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# Correlation of financial markets in times of crisis

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ABSTRACT

Using the eigenvalues and eigenvectors of correlations matrices of some of the main financial market indices in the world, we show that high volatility of markets is directly linked with strong correlations between them. This means that markets tend to behave as one during great crashes. In order to do so, we investigate financial market crises that occurred in the years 1987 (Black Monday), 1998 (Russian crisis), 2001 (Burst of the dot-com bubble and September 11), and 2008 (Subprime Mortgage Crisis), which mark some of the largest downturns of financial markets in the last three decades.



## 1. Introduction

The study of why many world financial markets crash simultaneously is of central importance, particularly after the recent worldwide downturn of the major markets in 2007 and 2008. Economists have been studying the reasons why markets crash, and why there is propagation of volatility from one market to another, since a long time. After the crash of 1987, many studies have been published on transmission of volatility (contagion) between markets using econometric models [1–15], on how the correlation between world markets change with time [16–19], and how correlation tends to increase in times of high volatility [20–34]. This issue is of particular importance if one wishes to build portfolios of international assets which can withstand times of crisis [35–47]. Many models were proposed by both economists and physicists in order to explain the correlation of international financial markets [48–66], which is considered a complex system with many relations which are difficult to identify and quantify.

One tool that was first developed in nuclear physics for studying complex systems with unknown correlation structure is random matrix theory [67–70], which confronts the results obtained for the eigenvalues of the correlation matrix of a real system with those of the correlation matrix obtained from a pure random matrix. This approach was successfully applied to a large number of financial markets [71–100], and also to the relation between world markets [101,102]. This approach was also used in the construction of hierarchical structures between different assets of financial markets [103–141].

Recently, time lagged random matrix theory was used in order to compute long-range cross-correlations of world stock indices [142], showing that the correlations of absolute returns decay much more slowly than the correlations of returns. In Ref. [143], the same results were reported for absolute values of returns between the Dow Jones and the S&P indices of the New York Stock Exchange. In Ref. [144], pronounced peaks were reported during the largest world market crashes of the last few decades. Cross-correlations between volume change and price change was studied in Ref. [145]. Other studies using cross-correlations were also done in Refs. [146–148].

Recent methods for studying random matrices obtained from non-Gaussian distributions, such as $t$-Student distributions, which represent more closely the probability density distributions obtained from financial data, were developed in Refs. [149–152]. Other studies focus on other measures of co-movement of financial indices that are more appropriate for systems with very strong correlations, as happens in times of financial crises [153,154].

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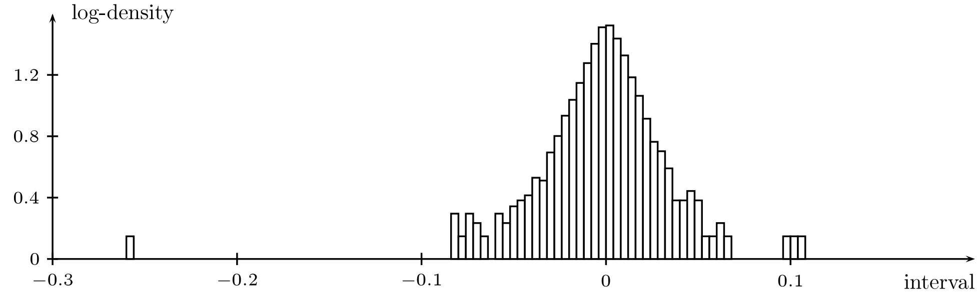


**Fig. 1.** Log-density distribution of the Dow Jones index of the NYSE, from 01/02/1985 to 12/31/2008.

Our work uses the tools of random matrix theory to analyze the correlation of world financial markets in times of crisis. In order to do so, we use data from some of the largest worldwide crashes since 1980, namely the 1987 Black Monday, the 1998 Russian Crisis, the burst of the dot-com bubble of 2001, the shock after September, 11, 2001, and the USA subprime mortgage crisis of 2008. We start by defining a global financial crisis (Section 2) based on evidence of some financial markets chosen from diverse parts of the world. Then, we discuss some of the main theoretical results on Random Matrix Theory (Section 3). Then, in Section 4, we make a quick discussion on how we collected the data and how it was treated.

In Sections 5–8 we study the correlation matrices between the log-returns of a number of financial market indices chosen so as to represent many geographical parts of the world and a diversity of economies. In each section, we calculate the eigenvalues of the correlation matrix of the chosen indices and then study the eigenvector that corresponds to its largest eigenvalue, which is usually related with a *market mode*, which is a co-movement of all indices. We then calculate correlation matrices in running windows and compare the average correlation between markets with the volatility and average volatility of the market mode obtained previously, showing that times of large volatility are strongly linked with strong correlations between world financial indices.

Section 9 looks more closely at the probability distribution of the correlation coefficients in different intervals of time and tests the hypothesis that it becomes closer to a Gaussian probability distribution during periods of crises.

Since the study of world stock exchanges involve dealing with different operating times, in Section 10 we compare the results obtained in the previous sections with results obtained by using the log-returns of Western markets with the log-returns of the next day in Asian markets. We also compare the results obtained in the main text of the article with those obtained by using Spearman's rank correlation instead of Pearson's correlation.

Since world stock market indices (countries) are easier to relate with than equities in a stock market (companies), one of the aims of this article is to be a pedagogical introduction to most of the techniques that are used when Random Matrix Theory is applied to financial data. Hence we also supply an ample bibliography on the subject.

## 2. Defining a global financial crisis

Before studying periods of financial crises, we must make it clear what we consider to be a global crash of the financial markets. In order to adopt a more precise definition, we considered the time series of 15 financial markets representing different regions of the world from the beginning of 1985 until the end of 2010. Looking at the closing indices of every day in which there was negotiation, we considered the log-returns, given by

$$S_t = \ln(P_t) - \ln(P_{t-1}) \approx \frac{P_t - P_{t-1}}{P_t}, \tag{1}$$

what makes it easier to compare the variations of the many indices. After that, the 10 most negative variations were chosen.

In order to illustrate the procedure, we consider the Dow Jones index of the New York Stock Exchange (NYSE). Fig. 1 shows the log-density distribution for this index with data from 01/02/1985 to 12/31/2008. The log-density, defined as

$$\text{log-density} = \ln(1 + \text{density}), \tag{2}$$

is used instead of simple density in order to better visualize the most extreme points.

The ten most negative values of the log-returns are below −0.07. These events occurred in the following occasions: 10/19/1987 (22.61%), 10/26/1987 (8.04%), 01/08/1988 (6.85%), 10/13/1989 (6.90%), 10/27/1997 (7.18%), 09/17/2001 (7.13%), 09/29/2008 (6.98%), 10/09/2008 (7.33%), 10/15/2008 (7.87%), and 12/01/2008 (7.70%). These dates include the 1987 Black Monday, part of the Asian Crisis of 1997, the 1998 Russian Crisis, the aftermath of September 11, 2001, and the Subprime Mortgage Crisis of 2008.

The same technique was used for the Nasdaq (USA), S&P/TSX Composite (Canada), Ibovespa (Brazil), FTSE 100 (UK), DAX (Germany), ISEQ (Ireland), AEX (Netherlands), SENSEX 30 (India), Colombo All-Share (Sri Lanka), Nikkei (Japan), Hang Seng (Hong Kong), TAIEX (Taiwan), Kospi (South Korea), Kuala Lumpur Composite (Malaysia), and Jakarta Composite (Indonesia).

**Table 1**
Number of occurrences per year of major drops in fifteen diverse stock markets in the world.

| Year | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Occurrences | 3 | 0 | 29 | 2 | 9 | 13 | 2 | 4 | 0 | 0 | 0 | 1 | 10 |
| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| Occurrences | 11 | 1 | 4 | 8 | 1 | 3 | 4 | 3 | 0 | 0 | 50 | 2 | 0 |

Table 1 displays the years of major drops (between the beginning of 1985, and the end of 2010) and the quantity of markets which presented those falls. When a market drops substantially more than once in the same year, these are counted more than once as well, in order to gauge the depth of the shocks. This helps identity the times where there were major crashes around the world.

It is possible to pinpoint two major crises in 1987 and 2008, and minor crises in 1989, 1990, 1997, 1998, and in 2001. The crisis of 1987 corresponds to the so called Black Monday, the one in 1989 is the USA saving and loan crisis, 1990 are the Japanese asset price bubble and the Scandinavian banking crisis, 1992 is the so-called Black Wednesday, 1997 is the Asian financial crisis, 1998 is the Russian crisis, 2000 and 2001 correspond to the Burst of the dot-com bubble, and 2008 corresponds to the Subprime Mortgage Crisis in the USA.

We shall apply a theory called *Random Matrix Theory* in order to analyze four of these crises. The next section gives a pragmatic introduction to this theory.

## 3. Random matrix theory

Random matrix theory had its origins in 1953, in the work of the German physicist Wigner [67,68]. He was studying the energy levels of complex atomic nuclei, such as uranium, and had no means of calculating the distance between those levels. He then postulated that those distances should mimic the distances between the eigenvalues of a random matrix (the Hamiltonian). Surprisingly, he could then be able to make sensible predictions about how the energy levels related to one another.

This method also found connections with the study of the Riemann zeta function, which is of primordial importance to the study of prime numbers, used for coding and decoding information, for example. The theory was later developed, with many and surprising results arising. Today, Random Matrix Theory is applied to quantum physics, nanotechnology, quantum gravity, the study of the structure of crystals, and may have applications in ecology, linguistics, and many other fields where a large amount of apparently unrelated information may be understood as being somehow connected (for a recent book on the subject, see Ref. [70]). The theory was also applied to finance in a series of works dealing with the correlation matrices of stock prices, and also to risk management in portfolios [71–100] (for recent reviews on the subject, see Refs. [93,94]).

In this section, we shall focus on the results that are most important to the present work, which is studying the correlations between world financial markets in times of crisis. The first result of the theory that we shall mention is that, given an $L \times N$ matrix with random numbers built on a Gaussian distribution with average zero and standard deviation $\sigma$, then, in the limit $L \to \infty$ and $N \to \infty$ such that $Q = L/N$ remains finite and greater than 1, the eigenvalues $\lambda$ of such a matrix will have the following probability density function, called a Marčenku–Pastur distribution [69]:

$$\rho(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}, \tag{3}$$

where

$$\lambda_- = \sigma^2\left(1 + \frac{1}{Q} - 2\sqrt{\frac{1}{Q}}\right), \qquad \lambda_+ = \sigma^2\left(1 + \frac{1}{Q} + 2\sqrt{\frac{1}{Q}}\right), \tag{4}$$

and $\lambda$ is restricted to the interval $[\lambda_-, \lambda_+]$.

Since the distribution (3) is only valid for the limit $L \to \infty$ and $N \to \infty$, finite distributions will present differences from this behavior. In Fig. 2, we compare the theoretical distribution for $Q = 10$ and $\sigma = 1$ to distributions of the eigenvalues of three correlation matrices generated from finite $L \times N$ matrices such that $Q = L/M = 10$, and the elements of the matrices are random numbers with mean zero and standard deviation 1.

Consequently, real data will deviate from the theoretical probability distribution. Nevertheless, the theoretical result may serve as a parameter to the results obtained experimentally.

Another source of deviations is the fact that financial time series are better described by non-Gaussian distributions, such as *t* Student or Tsallis distribution. This can be seen from Fig. 1: a Gaussian distribution would be represented by a parabola, what is clearly not the case. Recent studies [149–154] developed part of the theoretical framework in which finite series and series with fat tails, as is the case of financial time series of returns, can be studied.

Since Random Matrix Theory is based on random matrices with a single standard deviation $\sigma$, we must compensate the data obtained from the many indices so that all series have average zero and the same standard deviation, which we chose

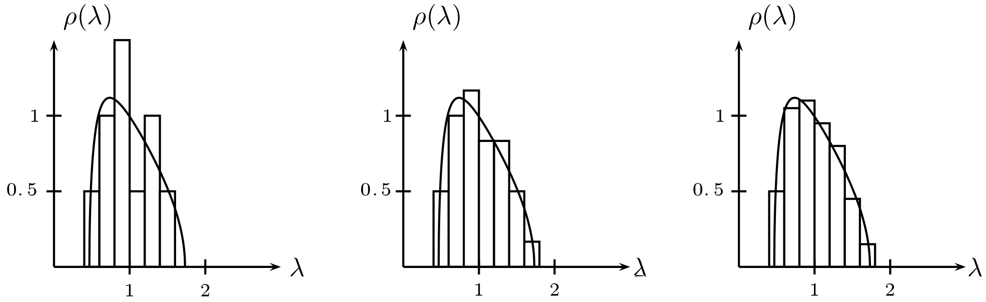


**Fig. 2.** Theoretical and sample finite distributions for a random matrix with $N = 10$ and $L = 100$, $N = 30$ and $L = 300$, and $N = 100$ and $L = 1000$.

to be equal to one. This can be done using the formula

$$X_t = \frac{S_t - \langle S\rangle}{\sigma}, \tag{5}$$

where $\langle S\rangle$ is the average of the time series used, and $\sigma$ is its standard deviation.

## 4. Data

We shall work with one stock market index of each country (with the exception of the USA, with two indices). The indices were chosen among the ones that are considered benchmarks in each stock market. The data were collected from Bloomberg, from 1980, or the first available data, until the end of 2010. For 1987, we collected 23 indices (4 from the Americas, 9 from Europe, and 10 from Asia); for 1998, we have 63 indices (12 from America, 24 from Europe, 19 from Asia, 1 from Oceania, and 7 from Africa); from 2001, 79 indices (13 from America, 29 from Europe, 26 from Asia, 2 from Oceania, and 9 from Africa); for 2008, we have 92 indices (14 from America, 35 from Europe, 30 from Asia, 2 from Oceania, and 11 from Africa). The number of indices collected grew in time both due to the adoption of the indices by their respective stock markets, the availability of data, and by the emergence of new countries.

This work is motivated by the will to understand how each index affects the others, so as to later attempt to build a model of how crises propagate in a network of indices. Thus, we do not consider here the indices normalized to a single currency, which would be useful, as an example, for building portfolios for investors. That is because we want the numbers to be the ones to which agents operating in their own stock markets react when they take decisions. That is also the reason we are not using indices that are standardized in terms of methodology, like the ones calculated by Morgan and Stanley Capital International (MSCI), which are used mainly by researchers and international investors, but that are not the ones usually published by the press, or seen on the news broadcasts. One of the authors (LSJr) has some research underway using MCSI indices.

When analyzing the data, we had to be careful with the differences in public holidays or weekends among countries. Particular care had to be taken with Israel, Palestine, Jordan, Saudi Arabia, Kuwait, Bahrain, Qatar, the United Arab Emirates, Ohman, Bangladesh, and Egypt, for which weekends do not occur on Saturdays and Sundays, but on Fridays and Saturdays, for example. For these countries, we shifted data so as not to lose information, making missing data due to weekends coincide with the other indices. Our general rule was that, when more than 30% of the markets did not open on a certain day, we removed that day from our data, and when that number was below 30%, we kept the existing indices and repeated the last computed index for each of the remaining ones. We did not make linear extrapolations of missing indices, for we could then lose the effects of drops, like the one that occurred after September 11, when the stock exchanges of the USA remained closed for some days.

Another pressing problem was that markets do not operate at the same time zones, so we had to decide whether to consider the data concerning American countries at the same day as Asian-Pacific countries, for example, or to shift the data for Asian countries so as to compare indices from the USA with the next day index from Japan, for example. There is even some evidence [27] that the correlations of Asian with the USA indices increase when one considers the correlation of the USA indices with the next day indices of the Asian markets.

We decided to consider all indices taken at the same date. This was motivated by some comparisons between the correlations among indices: we compared the individual correlations among most of the indices and checked if the correlation increased or decreased shifting the relevant data. The result was inconclusive, for there was an almost equal number of correlations which increased taking Western indices one day before their Eastern counterparts as there were other correlations that remained higher taking the same day indices (some correlations were higher when we took the Western indices one day after the Eastern ones). In order to gauge the effects of such a shift, we recalculated all our results by shifting the indices from Asia and Oceania in one day, without many changes in the outcomes. Those results are better explained in Section 10.

One option, frequently adopted, to avoid the problem of different operating hours between international markets is to consider weekly data instead of daily data. We did not adopt that approach so as not to miss major changes in markets,

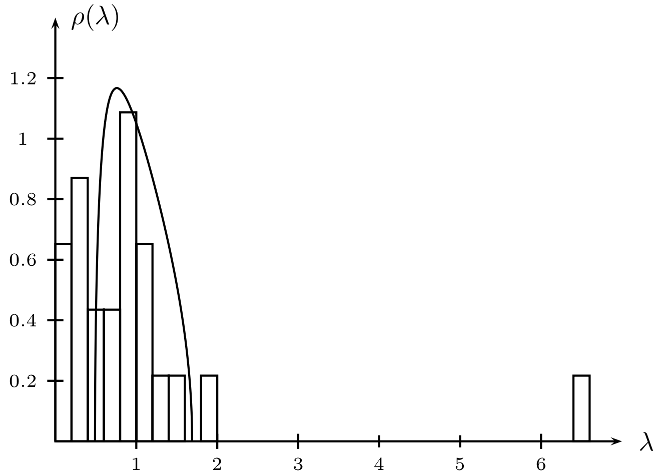


**Fig. 3.** Frequency distribution of the eigenvalues of the correlation matrix for 1987. The theoretical distribution for a random matrix is superimposed on it .

which tend to occur during a small interval of days. Instead, we preferred to compare results with and without shifting part of the data by one day.

In the next four sections, we shall consider the years 1987, 1998, 2001, and 2008 in the light of Random Matrix Theory. The indices we used, together with their countries of affiliation, the symbols used for them, and their codes in Bloomberg, are placed in Table 2, in the Appendix.

## 5. 1987, the Black Monday

In 1987, the financial world lived a time of panic, much like the one of the great crash of 1920. In a matter of 3 days, most of the stock markets in the world lost about 30% of their values and trillions of dollars evaporated, leaving a trace of destruction that affected what is referred to as real economy for many years. The day the first and major collapse occurred, a Monday, was later called the *Black Monday*.

In order to analyze that crisis, we consider now the correlation matrix of 23 indices of stock exchanges around the world: the S&P 500 from the New York Stock Exchange (S&P), and Nasdaq (Nasd), both from the USA, S&P TSX from Canada (Cana), Ibovespa from Brazil (Braz), FTSE 100 from the United Kingdom (UK), ISEQ from Ireland (Irel), DAX from Germany (Germ) – West Germany at the time – ATX from Austria (Autr), AEX from the Netherlands (Neth), OMX from Sweden (Swed), OMX Helsinki from Finland (Finl), IBEX 35 from Spain (Spai), ASE General Index from Greece (Gree), SENSEX from India (Indi), Colombo All Share from Sri Lanka (SrLa), DSE General Index from Bangladesh (Bang), Nikkei 25 from Japan (Japa), Hang Seng from Hong Kong (HoKo), TAIEX from Taiwan (Taiw), Kospi from South Korea (SoKo), Kuala Lumpur Composite from Malaysia (Mala), JCI from Indonesia (Indo), and the PSEi from the Philippines (Phil). So, we have three indices from North America, one from South America, nine from Europe, and ten from Asia, with a total of 23 indices. These offer a good variety of indices worldwide. In subsequent years, we shall increase this number, mainly due to the appearance of new indices and countries, and to the access to data about them.

We shall use 1987 as an example for the other years, and because of that we will be showing more details in the calculations for that year. Calculating the correlation matrix for the indices that are being considered, one obtains a $23 \times 23$ matrix. The average of the values of this correlation matrix is a good measure of the overall correlation between the many indices (the average is taken over the elements of the correlation matrix for which $i < j$). For the present correlation matrix, the average is given by $\langle C \rangle = 0.16$, with standard deviation $\sigma = 0.04$. Since the correlation matrix is $23 \times 23$, and the number of days considered in calculating it is 256, we then have $Q = L/M = 256/23 \approx 11.130$, and the upper and lower bounds

$$\lambda_- = 0.490, \qquad \lambda_+ = 1.689 \tag{6}$$

for the eigenvalues that constitute the bulk of the eigenvalue distribution due to noise.

A frequency distribution of the 23 eigenvalues of the correlation matrix is shown in Fig. 3, with the theoretical distribution of an infinite random matrix for $Q = 11.130$ with mean zero and standard deviation one superimposed on it. In Fig. 4, the eigenvalues are plotted in order of magnitude. The shaded area indicates the region predicted by theory were the data related with a purely random behavior of the normalized log-returns.

Only 60% of the eigenvalues fall inside the region predicted by Random Matrix Theory. Note that the highest eigenvalue stands out from all the others, being more than three times bigger than the uppermost limit $\lambda_+$ of the theoretical distribution. This is in agreement with many other results, obtained for a great number of financial institutions to which this same formalism has been already applied [71–100]. It is believed that this eigenvalue corresponds to the action of a single market,

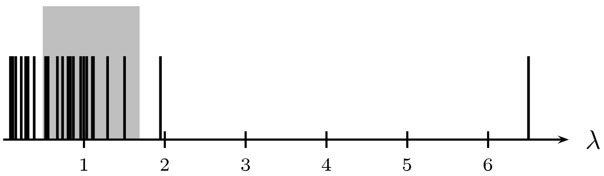


**Fig. 4.** Eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted for a random matrix.

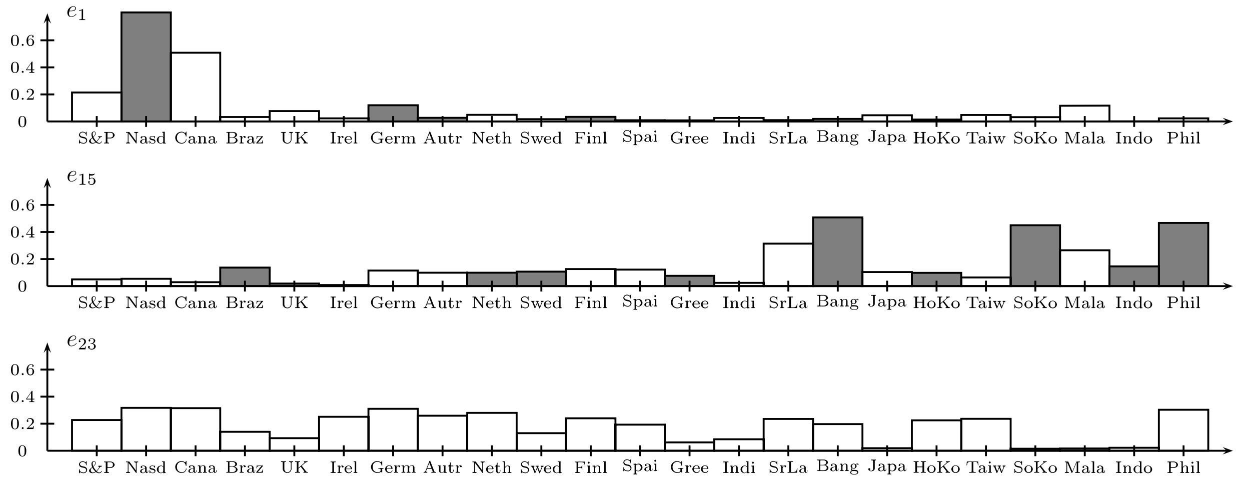


**Fig. 5.** Contributions of the stock market indices to eigenvectors $e_1$, $e_{15}$, and $e_{23}$. White bars indicate positive values, and gray bars correspond to negative values.

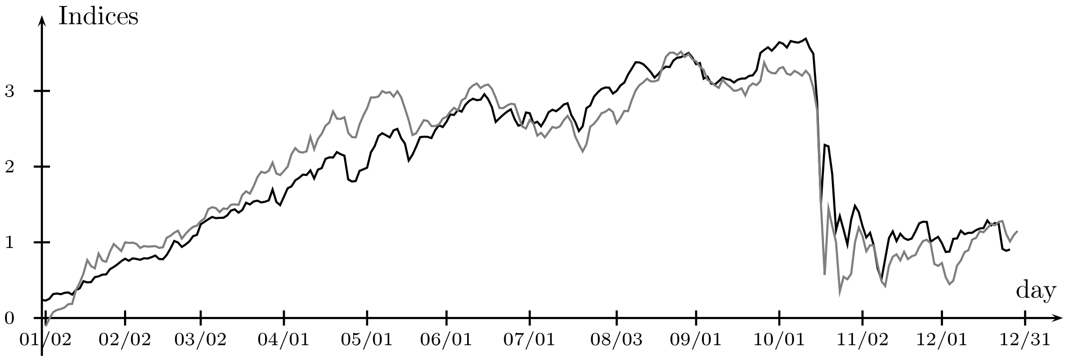


**Fig. 6.** Time series of the market mode calculated using the eigenvector related with the largest eigenvalue of the correlation matrix (black) plotted against the world index calculated by the MSCI (gray). Both indices are normalized so as to have mean two and standard deviation one.

which influences all the other members of the correlation matrix. Usually, for the correlation matrix of individual stocks in a single market, this eigenvalue is much larger, some times 25 times larger, than the largest eigenvalue predicted for the correlation matrix of a random time series, although the size of the sample directly influences that as well. In our case, it responds for about 28% of the collective behavior of the time series being considered, which is the ratio of the largest eigenvalue and the sum of the eigenvalues of the correlation matrix.

Fig. 5 shows the contributions of the many indices which we are considering in our study in some of the eigenvectors of the correlation matrix. The white bars represent positive values and the gray bars represent negative ones.

One can see that the eigenvector corresponding to the largest eigenvalue is qualitatively different from the others. Nearly all markets (with the exception of Bangladesh and Indonesia) have positive representations. That is a compelling reason to believe that it represents a global market that is the result of the interactions of all local markets, or may also be the result of external news on the market as a whole. Fig. 6 compares the time series of an index built using eigenvector 23 (in black) with the world index calculated by the MSCI (Morgan Stanley Capital International), in gray. Both indices are normalized so as to have mean two and standard deviation one.

In terms of portfolio theory, as stated by Markowitz' ideas [155,156], the eigenvector corresponding to the largest eigenvalue represents the riskier portfolio one may build, as most of the indices vary in the same way. In contrast, some

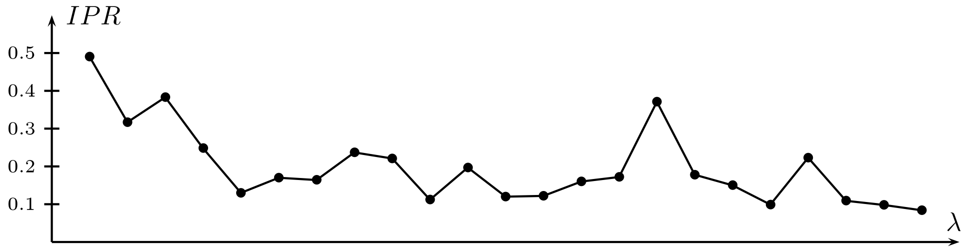


**Fig. 7.** Inverse participation ratio of the eigenvectors of the correlation matrix.

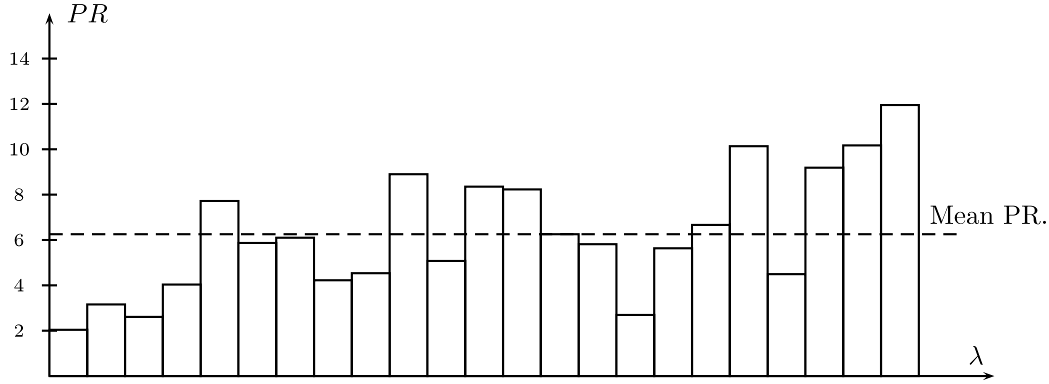


**Fig. 8.** Participation ratio of the eigenvectors of the correlation matrix.

of the smaller eigenvectors represent portfolios with less risk, as, for example, eigenvector $e_1$, which basically consists on "buying" S&P 500 (USA) and S&P TSX (Canada) and "short-selling" Nasdaq (USA), which are three very closely connected indices. Eigenvector $e_{15}$ corresponds to one of the eigenvectors that are within the region considered as noise, and should represent just a random combination of stock market indices.

More differences between eigenvector $e_{23}$ and the other eigenvectors can be seen if we build probability distribution of frequencies graphs for the twenty three eigenvalues. All distributions, except the one for eigenvector $e_{23}$, have average near zero and standard deviation around 0.21, while this is not the case for eigenvector $e_{23}$. The elements of eigenvector $e_{23}$ have mean 0.17 and standard deviation 0.13.

Some recent works discussed how finite sized data and log-return distributions that are not Gaussian could affect the probability distribution of the eigenvalues of an empirical correlation matrix. Some of the results imply that the usual Marĕnko–Pastur distribution acquires a fat tail in the direction of the largest eigenvalue.

A last analysis which shows the difference between the highest eigenvalues and the eigenvalues belonging to the range associated with noise may be done using the so called *Inverse Participation Ratio* (IPR),

$$\mathrm{IPR}_k = \sum_{i=1}^{N} \left(e_k^i\right)^4 , \tag{7}$$

where $e_k^i$ is the $i$-th element of eigenvector $e_k$, and $N$ is the total number of eigenvectors. Its inverse gives the average number of stocks which contribute significantly to a portfolio built with such eigenvector. Figs. 7 and 8 show the IPR for the 23 eigenvectors, in ascending order from the left to the right (Fig. 7), and its inverse, $\mathrm{PR}_k = 1/\mathrm{IPR}_k$, for Participation Ratio (Fig. 8).

Note that, for eigenvector $e_{23}$, the number of participating indices is larger than the average, which is about 6. Most of the eigenvectors corresponding to noise fall around that average number, but this is not true for the eigenvectors corresponding to the lowest eigenvalues, which have a very small number of participating indices.

One important result of this theoretical treatment is that the largest eigenvalue, associated with a *market mode*, is like another matrix that is added to the true correlation matrix of the log-returns. In order to study the remaining eigenvalues, one must first clean the empirical correlation matrix from the market mode. The process is known as *single index model*, and is widely used by theoreticians and practitioners of financial markets in order to remove the market mode of stocks negotiated in the same stock exchange [155]. This is done in order to study clusters of stocks that move together as blocks in stock markets.

We now measure the average of the correlation matrices in a moving window of 30 days, changing one day at a time. The results are displayed in Fig. 9, where the average correlation is plotted together with the average volatility of the market mode, which we consider here as the absolute value of $S_t$, where $S_t$ is a linear combination of all indices with the elements

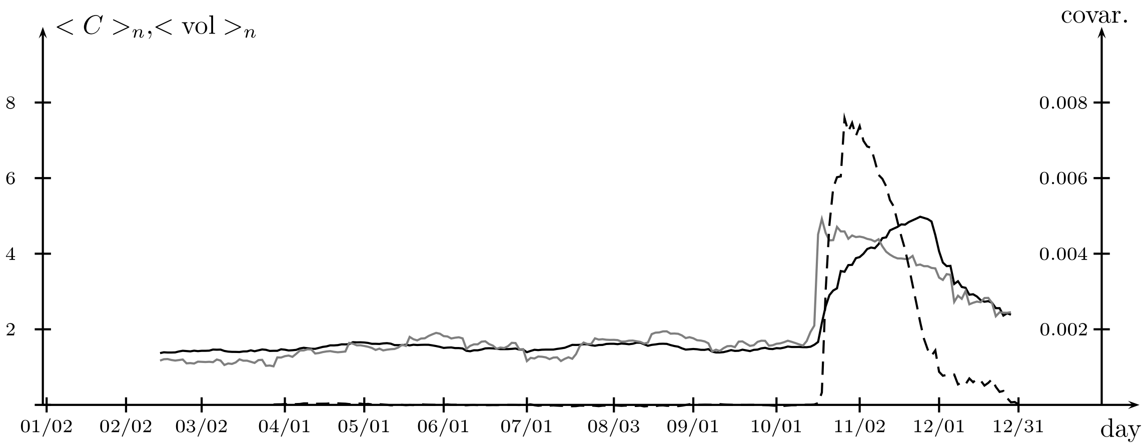


**Fig. 9.** Average volatility of the market mode (black) and average correlation (gray) based on the log-returns for 1987, both calculated in a moving window of 30 days and normalized so as to have mean two and standard deviation one. The covariance between volatility and average correlation is plotted in the same graph, in a dashed line.

of eigenvector $e_{23}$ as the coefficients. Both variables are normalized so as to have mean two and standard deviation one so that they can best be compared.

The plot represents the average correlation of each window as a function of the last day of the window, so that events that occur after the date to which the average correlation is assigned do not influence its value. The average volatility is in black, and the average correlation is in gray. In this picture, the rise of volatility seems to be preceded by a rise in the correlation between international stock market indices, although that is not a conclusion that may be taken, since we are using averages here over a large period of time.

It is quite clear that there is a strong correspondence between global market volatility and the correlation of the market indices. The correlation between the two variables along this period is 0.62. One can also note that markets are much more correlated after the period of crisis, and this behavior tends to endure for some time after the crash [157], although one must take into account that the averaging procedure for the average correlation makes the curve smoother and thus decreasing less steeply. Fig. 9 also shows, in a dashed line, the evolution of the covariance between volatility and $\langle C \rangle$ in time, calculated in a moving window of 30 days, starting from 02/12/1987 (the first day we assign an average correlation). A clear peak can be seen on the days of greatest volatility (we plot the covariance at the end of the time interval considered for each calculation). Although the covariance is influenced by the value of the volatility, so we expect to have large covariance when volatility is high, it has shown to be more efficient in determining periods of crisis than the correlation, that being the reason we are using it.

## 6. 1998, Russian crisis

The Asian Financial Crisis, which occurred in 1997, made the demand for raw materials fall worldwide, affecting Russia in particular, which is one of the major world exporters of commodities. With the war in Chechnya, and the transition to a capitalist economy, Russia showed signs of decline in its economy. By May, 1998, the fears concerning the Russian economy brought most of the world's financial markets down, since many countries had a good amount of money invested in that country.

In order to analyze that crisis, we added to the previous indices the following: IPC from Mexico (Mexi), BCP Corp Costa Rica from Costa Rica (CoRi), Bermuda SX Index (Bermuda), Jamaica SX Market Index from Jamaica (Jama), MERVAL from Argentina (Arge), IPSA from Chile (Chil), IBVC from Venezuela (Vene), IGBVL from Peru (Peru), CAC 40 from France (Fran), SMI from Switzerland (Swit), FTSE MIB from Italy (Ital), BEL 20 from Belgium (Belg), OMX Copenhagen 20 from Denmark (Denm), OBX from Norway (Norw), OMX Iceland All-Share Index from Iceland (Icel), PSI 20 from Portugal (Port), PX from the Czech Republic (CzRe), PX from Slovakia (Slok), Budapest SX Index from Hungary (Hung), WIG from Poland (Pola), BET 10 from Romania (Roma), OMXT from Estonia (Esto), PFTS from Ukraine (Ukra), MICEX from Russia (Russ), ISE National 100 from Turkey (Turk), TA 25 from Israel (Isra), BLOM from Lebanon (Leba), TASI from Saudi Arabia (SaAr), MSM 30 from Ohman (Ohma), Karachi 100 from Pakistan (Paki), SSE Composite from China (Chin), SET from Thailand (Thai), S&P/ASX 200 from Australia (Aust), CFG 25 from Morocco (Moro), EGX 30 from Egypt (Egyp), Ghana All Share from Ghana (Ghan), NSE ASI from Nigeria (Nige), NSE 20 from Kenya (Keny), FTSE/JSE Africa All Shares from South Africa (SoAf), and SEMDEX from Mauritius (Maur). So, now we have a total of 63 indices, 5 from North America (if we include Bermuda), 2 from Central America and the Caribbean, 5 from South America, 24 from Europe, 2 from Eurasia, 17 from Asia, 1 from Oceania, and 7 from Africa, where we are considering Russia and Turkey as part of Eurasia, for both countries are located in both continents. This offers a good degree of diversification, and includes Russia, which was of paramount importance in that particular crisis.

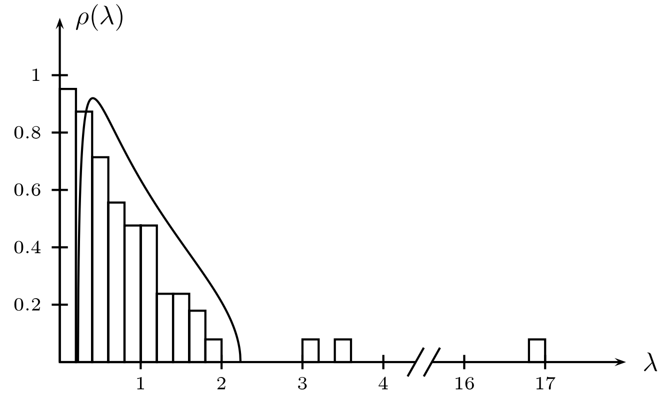


**Fig. 10.** Frequency distribution of the eigenvalues of the correlation matrix for 1998. The theoretical distribution is superimposed on it.

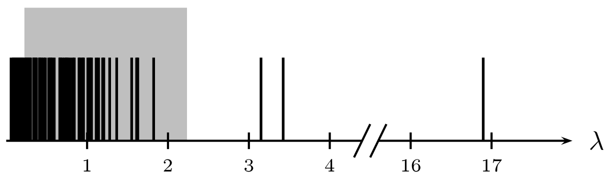


**Fig. 11.** Eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted for a random matrix.

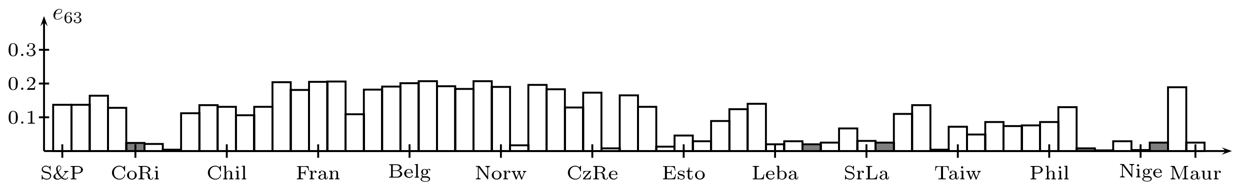


**Fig. 12.** Contributions of the stock market indices to eigenvector $e_{48}$, corresponding to the largest eigenvalue of the correlation matrix. White bars indicate positive values, and gray bars correspond to negative values. The indices are aligned in the following way: *S&P*, Nasd, Cana, Mexi, *CoRi*, Berm, Jama, Bra, Arg, *Chil*, Ven, Peru, UK, Irel, *Fran*, Germ, Swit, Autr, Ital, *Belg*, Neth, Swed, Denm, Finl, *Norw*, Icel, Spai, Port, Gree, *CzRe*, Slok, Hung, Pola, Roma, *Esto*, Ukra, Russ, Turk, Isra, *Leba*, SaAr, Ohma, Paki, Indi, *SrLa*, Bang, Japa, HoKo, Chin, *Taiw*, SoKo, Thai, Mala, Indo, *Phil*, Aust, Moro, Egyp, Ghan, *Nige*, Keny, SoAf, *Maur*.

Using the modified log-returns (5) based on the closing indices from 01/02/1998 to 12/30/1998, we built a $63 \times 63$ correlation matrix between those. This matrix has average correlation $\langle C \rangle = 0.17$, standard deviation $\sigma = 0.04$, and is based on $L = 257$ days for the $M = 63$ indices, which gives $Q = L/M = 257/63 \approx 4.079$.

The upper and lower bounds of the eigenvalues of the Marěnko–Pastur distribution (3) are

$$\lambda_- = 0.255 \quad \text{and} \quad \lambda_+ = 2.235. \tag{8}$$

The frequency distribution of the eigenvalues is displayed in Fig. 10, plotted against the theoretical Marěnko–Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1. Fig. 11 shows the eigenvalues in order of magnitude, with the area corresponding to noise shaded.

Note that the largest eigenvalue is completely out of scale. We also have several eigenvalues that are below the minimum theoretical eigenvalue and two other eigenvalues above the maximum theoretical eigenvalue.

Fig. 12 shows eigenvector $e_{63}$, which corresponds to a combination of all indices in a market movement that explains about 36% of the collective movement of all indices.

Note that most indices have similar participations, with the USA and European indices appearing with the largest components for the eigenvector. The smallest participations, some of them with very small negative values, are the ones from Costa Rica, Bermuda, and Jamaica (Central America and the Caribbean), Iceland and Slovakia (Europe), all the Arab countries and most of the Southern Asia ones, China, and the African countries, with the exception of South Africa.

Fig. 13 shows the average volatility of the market mode (black), together with the average correlation between the indices for 1998 (gray), using a running window of 70 days, and representing the averages of each window as a volatility or correlation of the last day of that window. The window has been enlarged due to the increase in the number of indices so as to avoid too much statistical noise. Both variables are normalized so as to have mean two and standard deviation one, what is done in order to better compare both measures.

Note that the average correlation is high throughout the period, and it increases beginning in August, 1998, which is the start of the Russian crisis. The average volatility of the market mode also grows higher during the same period, although it

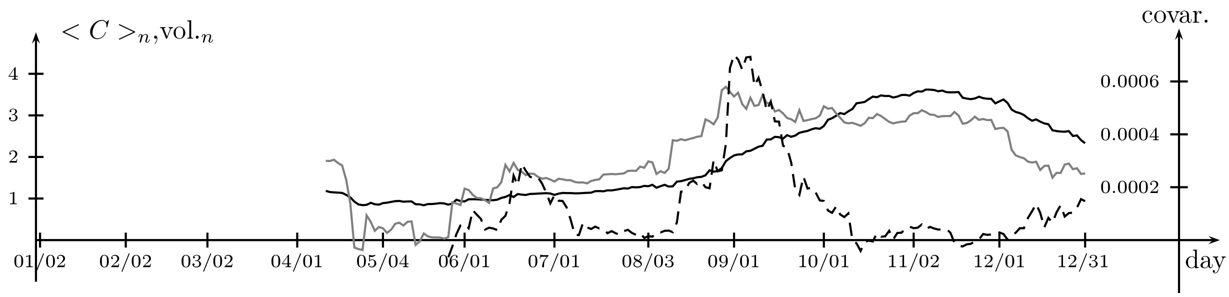


**Fig. 13.** Average volatility of the market mode (black) and average correlation (gray) based on the log-returns for 1998, both calculated in a moving window of 70 days and normalized so as to have mean two and standard deviation one. The covariance between volatility and average correlation as a function of time is plotted in the same graph, in a dashed line.

presents some peaks prior to that time. As the market was unstable due to the Asian crisis of the previous year, that can be explained as well, although there was a drop in correlation between the world stock markets around April, 1998. One can see that the average volatility is correlated with the average correlation between the indices during the times of crisis. This does not seem to be the case at the beginning of the year, when there was no crisis.

The covariance between the volatility (not the average volatility) and the average correlation, in a dashed line, calculated in a moving window of 30 days, is also plotted in Fig. 13. One can verify that the covariance between them increases during the Russian crisis.

## 7. 2001, burst of the dot-com bubble and September 11

On September, 11, 2001, the world was shocked, as the biggest terrorist attack in human history was perpetrated against the USA. The death toll was close to 3000, when two hijacked airplanes were thrown into the Twin Towers of the World Trade Center, in New York, one hit the Pentagon, in Virginia, and another fell in Pennsylvania. Financial markets all over the world plummeted, in an uncertainty crisis that lasted a few days.

On that same year, closer to March, there was the end of a financial bubble centered on internet-based companies, the so-called burst of the dot-com companies. That crash affected most markets in the world and is believed to be a result of an escalation of speculation with companies whose true values were much below the prices their stocks were being negotiated for.

Here we analyze these two crises, one (September 11) which is a good example of a crisis which is caused by a completely exogenous cause, and the other (burst of the dot-com bubble) which is the result of high speculation on stock prices. For 2001, we use 79 indices, adding the KSE 100 from Pakistan (Paki), the Tunindex from Tunisia (Tuni), the SOFIX from Bulgaria (Bulg), the KASE from Kazakhstan (Kaza), and the NZSX 50 from New Zealand (NZ) to the ones already used for 1998.

Using the modified log-returns (5) based on the closing indices from 01/02/2001 to 12/31/2001, we built a $79 \times 79$ correlation matrix between those. This matrix has an average correlation $\langle C \rangle = 0.11$, standard deviation $\sigma = 0.03$, and is based on $L = 260$ days for the $M = 79$ indices, which gives $Q = L/M = 260/79 \approx 3.29$.

The upper and lower bounds of the eigenvalues of the Marĕnko–Pastur distribution (3) are

$$\lambda_- = 0.295 \quad \text{and} \quad \lambda_+ = 2.122. \tag{9}$$

The frequency distribution of the eigenvalues is displayed in Fig. 14, plotted against the theoretical Marĕnko–Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1. Fig. 15 shows the eigenvalues in order of magnitude. The region associated with noise is shaded.

The largest eigenvalue is once more completely out of scale. We also have several eigenvalues that are below the minimum theoretical eigenvalue. Fig. 16 shows eigenvector $e_{79}$, which corresponds to a combination of all indices in a market movement that explains about 19% of the collective movement of all indices.

Many of the indices have very small participations, which amounts to no participation if we consider error bars. The indices that have participation smaller than 0.05 are the ones from Central America, Bermuda, Venezuela, Malta, Slovakia, Romania, Bulgaria, Latvia, Lithuania, Ukraine, Kazakhstan, all the Arab countries, with the exception of Saudi Arabia, Sri Lanka, Bangladesh, China, Mongolia, Vietnam, Malaysia, Indonesia, Philippines, and all the African countries, with the exception of South Africa. Given the size of those markets, this is within what was expected. The major contributions come from the North American countries, the major South American ones, most of Western and Central Europe, the Czech Republic, Hungary, Poland, Estonia, Russia, Israel, Hong Kong, South Korea, Singapore, and South Africa.

Fig. 17 shows the average correlation, calculated in a running window of 80 days, and the average volatility of the market mode, both normalized so as to have mean 2 and standard deviation 1, since the correlation between both measures becomes more transparent in this framework. The normalized average volatility is in black, and the normalized average correlation is in gray.

The figure shows a great increase in volatility just after September 11, followed by an increase in average correlation between the world stock market indices. This is expected from a crisis that was completely exogenous to the financial

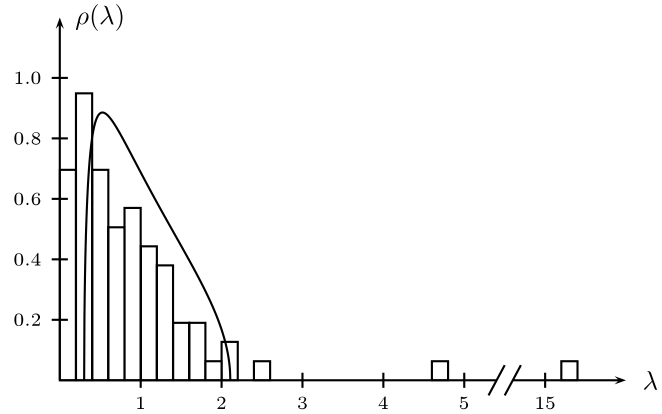

**Fig. 14.** Frequency distribution of the eigenvalues of the correlation matrix for 2001. The theoretical distribution is superimposed on it.

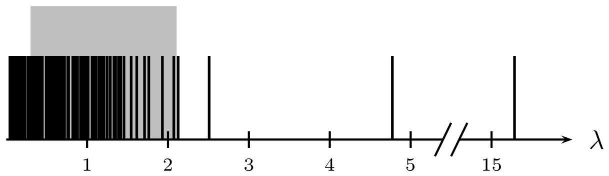

**Fig. 15.** Eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted for a random matrix.

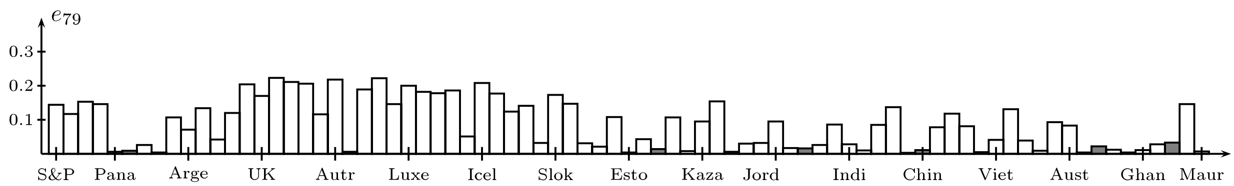

**Fig. 16.** Contributions of the stock market indices to eigenvector $e_{79}$, corresponding to the largest eigenvalue of the correlation matrix. White bars indicate positive values, and gray bars correspond to negative values. The indices are aligned in the following way: *S&P*, Nasd, Cana, Mexi, *Pana*, CoRi, Berm, Jama, Braz, *Arge*, Chil, Vene, Peru, *UK*, Irel, Fran, Germ, Swit, *Autr*, Ita, Malt, Belg, Neth, *Luxe*, Swed, Denm, Finl, Norw, *Icel*, Spai, Port, Gree, CzRe, *Slok*, Hung, Pola, Roma, Bulg, *Esto*, Latv, Lith, Ukra, Russ, *Kaza*, Turk, Isra, Pale, Leba, Jord, SaAr, Qata, Ohma, Paki, *Indi*, SrLa, Bang, Japa, HoKo, *Chin*, Mong, Taiw, SoKo, Thai, *Viet*, Mala, Sing, Indo, Phil, *Aust*, NeZe, Moro, Tuni, Egyp, *Ghan*, Nige, Keny, Bots, SoAf, *Maur*.

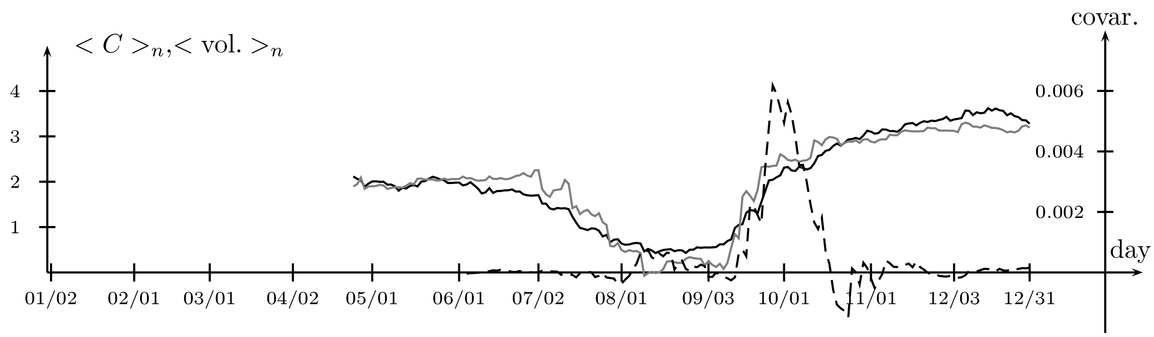

**Fig. 17.** Average volatility (black) and average correlation (gray) based on the log-returns for 2001, both calculated in a moving window of 80 days and normalized so as to have mean two and standard deviation one. The covariance between volatility and average correlation as a function of time is plotted in the same graph, in a dashed line.

markets. A similar increase of both volatility and average correlation occur close to the beginning of the year, related with the burst of the dot-com bubble.

The covariance between the volatility and the average correlation is also plotted in Fig. 17, calculated in a moving window of 30 days. One can readily identify a peak around September 11, but no peak related with the burst of the dot-com bubble, which was not a precisely defined event in time.

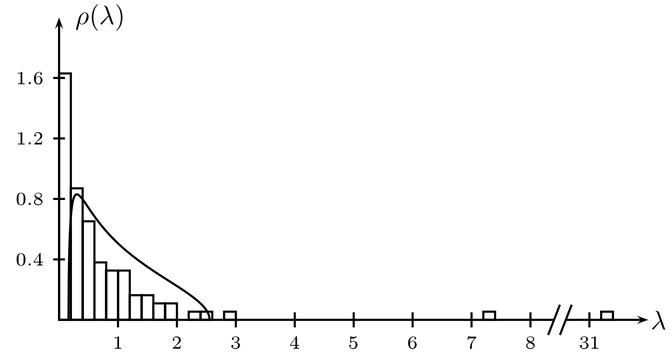


**Fig. 18.** Frequency distribution of the eigenvalues of the correlation matrix for 2008. The theoretical distribution is superimposed on it.

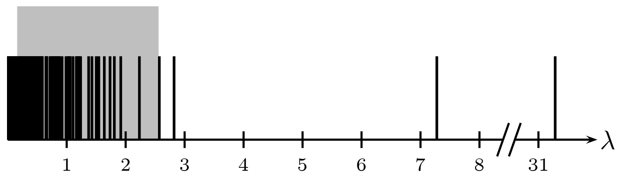


**Fig. 19.** Eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted for a random matrix.

## 8. 2008, subprime mortgage crisis

The last large financial crisis began in 2007, reached its peak in 2008, and is happening until now. This crisis was triggered by the default of a large number of mortgages in the USA. Subprimes are loans to borrowers who have low credit scores. Most of them had a small initial interest rate, adjustable for future payments, which led to many home foreclosures after the rates climbed substantially. Meanwhile, the loans were transformed into pools that were then resold to interested investors. Since the returns of such investments were high, a financial bubble was created, inflating the subprime mortgage market until the defaults started to pop up.

Because of their underestimation of risk, financial institutions worldwide lost trillions of dollars, and many of them declared bankruptcy. Because of that, credit lines tightened around the world, taking the financial crisis to the so called real economy. The world is yet to recover from this crisis, and many institutions are still to lose a good part of their assets in the following years.

Here we analyze the year 2008, which is considered the time when the subprime crisis reached its peak, marked by events like the Lehman Brothers' announcement of bankruptcy, and the liquidation of three of the largest investment banks in the USA. In our research, we add now 13 indices to the ones we used for 2001: IGBC from Colombia (Colo), BELEX 15 from Serbia (Serb), CROBEX from Croatia (Croa), SBI TOP from Slovenia (Slov), SASE 10 from Bosnia and Herzegovina (BoHe), MOSTE from Montenegro (Mont), MBI 10 from Macedonia (Mace), CSE from Cyprus (Cypr), Kuwait SE Weighted Index from Kuwait (Kuwa), Bahrain All Share Index from Bahrain (Baha), ADX General Index from the United Arab Emirates (UAE), DSEI from Tanzania (Tanz), and FTSE/Namibia Overall from Namibia (Nami). So, we use a total of 92 indices, 4 from North America, 2 from Central America, 2 from the islands of the Atlantic, 6 from South America, 35 from Europe, 2 from Eurasia, 28 from Asia, 2 from Oceania, and 11 from Africa. The sample became larger mainly because of the partition of Yugoslavia (which occurred years before 2008), and led to the posterior emergence of new financial markets.

Using the modified log-returns (5) based on the closing indices from 01/02/2008 to 12/31/2008, we built a $92 \times 92$ correlation matrix between those. This matrix has an average correlation $\langle C \rangle = 0.26$, standard deviation $\sigma = 0.05$, and is based on $L = 253$ days for the $M = 92$ indices, which gives $Q = L/M = 256/92 \approx 2.78$.

The upper and lower bounds of the eigenvalues of the Marĕnko–Pastur distribution (3) are

$$\lambda_- = 0.160 \quad \text{and} \quad \lambda_+ = 2.558. \tag{10}$$

The frequency distribution of the eigenvalues is displayed in Fig. 18, plotted against the theoretical Marĕnko–Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1. Fig. 19 shows the eigenvalues of the correlation matrix in order of magnitude (the region associated with noise appears shaded).

Note that the largest eigenvalue is even more out of scale than in previous crisis, what usually indicates a high level of correlation between the market indices and the presence of a powerful global market movement, although it is also influenced by the size of the sample of indices. We also have several eigenvalues that are below the minimum theoretical eigenvalue. Fig. 20 shows eigenvector $e_{92}$, which corresponds to a combination of all indices in a market movement that explains about 34% of the collective movement of all indices.

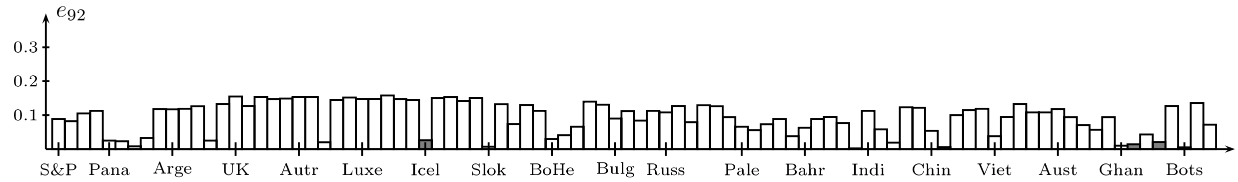


**Fig. 20.** Contributions of the stock market indices to eigenvector $e_{92}$, corresponding to the largest eigenvalue of the correlation matrix. White bars indicate positive values, and gray bars correspond to negative values. The indices are aligned in the following way: *S&P*, Nasd, Cana, Mexi, *Pana*, CoRi, Berm, Jama, Braz, *Arge*, Chil, Colo, Vene, Peru, *UK*, Irel, Fran, Germ, Swit, *Autr*, Ital, Malt, Belg, Neth, *Luxe*, Swed, Denm, Finl, Norw, *Icel*, Spai, Port, Gree, CzRe, *Slok*, Hung, Serb, Croa, Slov, *BoHe*, Mont, Mace, Pola, Roma, *Bulg*, Esto, Latv, Lith, Ukra, *Russ*, Kaza, Turk, Cypr, Isra, *Pale*, Leba, Jord, SaAr, Kuwa, *Bahr*, Qata, UAE, Ohma, Paki, *Indi*, SrLa, Bang, Japa, HoKo, *Chin*, Mong, Taiw, SoKo, Thai, *Viet*, Mala, Sing, Indo, Phil, *Aust*, NeZe, Moro, Tuni, Egyp, *Ghan*, Nige, Keny, Tanz, Nami, *Bots*, SoAf, Maur.

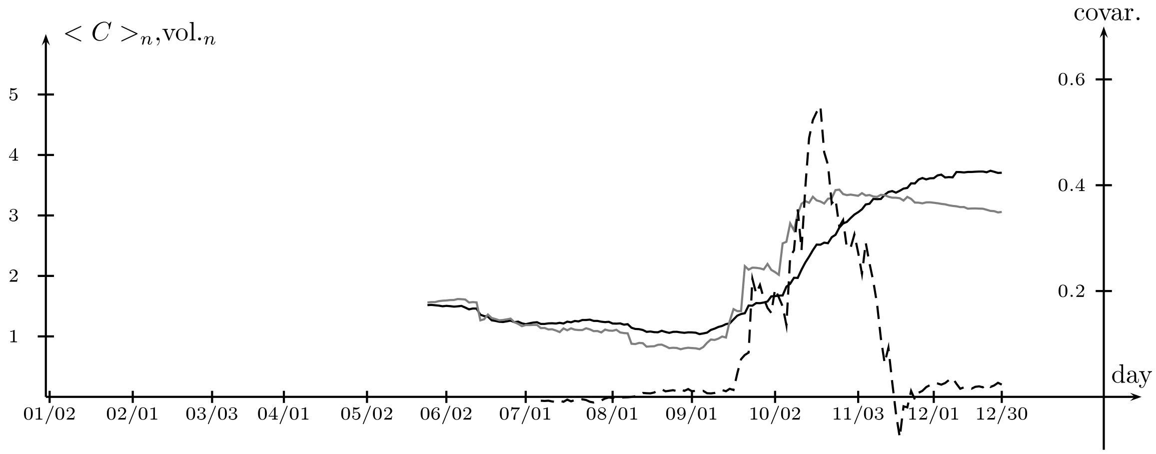


**Fig. 21.** Average volatility (black) and average correlation (gray) based on the log-returns for 2008, both calculated in a moving window of 100 days and normalized so as to have mean two and standard deviation one. The covariance between volatility and average correlation as a function of time is plotted in the same graph, in a dashed line.

Indices with small negative contributions are those from Iceland, which suffered the effects of the crisis with greater impact than most of the other countries, Mongolia, Nigeria, Tanzania, and Botswana. Very small participations (less than 0.050) are related with the indices from Central America, the Atlantic Islands, Venezuela, Malta, Slovakia, Bosnia and Herzegovina, Montenegro, Kuwait, Pakistan, Bangladesh, Vietnam, Ghana, and Kenya. Indices with strong participation (greater than 0.100) are those from Canada, Mexico, most South American countries, most of the European countries, Russia, Turkey, Cyprus, India, Japan, Hong Kong, Taiwan, South Korea, Thailand, Singapore, Indonesia, Philippines, Australia, Namibia, and South Africa. The surprise is the participations of the indices from the USA—S&P 500 (0.089), and Nasdaq (0.082), which are lower than expected.

Fig. 21 shows the average correlation (in gray) and the average volatility (in black), both calculated in a moving window of 100 days, and normalized so as to have mean two and standard deviation one.

One can see that the period of high volatility seems to be preceded by a period of high correlation between the stock markets of the world. Fig. 21 also shows the evolution of the covariance between the mean correlation and the mean volatility (in a dashed line), calculated in a moving window of 30 days.

## 9. Normality tests for the correlation matrix

In this section, we make tests in order to check whether the elements of the correlation matrix exhibit a normal or close to normal probability distribution or not. A first analysis of the data might lead us to believe it does. Observe the following graphics, with the skewness and kurtosis of the probability distribution obtained by considering the elements of correlations matrix (except those of the diagonal) calculated over moving windows (the size of the windows vary for each of the years that were considered).

Fig. 22 shows the skewness of the correlation matrix during the years 1987 (black), 1998 (gray), 2001 (dashed black), and 2008 (dashed gray) for correlation matrices calculated over running windows of, respectively, 30, 70, 80, and 100 days. The same is done in Fig. 23 for the kurtosis, where we also plotted a straight line at kurtosis 3.

For 1987, near the Black Monday, which occurred in October, kurtosis dropped substantially, what seems to imply that the distribution of the coefficients of the correlation matrix approach that of a normal curve. For 1998, the skewness remains nearly constant for most of the time, and the kurtosis of the same distribution stays near 3 during the same period. For 2001,

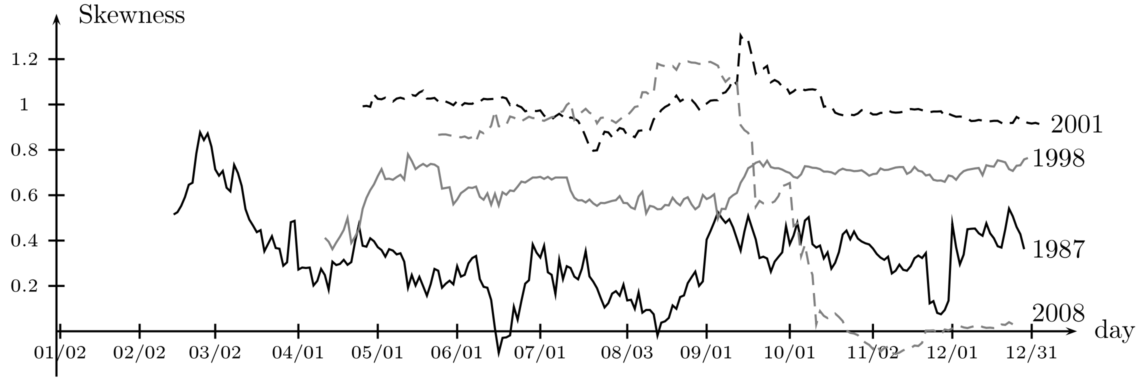


**Fig. 22.** Skewness of the correlation matrix during the years 1987 (black), 1998 (gray), 2001 (dashed black), and 2008 (dashed gray) for correlation matrices calculated over running windows of, respectively, 30, 70, 80, and 100 days.

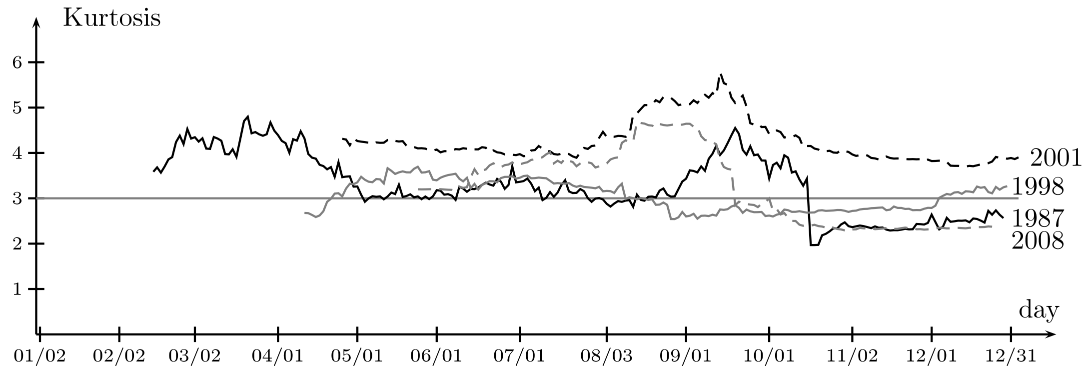


**Fig. 23.** Kurtosis of the correlation matrix during the years 1987 (black), 1998 (gray), 2001 (dashed black), and 2008 (dashed gray) for correlation matrices calculated over running windows of, respectively, 30, 70, 80, and 100 days.

both skewness and kurtosis present a peak in September 11, but otherwise remain nearly constant throughout the period. For 2008, the skewness becomes negative for the time after the beginning of the crisis, something that did not happen in the previous cases. The kurtosis drops to values below 3 for the period of crisis.

Since a perfect normal distribution would have skewness zero and kurtosis 3, we may see that the distribution of the elements of the correlation matrix of international indices in periods of crisis are not normal, although in the case of 1987 and 2008, it seems to be the case. This assumption is contradicted if one plots the probability distributions of the correlation matrix every two months (Figs. 24–27). During the months of highest volatility of each crisis (October for 1987, August for 1998, September for 2001, and October for 2008), the probability distributions deviate somewhat from a normal distribution.

One can see that the probability distributions for 2008 are less strongly peaked than for the other years, but this is mainly caused by the inclusion of a large number of weakly correlated indices. One can also notice that, in the months of crises, the average correlation grows, but the correlation gets more evenly distributed among the possible spectrum.

Our claim that the probability distributions are far from Gaussian during periods of high volatility may be substantiated by using two tests for normality of those distributions. The Jarque–Bera test [158] is based on the formula

$$\mathrm{JB} = \frac{N}{6}\left[s^2 + \frac{(k-3)^2}{4}\right], \tag{11}$$

where $N$ is the size of the sample, $s$ is its skewness, and $k$ is its kurtosis. The Lilliefors test [159], a variant of the Kolmogorov–Smirnov test, is based on the formula

$$L = \max\left|E(x) - N(x)\right|, \tag{12}$$

where $E(x)$ is the cumulative distribution function estimated from the data and $N(x)$ is the cumulative distribution function of a normal distribution with the same mean and standard deviation as the data.

The Jarque–Bera test rejects the null hypothesis that the distribution is normal at the 5% significance level for all months of the years we have studied. The Lilliefors test rejects the null hypothesis that the distribution is normal at the 5% significance level for all months except March/April and May/June, 1987. When applied to the whole years of data, both tests strongly reject the hypothesis that the distribution of the correlation matrix is similar to a normal distribution.

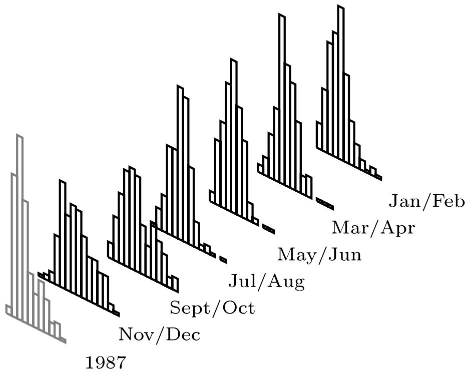


**Fig. 24.** Probability distributions of the correlation matrix calculated every two months in 1987. The probability distribution for the data for the whole year appears last, in gray.

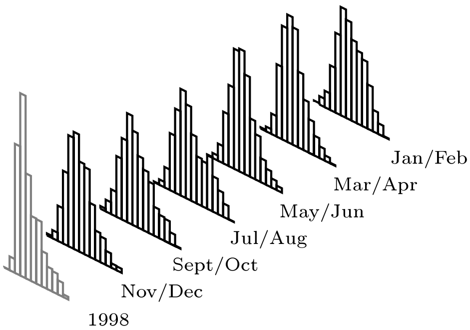


**Fig. 25.** Probability distributions of the correlation matrix calculated every two months in 1998. The probability distribution for the data for the whole year appears last, in gray.

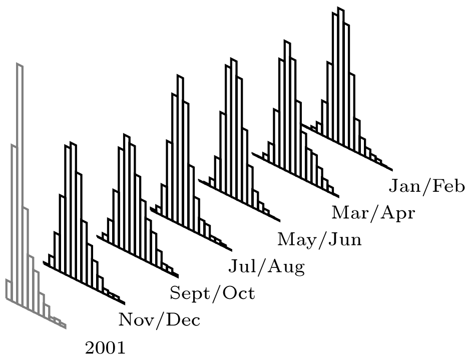


**Fig. 26.** Probability distributions of the correlation matrix calculated every two months in 2001. The probability distribution for the data for the whole year appears last, in gray.

## 10. Gauging the results

As we commented in the introduction of this article, one of the major concerns when dealing with data from stock markets all over the world is that most of them do not operate at the same hours. This leads to incorrections when one tries to study the correlations between them. Another source of concern is that sometimes the correlations between markets may not be measured correctly by the Pearson correlation coefficient, since it is better suited for linear correlation, which may not be the case. Other correlation coefficients, like Spearman's or Kendall's rank correlation coefficients, may capture relations which are not seen using Pearson's correlation.

In order to gauge the effect of these two possible problems, we did two additional analysis of the data. In the first one, we phased the data of Eastern markets (from Russia to the east) so that the data of Western stock markets were compared with data from the next day of Eastern markets. In the second one, we switched to Spearman's correlation whenever Pearson's correlation was used. We did all the calculations again for both cases and compared the results with the ones previously obtained. An account of the comparisons is given now for the four crises being considered.

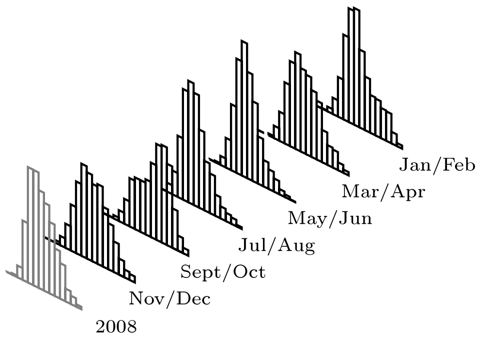


**Fig. 27.** Probability distributions of the correlation matrix calculated every two months in 2008. The probability distribution for the data for the whole year appears last, in gray.

For 1987 with phased data, the average correlation becomes $\langle C\rangle = 0.15$, slightly smaller than the value $\langle C\rangle = 0.16$ for the unphased data. Using Spearman's correlation, we obtain $\langle C\rangle_S = 0.07$ (remember it is a different type of correlation, and so it should not be compared numerically with the Pearson correlation). For the phased data, the maximum eigenvalue, which was $\lambda_{max} = 6.500$, becomes $\lambda_{max} = 6.135$, and for Spearman's correlation, it becomes $\lambda_{max} = 3.977$.

While for the original data Indonesia had a substantial negative participation in the eigenvector with the highest eigenvalue, no index has relevant negative participation for the phased data, and Taiwan and South Korea increase their participations, although Hong Kong decreases its own. For the eigenvector obtained with Spearman's correlation, Brazil, Finland, Bangladesh, and Taiwan acquire small negative participations, Sri Lanka and Indonesia maintaining their negative coefficients.

For the phased data, there is nearly no change in the relations between average correlation and volatility, or between average correlation and average volatility, and the skewness and kurtosis of the probability distributions for the correlation matrix are also very similar. For the results obtained using Spearman's correlation, the agreement between the average correlation and average volatility is much greater for the data concerned with the beginning of the crisis.

For 1998, the average correlation $\langle C\rangle = 0.17$ drops to $\langle C\rangle = 0.15$ for the phased data and is given by $\langle C\rangle_S = 0.16$ for the data related with Spearman's correlation. The maximum eigenvalue goes from $\lambda_{max} = 16.897$ to $\lambda_{max} = 15.511$ (phased data) and $\lambda_{max} = 16.022$ (Spearman's correlation). The participation of the Asian markets in the eigenvector corresponding to the largest eigenvalue grows for phased data and keeps essentially the same for Spearman's correlation. There are no substantial changes between average correlation and volatility and average volatility calculated in a moving window, nor in the skewness and kurtosis of the probability distribution of the off-diagonal elements of the correlation matrix, although for Spearman's correlation, the average correlation and the average volatility are slightly more connected.

For 2001, the average correlation $\langle C\rangle = 0.11$ remains $\langle C\rangle = 0.11$ for the phased data, and it is $\langle C\rangle_S = 0.07$ for the data obtained using Spearman's correlation. The maximum eigenvalue goes from $\lambda_{max} = 15.284$ to $\lambda_{max} = 15.052$ (phased data) and $\lambda_{max} = 10.577$ (Spearman's correlation). For the phased data, the number of participating Asian countries clearly grows, and the average participation, including those of some Western countries, also grows, but not substantially. For Spearman's correlation, the participations of indices in the market mode do not change substantially. There are no substantial changes to the skewness and kurtosis of the probability distribution of the off-diagonal elements of the correlation matrix if we use phased data or Spearman's correlation. For Spearman's correlation, the relation between average correlation and average volatility is even clearer.

For 2008, the average correlation $\langle C\rangle = 0.26$ drops to $\langle C\rangle = 0.21$ for the phased data and is $\langle C\rangle_S = 0.22$ for the data related with Spearman's correlation. The maximum eigenvalue goes from $\lambda_{max} = 31.284$ to $\lambda_{max} = 26.761$ (phased data) and $\lambda_{max} = 26.587$ (Spearman's correlation). The participation of Asian and African markets increase slightly in the eigenvector corresponding to the largest eigenvalue for the case of phased data. For Spearman's correlation, participations do not change significantly. The relation between average correlation and average volatility becomes stronger using phased data, and increases drastically for Spearman's correlation. There is nearly no change in the skewness and kurtosis of the probability distribution of the off-diagonal elements of the correlation matrix for phased data, but for Spearman's correlation the skewness and kurtosis curves become smoother.

What we may conclude from this analysis is that the use of phased data gives occasional better results, but in general makes the average correlation between indices lower. So, we do not really have compelling reasons to use phased data. Now, for the calculations using Spearman's rank correlation, the agreement between average correlation and average volatility increases, sometimes drastically, as may be seen by comparing Figs. 28–31 (shown next) with Figs. 9, 13, 17 and 21, respectively.

These four figures summarize what we have attempted here: to show that high correlation between world indices goes hand in hand with high volatility, possibly causing and definitely being caused by it.

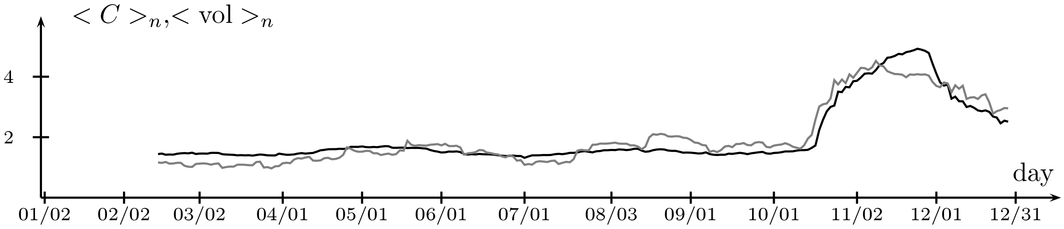


**Fig. 28.** Average volatility (black) and average correlation (gray) based on the log-returns for 1987, calculated in a moving window of 30 days and normalized so as to have mean two and standard deviation one, using Spearman's rank correlation.

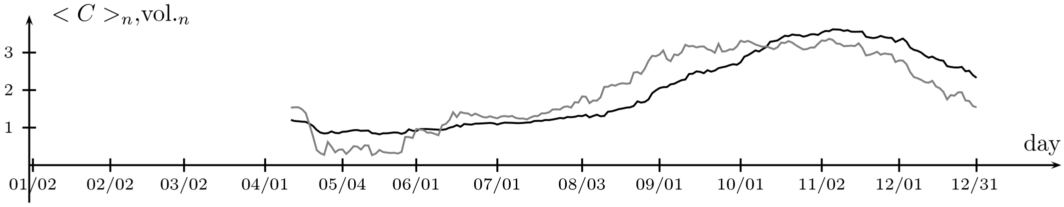


**Fig. 29.** Average volatility (black) and average correlation (gray) based on the log-returns for 1998, calculated in a moving window of 70 days and normalized so as to have mean two and standard deviation one, using Spearman's rank correlation.

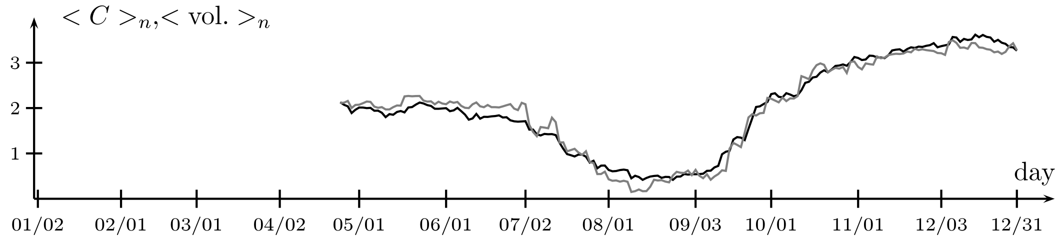


**Fig. 30.** Average volatility (black) and average correlation (gray) based on the log-returns for 2001, calculated in a moving window of 80 days and normalized so as to have mean two and standard deviation one, using Spearman's rank correlation.

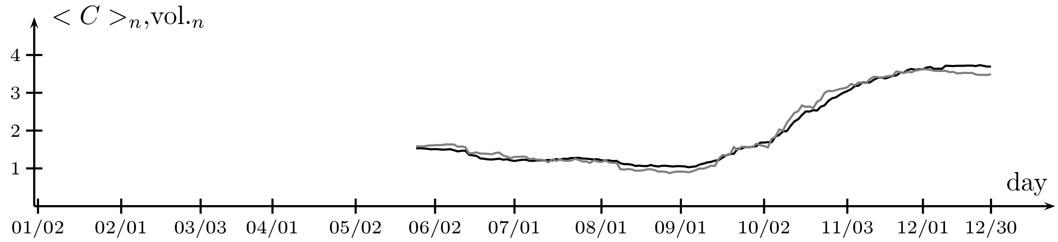


**Fig. 31.** Average volatility (black) and average correlation (gray) based on the log-returns for 2008, calculated in a moving window of 100 days and normalized so as to have mean two and standard deviation one, using Spearman's rank correlation.

## 11. Conclusion and future research

Using the correlation matrices of the log-returns of a diversity of market indices during times of crisis, we showed that markets tend to behave similarly during times of high volatility. In the process, we verified the results obtained in a diversity of articles, but now applied to world financial market indices, and not to equities. Some of those results are that the probability distributions of the eigenvalues of the correlation matrices show peaks that are far off the maximum values predicted by Random Matrix Theory. Another result was the presence of certain combinations of indices that emulate a joint movement of most indices in what is called a market mode. An analysis of the probability distributions of the correlation matrices obtained show that those distributions are not normal and tend to flatten (low kurtosis) in times of crisis.

We also showed that the relation of the average correlation and the average volatility (as calculated using the market mode) increases when one uses Spearman's rank correlation instead of Pearson's correlation, possibly highlighting nonlinear relations between them. The covariance between average correlation and the volatility of the market mode seems to be a good indicator of when periods of acute crises occur.

Some direction for future research is to analyze how the techniques used in this work are modified if we consider that the frequency distributions of the log-returns are not Gaussian. Another topic that is being pursued is to study the hierarchies between the many indices and its evolution in times of crisis. For that, we shall use a distance measure based on the correlation between indices and build Minimum Spanning Trees and also Asset Trees in order to study cluster formation between indices. Some of the results obtained here shall also be used in our studies of financial markets as coupled damped harmonic oscillators subject to stochastic perturbations [160].

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## Appendix. Stock market indices

The next table (Table 2) shows the stock market indices we used, their original countries, the symbols we used for them in the main text, and their codes in Bloomberg. In the tables, we use "SX" as short for "Stock Exchange". Some of the indices changed names and/or method of calculation and are designated by the two names, prior to and after the changing date.

**Table 2**
Names, codes, and abbreviations of the stock market indices used in this article.

| Index | Country | Symbol | Code in Bloomberg |
|---|---|---|---|
| *North America* | | | |
| S&P 500 | United States of America | S&P | SPX |
| Nasdaq Composite | United States of America | Nasd | CCMP |
| S&P/TSX Composite | Canada | Cana | SPTSX |
| IPC | Mexico | Mexi | MEXBOL |
| *Central America* | | | |
| Bolsa de Panama General | Panama | Pana | BVPSBVPS |
| BCT Corp Costa Rica | Costa Rica | CoRi | CRSMBCT |
| *Caribbean* | | | |
| Jamaica SX Market Index | Jamaica | Jama | JMSMX |
| *British overseas territories* | | | |
| Bermuda SX Index | Bermuda | Berm | BSX |
| *South America* | | | |
| Ibovespa | Brazil | Braz | IBOV |
| Merval | Argentina | Arge | MERVAL |
| IPSA | Chile | Chil | IPSA |
| IGBC | Colombia | Colo | IGBC |
| IBC | Venezuela | Vene | IBVC |
| IGBVL | Peru | Peru | IGBVL |
| *Western and Central Europe* | | | |
| FTSE 100 | United Kingdom | UK | UKX |
| ISEQ | Ireland | Irel | ISEQ |
| CAC 40 | France | Fran | CAC |
| DAX | Germany | Germ | DAX |
| SMI | Switzerland | Swit | SMI |
| ATX | Austria | Autr | ATX |
| FTSE MIB or MIB-30 | Italy | Ital | FTSEMIB |
| Malta SX Index | Malta | Malt | MALTEX |
| BEL 20 | Belgium | Belg | BEL20 |
| AEX | Netherlands | Neth | AEX |
| Luxembourg LuxX | Luxembourg | Luxe | LUXXX |
| OMX Stockholm 30 | Sweden | Swed | OMX |
| OMX Copenhagen 20 | Denmark | Denm | KFX |
| OMX Helsinki | Finland | Finl | HEX |
| OBX | Norway | Norw | OBX |
| OMX Iceland All-Share Index | Iceland | Icel | ICEXI |
| IBEX 35 | Spain | Spai | IBEX |
| PSI 20 | Portugal | Port | PSI20 |
| Athens SX General Index | Greece | Gree | ASE |
| *Eastern Europe* | | | |
| PX or PX50 | Czech Republic | CzRe | PX |
| SAX | Slovakia | Slok | SKSM |
| Budapest SX Index | Hungary | Hung | BUX |
| BELEX 15 | Serbia | Serb | BELEX15 |

Table 2 (*continued*)

| Index | Country | Symbol | Code in Bloomberg |
|---|---|---|---|
| CROBEX | Croatia | Croa | CRO |
| SBI TOP | Slovenia | Slov | SBITOP |
| SASE 10 | Bosnia and Herzegovina | BoHe | SASX10 |
| MOSTE | Montenegro | Mont | MOSTE |
| MBI 10 | Macedonia | Mace | MBI |
| WIG | Poland | Pola | WIG |
| BET | Romania | Roma | BET |
| SOFIX | Bulgaria | Bulg | SOFIX |
| OMXT | Estonia | Esto | TALSE |
| OMXR | Latvia | Latv | RIGSE |
| OMXV | Lithuania | Lith | VILSE |
| PFTS | Ukraine | Ukra | PFTS |
| *Eurasia* | | | |
| MICEX | Russia | Russ | INDEXCF |
| ISE National 100 | Turkey | Turk | XU100 |
| *Western and Central Asia* | | | |
| KASE | Kazakhstan | Kaza | KZKAK |
| CSE | Cyprus | Cypr | CYSMMAPA |
| Tel Aviv 25 | Israel | Isra | TA-25 |
| Al Quds | Palestine | Pale | PASISI |
| BLOM | Lebanon | Leba | BLOM |
| ASE General Index | Jordan | Jord | JOSMGNFF |
| TASI | Saudi Arabia | SaAr | SASEIDX |
| Kuwait SE Weighted Index | Kuwait | Kuwa | SECTMIND |
| Bahrain All Share Index | Bahrain | Bahr | BHSEASI |
| QE or DSM 20 | Qatar | Qata | DSM |
| ADX General Index | United Arab Emirates | UAE | ADSMI |
| MSM 30 | Ohman | Ohma | MSM30 |
| *South Asia* | | | |
| Karachi 100 | Pakistan | Paki | KSE100 |
| SENSEX 30 | India | Indi | SENSEX |
| Colombo All-Share Index | Sri Lanka | SrLa | CSEALL |
| DSE General Index | Bangladesh | Bang | DHAKA |
| *Asia-Pacific* | | | |
| Nikkei 25 | Japan | Japa | NKY |
| Hang Seng | Hong Kong | HoKo | HSI |
| Shanghai SE Composite | China | Chin | SHCOMP |
| MSE TOP 20 | Mongolia | Mong | MSETOP |
| TAIEX | Taiwan | Taiw | TWSE |
| KOSPI | South Korea | SoKo | KOSPI |
| SET | Thailand | Thai | SET |
| VN-Index | Vietnam | Viet | VNINDEX |
| KLCI | Malaysia | Mala | FBMKLCI |
| Straits Times | Singapore | Sing | FSSTI |
| Jakarta Composite Index | Indonesia | Indo | JCI |
| PSEi | Philippines | Phil | PCOMP |
| *Oceania* | | | |
| S&P/ASX 200 | Australia | Aust | AS51 |
| NZX 50 | New Zealand | NeZe | NZSE50FG |
| *Northern Africa* | | | |
| CFG 25 | Morocco | Moro | MCSINDEX |
| TUNINDEX | Tunisia | Tuni | TUSISE |
| EGX 30 | Egypt | Egyp | CASE |
| *Central and Southern Africa* | | | |
| Ghana All Share Index | Ghana | Ghan | GGSEGSE |
| Nigeria SX All Share Index | Nigeria | Nige | NGSEINDX |
| NSE 20 | Kenya | Keny | KNSMIDX |
| DSEI | Tanzania | Tanz | DARSDSEI |
| FTSE/Namibia Overall | Namibia | Nami | FTN098 |
| Gaborone | Botswana | Bots | BGSMDC |
| FTSE/JSE Africa All Share | South Africa | SoAf | JALSH |
| SEMDEX | Mauritius | Maur | SEMDEX |

## References

[1] M. King, S. Wadhwani, Transmission of volatility between stock markets, National Bureau of Economic Research Working Paper Series, Number 2910, 1989.
[2] M. King, E. Sentana, S. Wadhwani, Volatility and links between national stock markets, National Bureau of Economic Research Working Paper Series, Number 3357, 1990.
[3] J. Ammer, J. Mei, Measuring international economic linkages with stock market data, Board of Governors of the Federal Reserve System, International Finance Discussion Papers, Number 449, 1993.
[4] W.-L. Lin, R.F. Engle, T. Ito, Do bulls and bears move across borders? international transmission of stock returns and volatility as the world turns, Review of Financial Studies 7 (1994) 507–538.
[5] C.B. Erb, C.R. Harvey, T.E. Viskanta, Forecasting international equity correlations, Financial Analysts Journal (1994) 32–45.
[6] T. Baig, I. Goldfajn, Financial market contagion in the Asian Crisis, IMF Staff Papers 46 (1999).
[7] K. Forbes, R. Rigobon, No contagion, only interdependence: measuring stock market co-movements, Journal of Finance 57 (2002) 2223–2261.
[8] H. Jang, W. Sul, The Asian financial crisis and the co-movement of Asian stock markets, Journal of Asian Economics 13 (2002) 94–104.
[9] R. Rigobon, On the measurement of the international propagation of shocks: is the transmission stable? Journal of International Economics 61 (2003) 261–283.
[10] P. Hartmann, S. Straetmans, C.G. De Vries, Asset market linkages in crisis periods, The Review of Economics and Statistics 86 (2004) 313–326.
[11] G. Corsetti, M. Pericoli, M. Sbracia, Some contagion, some interdependence. More pitfalls in tests of financial contagion, Journal of International Money and Finance 24 (2005) 1177–1199.
[12] D. Baur, N. Schulze, Coexceedances in financial markets-a quantile regression analysis of contagion, Emerging Markets Review 6 (2005) 21–43.
[13] T.C. Chianga, B.N. Jeonb, H. Lic, Dynamic correlation analysis of financial contagion: evidence from Asian markets, Journal of International Money and Finance 26 (2007) 1206–1228.
[14] D. Baur, R.A. Fry, Multivariate contagion and interdependence, Journal of Asian Economics 20 (2009) 353–366.
[15] A.G. Orlov, A cospectral analysis of exchange rate comovements during Asian financial crisis, Journal of International Financial Markets, Institutions and Money 19 (2009) 742–758.
[16] F. Longin, B. Solnik, Is the correlation in international equity returns constant: 1960–1990? Journal of International Money and Finance 14 (1995) 3–26.
[17] G. Bekaert, C.R. Harvey, Time-varying world market integration, The Journal of Finance 1 (1995) 403–444.
[18] G. de Santis, B. Gerard, International asset pricing and portfolio diversification with time-varying risk, The Journal of Finance 52 (1997) 1881–1912.
[19] D.J. Fenn, M.A. Porter, S. Williams, M. McDonald, N.F. Johnsn, N.S. Jones, Temporal evolution of financial market correlations, Phys. Rev. E 84 (2011) 026109–026121.
[20] B. Solnik, C. Boucrelle, Y. Le Fur, International market correlation and volatility, Financial Analysts Journal 52 (1996) 17–34.
[21] I. Meric, G. Meric, Co-movements of European equity markets before and after the 1987 crash, Multinational Finance Journal 1 (1997) 137–152.
[22] F. Longin, B. Solnik, Correlation structure of international equity markets during extremely volatile periods, Le Cashiers de Reserche, HEC Paris, vol. 646, 1999.
[23] P. Hartmann, S. Straetmans, C.G. de Vries, Asset market linkages in crisis periods, Tinbergen Institute Discussion Paper, TI 2001-71/2, 2001.
[24] F. Lillo, G. Bonanno, R.N. Mantegna, Variety of stock returns in normal and extreme market days: the August 1998 crisis, in: H. Takayasu (Ed.), Proceedings of Empirical Science of Financial Fluctuations, Econophysics on the Horizon, 2001.
[25] A. Ang, J. Chen, Asymmetric correlations of equity portfolios, Journal of Financial Economics 63 (2002) 443–494.
[26] F. Longin, B. Solnik, Extreme correlation of international equity markets, The Journal of Finance 56 (2001) 649–675.
[27] I. Meric, S. Kim, J.H. Kim, G. Meric, Co-movements of US, UK, and Asian stock markets before and after September 11, 2001, Journal of Money, Investment and Banking 3 (2008) 47–57.
[28] P. Cizeau, M. Potters, J.-P. Bouchaud, Correlation structure of extreme stock returns, Quantitative Finance 1 (2001) 217–222.
[29] Y. Malevergne, D. Sornette, Investigating extreme dependences: concepts and tools, in: Extreme Financial Risks: From Dependence to Risk Management, Springer, Heidelberg, 2006.
[30] R. Marshal, A. Zeevi, Beyond correlation: extreme co-movements between financial assets, Working Paper, Columbia Business School, 2002.
[31] S.M. Bartram, Y.-H. Wang, Another look at the relationship between cross-market correlation and volatility, Finance Research Letters 2 (2005) 75–88.
[32] J. Knif, J. Kolari, S. Pynnönen, What drives correlation between stock market returns? IMF Working Paper WP/07/157, 2007.
[33] J. Maskawa, W. Souma, Large correlations as a signal of instability in stock market, Preprint, 2010.
[34] P.-A. Reigneron, R. Allez, J.-P. Bouchaud, Principal regression analysis and the index leverage effect, Physica A 390 (2011) 3026–3035.
[35] R. Campbell, K. Koedijk, Covariance and correlation in international equity returns: a value-at-risk approach, Erasmus University Rotterdam Working Paper No. 004, 2000.
[36] S. Pafka, I. Kondor, Noisy covariance matrices and portfolio optimization, The European Physical Journal B 27 (2002) 277–280.
[37] B. Rosenow, V. Plerou, P. Gopikrishnan, H.E. Stanley, Portfolio optimization and the random magnet problem, Europhysics Letters 59 (2002) 500.
[38] A. Ang, G. Bekaert, International asset allocation with regime shifts, The Reviews of Financial Studies 15 (2002) 1137–1187.
[39] S. Pafka, I. Kondor, Noisy covariance matrices and portfolio optimization II, Physica A 319 (2003) 487–494.
[40] J.-P. Onnela, A. Chakraborti, K. Kaski, Dynamics of market correlations: taxonomy and portfolio analysis, Physical Review E 68 (2003) 056110.
[41] S. Sharifi, M. Crane, A. Shamaier, H. Ruskin, Random matrix theory for portfolio optimization: a stability approach, Physica A 335 (2004) 629–643.
[42] S. Pafka, I. Kondor, Estimated correlation matrices and portfolio optimization, Physica A 343 (2004) 623–634.
[43] G. Papp, S. Pafka, N. Nowak, I. Kondor, Random Matrix filtering in portfolio optimization, Acta Physica Polonica B 36 (2005) 2757–2766.
[44] T. Conlon, H.J. Ruskin, M. Crane, Random matrix theory and fund of funds portfolio optimization, Physica A 382 (2007) 565–576.
[45] V. Tola, F. Lillo, M. Gallegati, R.N. Mantegna, Cluster analysis for portfolio optimization, Journal of Economic Dynamics and Control 32 (2008) 235–258.
[46] E. Pantaleo, M. Tumminello, F. Lillo, R.S. Mantegna, When do improved covariance matrix estimators enhance portfolio optimization? An empirical comparative study of nine estimators, Quantitative Finance 11 (2011) 1067–1080.
[47] F. Abergel, M. Politi, Optimizing a basket against the efficient market hypothesis, 2010. arXiv:1006.5230v1.
[48] S.D. Bekiros, D.A. Goergoutsos, Estimating the correlation of international equity markets with multivariate extreme and GARCH, CeNDEF Working Paper 06-17, University of Amsterdam, 2001.
[49] S. Drożdz, F. Grümer, R. Ruf, J. Speth, Dynamics of competition between collectivity and noise in the stock market, Physica A 287 (2000) 440–449.
[50] C.A. Ball, W.N. Torous, Stochastic correlation across international stock markets, Journal of Empirical Finance 7 (2000) 373–388.
[51] B. Solnik, J. Roulet, Dispersion as cross-sectional correlation, Financial Analysts Journal 56 (2000) 54–61.
[52] P.C. Ivanov, B. Podobnik, Y. Lee, H.E. Stanley, Truncaded Lévy process with scale-invariant behavior, Physica A 299 (2001) 154–160.
[53] T. Flavin, M.J. Hurley, F. Rousseau, Explaining stock market correlation: a gravity model approach, The Manchester School 70 (2002) 87–106.
[54] F. Michael, M.D. Johnson, Financial market dynamics, Physica A 320 (2003) 525–534.
[55] P. Gopikrishnan, B. Rosenow, V. Plerou, H.E. Stanley, Quantifying and interpreting collective behavior in financial markets, Physical Review E 64 (2001).
[56] V.D. Skintzi, Dynamic correlation models, Ph.D. thesis, Athens University of Economics & Business, Department of Management Science & Technology, 2003.
[57] W.-K. Wong, J. Penm, R.D. Terrell, K.Y.C. Lim, The relationship between stock markets of major developed countries and Asian emerging markets, Journal of Applied Mathematics and Decision Sciences 8 (2004) 201–218.
[58] J.D. Farmer, L. Gillemot, F. Lillo, S. Mike, A. Sen, What really causes large price changes? Quantitative Finance 4 (2004) 383–397.

[59] P. Repetowicz, P. Richmond, Removing noise from correlation in multivariate stock price data, 2004. arXiv:cond-mat/0403177v1.
[60] S.R.S. Durai, S.N. Bhaduri, Correlation dynamics in equity markets. Evidence from India, NSE, Working Paper No. 51, 2009.
[61] S.J. Hyde, D.P. Bredin, N. Nguyen, Correlation dynamics between Asia-Pacific, EU and US stock returns, in: Munich Personal RePEc Archive, vol. 9681, 2007.
[62] P. Ormerod, Random matrix theory and the evolution of business cycle synchronisation, 1886–2006, Economics E-Journal 2 (2008).
[63] M. Karanasos, The correlation structure of some financial time series models, Quantitative and Qualitative Analysis in Social Sciences 1 (2007) 71–87.
[64] A. Abdelwahab, O. Amor, T. Abdelwahed, The analysis of the interdependence structure in international financial markets by graphical models, International Research Journal of Finance and Economics 15 (2008) 291–306.
[65] C. Genovese, R. Renò, Modeling international market correlations with high frequency data, in: Correlated Data Modelling 2004, Franco Angeli Editore, Milano, Italy, 2008, pp. 99–113.
[66] T. Evans, D.G. McMillan, Financial co-movements and correlation: evidence from 33 international stock market indices, International Journal of Banking, Accounting and Finance (2009) 215–241.
[67] E.P. Wigner, Characteristic vectors of bordered matrices with infinite dimensions, Annals of Mathematics 62 (1955) 548–564.
[68] E.P. Wigner, On the distribution of the roots of certain symmetric matrices, Annals of Mathematics 67 (1958) 325–327.
[69] V.A. Marěnko, L.A. Pastur, USSR-Sb 1 (1967) 457–483.
[70] M.L. Mehta, Random Matrices, Academic Press, 2004.
[71] L. Laloux, P. Cizeau, J.-P. Bouchaud, M. Potters, Noise dressing of financial correlation matrices, Physical Review Letters 83 (1999) 1467–1470.
[72] L. Laloux, P. Cizeau, M. Potters, J.-P. Bouchaud, Random matrix theory and financial correlations, Mathematical Models and Methods in Applied Sciences (2000).
[73] V. Plerou, P. Gopikrishman, B. Rosenow, L.A.N. Amaral, H.E. Stanley, Universal and non-universal properties of cross-correlations in financial time series, Physics Letters 83 (1999) 1471–1474.
[74] V. Plerou, P. Gopikrishman, B. Rosenow, L.A.N. Amaral, H.E. Stanley, Scaling of the distribution of fluctuations of financial market indices, Physical Review E 60 (1999) 5306–5316.
[75] V. Plerou, P. Gopikrishman, B. Rosenow, L.A.N. Amaral, H.E. Stanley, A Random Matrix Theory approach to financial cross-correlations, Physica A 287 (2000) 374–382.
[76] B. Rosenow, V. Plerou, P. Gopikrishnan, L.A.N. Amaral, H.E. Sanley, Application of Random Matrix Theory to study cross-correlations of stock prices, International Journal of Theoretical and Applied Finance 3 (2002) 399–403.
[77] V. Plerou, P. Gopikrishman, B. Rosenow, L.A.N. Amaral, T. Guhr, H.E. Stanley, Random matrix approach to cross-correlations in financial data, Physical Review E 65 (2002) 066126.
[78] P. Ormerod, The convergence of European business cycles 1980–2004, Acta Physica Polonica B 36 (2005) 2747–2756.
[79] V. Kulkarni, N. Deo, Correlation and volatility of an Indian stock market: a random matrix approach, The European Physical Journal B 60 (2007) 101–109.
[80] J. Kwapièn, S. Drożdż, P. Oświęcimka, The bulk of the stock market correlation matrix is not pure noise, Physica A 359 (2006) 589–606.
[81] J. Kwapièn, S. Drożdż, A.Z. Górski, P. Oświęcimka, Asymmetric matrices in an analysis of financial correlations, Acta Physica Polonica B 37 (2006) 3039–3048.
[82] M. Potters, J.-P. Bouchaud, L. Laloux, Financial applications of Random Matrix Theory: old laces and new pieces, in: Proceedings of the Cracow Conference on Applications of Random Matrix Theory to Economy and Other Complex Systems, 2005.
[83] J.-P. Bouchaud, L. Laloux, M.A. Miceli, M. Potters, Large dimension forecasting models and random singular value spectra, European Physical Journal B 2 (2007) 201–207.
[84] R. Rak, S. Drożdż, J. Kwapien, P. Oświęcimka, Correlation matrix decomposition of WIG20 intraday fluctuations, Acta Physica Polonica B 37 (2006) 3123–3132.
[85] D. Wilcox, T. Gebbie, An analysis of cross-correlations in an emerging market, Physica A 375 (2007) 584–598.
[86] G. Birolli, J.-P. Bouchaud, M. Potters, On the top eigenvalue of heavy-tailed random matrices, Europhysics Letters 78 (2007) 10001.1–10001.5.
[87] A. Chakraborti, An outlook on correlations in stock prices, in: Econophysics of Stock and other Markets—Proceedings of the Econophys-Kolkata II, Springer, Milan, 2007.
[88] A.C.R. Martins, Non-stationary correlation matrices and noise, Physica A 379 (2007) 552–558.
[89] A.C.R. Martins, Random but not so much. A parametrization for the returns and correlation matrix of financial time series, Physica A 383 (2007) 527–532.
[90] R.K. Pan, S. Sinha, Collective behavior of stock price movements in an emerging market, Physical Review E 76 (2007) 1–9.
[91] A. Chakraborti, M. Patriarca, M.S. Santhanam, Financial time-series analysis: a brief overview, in: Proceedings of the International Workshop "Econophys-Kolkata III", Springer, Milan, 2007.
[92] I.I. Dimov, P.N. Kolm, L. Maclin, D.Y.C. Shiber, Hidden noise structure and random matrix models of stock correlations, Working Paper 2009-4, New York University, Courant Institute of Mathematical Sciences, 2009.
[93] A. Chakraborti, I.M. Toke, M. Patriarca, F. Abergel, Econophysics review: I. Empirical facts, Quantitative Finance 11 (2011) 991–1012;
A. Chakraborti, I.M. Toke, M. Patriarca, F. Abergel, Econophysics review: II. Agent-based models, Quantitative Finance 11 (2011) 1013–1041.
[94] J.-P. Bouchaud, M. Potters, Financial applications of Random Matrix Theory: a short review, in: G. Akemann, J. Baik, P. Di Francesco (Eds.), The Oxford Handbook of Random Matrix Theory, Oxford University Press, 2011.
[95] J.D. Noh, A model of correlations in stock markets, Physical Review E 61 (2000) 5981.
[96] R. Rak, J. Kwapień, S. Drożdż, P. Oświęcimka, Cross-correlations in Warsaw Stock Exchange, Acta Physica Polonica A 114 (2008) 561–568.
[97] T. Conlon, H.J. Ruskin, M. Crane, Cross-correlations dynamics in financial time series, Physica A 388 (2009) 705–714.
[98] M.C. Münnix, R. Schäfer, T. Guhr, Compensating asynchrony effects in the calculation of financial correlations, Physica A 389 (2010) 767–779.
[99] A. Namaki, G.R. Jafari, R. Raei, Comparing TEPIX as an emerging market with efficient market by Random Matrix Theory, Preprint, 2010.
[100] G. Oh, C. Eom, F. Wang, W.-S. Jung, H.E. Stanley, S. Kim, Statistical properties of cross-correlation in the Korean stock market, The European Physical Journal B 79 (2011) 55–60.
[101] S. Maslov, Measures of globalization based on cross-correlations of world financial indices, Physica A 301 (2001) 397–406.
[102] S. Drożdż, F. Grümmer, F. Ruf, J. Speth, Towards identifying the world stock market cross-correlations: DAX versus Dow Jones, Physica A 294 (2001) 226–234.
[103] R.N. Mantegna, Hierarchical structure in financial markets, The European Physical Journal B 11 (1999) 193.
[104] G. Bonanno, N. Vandewalle, R.N. Mantegna, Taxonomy of stock market indices, Physical Review E 62 (2000) R7615–R7618.
[105] G. Bonanno, F. Lillo, R.N. Mantegna, Levels of complexity in financial markets, Physica A 299 (2001) 16–27.
[106] G. Bonanno, F. Lillo, R.N. Mantegna, High-frequency cross-correlation in a set of stocks, Quantitative Finance 1 (2001) 96–104.
[107] P. Gopikrishnan, B. Rosenow, V. Plerou, H.E. Stanley, Identifying business sectors from stock price flutuations, Physical Review E 64 (2001) 35106.
[108] S. Micchichè, G. Bonanno, F. Lillo, R.N. Mantegna, Degree stability of a minimum spanning tree of price return and volatility, Physica A 324 (2003) 66–73.
[109] J.-P. Onnela, A. Chakraborti, K. Kaski, J. Kertész, Dynamic asset trees and Black Monday, Physica A 324 (2003) 247–252.
[110] J.-P. Onnela, A. Chakraborti, K. Kaski, Dynamics of market correlations: taxonomy and portfolio analysis, Physical Review E 68 (2003) 1–12.
[111] J.-P. Onnela, A. Chakraborti, K. Kaski, J. Kertész, A. Kanto, Asset trees and asset graphs in financial markets, Physica Scripta T 106 (2003) 48–54.
[112] J.-P. Onnela, A. Chakraborti, K. Kaski, J. Kertész, Dynamic asset trees and Black Monday, Physica A 324 (2003) 247–252.
[113] J.-P. Onnela, K. Kaski, J. Kertész, Clustering and information in correlation based financial networks, The European Physical Journal B 38 (2004) 353–362.

[114] G. Bonanno, G. Caldarelli, F. Lillo, S. Miccichè, N. Vandewalle, R.N. Mantegna, Networks of equities in financial markets, The European Physical Journal B 38 (2004) 363–371.
[115] C. Coronnello, M. Tumminello, F. Lillo, S. Micchichè, R.N. Mantegna, Sector identification in a set of stock return time series traded at the London Stock Exchange, Acta Physica Polonica B 36 (2005) 2653–2679.
[116] T. Aste, T. Di Matteo, Correlation filtering in financial time series, in: Noise and Fluctuations in Econophysics and Finance, Proceedings of the SPIE 5848 (2005) 100–109.
[117] S. Sinha, R.K. Pan, Uncovering the internal structure of the Indian financial market: cross-correlation behavior in the NSE, in: Econophysics of Markets and Business Networks, Springer, 2007, pp. 215–226.
[118] C. Coronnello, M. Tumminello, F. Lillo, S. Micchichè, R.N. Mantegna, Economic sector identification in a set of stocks traded at the New York Stock Exchange: a comparative analysis, Proceedings of the SPIE 6601 (2007) 66010T.
[119] M. Tumminello, T. Di Matteo, T. Aste, R.N. Mantegna, Correlation based networks of equity returns sampled at different time horizons, The European Physical Journal B 55 (2007) 209–217.
[120] R. Coelho, C.G. Gilmore, B. Lucey, P. Richmond, S. Hutzler, The evolution of interdependence in world equity markets—evidence from minimum spanning trees, Physica A 376 (2007) 455–466.
[121] C. Borghesi, M. Marsili, S. Miccichè, Emergence of time-horizon invariant correlation structure in financial returns by subtraction of the market mode, Physical Review E 76 (2007) 026104.
[122] M. Tumminello, F. Liloo, R.N. Mantegna, Kullback–Leiber distance as a measure of the information filtered from multivariate data, Physical Review E 76 (2007) 031123.
[123] M. Tumminello, F. Liloo, R.N. Mantegna, Shrinkage and spectral filtering of correlation matrices: a comparison via the Kullback–Leiber distance, Acta Physica Polonica B 38 (2007) 4079–4088.
[124] D. Garlaschelli, T. Di Matteo, T. Aste, G. Caldarelli, M.I. Loffredo, Interplay between topology and dynamics in the World Trade Web, The European Physical Journal B 57 (2007) 159–164.
[125] M. Ausloos, R. Lambiotte, Clusters or networks of economies? A macroeconomy study through gross domestic product, Physica A 382 (2007) 16–21.
[126] J.G. Brida, W.A. Risso, Multidimensional minimal spanning tree: the Dow Jones case, Physica A 387 (2008) 5205–5210.
[127] C. Eom, G. Oh, W.-S. Jung, H. Jeong, S. Kim, Topological properties of stock networks based on minimal spanning tree and random matrix theory in financial time series, Physica A 388 (2009) 900–906.
[128] M. Eryiğit, R. Eryiğit, Topological properties of stock networks based on minimal spanning tree and random matrix theory in financial time series, Physica A 388 (2009) 900–906.
[129] J.C. Wong, H. Lian, S.A. Cheong, Detecting macroeconomic phases in the Dow Jones Industrial Average time series, Physica A 388 (2009) 4635–4645.
[130] Y.W. Goo, T.W. Lian, W.G. Ong, W.T. Choi, S.A. Cheong, Financial atoms and molecules, Preprint, 2009. arXiv:0903.2009v1.
[131] P. Sieczka, J.A. Hołyst, Correlations in commodity markets, Physica A 388 (2009) 1621–1630.
[132] J. Kwapień, S. Gworek, S. Drożdż, A. Górski, Analysis of a network structure of the foreign currency exchange market, Journal of Economic Interaction and Coordination 4 (2009) 55–72.
[133] J. Kwapień, S. Gworek, S. Drożdż, Structure and evolution of the foreign exchange networks, Acta Physica Polonica B 40 (2009) 175–194.
[134] G. Fagiolo, The international-trade network: gravity equations and topological properties, Journal of Economic Interaction and Coordination 5 (2010) 1–25.
[135] M. Tumminello, F. Liloo, R.N. Mantegna, Correlation, hierarchies, and networks in financial markets, Journal of Economic Behavior & Organizations 75 (2010) 40–58.
[136] J. He, M.W. Deem, Structure response in the world trade network, Physical Review Letters 105 (2010) 198701-1–198701-4.
[137] K.M. Lee, J.-S. Yang, J. Lee, K.-I. Goh, I.-M. Kim, Impact of the topology of global macroeconomic network on the spreading of economic crises, PLoS ONE 6 (3) (2011) e18443. doi:10.1371/journal.pone.0018443.
[138] S. Drożdż, J. Kwapień, J. Speth, Coherent patterns in nulcei and in financial markets, AIP Conference Proceedings 1261 (2010) 256–264.
[139] D.J. Fenn, M.A. Porter, P.J. Mucha, M. McDonald, S. Williams, N.F. Johnson, N.S. Jones, Dynamical clustering of exchange rates, Preprint, 2010. arXiv:0905.4912v2.
[140] M. Keskin, B. Deviren, Y. Kocalkaplan, Topology of the correlation networks among major currencies using hierarchical structure methods, Physica A 390 (2011) 719–730.
[141] Y. Zhang, G.H.T. Lee, J.C. Wong, J.L. Kok, M. Prusty, S.A. Cheong, Will the US Economy Recover in 2010? A Minimal Spanning Tree Study, Physica A 390 (2011) 2020–2050.
[142] D. Wang, B. Podobnik, D. Horvatić, H.E. Stanley, Quantifying and modeling long-range cross correlation in multiple time series with applications to world stock indices, Physical Review E 83 (2011) 046121-1–046121-9.
[143] B. Podobnik, D.F. Fu, H.E. Stanley, P.Ch. Ivanov, Power-law autocorrelated stochastic processes with long-range cross-correlations, The European Physical Journal B 56 (2007) 47–52.
[144] B. Podobnik, D. Wang, D. Horvatić, I. Grosse, H.E. Stanley, Time-lag cross-correlations in collective phenomena, EPL 90 (2010) 68001-p1–68001-p6.
[145] B. Podobnik, D. Horvatić, A. Petersen, H.E. Stanley, Cross-correlation between volume change and price change, Proceedings of the National Academy of Sciences USA 106 (2009) 22079–22084.
[146] B. Podobnik, D. Horvatić, A.L. Ng, H.E. Stanley, P.Ch. Ivanov, Modeling long-range cross correlations in two-component ARFIMA and FIARCH processes, Physica A 387 (2008) 3954–3959.
[147] B. Podobnik, H.E. Stanley, Detrended cross-correlation analysis: a new method for analyzing two nonstationary time series, Physical Review Letters 100 (2008) 084102-1–084102-4.
[148] S. Arianos, A. Carbone, Cross-correlation of long-range correlated series, Journal of Statistical Mechanics: Theory and Experiment (2009) PO3037.
[149] B. Podobnik, P.C. Iavanov, Y. Lee, A. Chessa, H.E. Stanley, Systems with correlations in the variance: generating power-law tails in probability distributions, Europhysics Letters 50 (2000) 711–717.
[150] P. Weber, B. Rosenow, Large stock price changes: volume or liquidity? Quantitative Finance 6 (2006) 7.
[151] C. Biely, S. Thurner, Random matrix ensembles of time-lagged correlation matrices: derivation of eigenvalue spectra and analysis of financial time-series, Quantitative Finance 8 (2008) 705–722.
[152] G. Biroli, J.-P. Bouchaud, M. Potters, The student ensemble of correlation matrices: eigenvalue spectrum and Kullback–Leibler entropy, Acta Physica Polonica B 38 (2007) 4009–4026.
[153] Y. Malevergne, D. Sornette, Investigating extreme dependences: concepts and tools, in: Extreme Financial Risks: From Dependence to Risk Management, Springer, Heidelberg, 2006.
[154] L. Borland, Statistical signatures in times of panic: markets as a self-organizing system, 2010. arXiv:0908.0111v2.
[155] E.J. Elton, M.J. Gruber, S.J. Brown, W. Goetzmann, Modern Portfolio Theory and Investment Analysis, eigth ed., Wiley, 2009.
[156] Z. Bodie, A. Kane, A.J. Marcus, Investments, eigth ed., McGraww-Hill, Irwin, 2009.
[157] J.-P. Bouchaud, J.D. Farmer, F. Lillo, How markets slowly digest changes in supply and demand, in: H. Thorsten, K. Schenk-Hoppe (Eds.), Handbook of Financial Markets: Dynamics and Evolution, Elsevier: Academic Press, 2008.
[158] C.M. Jarque, A.K. Bera, A test for normality of observations and regression residuals, International Statistical Review 55 (2) (1987) 163–172.
[159] H.W. Lilliefors, On the Komogorov–Smirnov test for normality with mean and variance unknown, Journal of the American Statistical Association 62 (1967) 399–402.
[160] L. Sandoval Jr., I. De P. Franca, Shocks in financial markets, price expectation, and damped harmonic oscillators, 2011. arXiv:1103.1992.