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CPSC 313 - Assignment 1

Exercise 1

1. $\Sigma = \{a, b\}$ $\qquad L_a = \{aa\}$
$L_2 = \{a^i b^j \mid |i-j| \leq 2\}$ $\qquad L_b = \{bbbb\}$
Compute $L = L_a(L_2 L_b \cap L_2)$.
$L_2 L_b = \{a^i b^j bbbb \mid |i-j| \leq 2\}$
$L_2 L_b \cap L_2 = \emptyset$ (we always will have too many b's, ε is not possible for $L_2 L_b$)
$L_a(L_2 L_b \cap L_2) = L_a(\emptyset) = \emptyset$
$\therefore L = L_a(L_2 L_b \cap L_2) = \emptyset$ No 0.75 Good try

2. Prove $(A \cup B)^* = A^*(B A^*)^*$
Suppose A and B are languages and $x \in (A \cup B)^*$. Since Union is commutative, $x \in (B \cup A)^*$. Then, by definition, x is a string obtained by concatenating zero or more strings from A, or zero or more string from B. From this definition we can deduce that $x \in (B^* A^*)^*$, because we can still get any string from $(A \cup B)^*$ in $(B^* A^*)^*$ by using $(B^0 A^*)^*$ for all strings in A, $(B^* A^0)^*$ for all strings in B, can $(B^* A^*)^*$ for all possible strings from A and B. We can also manipulate $(B^* A^*)^*$ to become $A^*(B A^*)^*$, because since we can initially make $A^0(B A^*)^*$, and concatenate strings of the form $A^*(B A^*)^*$ afterwards, these are equivalent because $(B^* A^*)^* = (B^* A^*)^0 \cup (B^* A^*)^1 \cup (B^* A^*)^2 \ldots$ and $A^*(B A^*)^* = A^*((B A^*)^1 \cup (B A^*)^2 \cup \ldots)$ and we can use different concatenations to make the same strings. Thus, if $x \in (A \cup B)^*$, ~~then~~ $x \in A^*(B A^*)^*$.

$\subseteq$ $\supseteq$
1. Try to shorten it
$\Rightarrow$ $\Leftarrow$
= 0.25
$\Leftrightarrow$

Now, we let $x \in A^*(B A^*)^*$. For reasons stated above, and since equality works in both directions, we know that $A^*(B A^*)^*$ is ~~equate~~ equivalent to $(B^* A^*)^*$, and by definition of union and kleene star, $(B^* A^*)^* = (B \cup A)^*$, and because union is commutative, $(B \cup A)^* = (A \cup B)^*$.
Thus, if $x \in A^*(B A^*)^*$, then $x \in (A \cup B)^*$.
Therefore, $(A \cup B)^* = A^*(B A^*)^*$

Exercise 2

Let the holes be the number of ways we can color each column of the 3×9 grid. We have $2^3 = 8$ possible combinations of ways we can color the columns red or black. Let the columns be the pigeons. Since we have 9 columns (pigeons) and 8 different colorings (holes) so two of the columns must be colored the same way. Since each column has at least 2 red or 2 black points, we can form the 4 corners of a rectangle.

Exercise 3

Proof by Induction:

Base Cases: 1 can be written as the sum of 1
2 can be written as the sum of 2
3 can be written as the sum of 3
4 can be written as the sum of 4
5 can be written as the sum of 5
→ Since a sum may contain a single number.

Inductive step:

Suppose k is an integer with $k \geq 6$ such that k can be written as the sum of distinct numbers from the series (IH). Then we must show that $k+1$ can be written as the sum of distinct numbers from the series.

Good idea but not correct way of proof

For any arbitrary k, if we want to construct it using distinct numbers from our series, we first begin with the largest number in our series that is smaller than k, and add it to our sum, then we move to the next smallest number in the series and add it to our sum, if it makes the sum larger than k, we eliminate it, if it does not, we add it to the sum, and we continue this while the sum is less than k and until it is equal to k. Since we can do this for some arbitrary k, we can do this for $k+1$.

(For example, with 87, we begin with 80, 40 is too big, 20 is too big, 10 is too big, but 5 works so we add it, then we check 4, 3, 2, and add 2. Now our sum is 87, so we stop, so this works)

Exercise 4

1. $R(L_1) = a(a(a+b)^*a + b(a+b)^*b + a + b)a$
$+ b(a(a+b)^*a + b(a+b)^*b + a + b)b$
$+ aa + bb$

This regular expression captures the language because it covers all three of our cases with this languge:

1) Strings of length 2 that start and end with the same letter and whose second and second to last letter are the same is covered by the "aa + bb" portion of the expression.
2) strings of length 3, whose first and last letters are the same as well as the second and second to last letters. This is covered by the portion "$a(a(a+b)^*a + b(a+b)^*b + a + b)a$" by selecting 'a' or 'b' within the inner expression. Same applies for expressions that start & end with b, in $b(a(a+b)^*a + b(a+b)^*b + a + b)b$.
3) Strings of length 4 or more whose first and last letters are the same, as well as the second and second last letters. This is covered by the portion: $a(a(a+b)^*a + b(a+b)^*b + a + b)a$ and $b(a(a+b)^*a + b(a+b)^*b + a + b)b$, bey using the $a(a+b)^*a$ or $b(a+b)^*b$ portion of the inner expression to ensure we have the second and second to last letters being the same.

These 3 cases ensure all strings are of length 2 or greater, and the first and last letters are always the same, as well as the second and second to last letters.

2. $R(L_2) = ((a+b)(a+b))^* + a(a+b)^*$

The first half of the expression ensures the string is even because we must choose a or b twice, and the second half ensures the string will start with an a because we begin it with an a and concatenate it with ~~[illegible]~~ zero or more strings from $(a+b)$.

3. $R(L_3) = ((b^* a b^* a b^* a b^+ a b^+)^* (aaa + aa + a) b^*)^*$

This expression captures the language because after every 4th a, with any number of b's before or after the first 3, there is at least 1 b surrounding it on either side, we ensure this by using b^+, that way there is always exactly 1 b on both sides, and we allow for 1, 2, or 3 a's on the end, but not 4 because we again would have to make sure we have a b on both side of it. By putting a Kleene star around this entire expression we are able to concatenate it zero or more times, which is allowed in the language.

2.5 Good attempt

$(\,) \, (aa)^*$

$(aaaa)(aa\overset{bb}{a}aa \; aa)$

$$\left[(b^* a b^* a b^* a b^+ a b^+)^* (aaa + aa + a) b^* \right]^*$$