Mathematic for Computer Science: Hanoi's Tower Problem

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Hanoi Tower is a classical mathematics/computer science problem. The problem state 2 variables: n number of disks and k number of pegs; classically $k = 3$. The goal of the puzzle is to move a stack of disks from the initial (or Departure) peg to the target (or Arrival) peg. Furthermore, the disks must be stack in an increase order of weight.

Classical Problem

In order to solve the problem for $n = 5, k = 3$, we devide the problem into subproblems. First, we move $n - 1$ disks to the intermidiate peg. The *n-th* disk (the largest disk) can now be move to the destination peg. The $n - 1$ disks stack can now move on to the destination peg.

This sequence give us a recursive solution to Hanoi Tower problem. Every iteration give us 2 subproblems (one for move $n - 1$ disks to intermediate peg, and one more to move said stack to the destination). Thus, the complexity for this solution is $\Theta(2^n)$.

In fact, the number of move required to move disks from one peg to another is $2^n - 1$

Coding of the position

In order to encode a state of the game we can ultilize k linked list to decode the state of the pegs. The head of the list refer to the disk on top of said stack, thus one can avoid puting a larger disk on top of the smaller disk. In this case, memory space complexity is $\Theta(n)$

In order to encode a move we simply need the original and destination pegs, or 2 variables.

Variation: Unbounded number of pegs

For a variation where $k \geq n + 1$ the puzzle become trivial. A simple solution where all $n - 1$ disks are dispered throughout the pegs, leave room for the *n-th* disk to arrive at the destination peg before recollecting the stack at the destination peg is completed in 2n step, thus have time complexity of $\Theta(n)$.

General solution

The two cases $H(n, \Theta(n))$ and $H(n, \Theta(2^n))$ represent 2 extreme of the input space for Hanoi's Tower problem, the best and worst case respectively. The average case, however, lie somewhere between these two extreme.

Let us consider the case where $n > k > 3$