor attribute in the current f-tree.
The maximum amount of children (unique values) of any attribute under any other attribute is the amount its unique values since we have unique values in our union nodes.

Calculate averages

The previous algorithm that estimates factorization size assumes existence of the averages XuY for each pair of attributes in the same *hyper-edge* (relation/dependency).

A procedure was implemented that calculates this but it is code-specific to be included in the thesis so we only provide a pseudocode for it showing the idea behind it.

The function returns a two-dimensional matrix with size (N x N, where is the number of attributes. *Matrix[X][Y]* corresponds to the notation used above, XuY, which means that cell located at row X and column Y has the average number of children (unique values) among all unions of attribute X which are located below each value of the attribute Y.

Algorithm 3.3: Calculate averages

```
// @fTree: the factorization tree used for the representation '@fRep'
// @fRep: an input factorization of the database instance we examine
double[][] calculate_averages(FactorizationTree *fTree, FRepresentation *fRep) {
    double matrix[fTree->number_of_attributes()][fTree->number_of_attributes()];
    for each attribute A in fTree->nodes {
        // make the current attribute A root of the factorization
        make_root_attribute(A, fRep, fTree);
        // traverse the factorization in either DFS or BFS mode and calculate
        // all the averages where attribute A is the parent since now all
        // other attributes are below attribute A
        averages = calculate_averages_for_root(A, fRep);
        update_matrix(matrix, averages);
    }
    return matrix;
}
```

The above algorithm's runtime could be improved but it is orthogonal to the project and only used during the off-line pre-processing of the database instance to generate the averages, thus its sub-optimality is not a serious concern.

The real need was to provide a fast cost function that during runtime could determine the size of the factorization given an arbitrary f-tree.

Chapter 4

Serialization of Data Factorizations

4.1 Motivation

An important part of the project investigated ways to serialize, and possibly compress, factorizations (f-representations). It is very important to support serialization and deserialization of a factorization both in a centralized setting and in a distributed setting. For example, sometimes we want to save an instance of a database on disk to manipulate and further process it later. In some other cases we want to ship data over the wire to neighboring nodes which need the data for additional processing on their side.

In general, serialization is the method of efficiently converting an in-memory factorization into a byte stream which is stored or transferred and later can be deserialized into the exact source factorization.
An important aspect of serialization and deserialization is that they have to be efficient in both processing time and space since we want to retain the major benefit of factorizations, which is the compression factor compared to corresponding flat representations. Thus, having a serialization that would take a lot of space or requiring a lot of time to process would be inappropriate for our setting, especially for the distributed system that is the goal.

4.2 Contributions

The contributions made to the project out of this chapter are four (4) serialization techniques for Data Factorizations and one (1) serialization technique for Factorization Trees, namely:

1. *Factorization Tree (De)serializer* - this is the only serialization technique for f-trees and is used in conjuction any of the factorization serialization techniques.
2. *Simple Raw (De)Serializer* - a simple serialization technique that is fast and retains the compression factor over flat representations.
3. *Byte (De)Serializer* - an extension to the Simple serialization technique to only store the required number of bytes for each value.
4. *Bit (De)Serializer* - a further extension to Byte serialization to only store the required number of bits for each value, with specialized methods that can be extended in the future to better support more values to allow better compression.
5. *Bit Serializer HyperCube* - the serialization technique that is used in the distributed system which differs from the normal *Bit Serializer* in that it does not ship all the values of a union but only those that should be shipped based on the Dimensions ID given (this will be explained thoroughly in Chapter 5).

During the preliminary stages, I also implemented a Boost-based serialization technique using *Boost::Serialization* library but it has been abandoned because it turned out to be very bloated and did not satisfy our requirements (explained in-detail later).

4.3 Factorization Serializations

In this section I will describe the different approaches I have taken for the serialization leading to the final version used in the distributed system.

4.3.1 Example

Let us first define an example scenario that we use throughout the chapter.

Assume that we started with four (4) relations, *R(A,B,C)*, *S(A,B,D)*, *T(A,E)* and *U(E,F)*, and applied a *NATURAL JOIN* operator on all of them, resulting in the final table shown below.

A	B	C	D	E	F
1	1	1	1	2	1
1	1	1	1	2	2
1	1	1	2	2	1
1	1	1	2	2	2
1	2	2	1	2	1
1	2	2	1	2	2
2	1	2	1	1	1
2	1	2	1	2	1
2	1	2	1	2	2

Figure 4.1: Result after Natural JOIN on R, S, T, U relations

Show the relation tables too // TODO

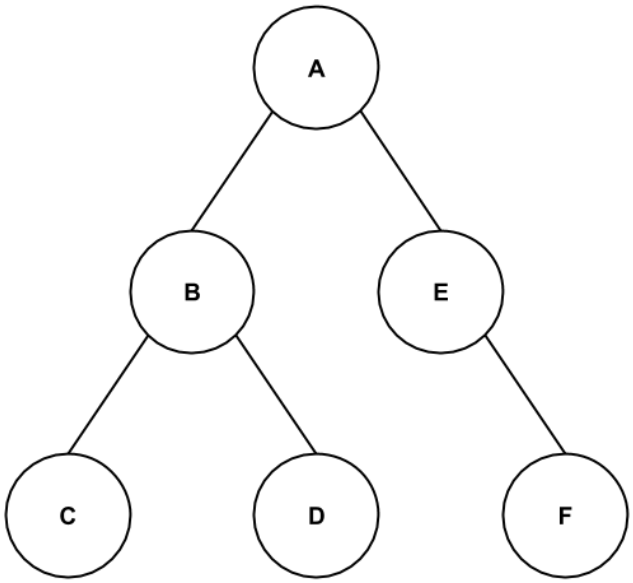


Figure 4.2: Example f-tree

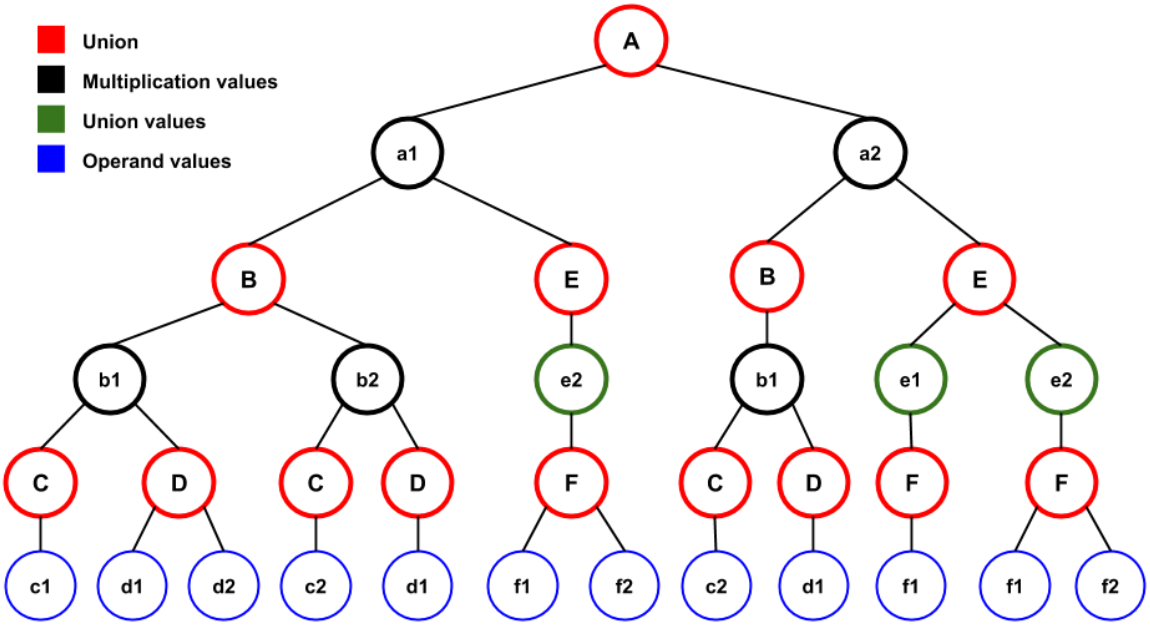


Figure 4.3: Example factorization

We will use the f-tree in Figure 4.2, named *Example f-tree* to factorize the result of the JOIN (see relational table in Figure 4.1) with the Data Factorization based on that f-tree being shown in Figure 4.3.

I will abstractly explain the in-memory representation of Data Factorizations as implemented at the moment. A factorized representation contains the following types of nodes:

1. *Union* nodes just contain a list of the values for that specific attribute union
2. *Multiplication values* are value nodes that act like *Multiplication* nodes since their attribute is a multiplication attribute (based on f-tree) and they have two or more Union nodes as children
3. *Union values* are value nodes that just have one Union node as a child
4. *Operand values* is just another node type to denote leaf values

4.3.2 F-Ttree serialization

An f-tree is the back-bone component of a factorization since it defines the structure of the representation and all the relations between the attributes of the query.
The serialization of an f-tree is the same for all the different factorization serialization techniques and is implemented as a separate module since it is a very small data structure (a few KBs) and we do not mind using the simplest serialization for it.

We use the same structure as an f-tree definition file for its serialization. As a result, the serialization of the f-tree show in Figure 4.2 is as follows:

```
6 4
A int
B int
C int
D int
E int
F int
-1 0 1 1 0 4
R S T U
```

```
2 3 4 5
A,B,C
A,B,D
A,E
E,F
```

The first line defines the number of attributes N and relations M in the f-tree, followed by N lines declaring the name of each attribute and its data type. The current FDB implementation assigns IDs to the attributes in the order they are defined here with the first attribute (A in this example) being given ID zero (0) and the last attribute (F in this example) being given ID five (*N-1*).
The next line defines the tree-relationship since for each attribute we specify the ID of its parent attribute. A has parent ID -1 which means A is root, then B has parent ID 0, C has parent ID 1, D has parent ID 1, E has parent ID 0 and F has parent ID 4.

Then we similarly have a line containing the relation names, again being given IDs internally, with the following line specifying for each relation its parent attribute node. The last M lines are just the relations enumeration with their attributes.

The serialization of an f-tree uses **Text** format and it is prefixed with its size length to allow the deserializer to know up-front the total f-tree serialization size in order to read all the information at once (prefixing messages with their total size is very common in message passing protocols).

The serialized f-tree (including its size header) is prefixed in the final serialization of the Data Factorization such that it can be deserialized first and allow us to use it during the factorization deserialization.

4.3.3 Boost Serialization

As a first attempt to provide serialization/deserialization we decided to use *Boost::Serialization* library since it gathered high rating reviews among the online community and since *Boost* was already being used for the networking modules of the system it seemed to be a great fit.

The purpose of *Boost::Serialization* library is to allow developers to provide an easy way to add serialization to their *existing* data structures without writing a lot of boilerplate code since it can be described more or less like a memory dump of a data structure into a stream (file, socket, etc.).

The integration of the library in FDB and the actual implementation was pretty straightforward. There were some *special* methods required to be added in each class we wanted to be serializable according to certain library rules. However, the end result was really disappointing due to very big serialization size.

As I mentioned, this is more of a memory dump of the structure, including any pointers and their destination objects, in order to easily allow the deserializer to create the exact data structure. The major problem here and the reason of the bloated serialized output is that the existing *FDB* implementation is not as space-efficient as it should be and that overhead is transferred into the serialization.

The current data structure of a factorization has a lot of overhead, like keeping all the values of a Union as a Double-Linked-List thus introducing excessive amount of pointers. As a result, the serialization module was dumping everything, more importantly the pointer references, to allow re-creation during deserialization leading to a bloated outcome, both in terms of raw size in bytes but also in long serialization times.

In our first preliminary experiments the serialized representation was almost the same size as the flat-relational representation, thus completely eliminating the compression factor of FDB over flat databases, which was unacceptable!

In order to use *Boost::Serialization* and at the same time having quality serialization we had to write custom code for each implementation class for every data structure we use to omit certain fields or doing my own book-keeping for the pointers and references to avoid all this going into the serialized output. Eventually, this was not worth it since Boost was still going to add some overhead which cannot be removed, like class versioning.

The *first attempt failed* but led to some interesting observations. Although the current implementation was poorly done, a good serialization does not need all that information and we could also take advantage of the special structure of a factorization to make it as succinct as possible.

4.3.4 Simple Raw (De)Serializer

Before going into details for this serialization technique we state some observations made after investigating the reasons that led to failure of the previous attempt for serialization.

- Each factorization is strictly associated with a factorization tree (f-tree) that defines its structure.
- The main types of a node in a factorization are the *Multiplication* (cross product) and the *Summation* (union) node types.
- The values inside a union node can be stored in continuous memory, thus avoiding the excessive overhead of Double-Linked-Lists due to the pointers for each value.
- There is necessity to de-couple the data, values, from the factorization structure since a lot of overhead comes with the representation and not the data.

Apart from the above observations, the trick that led to this serialization method is that the only nodes required to be serialized are the *Union* nodes along with their values. Since each factorization strictly follows an f-tree, it became obvious to use the f-tree as a guide during serialization and deserialization leading to a more succinct outcome which just contains the absolute minimum of information, *the values!*

The problem with generic serialization techniques, like *Boost* described above is that all information goes into the serialized outcome to allow correct deserialization. We can avoid this overhead in our case since we know the special structure of the factorization and therefore we can use the f-tree to infer the structure of the representation and load the values from the serialized form as we go along during deserialization.

4.3.4.1 Idea

The main idea of *Simple Serializer* is that we traverse the factorization in a DFS (Depth-First-Search) order and every time we find a *Union* node we serialize it, then recurse to the next. The serialization of a union node is simple and just contains a number N indicating the number of values in that specific union, followed by N values of the attribute represented by that union.

For example, if a specific union of attribute A (of type *int*) has the values *[3, 6, 7, 8, 123, 349]*, its serialization would be:

6 3 6 7 8 123 349

It is important to mention that we use **Binary** read and write methods during serialization and deserialization and for each children count we use 32-bit unsigned integer values whereas for the actual values the corresponding number of bytes required for that attribute data type (i.e. *double* = *sizeof(double)* = 8 bytes) is used.

The serialization of a factorization is just a sequence of *children counts* followed by their *corresponding values*. The important benefit of this serialization technique is that only the absolute minimum information required to recover the representation is stored.

Moreover, *Simple Serializer* assumes that we already deserialized the f-tree (discussed previously) and we can use it to infer the structure of the representation.

4.3.4.2 Algorithms

Algorithm 4.1: Simple Serializer

```
// @node: the starting node of our serialization (initially root of factorization)
// @fTree: the factorization tree to be used as guide
// @out: the outpout stream to write the serialization
dfs_save(Operation *op, FactorizationTree *fTree, ostream *out) {
    if (is_multiplication(op)) {
        // in multiplication nodes we just recurse without serializing
        for each child attribute CA in op->children { dfs_save(CA, fTree, out); }
    } else if (is_union(op)) {
```

```
        // in union we serialize the number of children and values
        write_binary(out, op->childrenCount);
        // serialize union values
        for each child value V in op->children { write_binary(out, V); }

        // recurse only if the union's attribute is not leaf in the f-tree
        if (!is_leaf_attribute(fTree, node->attributeID)) {
            for each child value CV in op->children { dfs_save(CV, fTree, out); }
        }
    }
}
```

Simple Serializer, see Algorithm 4.1, is an extension of the well-known DFS traversal algorithm for trees with in-order value processing.

The representation has two types of nodes, thus leading to two different treatments in serialization. When a *multiplication* node is encountered, the algorithm recurses on its descendants without serializing anything since the multiplication information can be inferred from the f-tree. When a *union* node is encountered we first serialize the number of values in that union, followed by the serialization of all the values. At the end we recurse on each of child to complete the DFS-traversal.

The algorithm iterates over the values twice since all the values of a union have to be serialized completely and *then* move on to the next union, like in an in-order traversal. Additionally, the f-tree is used to determine if a union belongs to an attribute which is *leaf* in the f-tree to avoid unnecessarily recursions.

Simple Deserializer, see Algorithm 4.2, is not as simple as its counterpart but it is easy as soon as some key things are explained.
First of all, only factorization nodes of type *Union* are serialized, so we know that during the deserialization phase we only deserialize union nodes, hence the creation of a Union node just from the start of the function (*opSummation*). Then we read the children counter for this union and such many values from the input stream (note that we use *binary* format in deserializer too to match the serializer).

Now that the values for the union are read we have to use the f-tree to determine what type of factorization node each value should represent. If the current union being deserialized represents a leaf attribute (*currentAddr*) (like C, D and F in the example) we just append the values in the union node using a special *Operand* node.

If the current union represents an internal attribute node (like A, B, E) we have to check if this is a multiplication attribute, meaning that it has 2 or more child attributes in the f-tree (like A and B). If the current attribute is not product/multiplication we just add the values to the current union (*opSummation*) and as a *subtree* node we add whatever the recursion on that value will return. If the current attribute is a multiplication then we need to create a factorization node of type *Multiplication* for each value and each child of this multiplication will be the recursion result on each of the current attribute's children. For example, if the current attribute (*currentAttr*) is B it means that is has two children, attribute C and attribute D. Therefore each value of union B will have a node of type *Multiplication* that has two subtrees, one for each of the C and D attributes and their subtree nodes will be the respective recursion result.

Algorithm 4.2: Simple Deserializer

```
// @in: the input stream from which we deserialize the factorization
// @currentAttr: the current attribute node in the f-tree (initially the root)
FRepNode* dfs_load(istream *in, FTreeNode *currentAttr) {
    // we know that we only deserialize unions so create a new union
    Operation *opSummation = new Summation(currentAttr->name,
                                           currentAttr->ID,
                                           currentAttr->value_type);
    // deserialize the children count and the values for this union
    unsigned int childrenCount = read_binary(in);
    vector<Value*> values = read_binary_many(in, value_type, childrenCount);

    // now use the f-tree to infer factorization structure
    if (is_leaf_attribute(currentAttr)) {
        // just append the values to the current union and return
        for each value V in 'values' { opSummation->addChild(V, new Operand(...)); }
        return opSummation;
    } else {
```

```
        // this is an internal attribute node therefore we need to check if
        // we have to create a multiplication node for each of child values
        if (!is_multiplication_attribute(currentAttr)) {
            // not a product attribute so just store children and recurse
            for each value V in 'values' {
                opSummation->addChild(V, dfs_load(in, currentAttr->firstChild));
            }
            return opSummation;
        } else {
            // each value of the union is a multiplication operation
            for each value V in 'values' {
                Operation *opMult = new Multiplication();
                opSummation->addChild(V, opMult);
                // recurse on each attribute child and add it to this multiplication
                for each child attribute CA in currentAttr->children {
                    opMult->addChild(new Value(CA->attributeID), dfs_load(in, CA));
                } // end for each attribute in the product
            }
            return opSummation;
        } // end of if product attribute
    } // end of if leaf_attribute
}
```

4.3.5 Byte (De)Serializer

Byte (De)Serializer is an extension of the *Simple Raw (De)Serializer* technique where the only difference is that it just stores required bytes only for each value and not all the number of bytes of each data type.

4.3.5.1 Idea

If we really wanted each value to have only the required amount of bytes then somehow we would need to store that amount somewhere in the serialization in order to allow the deserializer to know how many bytes to read. It is easy to see that with millions of values, having a companion byte indicating the number of required bytes for each value

could be excessive. Therefore, we decided for each attribute to use the required amount of bytes to cover the maximum value occurred for that attribute. Therefore, we record different required-bytes for each attribute and we avoid the overhead of having them for each value since we just store them once as a serialization header at the very beginning.

We also apply the same logic to the union children counts, thus for each attribute we store two values, required-bytes for union children and required-bytes for union values. These two counters for each attribute are serialized in full binary format (8-bit unsigned numbers) at the beginning of the serialization. Therefore the deserializer will read these counters and then it will know exactly the amount of bytes to read for each union node.

4.3.5.2 Algorithms

Byte Serializer

The *dfs_save()* method is the same as the *Simple Serializer* with the only difference that the 2 lines writing to the output stream *a)* the children count and *b)* the actual values, use a special variant of the *write_binary()* method that accepts a third argument denoting the number of bytes to write from the given value.

However, in order to know this required-bytes for each attribute union children and values we have to do a pass over the factorization and gather statistics around the actual values. This means that *Byte Serializer* traverses the whole factorization twice, but as the experiments show it does not hurt a lot in processing time and helps a lot in space-efficiency.

We skip the *dfs_save()* method code since it is exactly the same as described above and we provide the first pass algorithm that gathers statistics about the unions

Algorithm 4.3: Byte Serializer – statistics gathering

```
// @attribute_info: used below = it is a field of the Byte Serializer class
// @node: the node to start gathering statistics (initially the factorization root)
// @fTree: the factorization tree of the representation
```

```
void dfs_statistics(Operation *op, FactorizationTree *fTree) {
   if (is_multiplication(op)) {
        // multiplication nodes children are unions so just recurse on them
        for each child union CU in op->children { dfs_statistics(CU, fTree); }
    } else {
        // check if the current attribute required-bytes need to be updated
        children_bytes = required_bytes(op->childrenCount);
        if (attribute_info[op->attributeID].required_union_bytes < children_bytes)
            attribute_info[op->attributeID].required_union_bytes = children_bytes;
        // check value bytes
        for each child value CV in op->children {
            val_bytes = required_bytes(CV);
            if (attribute_info[op->attributeID].required_value_bytes < val_bytes)
                attribute_info[op->attributeID].required_value_bytes = val_bytes;
            // recurse if this is not a leaf attribute in the f-tree
            if (!is_leaf_attribute(fTree, op->attributeID)) {
                dfs_statistics(CV, fTree);
            }
        } // end for each child value
    }
}
```

The statistics gathering procedure is pretty straight-forward. We do a DFS-traversal on the factorization and whenever we are at a union node we update the required bytes for the number of children and for the values of that specific attribute represented by that union node. The *required_bytes()* method returns the number of active value bytes starting from the LSB (least significant byte) to the MSB (most significant byte).

In the pseudocode above *attribute_info* is a field of the *Byte Serializer* class and its type is as shown below:

```
struct AttrInfo {
    uint8_t required_value_bytes;
    uint8_t required_union_bytes;
}
```

This structure represents the header for each attribute that is written before the actual factorization serialization as part of the header and each counter is an 8-bit unsigned integer (thus 2 bytes per attribute required).

Byte Deserializer

The *Byte Deserializer* is exactly the same as the *Simple Deserializer* with the only difference that the 2 lines where it reads from the input stream the number of children and the values themselves it uses a third argument to the *read_binary()* method that specifies the number of bytes to read.

Before calling the method *dfs_load()* we separately read the counters for the required-bytes needed for union children and values respectively.

4.3.6 Bit (De)Serializer

The final version of the serialization technique is *Bit Serializer*. As the name suggests it follows the same idea as the *Byte Serializer* but instead of working at byte-level, it works at bit-level. Therefore, instead of storing the minimum amount of required bytes for each union count and each value, it stores only the required *bits*.

4.3.6.1 Idea

The idea of this serialization technique came up after we tested applying state-of-the-art compression algorithms like *GZIP* and *BZIP2* upon our own *Simple* and *Byte* serializers. We saw that applying these compression algorithms reduced the output size by a constant factor ranging from 1-4x while at the same time increased the processing (serialization and deserialization) time significantly!

Although *serialization* is different than compression and should not be mixed (serialization is used for saving and loading a structure whereas compression is used to

exploit values to reduce size), in our case it was obvious that we could be more space-efficient by exploiting the data in our factorizations. We achieved similar or close enough compression on our factorizations in a fraction of the time required by *BZIP2* compression for example, which provides the best compression at the cost of slow processing.

I want to emphasize that serialization is different than compression and that this chapter aimed at serialization of data factorizations. But, the knowledge of our structure allows us to exploit certain factorization properties and at the same time be more space-efficient without increasing processing time significantly. Additionally, although we have some kind of compression, we do not have the drawback of standard compression algorithms (*GZIP, BZIP2*) that need the decompress the whole fragment first and then do any processing, since we are still able to deserialize each union separately and process it before we move to the next one.

4.3.6.2 Algorithms

The algorithms are identical to those of *Byte Serializer* and *Byte Deserializer* with the exception that instead of using the *required_bytes()* method it uses the *required_bits()* method to only write the specific bits required to the output stream.

4.3.6.3 Bit Stream

This serialization technique requires bit-level precision when reading and writing values, but as we know all system calls and existing functionality provided by the standard libraries work at byte-level precision.

Therefore, in order to provide this functionality we implemented custom input and output streams (*obitstream* and *ibitstream* classes) that are used upon the underlying standard binary byte streams and use those in the *Bit Serializer* and *Bit Deserializer*. These custom bit streams basically allow for a given value to write only certain bits of its memory representation and respectively can read a certain number of bits from an input stream and reinterpret them as a data type in memory.

Briefly an explanation how the bitstreams work. When a value is written or read we use internally an in-memory bytes buffer to write and read from certain amount of bits. Whenever the bytes available in the internal buffer are insufficient to satisfy a read operation it is refilled by reading bytes from the underlying input stream. Whenever the internal buffer fills (or at user's request) the internal buffer is flushed to the underlying output stream. Therefore, this implementation of bit streams works upon the underlying standard binary streams of C++ and use buffers to handle the required read and write operations.

In addition, an important feature that makes this serializer *great* is that in future work specialized read/write methods could be provided for certain data types (floats, doubles, strings) and further increase compression without adding processing overhead by applying compression algorithms. The current implementation of bit streams heavily uses C++ templates therefore this extension should be trivial to implement in a future project.

4.4 Final remarks

We described a serialization for f-trees and three serialization techniques for Data Factorizations. The serialization module provides helper methods inside the package *fdb::serialization* that allows a user of the library to serialize and deserialize a full factorization with its f-tree easily.

Namely the ***fdb::serialization::serialize(FRepTree*, ostream&)*** receives a data factorization, *FRepTRee*, and a reference to an output stream and serializes both f-tree and representation into the stream.
Its counterpart function ***fdb::serialization::deserialize(istream&)*** deserializes from the input stream and returns an *FRepTRee*.

4.4.1 Serializations illustrated

In this section we provide an illustration of the aforementioned serialization techniques and how they compare against the binary flat table serialization. The example factorization is used, see Figure 4.3.

The separator | is just used for illustration purposes to separate the different fragments of each serialization. In real-world it does not exist and the bytes of each fragment are contiguous.

Flat tuples binary serialization

```
1 1 1 1 2 1 | 1 1 1 1 2 2 | 1 1 1 2 2 1 | 1 1 1 2 2 2 | 1 2 2 1 2 1 | 1 2 2 1 2 2 | 2 1 2 1
1 1 | 2 1 2 1 2 1 | 2 1 2 1 2 2
```

total bytes = *number of tuples* * *number of attributes* * *sizeof(int)* = 9 * 6 * 4 = **216**

Simple Serializer

```
2 a1 a2 | 2 b1 b2 | 1 c1 | 2 d1 d2 | 1 c2 | 1 d1 | 1 e1 | 2 f1 f2 | 1 b1 | 1 c2 | 1 d1 | 2
e1 e2 | 1 f1 | 2 f1 f2
```

total bytes = (*number of unions* * *sizeof(uint_32)*) + (*sizeof(int)* * *number value nodes*)
= (14 * 4) + (20 * 4) = **136 bytes**

Bit / Byte Serializer

Recall that *Byte Serializer* and *Bit Serializer* use the same serialization form as the *Simple Serializer* but store only the required amount of bytes and bits respectively for each attribute union children count and union max value.

For the example we illustrate here the *Byte Serializer* just needs **1 byte** for both the union children counts and for the max value occurred in each attribute. Therefore its total serialization size would be **34 bytes**.
Bit Serializer needs **2 bits** to represent max values and max number of children occurred in each attribute so the serialization size is reduced to 68 bits, thus requiring **9 bytes**.

We should note that both these serializers require a header that for each attribute has **2 bytes** denoting the max number of bytes/bits used in each union or value (i.e. 6

attributes * 2 bytes each in the header). Thus, the total serialization size for *Byte* and *Bit Serializers* is **46** and **21 bytes** respectively.

Chapter 5

Distributed Query Processing in FDB

Mini TOC

Chapter 6

Experimental Evaluation

In this section we will present experimental evaluation for the main contributions of this project, namely the *COST* function for finding good f-trees explained in Chapter 3, the serialization techniques detailed in Chapter 4 and D-FDB, the distributed query engine as presented in Chapter 5.

6.1 Datasets and evaluation setup

This section contains information regarding datasets used and the evaluation setup used to record the reported times and sizes.

6.1.1 Datasets

We used two different datasets throughout the development and evaluation of the above contributions, both described below.

1. *Housing*

   This is a synthetic dataset emulating the textbook example for the house price market.

   It consists of six tables:

   - *House* (postcode, size of living room/kitchen area, price, number of bedrooms, bathrooms, garages and parking lots, etc.)
   - *Shop* (postcode, opening hours, price range, brand, e.g. Costco, Tesco, Sainsbury's)
   - *Institution* (postcode, type of educational institution, e.g., university or school, and number of students)
   - *Restaurant* (postcode, opening hours, and price range)
   - *Demographics* (postcode, average salary, rate of unemployment, criminality, and number of hospitals)
   - *Transport* (postcode, the number of bus lines, train stations, and distance to the city center for the postcode).

The scale factor s determines the number of generated distinct tuples per postcode in each relation: We generate s tuples in *House* and *Shop*, $log2(s)$ tuples in *Institution*, $s/2$ in *Restaurant*, and one in each of *Demographics* and *Transport*. The experiments that use the *Housing* dataset will examine scale factors ranging from 1 to 15.

2. *US retailer*

   The dataset consists of three relations:

   - *Inventory* (storing information about the inventory units for products in a location, at a given date) (84M tuples)
   - *Sales* (1.5M tuples)
   - *Clearance* (370K tuples)
   - *ProMarbou* (183K tuples)

6.1.2 Evaluation setup

The reported times for the *COST* function and the serialization techniques were taken on a server with the following specifications:

- Intel Core i7-4770, 3.40 GHz, 8MB cache
- 32GB main memory
- Linux Mint 17 Qiana with Linux kernel 3.13

The experiments to evaluate the distributed query engine D-FDB were run on a cluster of 10 machines with the following specifications:

- Intel Xeon E5-2407 v2, 2.40GHZ, 10M cache
- 32GB main memory, 1600MHz
- Ubuntu 14.04.2 LTS with Linux kernel 3.16

All experiments were run after the application was compiled with optimization flags turned on (i.e. O3, ffastmath, ftree-vectorize, march=native) and with *C++11* enabled.

6.2 COST – Finding good f-trees

In this section we evaluate the *COST* function, analyzed in Chapter 3. Through the following experiments we try to decide whether having a function that *estimates* the factorization size, in number of singletons, using statistics (i.e. unique values per attribute, number of unique values of attribute under another attribute's single value) derived from off-line preprocessing will give us better insights on f-tree selection compared to the existing work that uses the theoretical size bounds of FDB, parameter *s(Q)*.

For this experiment we will only use the *Housing* dataset for which we devised the optimal f-tree by hand, let's call it *Tree-O*. Recall that *Housing* dataset JOINs six relations on their common attribute *postcode*.
The optimal f-tree *Tree-O* is as shown in Figure 6.1.

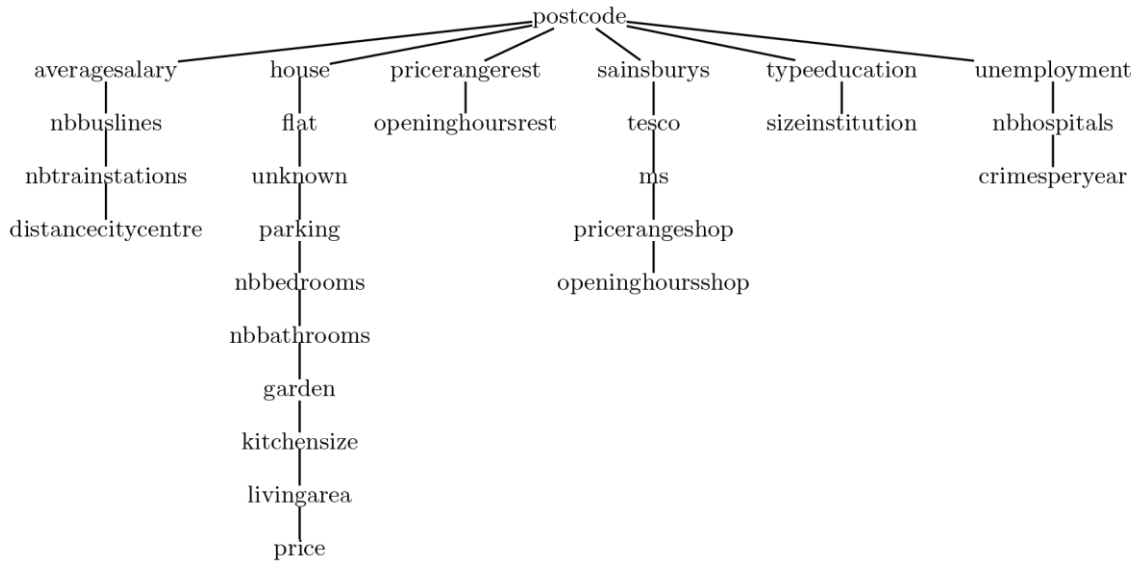


Figure 6.1: Optimal f-tree for Housing

This f-tree has a parameter $s(Q) = 2$ and it is optimal, which means FDB cannot find any f-tree asymptotically better than *Tree-O*. In order to evaluate our *COST* function we change the order of some attributes in their relation paths and compare the real factorization size in number of singletons with the estimation by *COST*.

The biggest desire here is for the estimations to follow the trend of real size with each f-tree, if not predict exactly. All these f-trees have $s(Q) = 2$ therefore they are indistinguishable by FDB.

From the experiments we made we noticed that while the scale factor increases the behavior of the *COST* differs. We will present results for Housing scale factors 1, 5 and 9 that show this variance in accuracy and ten different f-trees. The f-trees can be found in Appendix A.

In Figures 6.2, 6.3 and 6.4 we present the relation between real size and estimated size for each of the 11 f-trees we examine (optimal and its varieties) for scale factors one, five and nine respectively. In small datasets, see Figure 6.2, we see that the *COST* function estimates exactly the number of singletons in the factorization, which is the same for all f-trees. This leads us to believe that the branches in the factorization become

single paths very early and the COST restricts its estimation by the total size of a relation, hence always matching the real case.

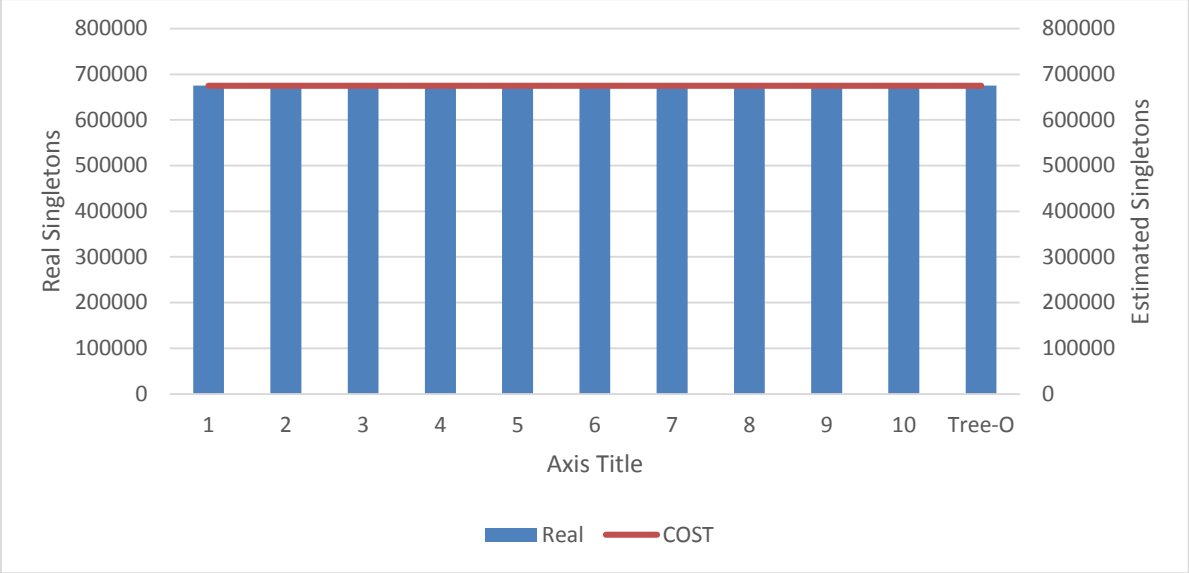


Figure 6.2: Real vs COST (Housing - 1)

Moving to scale factor five, see Figure 6.3, we see that the real size now differs up to 150 000 singletons among some f-trees. The estimated size is very high sometimes due to excessive usage of averages which can be misleading in many cases. However, we can see that the trend of the estimated size follows the real size which shows that it could be very useful to at least be able to eliminate very bad f-trees, always among those that have the optimal $s(Q)$ parameter.

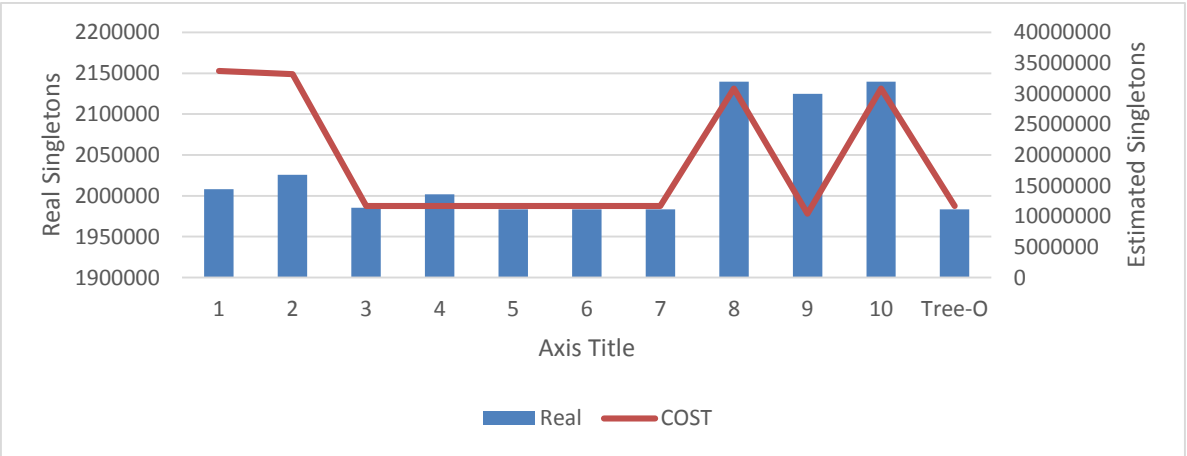


Figure 6.3: Real vs COST (Housing - 5)

Really interesting is the fact that f-tree 9 is always estimated wrongly by *COST*, which shows the weakness in using global averages, specifically the number of unique values of an attribute X under any other attribute Y. F-Tree 9 modifies the optimal f-tree in its fourth subtree (sainsburys, ..., *openinghoursshop*) such that it is completely reverse, thus having *openinghoursshop* on top, as child of *postcode*.

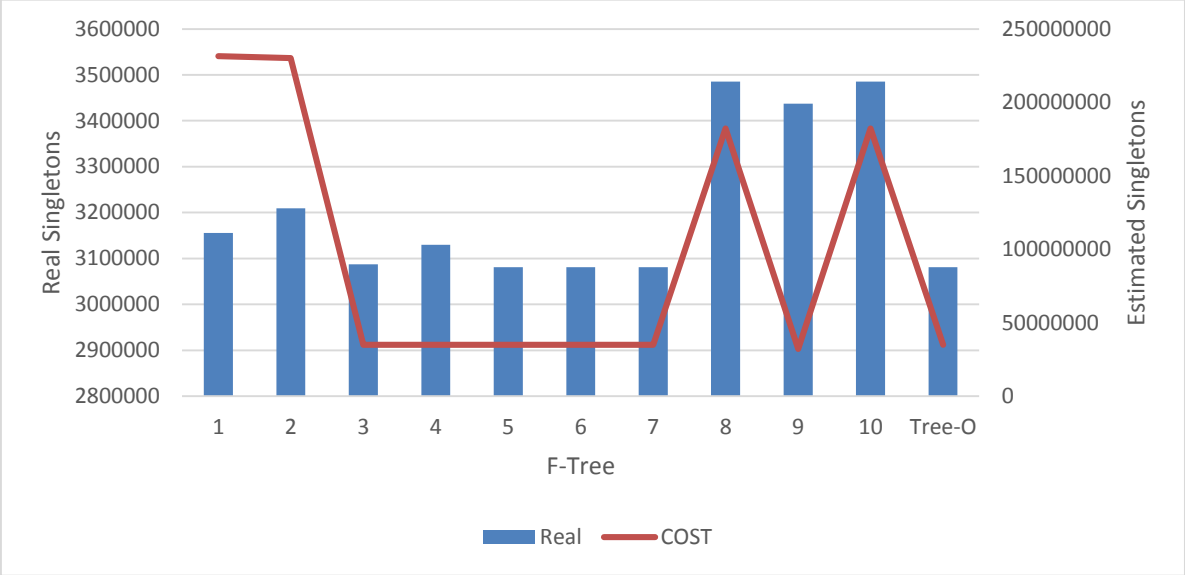


Figure 6.4: Real vs COST (Housing - 9)

Similar results can be seen in Figure 6.4, where again the *COST* function overestimates the size with some f-trees. F-Tree 1 swaps *house* with *flat* and f-tree 2 takes *house* as leaf of its branch as seen in the optimal f-tree.

In conclusion, although the COST is overestimating with some f-trees it can be used to reject some *bad* f-trees.

6.3 Serialization of Data Factorizations

In this section, we evaluate each serialization technique examined and described in Chapter 4. The factorizations we use to evaluate the serialization techniques are the result of applying *NATURAL JOIN* on all the relational tables of the two datasets, *Housing* and *US retailer*.

6.3.1 Correctness of serialization

The correctness test of each serialization was done both in-memory and off-memory (using secondary storage). For equality comparison between two factorizations we use a special function *toSingletons()* that traverses the factorization, encoding the singletons into a string representation that contains *a)* attribute name, *b)* value and *c)* attribute ID in text format, thus creating a huge string that contains the whole data of the factorization.

For the *in-memory* tests we performed the following steps:

1. Load the factorization from disk, let's call it *OriginRep*
2. Serialize it in memory writing into a memory buffer (array of bytes)
3. Deserialize the buffer into a new instance of a factorization, let's call it *SerialRep*
4. Check that the fields of *SerialRep* have valid values
5. Use the *toSingletons()* method and create the string representation for *OriginRep* and *SerialRep* and compare the two strings for equality. This ensures that not only we recover the same number of singletons properly but also that the IDs and values of those singletons are preserved during serialization and de-serialization, even with problematic datatypes like floating point values.

For the *off-memory* tests we performed similar steps as in-memory with an extra additional test to further prove correction.

1. Load the factorization from disk, let's call it *OriginRep*
2. Serialize it to a file on disk (binary file mode)
3. Open the file in read mode and de-serialize it into a new instance of a factorization, let's call it *SerialRep*
4. Check that the fields of *SerialRep* have valid values
5. Use the *toSingletons()* method and create the string representation for *OriginRep* and *SerialRep* and compare the two strings for equality.
6. Enumerate the tuples encoded by the factorizations *OriginRep* and *SerialRep* into two files. Compare the two files for equality using the standard command line tool *diff*.

6.3.2 Serialization sizes

In this section we will examine the size of the serialization output against the flat size of the input factorization (number of tuples).

The **Flat** serialization mentioned in some plots is the simplistic serialization of a flat relational table into bytes. That is by writing the bytes of each value in each tuple one after the other. Therefore, the total size would be equal to $\textit{number of tuples} * \textit{number of attributes} * 4\ \text{bytes}$ if for example all values are of the data type integer.

Additionally, we used the standard compression algorithms *GZIP* and *BZIP2* to compress *a)* the output serializations and *b)* the flat serialization. We incorporated compression in our experiments to investigate if applying these algorithms on the flat serialization would reduce the size close to our serializations, and also we apply them on the factorization serializations to analyze if there is still improvement to be made regarding value compression as part of our serialization techniques. We will use the notation *GZ1* and *GZ9* to denote compression using *GZIP* at minimum (1) and maximum (9) compression levels respectively. Similarly for *BZIP2* compression using *BZ1* and *BZ9*. The reason we have chosen these two compression techniques is because a) they are widely available and used in almost all web services (e.g. HTTP, REST APIs) and b) *GZIP* is a very fast algorithm with good compression, whereas *BZIP2* is slower but with much better compression, so we can have both choices tested.

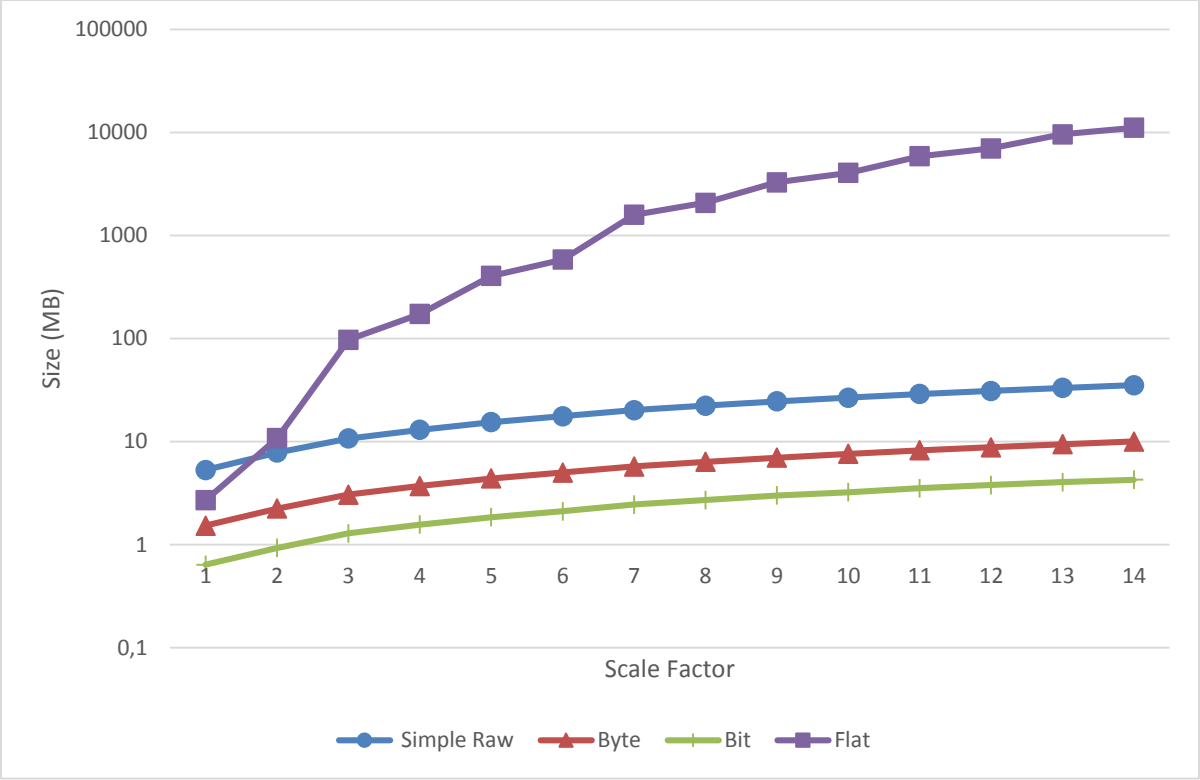


Figure 6.5: Serialization sizes against Flat serialization (Housing)

In Figure 6.5, we present the sizes of the serializations after using each one of our serialization techniques, *Simple* for Simple Raw Serializer, Byte for *Byte* Serializer and *Bit* for Bit Serializer, against the flat serialization for the *Housing* dataset. As expected, the flat serialization size is increasing by several orders of magnitude more than our serializations. This confirms that our serializations retain the theoretical compression factor brought by factorizing the relational table. Moreover, the figure shows that each extension of our serialization brings some additional reduction in the total size with the *Bit Serializer* being the best performing.

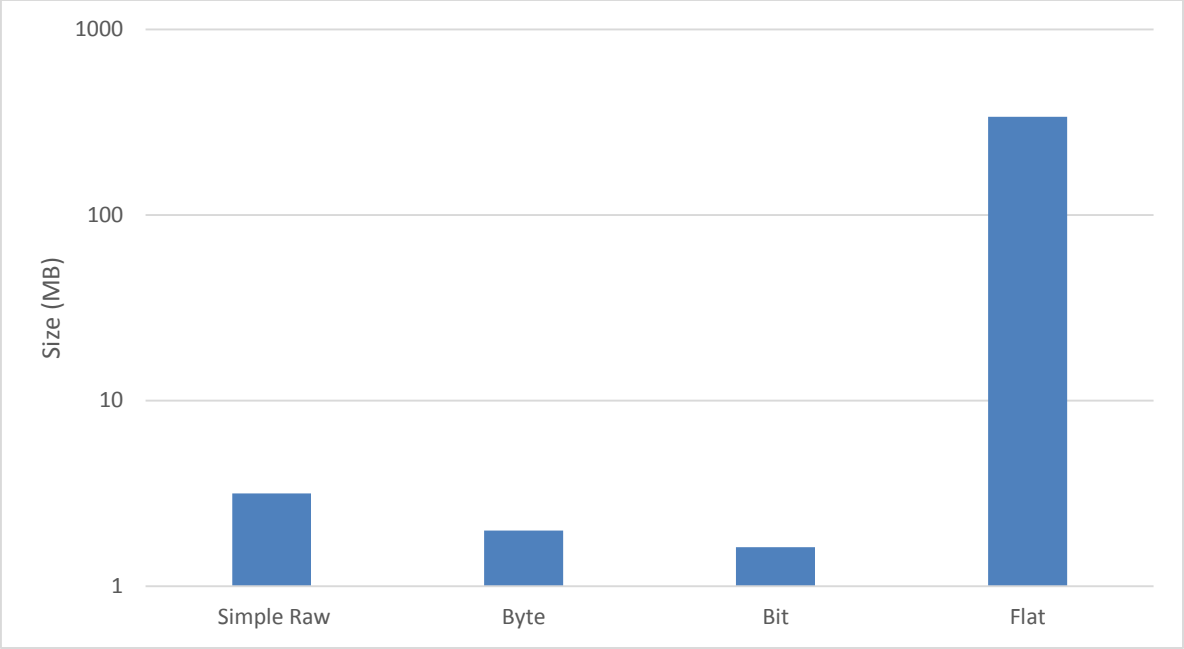


Figure 6.6: Serialization sizes against Flat serialization (US retailer)

The same results are shown in Figure 6.6, where all the serialization sizes follow the same pattern as the *Housing* dataset. The flat serialization is more than two orders of magnitude larger than our fatter serializer, Simple Raw, with Byte and Bit following with smaller output sizes and *Bit* being the best.

In addition, Figure 6.7, presents all three serialization techniques along with compression algorithms applied on their output for additional compression. It is clear that *Simple Raw* serialization which is just the byte enumeration for the values in the factorization grows linearly as the scale factor increases. The second worst serialization is of *Byte Serializer* without any compression applied, but it is very far from the worst and close to the rest of the sizes. A worthy observation is that after applying compression algorithms on-top of *Simple Raw* we get smaller serialization than that of *Byte*'s, which means that the values in this dataset are great candidates for compression. This can be also inferred by the difference in the sizes between the Simple, Byte and Bit outputs since each one uses a more refined technique to use as much less bytes as possible.
Another important point is that Bit serialization is almost perfect, since even when the compression algorithms were applied on it its size did not reduce at all, which means that for this dataset we already do sufficient compression to the values.

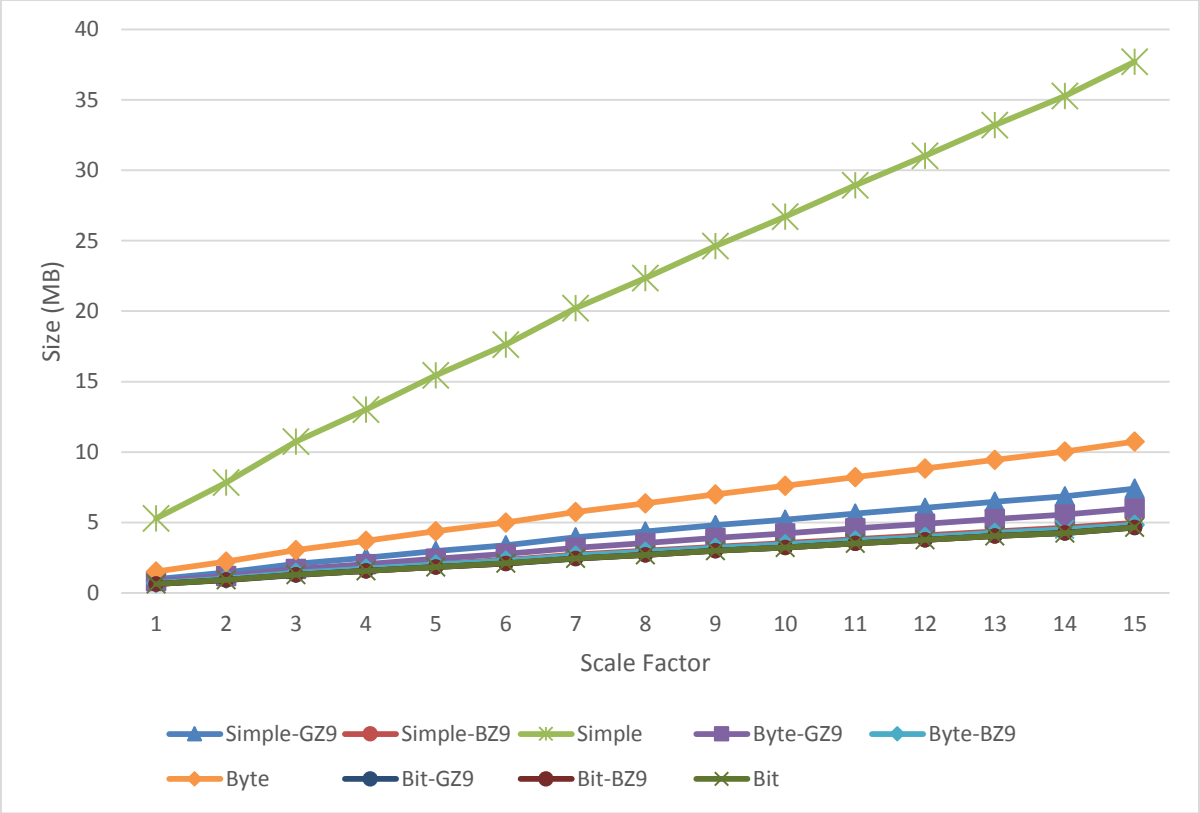


Figure 6.7: Compression GZIP and BZIP2 applied on our serializers

In Figures 6.8 and 6.9 we further explore the effect of additional compression on our serializations. It is clear that the flat serialization can benefit significantly from compression which is expected, but still Figure 6.8 shows that for *Housing* dataset there is a difference between the maximum compression of BZIP2 and GZIP on flat serialization and Bit serialization of two orders of magnitude.

In Figure 6.9 we have different results, which arise due to different datasets. In *US retailer* dataset Bit serialization is still the best performing in terms of output-size but the difference from the flat serialization having applied any of the compression algorithms is not as big as with *Housing* dataset (only around one order smaller). Additionally, the difference between our serializations is also smaller. Having investigated the datasets better, we found that large amount of values in *Housing* dataset are only single-digit numbers, therefore they have many leading zero-bits in their representation in memory (sometimes 31 out of 32), hence the big gain using Bit serialization. However, in *US retailer* dataset the values are more random and there are less such values.

Although, the advantage is smaller in *US retailer* using our serialization technique is still more preferable because as we will show later it is considerably faster.

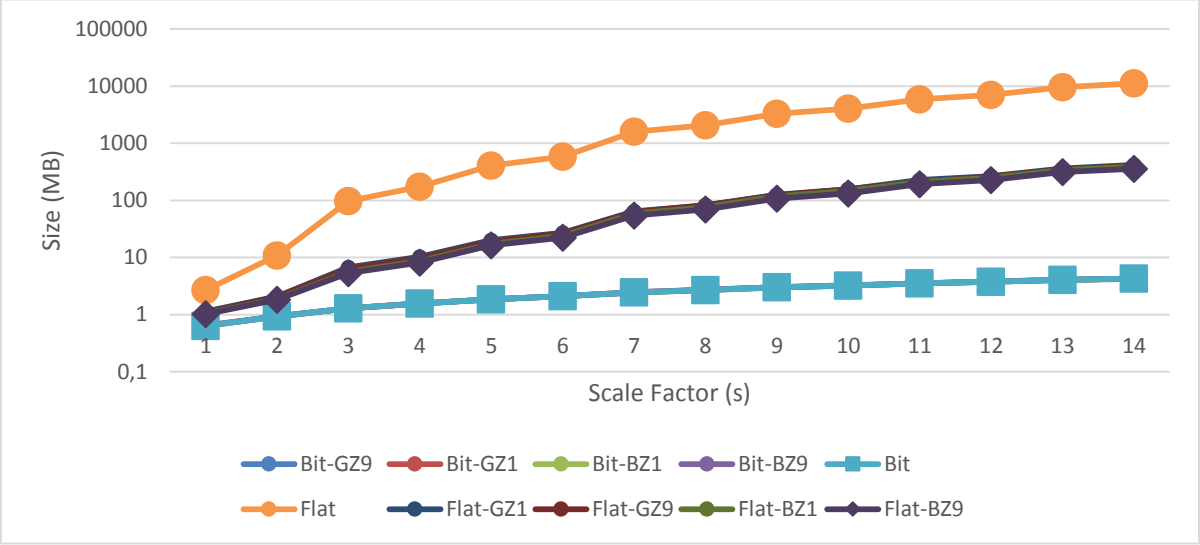


Figure 6.8: Compression GZIP and BZIP2 applied on Bit and Flat serializations (Housing)

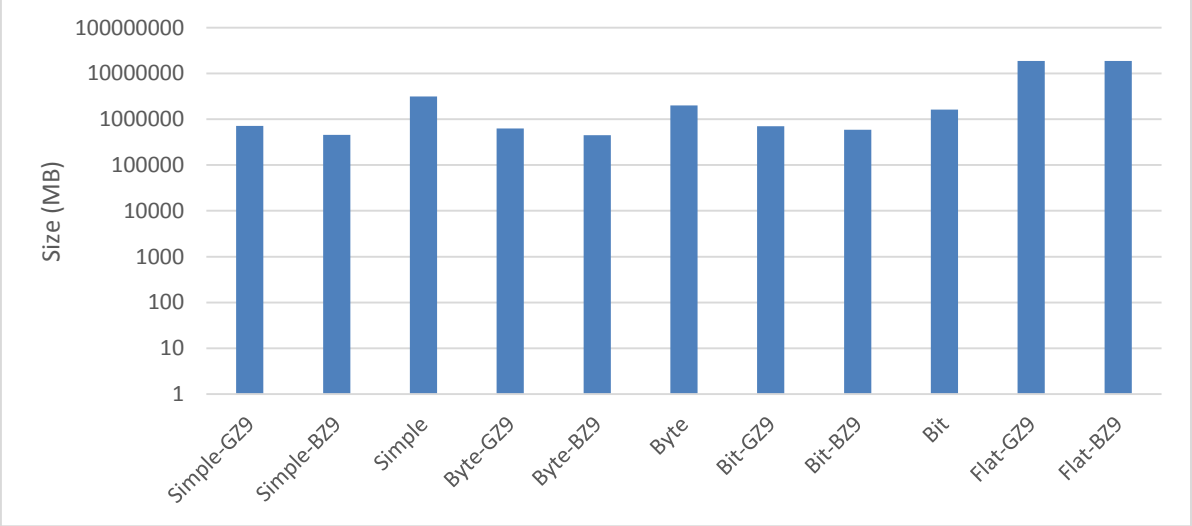


Figure 6.9: Compression GZIP and BZIP2 applied on Bit and Flat serializations (US retailer)

6.3.3 Serialization times

In this section we evaluate the time required to serialize factorizations using our serializers with and without compression techniques on-top.

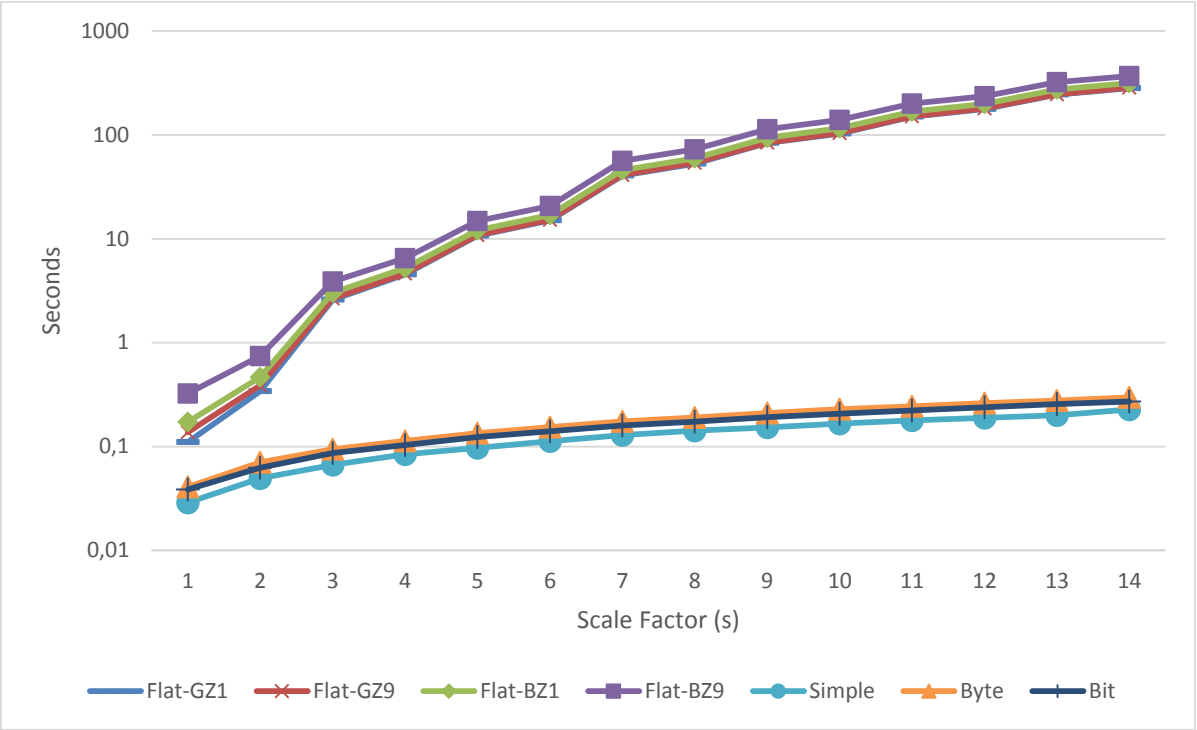


Figure 6.10: Serialization times with compression only on Flat (Housing)

First of all, Figure 6.10, presents the serialization times for our serialization techniques *without* any compression applied and the flat serializations with both compressions. The reason that we decided to show ours without and flat with compression is that we will never ship data over the network *as is* without compressing them due to the huge size, therefore the default choice for a real-world application would be either *GZIP* or *BZIP2* or some other algorithm with similar properties.
The performance of our serialization techniques is more than two orders of magnitude even when applying minimum compression level on flat serialization with both *GZIP* and *BZIP2*.

Figure 6.11 presents the times for all our serializations with and without compression applied on-top. There is significant overhead added, as seen by comparing *Bit* serialization without compression and *Bit-BZ9* for example, or *Byte* with *Byte-GZ9*, but even with compression added the serialization times are significantly faster than compressing the flat serialization.

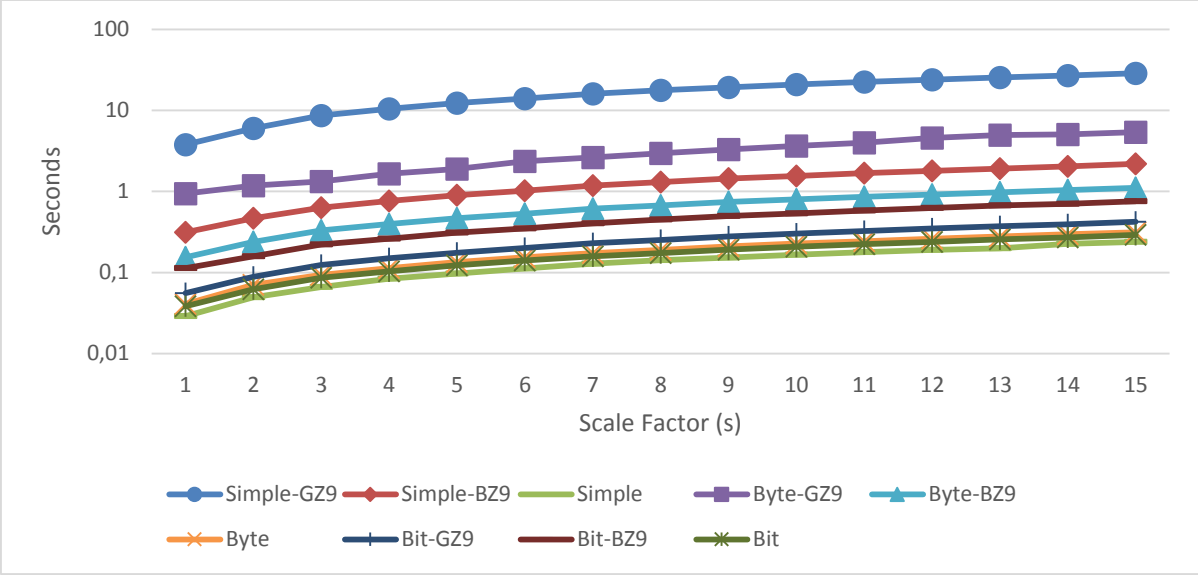


Figure 6.11: Serialization times with compression (Housing)

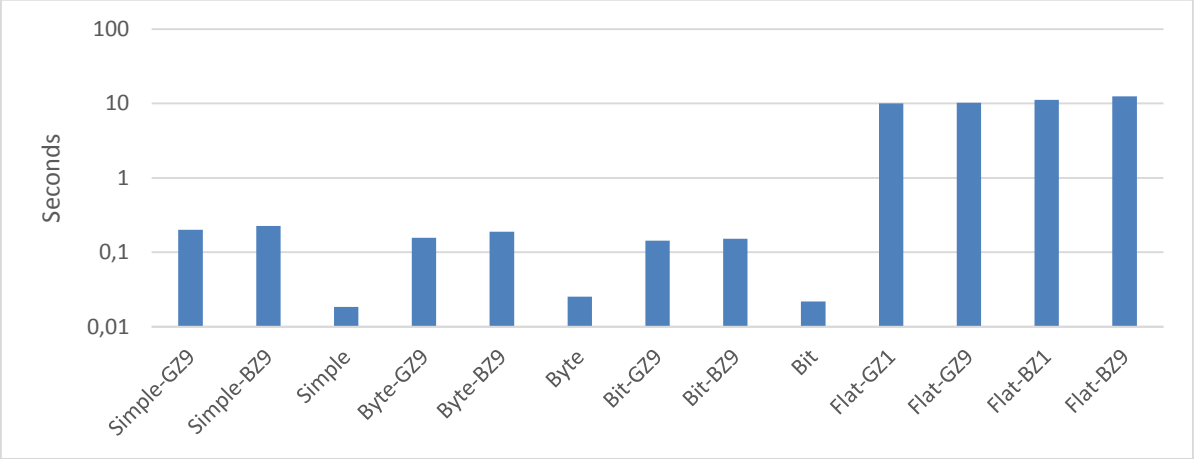


Figure 6.12: Serialization times with compression (US retailer)

Figure 6.12, shows the same experiment, compression applied on-top of the serialization and we see very similar results. Compression upon the flat serialization is a lot slower than compression upon our serializations, which in turn is slower than our serialization without compression.

6.3.4 Deserialization times

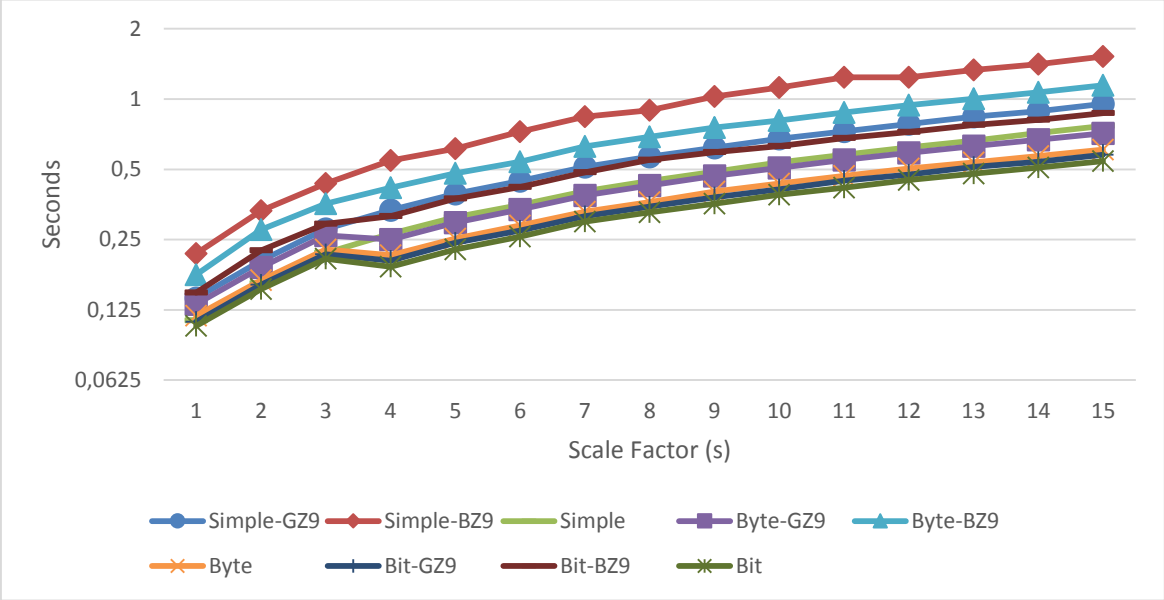


Figure 6.13: Deserialization times for our de-serializers (Housing)

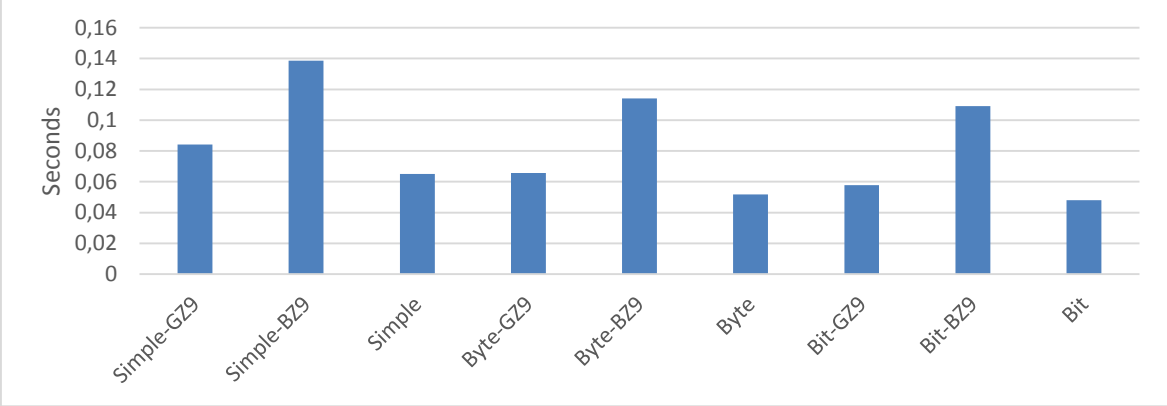


Figure 6.14: Deserialization times for our de-serializers (US retailer)

In this section we examine the time needed to de-serialize a serialized factorization back into a factorization in memory.

Both datasets have similar results, as seen in Figures 6.13 and 6.14. It is obvious that *BZIP2* is the slowest in all three de-serializers. *GZIP* compression is fast and this is shown in our results since the difference between de-serializing with and without this compression is small, however it is still an overhead. It is remarkable to that even though

Byte and *Bit* have additional complexity compared to *Simple Raw* de-serializer they both have faster times, which is due to the smaller total size they process.

6.3.5 Conclusions

We performed a variety of experiments with all three serializations against two datasets with different characteristics (one artificial with a lot of single-digit values, one real-world dataset with complex values). We also compared our serialization against the flat serialization with and without compression.

Analyzing the results of these experiments led to the following conclusions:

- The three serializers retain the theoretical compression of factorizations against flat relational tables into their serializations.
- The flat serialization requires significantly more time to apply compression on its data than our serializers with and without compression applied on them.
- The benefit of applying additional compression over the three serialization techniques depends mostly on the actual factorization values, but especially with *Bit Serializer,* which is the final version, it is questionable whether the additional overhead to compress is worthy.
- We showed that it would be very interesting to explore additional extensions to Bit Serializer in order to enhance its compression capabilities. A very important feature of our serialization algorithms is that during de-serialization we *do not have to* process *all* the data as is the case with standard compression algorithms that process large blocks each time.
- Overall, we conclude that Bit Serializer can be the basis of more advanced serialization techniques for factorizations and that even at this stage it can be a great alternative to standard compression algorithms for systems that use factorizations as a means of data communication.

Chapter 7

Conclusions and Future Work

Mini TOC

A player faces a dynamic optimization problem of 5 periods. Let a_t denotes the player's action in period t,

$$a_t \in \{P, N\} \tag{2}$$

We denote the vector of action choices by $\boldsymbol{a} = (a_1, a_2, a_3)$. Playing in a period yields an immediately consumption level of x at a certain future cost, to be paid at period 4, while not playing yields no consumption and incurs no cost, so

$$x_t = \begin{cases} x & if\ a_t = P \\ 0 & if\ a_t = N \end{cases} \tag{3}$$

The player observe x in period 1 before she pick her action.
Let C_s denotes total cost for playing s games and S_t the number of games played up till and including time t.

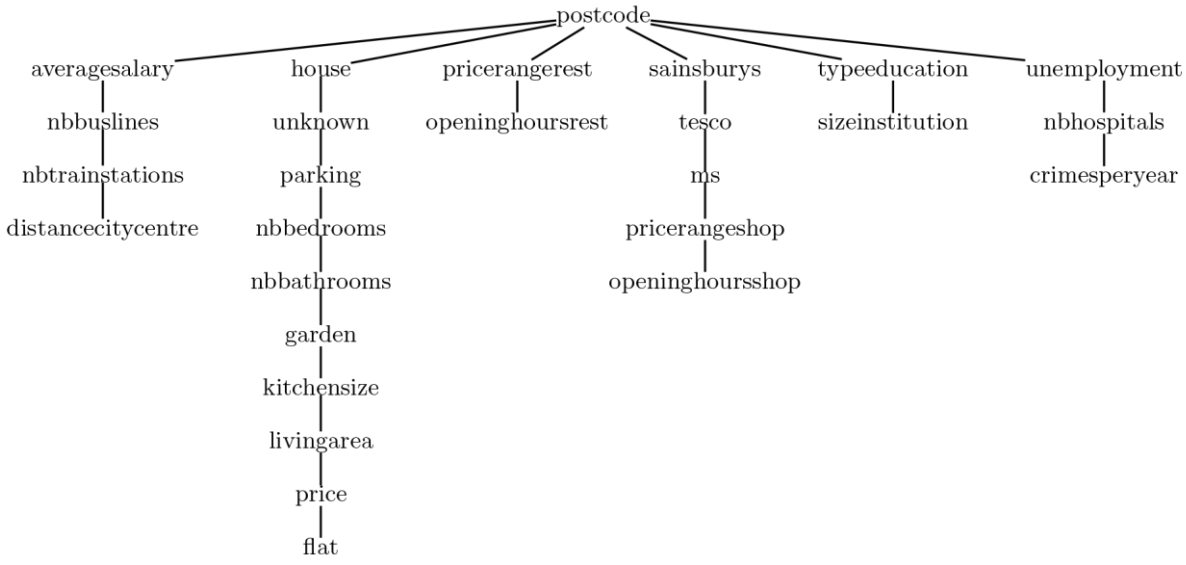
This paper.[1] Theoretically, ...
The issue of ...
This paper is organized as follows. The next section presents ... Then, Section 3 discusses

[1] Ashraf et. al [1] uses a ...

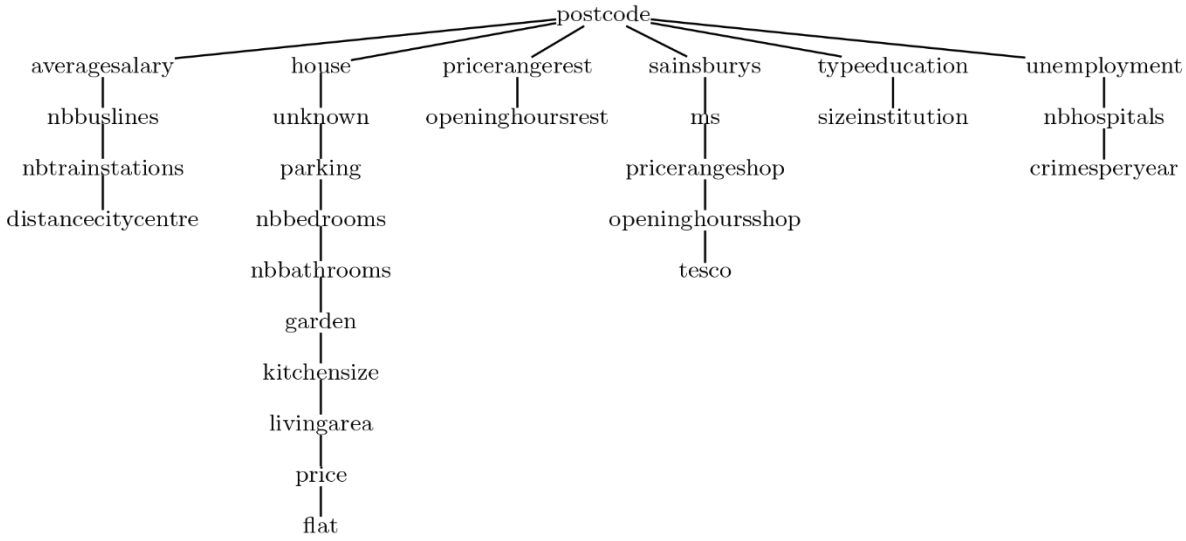
the ... Section 4 analyzes the ... Concluding remarks are offered in Section 5.

Algorithm 3.1: Calculate estimated size for factorization using given f-tree	
	`// @fTree: the f-tree to estimate the size for, if used for factorization` `// @FLATSIZE: the flat size in number of tuples`

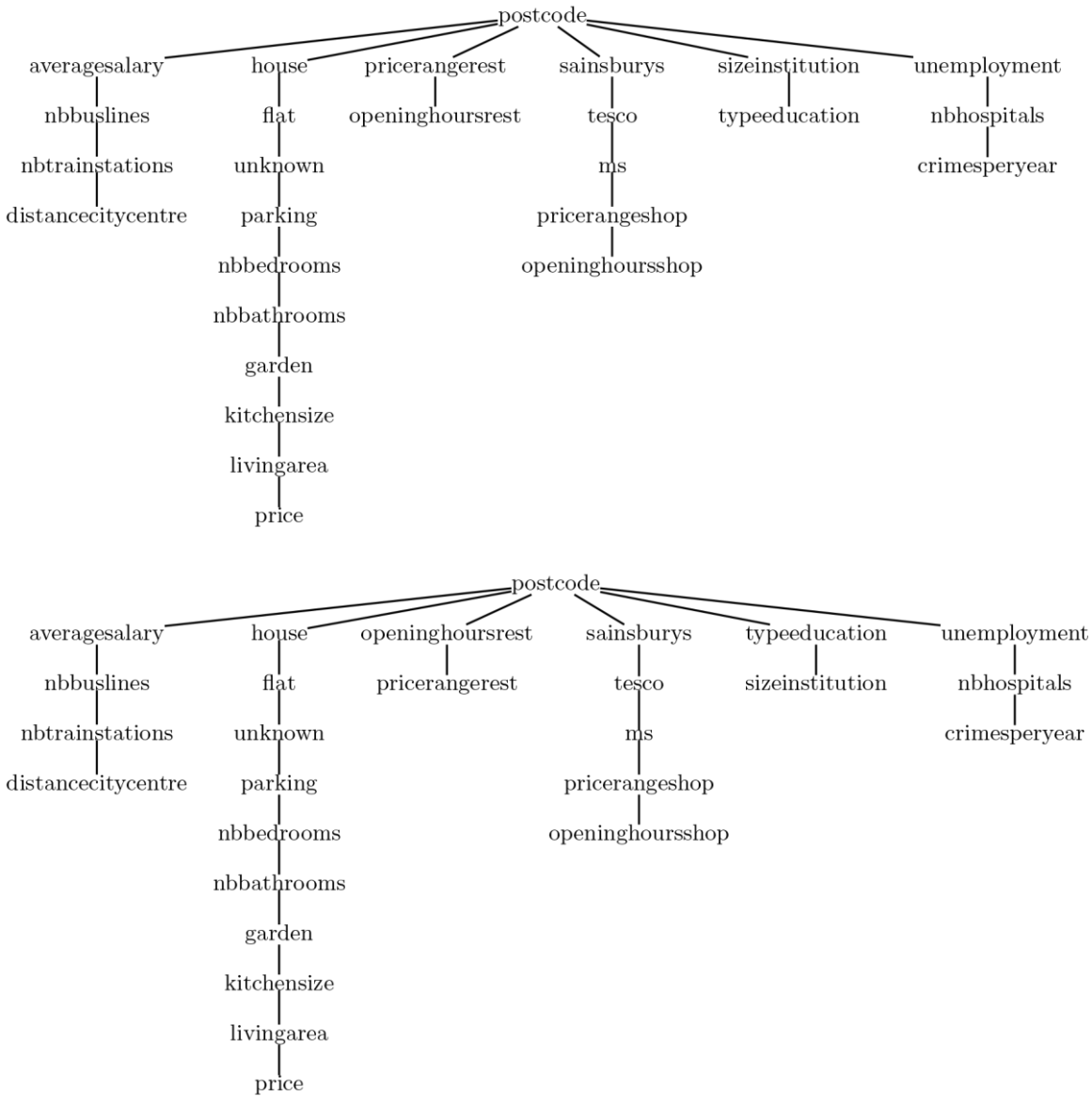
Appendix A



Appendix 1: F-Tree 1



Appendix 2: F-Tree 2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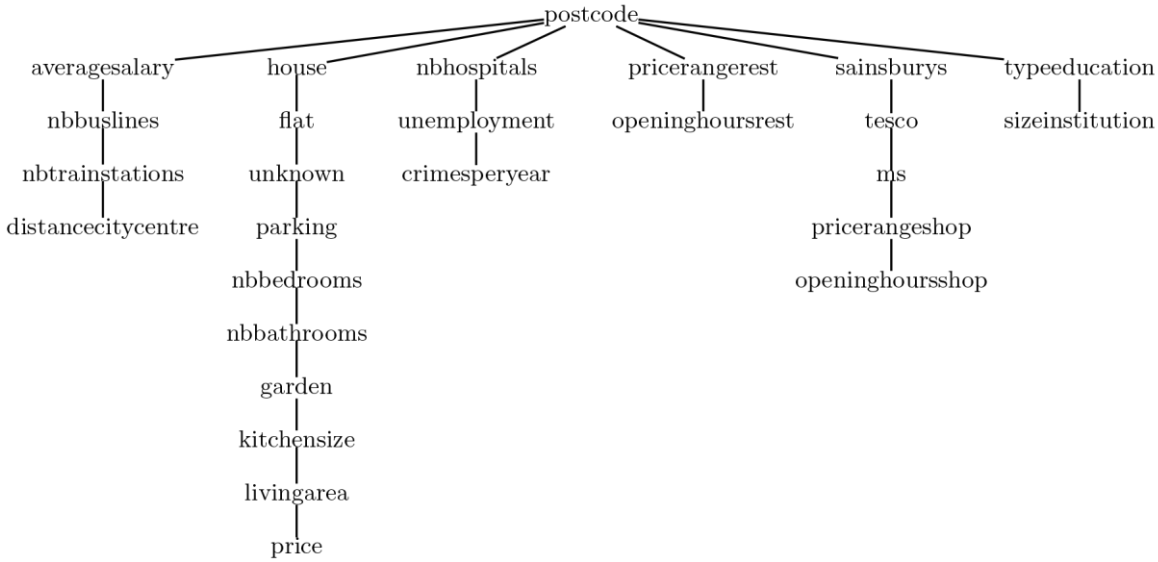
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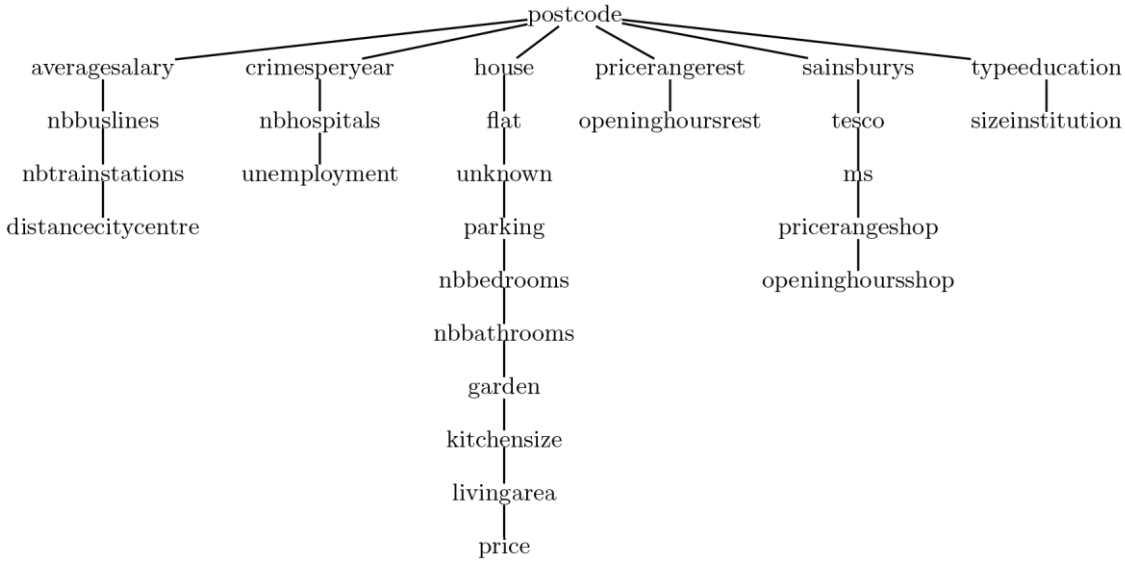
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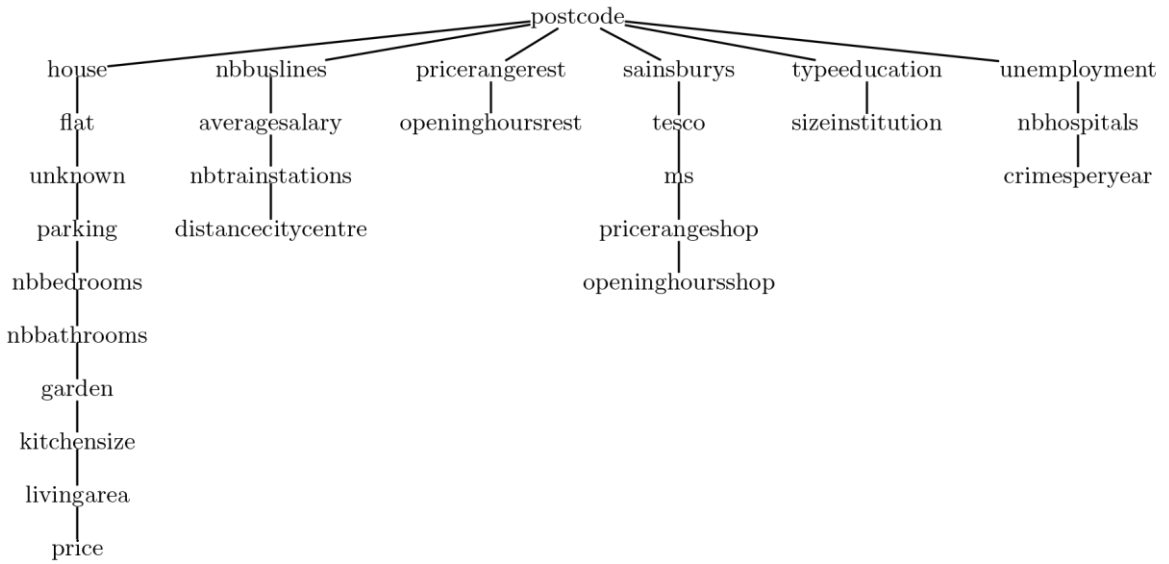
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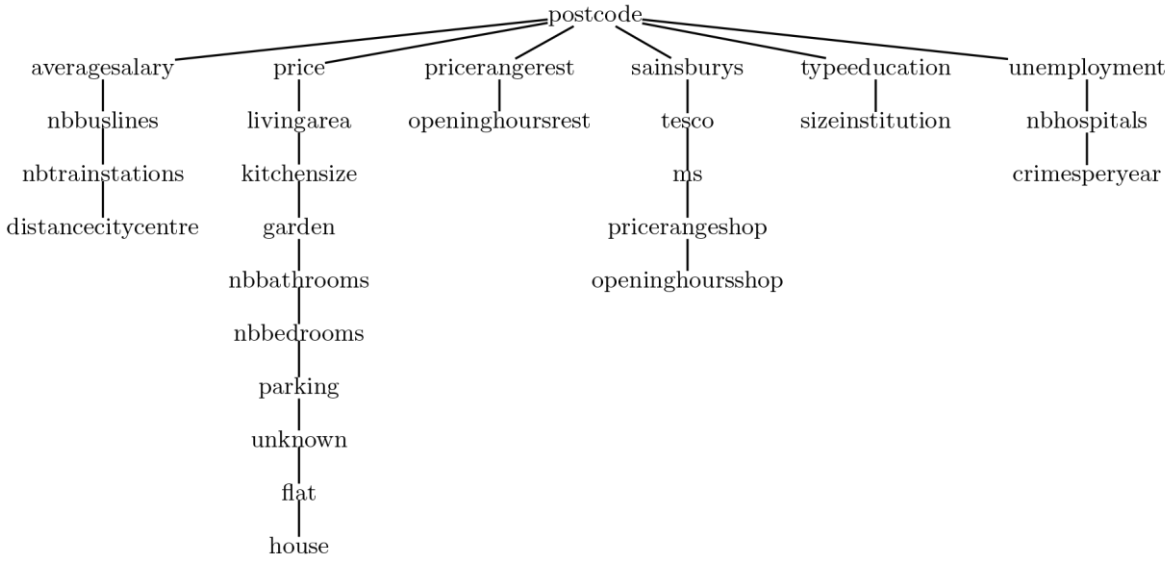
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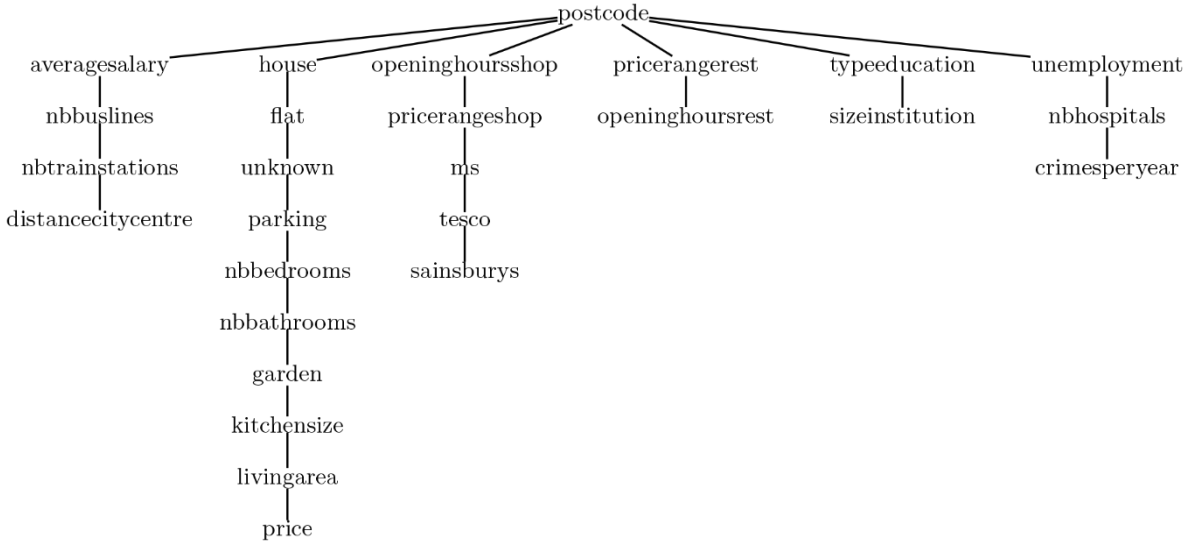
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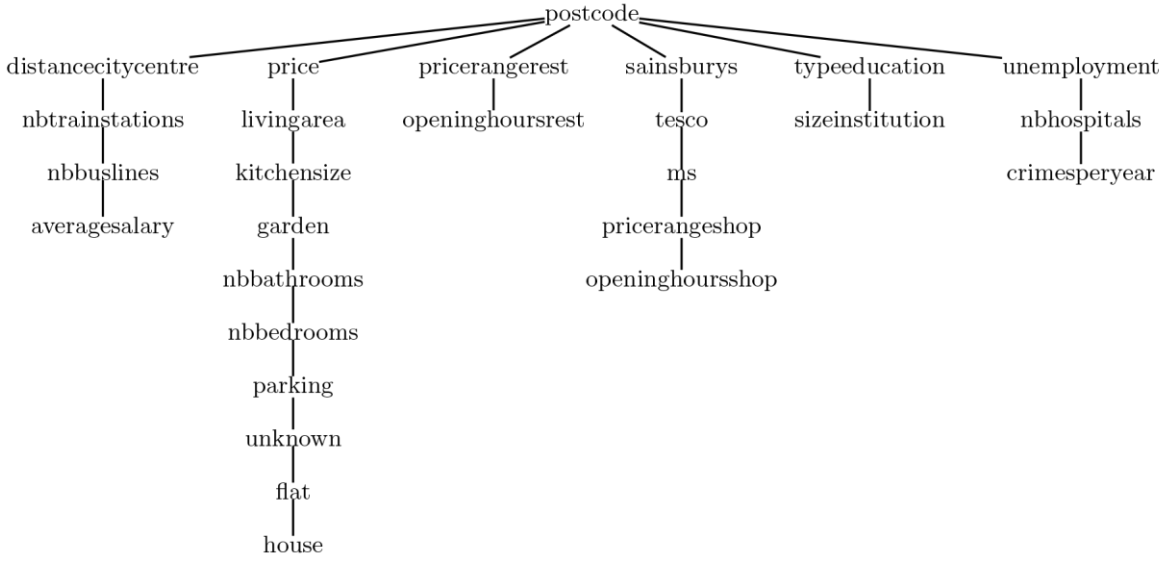
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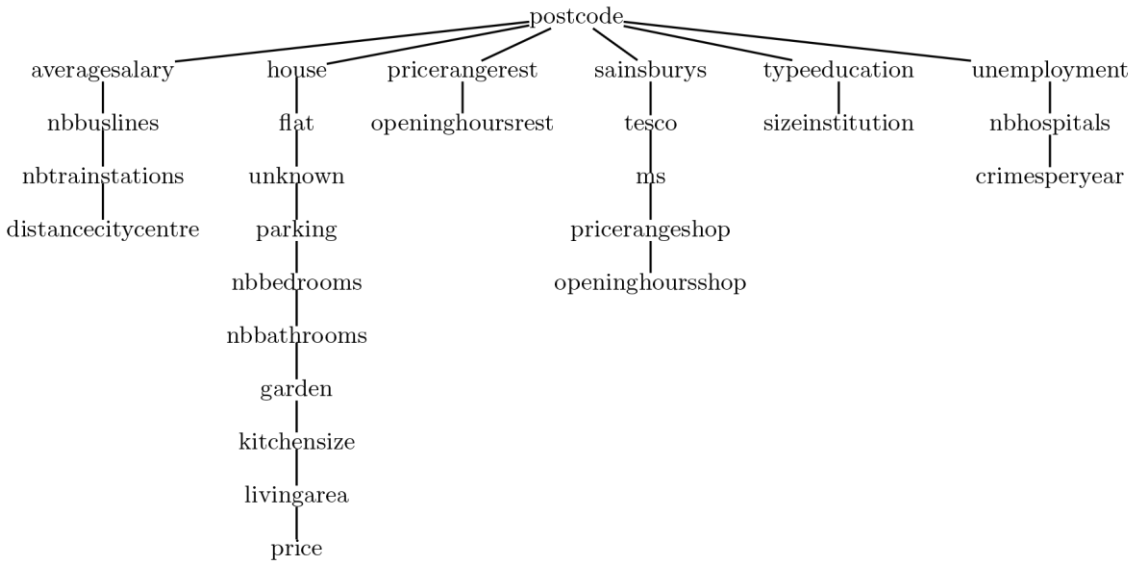
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Ashraf, Nava, Dean Karlan and Wesley Yin. "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines." Quarterly Journal of Economics. Vol. 121, No. 2, pp. 635-672. May 2006.