Analysis of Perceptron Algorithm

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Algorithm

The perceptron algorithm is one of the first supervised models proposed *(Rosenblatt, 1957)* . The algorithm is trained by finding the parameters of a linear function defined by

$$h_w : \mathbb{R}^d \to \mathbb{R} \quad (1)$$
$$x \mapsto \langle w, x \rangle \quad (2)$$

using a training set $S = ((x_i, y_i))_{i=1}^m$ of size m where, $\langle ., . \rangle$ denotes the dot product and the classes verify $\forall i \in \{1, ..., m\}, y_i \in \{-1, +1\}$. The training of the model is done on-line following the perceptron update algorithm.41 mins ·

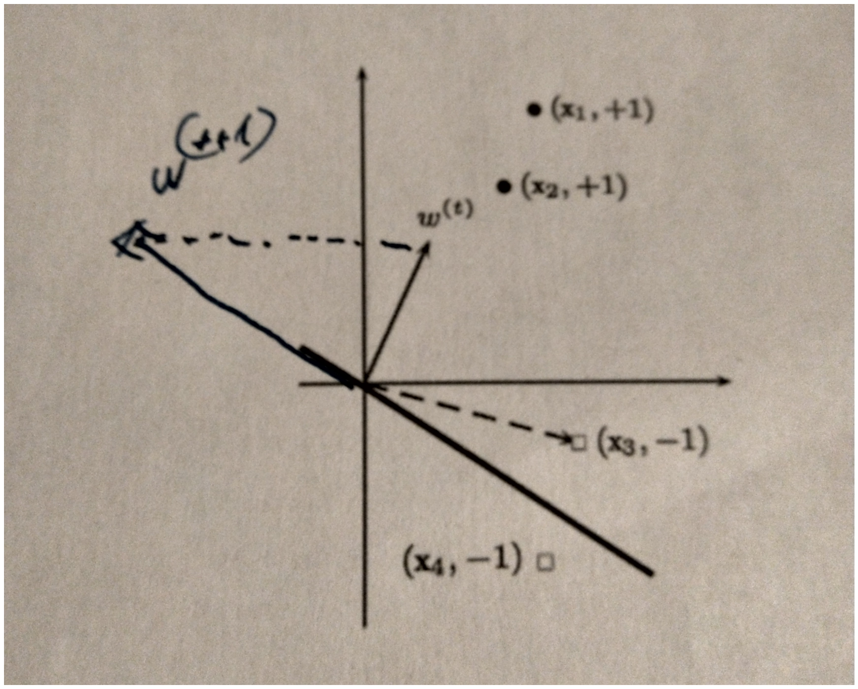
1. The algorithm randomly select a datapoint (x_i, y_i) and check with the current model (w^t) if the model correctly predict the outcome (y_i). $y \cdot (w \cdot x) = 1$ iff $y = (w \cdot x)$. Indeed, if $y = 1$, $y \cdot (w \cdot x) = 1$ if and only if $w \cdot x = 1$ too, and vise versa for $y = -1$. Otherwise, $y \cdot (w \cdot x) = -1$.

2. When a datapoint is found to disagree with the model, the model is updated by
$$w^{(t+1)} = w^{(t)} + \epsilon \times y \times x$$
where ϵ is the learning rate; $0 \leq \epsilon \leq 1$. This mean that the weight is modified by a scalar of $y \times x$.

   $y \times x$ is a scalar product of x itself. As $y \in \{-1, +1\}$, this essentially meant that depend on the intended outcome, the model should be the align or opposite to the direction of $\vec{x}$.

3. According to fig. 1, $w^t \cdot x_3 > 0$; however $y_3 = -1$. Therefore, $w^{(t+1)}$ is updated as:

$$\begin{aligned} w^{(t+1)} &= w^t + \epsilon \times y \times x \\ &= w^t - x_3 \end{aligned}$$



4. $(y_i \times (w*, x_i)) > 0$ mean the dot product of $w*$ and x_i correctly predict the class of the output y_i.

5. Since $\frac{w}{||w||}$ is a scalar product of w (also know as normalization), the product $y_i \times \langle \frac{w}{||w||}, x_i \rangle > 0$ for all i (since $\forall i, y_i \times \langle w, x_i \rangle > 0$). Therefore $min_{i \in [1,..,m]}(y_i \times \langle \frac{w}{||w||}, x_i \rangle)$ is strickly positive.