Assignment 1
CAP 6419 - 3D Computer Vision

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Q1. (10 pts.) We learned about the duality of points and lines in the projective space of dimension 2 ($\mathbb{P}^2$). Therefore, a 3-vector $\begin{bmatrix} a & b & c \end{bmatrix}^T$ can be either a point in $\mathbb{P}^2$ or a line in $\mathbb{P}^2$. What is the relationship between the point $\boldsymbol{m} \sim \begin{bmatrix} a & b & c \end{bmatrix}^T$ and the line $\boldsymbol{l} \sim \begin{bmatrix} a & b & c \end{bmatrix}^T$? (explain) (Hint: find the distances from the origin to both $\boldsymbol{m}$ and $\boldsymbol{l}$ and try to see if you can identify a relationship.)

Q2. (10 pts.) Determine the point at infinity (the ideal point) on the line $\boldsymbol{l} \sim \begin{bmatrix} 5 & -7 & 3 \end{bmatrix}^T$. What is the dual line to this ideal point? What can you say about this line? What is the dual point to the line at infinity? (You must show all your work.)

Q3. (10 pts.) (Isomorphism of Incidence) Consider the two lines $\boldsymbol{l}_1 \sim \begin{bmatrix} 5 & -7 & 3 \end{bmatrix}^T$ and $\boldsymbol{l}_2 \sim \begin{bmatrix} -3 & -5 & 2 \end{bmatrix}^T$. What is the dual point $\boldsymbol{m}$ corresponding to the line through the points dual to the lines $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$? Show how you would find the answer and verify mathematically based on your answer in Q1. What can you say about $\boldsymbol{m}$? (Explain)
How does your answer to Q3 explain the answer to Q2? (Hint: Use the x and y axes as the two lines.)

Q4. (20 pts.) Five points in $\mathbb{P}^2$ uniquely define a general conic. Therefore a general conic is fully specified by a symmetric 3x3 matrix **C** that in general has 5 d.o.f.:

$$\mathbf{C} \sim \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$$

As we learned in the lectures a point $\boldsymbol{m} \sim \begin{bmatrix} x & y & 1 \end{bmatrix}^T$ is on **C** *iff* $\boldsymbol{m}^{\mathrm{T}}\mathbf{C}\boldsymbol{m} = 0$. If $a = c$ and $b = 0$, then **C** has only 3 d.o.f. and becomes a circle (in the usual Euclidean sense that you are familiar with).

For this question, you are required to write a Matlab function that would allow the user to input random values for the parameters $a = c$, d, e, and f (with b assumed equal to zero), and would output the intersection points of the circle **C** with the line at infinity $\boldsymbol{l}_\infty \sim [0 \quad 0 \quad 1]^T$.

Experiment with many different values of the input parameter set and answer the following questions:
What do you notice?
What conclusion can you draw from this experiment?

Do you have any explanation?
(If not, do not worry…you will not lose points for not giving an explanation.)

Reminder: The intersection of a line $\boldsymbol{l}$ and a conic **C** can be determined as follows: Let $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ be two points on the line $\boldsymbol{l}$. Then, any arbitrary point $\boldsymbol{m}$ on the line can be specified parametrically by $\boldsymbol{m} = \boldsymbol{m}_1 + \lambda \boldsymbol{m}_2$. Point $\boldsymbol{m}$ is on the intersection of the line with the conic **C**, *iff*

$$\boldsymbol{m}^T \mathbf{C}\, \boldsymbol{m} = (\boldsymbol{m}_1 + \lambda \boldsymbol{m}_2)^T\, \mathbf{C}\, (\boldsymbol{m}_1 + \lambda \boldsymbol{m}_2) = 0$$

This yields the following quadratic equation in terms of λ

$$\lambda^2 \boldsymbol{m}_2^T \mathbf{C}\, \boldsymbol{m}_2 + 2\lambda\, \boldsymbol{m}_2^T \mathbf{C}\, \boldsymbol{m}_1 + \boldsymbol{m}_1^T \mathbf{C}\, \boldsymbol{m}_1 = 0$$

From which we get the two values λ_1 and λ_2 , and hence the two intersection points $\boldsymbol{m}_1 + \lambda_1 \boldsymbol{m}_2$ and $\boldsymbol{m}_1 + \lambda_2 \boldsymbol{m}_2$. Note that in general a line intersects a conic at two points, which may be distinct or not and real or complex.