Funzione di verosimiglianza per fp-HDGM

21 November 2023 21:07

$\underset{nq\times1}{y_t} = \underset{nq\times1}{\mu_t} + \underset{nq\times1}{\varepsilon_t} \xrightarrow{\text{quindi}} y_t \mid z_t \sim N_{nq}(\mu_{nq}, \Sigma_\varepsilon)$

con n = num di posizioni spaziali
q = risoluz. alta frequenza (es. 24 ore)

Σ_ε: diagonale con σ_ε^2 sulla diagonale principale

b = numero di regressori

stesso numero di basi sia per β sia per z (p)

$\underset{nq\times1}{\mu_t} = \underset{nq\times b}{X_t}\cdot\underset{b\times bp}{\Phi_{\beta t}}\cdot\underset{bp\times1}{c_\beta} + \underset{nq\times np}{\Phi_{zt}}\cdot\underset{np\times1}{z_t}$

Φ_{zt}: diagonale a blocchi come $\Phi_{\beta t}$ (rif all'uscita e NON al singolo regressore)

con $\Phi_{\beta t} = \underbrace{\begin{bmatrix} \overbrace{\operatorname{sen}(h)\ \operatorname{sen}(2h)}^{p} & \emptyset\ \ \emptyset & \cdots & \emptyset\ \ \emptyset \\ \emptyset\ \ \emptyset & \operatorname{sen}(h)\ \operatorname{sen}(2h) & \cdots & \emptyset\ \ \emptyset \\ \vdots & \vdots & \ddots & \vdots \\ \emptyset\ \ \emptyset & \emptyset\ \ \emptyset & \cdots & \operatorname{sen}(h)\ \operatorname{sen}(2h) \end{bmatrix}}_{bp}\ b$ $(p=2)$

(p basi con spec. coef $\forall$ regr.)

e $c_\beta = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \\ \vdots \\ c_{b1} \\ c_{b2} \end{bmatrix}$ $bp \times 1$ $(p=2)$

$\underset{np\times1}{z_t} = \underset{np\times np}{\tilde{G}}\cdot \underset{np\times1}{z_{t-1}} + \underset{np\times1}{\eta_t}$ con $\eta_t \sim N_{np}(\tilde{G} z_{t-1}, \Sigma_\eta)$

con $\tilde{G} = \underbrace{\begin{bmatrix} G & \emptyset & \emptyset & \cdots & \emptyset \\ \emptyset & G & \emptyset & \cdots & \emptyset \\ \emptyset & \emptyset & G & \cdots & \emptyset \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \emptyset & \emptyset & \emptyset & \cdots & G \end{bmatrix}}_{np}\ np$ e $G = \begin{bmatrix} g_1 & \emptyset \\ \emptyset & g_2 \end{bmatrix}$ $p \times p$ $(p=2)$

NB: da adesso in poi y_t diventa w_t

$\underset{nq\times1}{y_t} = \underset{nq\times nq}{H}\cdot\underset{nq\times1}{w_t}$ con $H = \underbrace{\begin{bmatrix} h_1 & \cdots & \emptyset & \emptyset & \cdots & \emptyset & \cdots & \emptyset \\ \vdots & \ddots & \vdots & \vdots & & \vdots & & \vdots \\ \emptyset & \cdots & h_1 & \emptyset & \cdots & \emptyset & \cdots & \emptyset \\ \emptyset & \cdots & \emptyset & h_2 & \cdots & \emptyset & \cdots & \emptyset \\ \vdots & & \vdots & \vdots & \ddots & \vdots & & \vdots \\ \emptyset & \cdots & \emptyset & \emptyset & \cdots & h_2 & \cdots & \emptyset \\ \vdots & & \vdots & \vdots & & \vdots & \ddots & \vdots \\ \emptyset & \cdots & \emptyset & \emptyset & \cdots & \emptyset & \cdots & h_n \end{bmatrix}}_{nq}\ nq$ (h_1 rip. q volte; q)

dove h_i, $i = 1 \ldots n$ è $h_{0,i}(s_i, S/s_i) = \left(1 + \sum_{s \in S/s_i} e^{-\frac{\|s_i - s\|}{\theta}}\right)^{-1}$

$\xrightarrow{\text{quindi}} y_t = H(\underbrace{\mu_t + \varepsilon_t}_{w_t})$ output F-HDGM

$\xrightarrow{\text{quindi}} \underset{nq\times nq}{H^{-1}}\underset{nq\times1}{y_t} = \underset{nq\times1}{\mu_t} + \underset{nq\times1}{\varepsilon_t}$ con $\underbrace{H^{-1}y_t}_{w_t} \sim N_{nq}(\mu_t, \Sigma_\varepsilon)$

Funzione di verosimiglianza

$L(\Psi; Y, Z, X) = L(\Psi; Y|Z)\cdot L(\Psi; Z)$

$L(\Psi; Y|Z) = \prod_{t=1}^{T} \left(|\Sigma_\varepsilon|^{1/2}\cdot(2\pi)^{nq/2}\right)^{-1} \exp\left(-\tfrac{1}{2}(H^{-1}y_t - \mu_t)'\Sigma_\varepsilon^{-1}(H^{-1}y_t - \mu_t)\right)$

$= \left(|\Sigma_\varepsilon|^{1/2}\cdot(2\pi)^{nq/2}\right)^{-T} \prod_{t=1}^{T} e^{-\frac{1}{2}(H^{-1}y_t - \mu_t)'\Sigma_\varepsilon^{-1}(H^{-1}y_t - \mu_t)}$

$\xrightarrow{\text{quindi}} -2\log L(\Psi; Y|Z) = T\log|\Sigma_\varepsilon| + \cancel{Tnq\log 2\pi} + \sum_{t=1}^{T}(H^{-1}y_t - \mu_t)'\Sigma_\varepsilon^{-1}(H^{-1}y_t - \mu_t)$

$L(\Psi; Z) = L(\Psi; z_0)\cdot L(\Psi; z_1|z_0)\cdot L(\Psi; z_2|z_1)\cdot \ldots \cdot L(\Psi; z_T|z_{T-1})$

$L(\Psi; z_0) = \left(|\Sigma_0|^{1/2}(2\pi)^{np/2}\right)^{-1}\exp\left(-\tfrac{1}{2}(z_0 - \mu_0)'\Sigma_0^{-1}(z_0 - \mu_0)\right)$

$\xrightarrow{\text{quindi}} -2\log L(\Psi; z_0) = \log|\Sigma_0| + \cancel{np\log 2\pi} + (z_0 - \mu_0)'\Sigma_0^{-1}(z_0 - \mu_0)$

$\prod_{t=1}^{T} L(\Psi; z_t|z_{t-1}) = \prod_{t=1}^{T}\left(|\Sigma_\eta|^{1/2}(2\pi)^{np/2}\right)^{-1}\cdot\exp\left(-\tfrac{1}{2}(z_t - \tilde{G}z_{t-1})'\Sigma_\eta^{-1}(z_t - \tilde{G}z_{t-1})\right)$

$= \left(|\Sigma_\eta|^{1/2}(2\pi)^{\frac{np}{2}}\right)^{-T}\cdot\prod_{t=1}^{T} e^{-\frac{1}{2}(z_t - \tilde{G}z_{t-1})'\Sigma_\eta^{-1}(z_t - \tilde{G}z_{t-1})}$

$\xrightarrow{\text{quindi}} -2\log\prod_{t=1}^{T} L(\Psi; z_t|z_{t-1}) = T\log|\Sigma_\eta| + \cancel{Tnp\log 2\pi} + \sum_{t=1}^{T}(z_t - \tilde{G}z_{t-1})'\Sigma_\eta^{-1}(z_t - \tilde{G}z_{t-1})$

$-2\log L(\Psi; Y, Z, X) = \underset{1\times1}{T\log|\Sigma_\varepsilon(\sigma_\varepsilon^2)|} + \sum_{t=1}^{T}\underset{1\times nq}{(H^{-1}(\theta)\cdot y_t - \mu_t(c_{11},\ldots,c_{ij},\ldots,c_{bp}))'}\underset{nq\times nq}{\Sigma_\varepsilon^{-1}(\sigma_\varepsilon^2)}\underset{nq\times1}{(H^{-1}(\theta)\cdot y_t - \mu_t(c_{11},\ldots,c_{ij},\ldots,c_{bp}))} + \underset{1\times1}{\log|\Sigma_0|} + \underset{1\times np}{(z_0 - \mu_0)'}\underset{np\times np}{\Sigma_0^{-1}}\underset{np\times1}{(z_0 - \mu_0)} + \underset{1\times1}{T\log|\Sigma_\eta(v_{11},\ldots,v_{ij},\ldots,v_{pp},\theta)|} + \sum_{t=1}^{T}\underset{1\times np}{(z_t + \tilde{G}(g_1,\ldots,g_i,\ldots,g_p)z_{t-1})'}\underset{np\times np}{\Sigma_\eta^{-1}(v_{11},\ldots,v_{ij},v_{pp},\theta)}\underset{np\times1}{(z_t + \tilde{G}(g_1,\ldots,g_i,\ldots,g_p)\cdot z_{t-1})}$

con $\underset{nq\times1}{\mu_t} = \underset{nq\times b}{X_t}\cdot\underset{b\times bp}{\Phi_{\beta t}}\cdot\underset{bp\times1}{c_\beta}$